Cheating as a dynamic marketing strategy in monopoly, cartel and duopoly

Markus Eigruber¹ · Franz Wirl¹

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Abstract

In 2015 it was discovered that Volkswagen had manipulated the exhaust emissions of its (diesel) cars. Since then, numerous other automotive car manufacturers were strongly suspected to violate against the same emission standards. This paper investigates how and why firms (monopoly, cartel and duopoly) engage in cheating, more precisely, promising attributes that are actually not part of the product. Firms make claims in order to better market their product but risk damaging their future reputation. The upshot of the paper is the stark difference between open loop and Markov perfect oligopolistic equilibrium outcomes. More precisely, the latter mitigates cheating substantially even below the levels attained by monopolies and cartels (unless consumers have a very short memory), which is contrary to the outcome in the limiting static version of the game. Therefore, revealing the true state (e.g., by mandating strict inspections) could force firms to use this information and play in Markov instead of open loop strategies.

Keywords Cheating · Differential game · Competition · Dieselgate

JEL Classification C71 · C72 · D25 · D43 · L10 · M30

Incidentally, the authors as well as the laureate Gustav Feichtinger and some of his team were also thinking about a model to understand Volkswagen's strategy. We thank for the brief exchange of ideas and our contribution is to bring a game to this topic. Franz Wirl also uses this opportunity to thank Gustav Feichtinger for his long-standing support, the many interesting discussions and the past and hopefully future cooperations.

✉ Markus Eigruber
markus.eigruber@univie.ac.at

Franz Wirl
franz.wirl@univie.ac.at

¹ Faculty of Business, Economics and Statistics, University of Vienna, Oskar Morgenstern Platz 1, 1090 Vienna, Austria

1 Introduction

Economic agents often face strong incentives to engage in immoral/non-conforming/deceptive or fraudulent behaviour in the marketplace. Such decisions to cheat may be made at small-scale bilateral trade relationships but can also be made at the highest corporate level, as the recent Volkswagen emissions scandal has shown. In this case, one of the leading automotive manufacturing companies in the world, the Volkswagen Group, misled customers by manipulating emissions, in particular of nitrogen oxides (NOx), of their cars during testing. Dubbed, inter alia, ‘diesel dupe’ (Hotten 2015), ‘diesel fraud’ (Smith and Parloff 2016), or ‘Dieselgate’ (Gorzelay 2015) by the media, and referred to as ‘diesel issue’ by the company (Volkswagen 2015) this cheating scandal had substantial effects not only on the perpetrator itself, but also on industry standards as well as other competitors in the marketplace (who, after the scandal was made public at the end of September 2015 also became the subject of investigations due to suspected cheating). Figure 1 compares the evolution of cumulative fractional changes in the share prices for the Volkswagen with its main German competitors (BMW AG, Daimler AG) during 2012–2018.

The diesel scandal had a substantial (at least short term) impact on the whole industry. Of course, this is not surprising since the current stock price approximates the present value of future cash flows, which are influenced by information on the company and its assets, including consumers’ goodwill and trust. The combination of negative publicity due to the disclosed scandal and legal consequences, with their inherent uncertain outcomes, affected the stock price substantially. However, despite this major event the industry has rebounded surprisingly strongly, as recent performance indicators show [2017 was Volkswagen’s best year in terms of sales, only to be topped by the numbers of 2018, Volkswagen (2017, 2018)]. Still, since firms knowingly engaged in inappropriate or illegal activities towards their customers by promising non-existent product features, their so far excellent brand’s reputation is being tarred over time.

So why do companies occasionally trade off reputation capital for increased profits? Corporate reputation can be defined as “... a stakeholder’s overall evaluation of a company over time. This evaluation is based on the stakeholder’s direct experiences with the company, any other form of communication and symbolism that provides information about the firm’s actions and/or a comparison with the actions of other leading rivals” (Gotsi and Wilson 2001, p. 29). It is therefore a critical measure of

Fig. 1 Normalized share prices of the main German automobile manufacturers since 2012

trust for agents as well as a valuable intangible asset for corporations (Coombs 2010, 2015; Haskel and Westlake 2017). Reputation is also rivalry, since it can be represented as a continuum, where each subject can be compared and ultimately ranked to another one. Generally, firms recognize this fact and focus on taking actions that increase their respective image, which is well researched in the literature on advertising for goodwill (Nerlove and Arrow 1962; Fershtman 1984; Cellini and Lambertini 2003). However, just like most other resources, reputation capital can not only be built up, but can also be consumed and restored. Stakeholders have certain expectations with regard to the organization's actions, which goes hand in hand with their perception of the firm. Clearly, organizational reputation suffers when such expectations are not fulfilled (Coombs 2015). This link is mediated by how much the organization is perceived to be responsible for a scandal. According to attribution theory, intentional corporate misbehavior leading to preventable scandals with substantial damages evoke some of the strongest perceptions of blame and responsibility, which results in a high potential for reputational damage (Coombs and Holladay 1996; Weiner 2013). Furthermore, one company's crisis may spill over to the entire industry (compare Fig. 1). Such externalities may be positive or negative in nature. It is possible that exposed cheating weakens the cheater's market position and hence favors his competitors. On the other hand, detected cheating could lead deceived victims to put whole industries under general suspicion, or at least increased scrutiny and, hence, to negative externalities. The case of Volkswagen highlights the Glauconian dilemma in business and here in particular with reference to the issue of Corporate Social Responsibility. Plato reports in *The Republic* how Glaucon challenges Socrates' view by the thought experiment in which a ring (Gyges) makes the person invisible [the following is quoted from Haidt (2012)], "... no one, it seems, would be so incorruptible that he would stay on the path of justice or away of other people's property Rather his actions would be in no way different from those of an unjust person and both would follow the same path." In short, having the choice between doing good even at the risk of being considered a scoundrel, or appearing virtuous but actually being a scoundrel, most economic agents, and firms in particular, would presumably have to opt for the second. The contribution of this paper is to investigate the incentives for firms to cheat and in particular how competition induces or mitigates cheating. Of course, intensity of competition matters, however, the evidence in which direction this effect holds is mixed. Cartwright and Menezes (2014) report that intermediate levels of competition induce the highest level of cheating (misreporting rate in their study), whereas Kopel and Lamantia (2018) roughly report the opposite.¹ Schwieren and Weichselbaumer (2010) argue that competitive pressures increase nonethical conduct (compare Shleifer 2004). In this paper we emphasize the dynamics of firm cheating, accounting for the trust or mistrust consumers have about a particular firm and its relation to its competitors. The states of trust depend on past actions, deceptions or honesty, of the firms. In short, we propose a differential game setup to study and compare outcomes under different market and information structures. A surprising finding of this paper is that dynamic strategic oligopolistic interactions (i.e., a Markov perfect equilibrium) mitigate cheating and in fact lead to a ranking

¹ In terms of long run equilibrium share (they use an evolutionary setup) of companies which also include consumer surplus in their objