Central European Journal of Operations Research
https://doi.org/10.1007/s10100-023-00902-z

MILP models of a patient transportation problem

Martin Tóth[1] · **Tamás Hajba**[2] · **Adrián Horváth**[2]

Accepted: 21 December 2023

Abstract

With ageing societies and increasing number of patients, there is a growing need for quality services that help transporting non-urgent patients to hospitals. In logistics, patient transportation problems are usually modeled as a dial-a-ride problem. In a Dial-a-Ride problem, a fleet of vehicles is providing the delivery services between the loading points and the delivery destinations. The demands are known in advance. In most cases the total travel distance of the vehicles is to be minimized. In this paper, we consider a specific dial-a-ride problem, where a single vehicle is used to transport patients to the same hospital. In determining the optimal route, the multiple and different travel needs of patients, such as their maximum travel time, are also taken into consideration. We introduce 4 different mixed integer linear programming models of the routing problem. Finally, the efficiency of the four models was compared using some real-life problems by solving them with a commercial solver.

Keywords Vehicle routing · On demand · Patient transport · Mixed integer linear program

All authors have contributed equally to this work.

✉ Tamás Hajba
hajbat@math.sze.hu

Martin Tóth
tothmartin0127@gmail.com

Adrián Horváth
hadrian@sze.hu

[1] FERCAM GmbH, Brunn am Gebirge, Austria

[2] Széchenyi István University, Győr, Hungary

Published online: 19 February 2024

1 Introduction

Due to continuously developing technological solutions, transportation companies can offer more personalized services. Passengers have the opportunity to determine their pickup time and location, and if they share their journey with others, they can also save on travel costs. However, route planning is still solved by classical methods in the transportation sector, mostly managed by a dispatcher. Thus, avoidable costs can occur due to this old type of route planning. Ride-sharing services, such as Uber, have been trying to maximize the capacity utilization of the vehicles, while reducing the total distance traveled by the vehicles and the size of the vehicle fleet. Still, there are areas where modernization of existing services is necessary, such as at patient transportation.

There is an aging society in Hungary, which has resulted in a growing number sick people. As a consequence the number of visits to various healthcare centers is also high. However, there is no convenient and cheap service that would guarantee the transportation of these people at particular healthcare points. The current patient transportation services focus on only one patient, providing door-to-door services. These vehicles are equipped with the necessary medical equipment and nurses are also on board. These services represent a high level of quality, but they are very expensive. Patients that do not require special travel conditions, usually do not choose this service.

Currently, the most used transportation service for patients, who want to go to hospital, is the public transport service. This is due to the fact that it is considered to be one of the cheapest personal transportation services. Due to the ample number bus stops, everybody can easily reach them. Despite the condition of the existing fleet and low number of direct routes to the healthcare centers, this mode of traveling is most often used. Taxi services also provide home-to-home services, but due to their expensive costs, they are used less by patients. Another opportunity is to use personal vehicles, but not all patients are in a condition to drive. Relatives and acquaintances cannot always help. Furthermore, parking places at healthcare centers are often overcrowded, which can cause further inconvenience. It can be stated that patients need such a service that combines the advantages of taxi services and public transportation and help them to get to the hospitals.

The healthcare system in Hungary is divided into districts, which means that patients from the same zone typically have to go to the same healthcare center. Furthermore, during the route planning of non-urgent patients, we do not have to use dynamic routing systems, because people usually know their transport needs at least one day in advance, while emergency cases are generally served by ambulances.

The Dial-a-Ride Problem (DARP) is a transportation optimization problem in which a fleet of vehicles serves a set of request. Each request has a pickup point, a delivery point and a number of passengers (or a quantity of some goods). The problem may involve some other constraints, such as time windows or maximum riding time as well. The vehicles have to transport the passengers from their pickup points to their destination while satisfying other side constraints. The goal of the problem is to find the optimal routes for the vehicles in which the total distance travelled by the vehicles is minimized. DARP is an NP-hard problem hence for large-scale systems the optimal solution cannot be determined in a reasonable amount of time.

DARP system is usually used to optimize the efficiency of transportation systems, which can lead to cost savings and improved service. Furthermore it can be used to model a wide range of transportation systems, from public transportation to parcel delivery, because of the possibility of building more and more difficult and specific system.

In this article we are focusing only on the non-urgent patients, of which there are 2 types. Based on the previously discussed points, in this article a logistics system in which the customers are served by a small bus is studied. This bus can transport multiple patients at the same time and people in wheelchairs can travel more comfortably. Due to ridesharing and home-to-home delivery, we can offer lower rates than taxi companies, and a more comfortable service than public transportation. Furthermore, patients have the ability to determine the earliest pick up time, the latest arrival time at the hospital and the maximum travel time. We also take into account the interests of the operating company by minimizing the distance traveled by the vehicle.

In the article, we present 4 mixed-integer linear programming models of the problem. One can determine the optimal solution of the problem by solving the models with the appropriate software (CPLEX, GUROBI). Due to the difficulty of the problem, it cannot be expected to solve large-scale problems using this approach, but due to the rapid increase in performance of softwares and computers, it can be expected that larger and larger problems could be solved in this way. We have compared the effectiveness of the models on real-life problems.

2 Literature overview

Scientists have been working on DARP for a long time. Several methods, including branch and bound algorithms, complex heuristic and meth-heuristic methods have been developed to solve the problem. A summary of the different variants and solution approaches to the problem can be found in Ho et al. (2018).

Beside patient transportation, DARP can arise in other fields, such as parcel delivery and inventory routing as well. Madankumar and Rajendran (2019) studied a parcel delivery problem with heterogeneous fleet of vehicles and time windows, in which the courier not only delivers a package at a given point, but also receives a package. The authors introduced a MILP model of the problem that was tested on some test problems.

Agra et al. (2022) examined a single vehicle inventory routing problem. The problem requires the transportation of a commodity from pickup points to delivery points, while preserving the inventory at each location between given limits. The locations can be visited multiple times during the period, and the vehicle can visit them in any order. Two mathematical models and two branching algorithms were presented by the authors to solve large-scale instances.

In the beginning, the main aspect of research related to patient transport was the solution of the transportation problem with soft time windows (clients requests for pickup and delivery time were known in advance). Melachrinoudis et al. (2007) studied one such problem. Parragh (2011) examined factors, such as the capacities of the vehicles; furthermore, this research already took into account space requirements of staff and wheelchairs users, and even paid attention to the possibility of using a stretcher. In this system several vehicles were used to deliver different types of patients. For this standard DARP, a Branch and Cut algorithm was presented. Qu and Bard (2015) investigated the different types of needs, not with vehicles with multiple characteristics, but rather with vehicles with configurable decks. In recent years, the problem of patient transportation has received more and more attention and more realistic variants of the problem have appeared.

In the Hong-Kong healthcare system, there is usually a nurse in the vehicle in case of a problem. Furthermore, the patients have the possibility to determine their arriving time. Several vehicles satisfy the requests, which makes the system more complicated. Lim et al. (2017) proposed a metaheuristic for this problem that uses a variable neighborhood search and 2 years later this system was expanded to consider the lunch break of the staff. Luo et al. (2019) solved this new problem using a two-phase branch-and-price-and-cut algorithm.

Chane-Haï et al. (2020) studied a patient transportation problem in which each patient has two requests, one request to get to the hospital and one request to get from the hospital to home. In this article, the demands are selected to home delivery next to arriving at hospital, in the interest of getting 2 smaller single DARP instead of 1 complex DARP. In a similar problem, Büsing et al. (2021) proposed two techniques for managing both requests. The first approach involves identifying and creating mini-clusters of outbound requests, which are then connected by solving a traveling salesman problem and creating routes using a splitting procedure. The second approach uses a rolling horizon approach to match vehicles with requests by solving bipartite matching problems.

Several countries use two different fleets of vehicles to meet patients' transportation needs, one for the non-urgent patients and one for the urgent patients. Unlike the classical dial-a-ride problem, where multiple passengers can be transported simultaneously and all requests are pre-known, den Berg and van Essen (2019) investigated a problem in which vehicles can transport only one patient at any given time and new requests arise throughout the day. To address this problem, an online model was proposed to handle these requests in real-time with the purpose of managing these demands without buying more vehicles.

Souza et al. (2022) studied a similar problem, in which a heterogeneous fleet of vehicles is used to serve static and dynamic requests of patients was examined. The non-urgent patients were included using a static system that was solved using a large local neighborhood search and the transportation of urgent patients was determined using an insertion heuristic in a dynamic system. This article also shows that the growing number of patients makes the problem more complex hence time windows cannot be always respected.

During Covid-19 a lot of emphasis was placed on the separate transport of patients. Molenbruch et al. (2017) proposed a model that could manage separate delivery with a fleet of vehicles. The routes of the vehicles were determined by a multi-directional local search algorithm. Gkiotsalitis (2022) investigated a similar problem in which the focus was to avoid crowded vehicles. To achieve this goal, the capacity of small buses was limited in order to guarantee a safe distance between passengers, and a MILP model of the problem was introduced.

In an Austrian research, Armbrust et al. (2022) studied a patient transportation problem. In this problem patients were in different regions and each region had its own hospital (delivery point). A heterogeneous fleet of vehicles was used to deliver the patients and disabled patients, while taking into account the capacity of the vehicles, the rest periods of the drivers, and requests of pick up and delivery time. A MILP formulation of the problem was established and a Large Neighborhood Search method was used to solve the problem.

Recently, the P-graph method, developed by Friedler et al. (1992) for the investigation and optimization of chemical processes, was also successfully applied to solve routing tasks. Frits and Bertok (2021) studied the field service operation problem in which tasks at different locations had to be completed by service groups. The conditions of the tasks, such as the time required for completion, the deadline for completing tasks, the tools, and the quality of the service groups required for completion of the tasks, are known. The service groups visit the locations by vehicles (e.g., car) from the company's depot. The goal is to minimize the total costs necessary to complete all tasks, including the wages of service groups and travel costs. The authors developed a 2-phase algorithm to solve the problem. In the first phase, the tasks are assigned to time intervals, while in the second phase, the routes of the service groups and the schedule for the execution of the tasks are determined using the P-graph approach.

Nagy et al. (2019) studied the bus transportation problem using a periodic timetable. The authors developed a procedure based on the P-graph method, which determines the optimal timetable by considering several conditions, including the minimum and maximum working hours of the bus drivers, rest time of the drivers, and frequency of bus departures within each period. The method was later successfully applied by the authors (Ercsey et al. 2021) to problems in which the bus departures were arbitrary (i.e., there was no periodic timetable). A survey of graph based methods for routing problems can be found in Wang et al. (2019).

In recent years, machine learning has been applied to solve routing problems. Dornemann (2023) presented two deep learning methods for the Capacitated Vehicle Routine Problem with Time Window in which vehicles have predetermined capacities and nodes have time windows. The first method is a combination of deep learning and tree search, while the second approach solves the problem's quadratic integer programming model using deep learning.

As it will be shown later, the problem investigated in this article can be transformed to a Traveling Salesman Problem (TSP) with special conditions. TSP can be considered as a route planning task in which one vehicle has to visit a set of customers at different locations. The goal is to find the optimal route that minimizes the total distance run by the vehicle. Due to the practical importance of the problem, it has been studied intensively. MILP models of the TSP have been introduced in Öncan et al. (2009), Matai et al. (2010), Roberti and Toth (2012) and Cacchiani et al. (2020). Exact methods, such as branch and bound, and branch and cut, have been proposed in Fischetti et al. (2003, 2007) and Ascheuer et al. (2001). In addition to exact procedures, many heuristics, such as tabu search (Gupta 2013), genetic algorithms (Razali and Geraghty 2011; Hussain et al. 2017; Juneja et al. 2019), ant colony algorithms (Yang et al. 2008; Gan et al. 2010), bacterial algorithms (Kóczy et al. 2018), and metaheuristics (Erdogan et al. 2012; Antosiewicz et al. 2013; Fogarasi et al. 2022) were developed to solve the problem. A survey on the variants and solution methods of the TSP can be found in Chauhan et al. (2012) and Cheikhrouhou and Khoufi (2021).

Despite the fact that the problem of transporting patients to the same hospital has had a growing importance in this topic, not much attention has been paid to it so far. In this article we studied a single vehicle patient transportation problem in which the vehicle had multiple capacity, patients at the pick-up points had different requirements (maximum traveling time, earliest pickup time, latest arrival time), but the endpoint of all pick-up points was the same. Although this problem can be connected to the many-to-many DARP, we determine that the solution obtained by solving a many-to-many DARP was not necessarily a feasible solution to the original problem. Therefore, many-to-many DARP models cannot be applied to one-to-one in our task. In this article, we present 4 MILP models that have been compared through several real-world test cases.

3 Problem description

Let G be a directed graph with $N + 2$ vertices where N denotes the number of the pickup locations. The vertices of G are numbered from 0 to $N + 1$. The vertex 0 represents the depot, vertex $N + 1$ represents the hospital while the other vertices represent the pickup points. Each arc (i, j) of the graph has two weights representing the traveling time from i to j and the traveling distance between the points i and j. A single vehicle transports the passengers to the hospital. The vehicle starts its tour from the depot and ends its tour at the depot. The depot has opening hours; the vehicle can start and end its route in this time interval. The vehicle has two capacities: one for the patients and one for the disabled patients. For each pickup point i the following pieces of information are known:

- The number of patients at point i.
- The number of disabled patients at point i.
- The earliest pickup time of the passengers at point i.
- The maximum riding time of passengers at point i.
- The latest arrival time of the passengers at point i at the hospital.
- The time it takes that all passengers at point i to get on the vehicle.

We assume that at each point neither the number of patients nor the number of disabled patients exceeds the capacity of the vehicle. Furthermore, if a vehicle arrives at a pickup point, then all passengers at that point have to get on the vehicle, which implies that each pickup point is visited by the vehicle exactly once. Finally, it is assumed that if the vehicle arrives at the hospital, then all passengers transported by the vehicle get off the vehicle. The goal is to find the optimal route of the vehicle that minimizes the total distance traveled by the vehicle.

4 Mathematical models

In the first model, the route of the vehicle is broken down to rounds. In the first round, the vehicle leaves the depot and after visiting some pickup points it arrives at the hospital. In the last round the vehicle directly travels from the hospital to the depot. In the remaining rounds the vehicle starts from the hospital then visits some pickup points and travels back to the hospital.

4.1 Notations of model 1

Parameters:

N Number of pickup locations
K Number of rounds
D_{ij} The traveling distance between points i and j; $0 \leq i,j \leq N+1$
T_{ij} Traveling time from point i to j; $0 \leq i,j \leq N+1$
C_1 Maximum capacity of the vehicle for patients
C_2 Maximum capacity of the vehicle for disabled patients
p_i Number of patients at point i; $1 \leq i \leq N$
dp_i Number of disabled patients at point i; $1 \leq i \leq N$
I_i Maximum traveling time for patients at point i; $\leq i \leq N$
a_i Boarding time of the patients at point i; $1 \leq i \leq N$
tr_i Earliest pickup time for patients at point i; $1 \leq i \leq N$
ta_i The latest arrival time at the hospital for patients at point i; $1 \leq i \leq N$
A_0 Opening time of the depot
B_0 Closing time of the depot

Continuous variables:

s^k Starting time of vehicle in the k-th round; $1 \leq k \leq K$
e^k Finishing time of the vehicle in the k-th round; $1 \leq k \leq K$
pn_i Number of patients in the vehicle after visiting point i; $0 \leq i \leq N+1$
dpn_i Number of disabled patients in the vehicle after visiting point i; $0 \leq i \leq N+1$
w_i Traveling time of the passengers at point i; $1 \leq i \leq N$
m_i Arrival time of the vehicle at point i; $1 \leq i \leq N$

Binary variables:

x_{ij}^{k} Equals 1, if in the k-th round, the vehicle travels from point i to j;
$0 \leq i, k \leq N+1,\ 1 \leq k \leq K$

4.2 Constraints of model 1

The mathematical model of the problem contains the following constraints:

- Each pickup point is visited by the vehicle in exactly one round.

$$\sum_{i=0}^{N+1} \sum_{k=1}^{K} x_{ij}^{k} = 1 \qquad 1 \leq j \leq N \tag{1}$$

- The vehicle enters a point in some round iff the vehicle leaves that point in the same round.

$$\sum_{j=0}^{N+1} x_{ij}^{k} = \sum_{j=0}^{N+1} x_{ji}^{k} \qquad 1 \leq i \leq N; 1 \leq k \leq K \tag{2}$$

- The vehicle starts from the depot and arrives at the hospital in the first round.

$$\sum_{i=1}^{N} x_{0i}^{1} = 1 \tag{3}$$

$$x_{i0}^{1} = 0 \qquad 1 \leq i \leq N+1 \tag{4}$$

$$\sum_{i=1}^{N} x_{i,N+1}^{1} = 1 \tag{5}$$

$$x_{N+1,i}^{1} = 0 \qquad 0 \leq i \leq N \tag{6}$$

- Except for the first round, the vehicle does not leave the depot.

$$\sum_{i=1}^{N+1} x_{0,i}^{k} = 0 \qquad 2 \leq k \leq K \tag{7}$$

- Except for the last round, the vehicle does not enter the depot.

$$\sum_{i=1}^{N+1} x_{i0}^{k} = 0 \qquad 1 \leq k \leq K-1 \tag{8}$$

- Except for the first and last rounds, if the vehicle leaves the hospital then the vehicle enters the hospital in the same round.

$$\sum_{i=0}^{N} x_{N+1,i}^{k} = \sum_{i=0}^{N} x_{i,N+1}^{k} \qquad 2 \le k \le K-1 \tag{9}$$

- From round 2 on, the vehicle leaves the hospital at most once.

$$\sum_{i=0}^{N} x_{N+1,i}^{k} \le 1 \qquad 2 \le k \le K \tag{10}$$

- In the last round, the vehicle travels from the hospital directly to the depot.

$$x_{N+1;0}^{K} = 1 \tag{11}$$

- The number of the patients/disabled patients on the vehicle after leaving the depot or the hospital is 0.

$$pn_0 = 0 \tag{12}$$

$$pn_{N+1} = 0 \tag{13}$$

$$dpn_0 = 0 \tag{14}$$

$$dpn_{N+1} = 0 \tag{15}$$

- If in any round the vehicle travels from point i to a pick up point j, then the number of patients/disabled patients on the vehicle after leaving j is at least the sum of the number of the patients/disabled patients on the vehicle after leaving point i and the number of patients/disabled patients at the pickup point j.

$$\begin{gathered} pn_i + p_j + M(1 - x_{ij}^{k}) \le pn_j \\ 0 \le i \le N+1,\ 1 \le j \le N,\ 1 \le k \le K \end{gathered} \tag{16}$$

$$\begin{gathered} dpn_i + dp_j + M(1 - x_{ij}^{k}) \le dpn_j \\ 0 \le i \le N+1,\ 1 \le j \le N,\ 1 \le k \le K \end{gathered} \tag{17}$$

- The number patients/disabled patients in the vehicle can not exceed the capacity of the vehicle,

$$dp_i \le C_1 \qquad 1 \le i \le N \tag{18}$$

$$dpn_i \le C_2 \qquad 1 \le i \le N \tag{19}$$

- If the vehicle travels from point i to point j in some round, then the arriving time of the vehicle at point j is at least the sum of the arriving time of the vehicle at point i the traveling time from i to j and the boarding time of the passengers at point i (if i is a pickup point).

$$s^1 + T_{0,j} + M(1 - x^1_{0,j}) \leq m_j \qquad 1 \leq j \leq N \tag{20}$$

$$\begin{gathered} s^k + T_{N+1,j} + M(1 - x^k_{N+1,j}) \leq m_j \\ 1 \leq j \leq N,\ 2 \leq k \leq K-1 \end{gathered} \tag{21}$$

$$\begin{gathered} m_i + T_{i,j} + M(1 - x^k_{i,j}) \leq m_j \\ 1 \leq i,j \leq N,\ 1 \leq k \leq K-1 \end{gathered} \tag{22}$$

$$\begin{gathered} m_i + T_{i,N+1} + M(1 - x^k_{i,N+1}) \leq e^k \\ 1 \leq i \leq N,\ 1 \leq k \leq K-1 \end{gathered} \tag{23}$$

$$s^K + T_{N+1,0} = e^K \tag{24}$$

- The vehicle has to arrive at a pickup point after the prescribed earliest pickup time at that point.

$$tr_i \leq m_i \qquad 1 \leq i \leq N \tag{25}$$

- The vehicle has to leave and enter the depot in the depot's opening time.

$$A_0 \leq s^1 \tag{26}$$

$$e^K \leq B_0 \tag{27}$$

- The ending time of a round cannot be less than the starting time of the same round.

$$s^k \leq e^k \qquad 1 \leq k \leq K \tag{28}$$

- A vehicle can start a round after it finishes the previous round.

$$e^k \leq s^{k+1} \qquad 1 \leq k \leq K-1 \tag{29}$$

- If in any round the vehicle travels from a pickup point i to the hospital, then the traveling time of the passengers at point i is the boarding time at point i plus the traveling time from point i to the hospital.

$$a_i + T_{i,N+1} - M \cdot \left(1 - \sum_{k=1}^{K-1} x^k_{i,N+1}\right) \leq w_i \qquad 1 \leq i \leq N \tag{30}$$

- If in any round the vehicle travels from a pickup point i to pickup point j, then the traveling time of the passengers at point i is at least the sum of the traveling time of the passengers at point j and difference between the arrival time of the vehicle at point i and point j.

$$m_j - m_i + w_j - M \cdot \left(1 - \sum_{k=1}^{K-1} x_{i,j}^k\right) \leq w_i \qquad 1 \leq i,j \leq N \tag{31}$$

- The traveling time of the patients at any point i cannot exceed the prescribed maximum traveling time at that point.

$$w_i \leq I_i \qquad 1 \leq i \leq N \tag{32}$$

- Each passenger has to arrive at the hospital in time.

$$m_i + w_i \leq ta_i \qquad 1 \leq i \leq N \tag{33}$$

- The goal is to minimize the total distance traveled by the vehicle.

$$\sum_{i=0}^{N+1} \sum_{j=0}^{N+1} \sum_{k=1}^{K} x_{ij}^k \cdot D_{ij} \rightarrow \min \tag{34}$$

The first MILP model of the problem is defined by constraints (1-33) and objective function (34).

4.3 Model 2

To formulate the second MILP model of the problem, first the original model is transformed to a special traveling salesman problem (TSP) with pickup and delivery points. Let G' be a directed graph with $2N+1$ vertices. Vertex 0 represents the depot, the vertices $1, 2, \dots N$ represent the pickup points and the vertices $N+1, N+2, \dots 2N$ represent the hospital. The patients at point i have to be transported to the hospital point $i+N$, so each request has a pickup point and a delivery point. Since the vertices $N+1, N+2, \dots 2N$ are all hospital points, hence the traveling time and the traveling distance between two such points is 0. Furthermore, for each pickup point i, the traveling time (traveling distance) from i to a hospital point is the same for all hospital points. Then the optimal solution of the original problem can be found by solving a TSP on this new graph with the following constraints:

- The capacities (patients, disabled patients) of the vehicle are not violated.
- The special requests of the passengers (earliest pickup time, maximum traveling time, latest arrival time) are satisfied.
- Every pickup point is visited by the vehicle earlier than its requested point, i.e., pickup point i precedes delivery point $i+N$ in the tour of the vehicle.
- If the vehicle arrives at a hospital point, then every passenger on the vehicle gets off the vehicle. This means that if the vehicle visits pickup points $i_1, i_2, \dots, i_j$ before arriving at a hospital point then the vehicle must arrive at one of the hos-

pital points $i_{1+N}, i_{2+N}, \ldots, i_{j+N}$ and then the route of the vehicle has to be followed by visiting the other hospital points of the points $i_{1+N}, i_{2+N}, \ldots, i_{j+N}$.

It is important to mention that without the last constraint a feasible solution of the transformed problem may not be a feasible solution of the original problem. For example if we have 3 pickup points, then in the transformed problem the pickup points are numbered 1, 2, 3, the hospital points are numbered by 4,5,6 and the depot is denoted by 0. The feasible tour 0–2–3–5–1–4–6–0 of the transformed problem is not a feasible tour of the original problem since the vehicle picks up the passengers at pickup points 2 and 3 but after arriving at the hospital (at hospital point 5, which is the delivery point of pickup point 2) the patients of pickup point 3 do not get off the vehicle (the tour is not followed by the hospital point 6).

Our second model is based on the MTZ formulation of the traveling salesman problem (Miller et al. 1960). Model 2 contains the same parameters as Model 1. Model 2 contains the following variables:

Continuous variables:
pn_i Number of patients in the vehicle after visiting point i; $0 \le i \le 2N$
dpn_i Number of disabled patients in the vehicle after visiting point i; $0 \le i \le 2N$
m_i Arrival time of the vehicle at point i; $1 \le i \le N$
Integer variables:
u_i $0 \le i \le 2N$
Binary variables:
x_{ij} Equals 1, if the vehicle travels from point i to j; $0 \le i,j \le 2N$

Model 2 contains the following constraints:

- The vehicle enters and leaves each point exactly once.

$$\sum_{i=0}^{2N} x_{ij} = 1 \qquad 0 \le j \le 2N \tag{35}$$

$$\sum_{j=0}^{2N} x_{ij} = 1 \qquad 0 \le i \le 2N \tag{36}$$

- Subtour elimination constraints of the classical MTZ model:

$$u_0 = 1 \tag{37}$$

$$2 \le u_i \qquad 1 \le i \le 2N \tag{38}$$

$$u_i \le 2N + 1 \qquad 1 \le i \le 2N \tag{39}$$

$$u_i + 1 \le u_j + 2N \cdot (1 - x_{i,j}) \qquad 1 \le i,j \le 2N \tag{40}$$

- Vehicle has to visit a pickup point before it travels to the corresponding hospital point.

$$u_i + 1 \leq u_{i+N} \qquad 1 \leq i \leq N \tag{41}$$

- The vehicle travels from pickup point i to pickup point j iff the vehicle travels from hospital point $i + N$ (which corresponds to pickup point i) to hospital point $j + N$ (which corresponds to pickup point j).

$$x_{ij} = x_{i+N,j+N} \qquad 1 \leq i,j \leq N \tag{42}$$

- The number of the patients/disabled patients on the vehicle after leaving the depot or a hospital point is 0.

$$pn_0 = 0 \tag{43}$$

$$pn_i = 0 \qquad N + 1 \leq i \leq 2N \tag{44}$$

$$dpn_0 = 0 \tag{45}$$

$$dpn_i = 0 \qquad N + 1 \leq i \leq 2N \tag{46}$$

- If the vehicle travels from point i to a pick up point j, then the number of patients/disabled patients on the vehicle after leaving j is at least the sum of the number of the patients/disabled patients on the vehicle after leaving point i and the number of patients/disabled patients at the pickup point j.

$$\begin{aligned} &pn_i + p_j + M(1 - x_{ij}) \leq pn_j \\ &0 \leq i \leq 2N,\ 1 \leq j \leq N \end{aligned} \tag{47}$$

$$\begin{aligned} &dpn_i + dp_j + M(1 - x_{ij}) \leq dpn_j \\ &0 \leq i \leq 2N,\ 1 \leq j \leq N \end{aligned} \tag{48}$$

- The number patients/disabled patients in the vehicle can not exceed the capacity of the vehicle,

$$dp_i \leq C_1 \qquad 1 \leq i \leq N \tag{49}$$

$$dpn_i \leq C_2 \qquad 1 \leq i \leq N \tag{50}$$

- If the vehicle travels from point i to point j, then the arriving time of the vehicle at point j is at least the sum of the arriving time of the vehicle at point i the traveling time from i to j and the boarding time of the passengers at point i (if i is a pickup point).

$$m_0 + T_{0,j} + M(1 - x_{0,j}) \leq m_j \qquad 1 \leq j \leq 2N \tag{51}$$

$$\begin{aligned} m_i + a_i + T_{i,j} + M(1 - x_{i,j}) \leq & m_j \\ 1 \leq i \leq N,\ 1 \leq j \leq & 2N \end{aligned} \tag{52}$$

$$\begin{aligned} m_i + T_{i,j} + M(1 - x_{i,j}) \leq & m_j \\ N + 1 \leq i \leq 2N,\ 1 \leq j \leq & 2N \end{aligned} \tag{53}$$

- The vehicle has to arrive at a pickup point after the prescribed earliest pickup time at that point.

$$tr_i \leq m_i \qquad 1 \leq i \leq N \tag{54}$$

- The traveling time of the patients at any point i cannot exceed the prescribed maximum traveling time at that point.

$$m_{i+N} - m_i \leq I_i \qquad 1 \leq i \leq N \tag{55}$$

- Each passenger has to arrive at the hospital in time.

$$m_{i+N} \leq ta_i \qquad 1 \leq i \leq N \tag{56}$$

- The vehicle has to leave and enter the depot within the depot's opening time.

$$A_0 \leq m_0 \tag{57}$$

$$m_{i+N} + T_{i+N,0} \leq B_0 \qquad 1 \leq i \leq N \tag{58}$$

- The goal is to minimize the total distance traveled by the vehicle.

$$\sum_{i=0}^{2N} \sum_{j=0}^{2N} x_{ij} \cdot D_{ij} \rightarrow \min \tag{59}$$

Constraints (51) and (52) together imply that if the vehicle visits some pickup points before traveling to a hospital point, the vehicle visits the corresponding hospital points in the same order, which implies that all passengers will get off of the vehicle at the hospital.

Our second MILP model consists of constraints (35–58) and objective function (59).

4.4 Complexity of models 1 and 2

Table 1 contains the number of continuous, integer and binary variables, and the number of the constraints of Models 1 and 2 depending on the number of pickup points (N). In model 1 the number of rounds (K) was set to $N + 1$.

It can be seen from Table 1 that Model 2 contains less continuous variables than Model 1 and it can be easily calculated that Model 2 contains less binary variables and constraints than Model 1 if $N \geq 4$.

Table 1 Complexity of models 1 and 2

	Model 1	Model 2
Continuous variables	$6N + 2$	$4N$
Integer variables	0	$2N$
Binary variables	$N^3 + N^2$	$4N^2 + 2N$
Constraints	$3N^3 + 7N^2 + 10N + 3$	$13N^2 + 15N + 3$

4.5 Strengthening the MILP models

If the latest arrival time of the patients at pickup point i at the hospital is less than or equal to the earliest pickup time of patients at pickup point j, then in any feasible solution the vehicle cannot travel from i to j, or from j to i. Furthermore, the vehicle has to visit j in a later round than i. Hence, the following valid inequalities can be added to Model 1:

$$\begin{gathered} x_{ij}^k = 0 \text{ and } x_{ji}^k = 0 \\ 1 \le i,j \le N,\ 1 \le k \le K, \qquad ta_i \le tr_j \end{gathered} \tag{60}$$

$$\begin{gathered} \sum_{l=0}^{N} x_{li}^k + \sum_{l=0}^{N} x_{lj}^{k_1} \le 1 \\ 1 \le i,j \le N,\ 1 \le k_1 \le k \le K, \qquad ta_i \le tr_j \end{gathered} \tag{61}$$

So the strengthened version of Model 1, called Model 1 S contains the constraints (1-33,60,61) and objective function (34).

Constraint (60) is a valid inequality for Model 2, too. Furthermore, if the latest arrival time of the patients at pickup point i at the hospital is less than or equal to the earliest pick up time of patients at pickup point j, then after leaving pickup point i, the vehicle has to visit at least one more point before visiting pickup point j. So Model 2 can be extended by the following valid inequality:

$$\begin{gathered} u_i + 2 \le u_j \\ 1 \le i,j \le N,\ ta_i \le tr_j \end{gathered} \tag{62}$$

Model 2 S, which is the strengthened version of Model 2, consists of constraints (35–58,60,62) and objective function (59).

4.6 Numerical results

To compare the models, 5 test problems were created. The location of the depot, the hospital, and the pickup points were chosen from streets in the town of Sopron while the distances and traveling times between the points were determined by using a route planning program. Sopron is a small town on the border of west Hungary with

Table 2 Main characteristics of the test problems

Instance	Number of pickup points	Total number of patients	Total number of disabled patients
1	10	20	4
2	15	30	5
3	20	40	11
4	25	50	9
5	30	60	9

Table 3 Comparison of the MILP models

Instance	Running time (in s)				Best solution (in km)			
	Model				Model			
	1	2	1 S	2 S	1	2	1 S	2 S
1	8	4	2	2	31.1	31.1	31.1	31.1
2	829	*	835	5	63.7	∘	63.7	63.7
3	*	*	*	7	84.5	∘	84.5	84.5
4	*	*	*	42	103	∘	103	103
5	*	*	*	50	∘	∘	103.95	103.95

1 h running time; *: the optimal solution was not found; ∘: no feasible solution was found

an area of 169 square kilometer and an official population of 69,000. It has mountainous features. The hospital can be reached by car even from the furthest point in 15 min.

The number of patients, patients with reduced mobility and the requested time of arrival at the hospital were randomly determined for the pickup points. The earliest admission time was set to 1 h before the requested arrival time at the hospital. Each patient who was not limited in mobility was assigned a 1-minute boarding time while each patient with limited mobility was assigned a 2-minute boarding time. To determine the maximum travel time of passengers at point i, we first added up the travelling time from point i to the hospital and the boarding time of the passengers, and then an extra 30 min was added to this value. The main properties of the 5 problems, including the number of the pickup points and the total number of patients and disabled patients, are summarized in Table 2.

The corresponding MILP models of the problems were formulated and solved using IBM Ilog Cplex Optimization Studio on an Intel(R) Core(TM) i7-7700HQ personal computer equipped with 8 GB RAM and 256 GB SSD. The time limit was set to 1 h. Except for the variable selection the default settings of CPLEX were used. In the case of variable selection the strong branching option of CPLEX was applied.

Table 3 contains the results of the numerical tests, including the time required by the models to find the optimal solution and the objective values of the best solutions.

Comparing the basic models (Model 1 and Model 2), it can be established that Model 1 worked better on the test problems than Model 2. Both models found the optimal solution in the smallest problem in a few seconds. For problems with at least 15 pickup points, Model 2 did not even find a feasible solution in the 1 h running time. Model 1 provided an optimal solution for the second problem and it found a feasible solution for problems 3 and 4. For the largest problem Model 1 also did not find a feasible solution.

Comparing a basic model with its strengthened version it can be also observed that the strengthened version of a basic model provided better solutions than the original model. Model 1 and its strengthened version, Model 1 S found the optimal solution of problems 1 and 2 in about the same time. For problems 3 and 4, both models found a feasible solution with the same objective value. Actually, it can be seen from Table 3 that these solutions are optimal solutions (as Model 2 S did find the optimal solution in these cases), but CPLEX was not able to prove it in the 1 h running time. In the case of the largest problem, Model 1 did not find a feasible solution, while the strengthened version found a feasible solution (which, again, is the optimal solution, but CPLEX could not prove it in the 1 h running time). There were bigger differences between the second and the fourth models. Model 2 found the optimal solution in the smallest example but it did not even find a feasible solution to the other problems. In contrast, the strengthened version found the optimal solution in every test problem in less than 1 min.

Comparing all of the 4 models it can be said that Model 2 S performed the best on the test problems as it found the optimal solution to every problem. Model 1 S was the second best model providing the optimal solution to the first two problems and a feasible solution for the remaining problems. The third model was Model 1, while Model 2 was the worst, because it provided a solution only for the first test, but in the other tests it was not able to find a feasible solution.

5 Conclusion

The patient transportation services in Hungary (as well as in many other countries) use outdated routing methods, mainly focusing on one patient at a time. The service itself is of high quality, as well-equipped vehicles ensure the patients' arrival at various health points, but it is extremely costly and less utilized. Patients who do not require emergency intervention opt for alternative methods of travel. These transportation possibilities can be taxi services, public transportation or even personal vehicle travel. However, none of these solutions fully cover the area of passenger transportation.

In this article we investigated a single vehicle DARP problem arising from patient transportation. In this problem a vehicle with multiple capacity transports the patients to the same hospital. We introduced 2 MILP models of the problem (Model 1 and 2). To improve the efficiency of these models they were strengthened by adding valid inequalities to them. The strengthened versions are called Model 1 S and 2 S. To compare the models 5 test examples were created. The tests showed that model 2 S was the best as it found the optimal solution for each test problem in less than 1 min. The logistical applicability of this model could be done in the future by running the program on the requests received up to the day before the delivery. Based on real-life experience, many logistics companies (mainly parcel delivery companies) operate on a similar principle. European regulations stipulate customers must be notified in time about next-day logistics delivery services. The running time would be a maximum of 1 h based on the testing tasks, after that the company would have time for the subsequent additional organizational and management tasks. In this way, all customers would be notified about the delivery in a timely manner.

In the future, we would like to further expand the models by taking into account additional aspects. We would increase the number of vehicles, in order to be able to solve systems with a larger number of patients, so that the needs of the patients are not harmed. Furthermore, we intend to increase the number of possible delivery locations in order to be able to use the model in larger cities as well. Based on what we have experienced after Covid 19, we would also like to take into account the possibility of passenger conflicts of interest. For example, if an infectious patient needs to be transported to the hospital, this patient should be transported separately. Furthermore, a cleaning period of the vehicle may also be required when the vehicle is disinfected after transporting infectious patients.

The definition of these conditions and the expansion of the MILP models require further research. Also, by increasing the complexity of MILP models, the time required for solving the models will increase. Therefore, for large scale problems, beside the MILP models we would like to investigate heuristic approaches as well.

Acknowledgements Project No. TKP2021-NKTA-48 has been implemented with the support provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund, financed under the TKP2021-NKTA funding scheme.

Funding Open access funding provided by Széchenyi István University (SZE).

Data availibility The datasets generated and analyzed during the current study are available from the corresponding author upon request.

Declarations

Conflict of interest The authors declare no competing interests.

References

Agra A, Christiansen M, Wolsey L (2022) Improved models for a single vehicle continuous-time inventory routing problem with pickups and deliveries. Eur J Oper Res 297(1):164–179

Antosiewicz M, Koloch G, Kaminski B (2013) Choice of best possible metaheuristic algorithm for the travelling salesman problem with limited computational time: quality, uncertainty and speed. J Theor Appl Comput Sci 7(1):46–55

Armbrust P, Hungerländer P, Maier K et al (2022) Case study of dial-a-ride problems arising in Austrian rural regions. Transp Res Procedia 62:197–204

Ascheuer N, Fischetti M, Grötschel M (2001) Solving the asymmetric travelling salesman problem with time windows by branch-and-cut. Math Program 90:475–506

Büsing C, Comis M, Rauh F (2021) The dial-a-ride problem in primary care with flexible scheduling. arXiv:2105.14472

Cacchiani V, Contreras-Bolton C, Toth P (2020) Models and algorithms for the traveling salesman problem with time-dependent service times. Eur J Oper Res 283(3):825–843

Chane-Haï T, Vercraene S, Monteiro T (2020) Sharing a ride time constraint in a multi-trip dial-a-ride problem. An application to the non-urgent patient transportation problem. In: MOSIM 2020-13th international conference on modeling, optimization and simulation

Chauhan C, Gupta R, Pathak K (2012) Survey of methods of solving tsp along with its implementation using dynamic programming approach. Int J Comput Appl 52(4)

Cheikhrouhou O, Khoufi I (2021) A comprehensive survey on the multiple traveling salesman problem: applications, approaches and taxonomy. Comput Sci Rev 40:100369

den Berg PV, van Essen JT (2019) Scheduling non-urgent patient transportation while maximizing emergency coverage. Transp Sci 53(2):492–509

Dornemann J (2023) Solving the capacitated vehicle routing problem with time windows via graph convolutional network assisted tree search and quantum-inspired computing. Front Appl Math Stat 9

Ercsey Z, Nagy A, Tick J, Kovács Z (2021) Bus transport process networks with arbitrary launching times. Acta Polytech Hung 18(4)

Erdogan G, Battarra M, Laporte G, Vigo D (2012) Metaheuristics for the traveling salesman problem with pickups, deliveries and handling costs. Comput Oper Res 39(5):1074–1086

Fischetti M, Lodi A, Toth P (2003) Solving real-world ATSP instances by branch-and-cut. In: Combinatorial optimization-Eureka, You Shrink! Papers dedicated to jack edmonds 5th international workshop Aussois, France, March 5–9, 2001, Revised papers. Springer, Berlin, Heidelberg, pp 64–77

Fischetti M, Lodi A, Toth P (2007) Exact methods for the asymmetric traveling salesman problem. In: The traveling salesman problem and its variations, pp 169–205

Fogarasi G, Tüü-Szabó B, Földesi P, Kóczy LT (2022) Comparison of discrete memetic evolutionary metaheuristics for TSP. In: Computational intelligence and mathematics for tackling complex problems 2. Springer, Cham, pp 29–37

Friedler F, Tarjan K, Huang YW, Fan LT (1992) Graph-theoretic approach to process synthesis: axioms and theorems. Chem Eng Sci 47(8):1973–1988

Frits M, Bertok B (2021) Routing and scheduling field service operation by P-graph. Comput Oper Res 136:105472

Gan R, Guo Q, Chang H, Yi Y (2010) Improved ant colony optimization algorithm for the traveling salesman problems. J Syst Eng Electron 21(2):329–333

Gkiotsalitis K (2022) Scheduling on-demand minibuses considering the in-vehicle crowding inconvenience due to covid-19. In: 101st Transportation research board (TRB) annual meeting 2022

Gupta D (2013) Solving tsp using various meta-heuristic algorithms. Int J Recent Contrib Eng Sci IT (iJES) 1(2):22–26

Ho SC, Szeto WY, Kuo YH et al (2018) A survey of dial-a-ride problems: literature review and recent developments. Transp Res Part B Methodol 111:395–421

Hussain A, Muhammad YS, Nauman Sajid M, Hussain I, Mohamd Shoukry A, Gani S (2017) Genetic algorithm for traveling salesman problem with modified cycle crossover operator. Comput Intell Neurosci

Juneja SS, Saraswat P, Singh K, Sharma J, Majumdar R, Chowdhary S (2019) Travelling salesman problem optimization using genetic algorithm. In: 2019 Amity international conference on artificial intelligence (AICAI). IEEE, pp 264–268

Kóczy LT, Földesi P, TüűSzabó B (2018) Enhanced discrete bacterial memetic evolutionary algorithm-An efficacious metaheuristic for the traveling salesman optimization. Inf Sci 460:389–400
Lim A, Zhang Z, Qin H (2017) Pickup and delivery service with manpower planning in Hong Kong public hospitals. Transp Sci 51(2):688–705
Luo Z, Liu M, Lim A (2019) A two-phase branch-and-price-and-cut for a dial-a-ride problem in patient transportation. Transp Sci 53(1):113–130
Madankumar S, Rajendran C (2019) A mixed integer linear programming model for the vehicle routing problem with simultaneous delivery and pickup by heterogeneous vehicles, and constrained by time windows. Sādhanā 44(2):1–14
Matai R, Singh SP, Mittal ML (2010) Traveling salesman problem: an overview of applications, formulations, and solution approaches. Travel Salesm Prob Theory Appl 1(1):1–25
Miller CE, Tucker AW, Zemlin RA (1960) Integer programming formulation of traveling salesman problems. J ACM (JACM) 7(4):326–329
Melachrinoudis E, Ilhan AB, Min H (2007) A dial-a-ride problem for client transportation in a healthcare organization. Comput Oper Res 34(3):742–759
Molenbruch Y, Braekers K, Caris A et al (2017) Multi-directional local search for a bi-objective dial-a-ride problem in patient transportation. Comput Oper Res 77:58–71
Nagy A, Ercsey Z, Tick J, Kovács Z (2019) Bus transport process network synthesis. Acta Polytech Hung 16(7)
Öncan T, Altınel IK, Laporte G (2009) A comparative analysis of several asymmetric traveling salesman problem formulations. Comput Oper Res 36(3):637–654
Parragh SN (2011) Introducing heterogeneous users and vehicles into models and algorithms for the dial-a-ride problem. Transp Res Part C Emerg Technol 19(5):912–930
Qu Y, Bard JF (2015) A branch-and-price-and-cut algorithm for heterogeneous pickup and delivery problems with configurable vehicle capacity. Transp Sci 49(2):254–270
Razali NM, Geraghty J (2011) Genetic algorithm performance with different selection strategies in solving TSP. In: Proceedings of the world congress on engineering, vol 2, No 1. International Association of Engineers, Hong Kong, pp 1–6
Roberti R, Toth P (2012) Models and algorithms for the asymmetric traveling salesman problem: an experimental comparison. EURO J Transp Log 1(1–2):113–133
Souza AL, Bernardo M, Penna PH et al (2022) Bi-objective optimization model for the heterogeneous dynamic dial-a-ride problem with no rejects. Optimiz Lett 16(1):355–374
Wang Y, Yuan Y, Ma Y, Wang G (2019) Time-dependent graphs: definitions, applications, and algorithms. Data Sci Eng 4:352–366
Yang J, Shi X, Marchese M, Liang Y (2008) An ant colony optimization method for generalized TSP problem. Prog Nat Sci 18(11):1417–1422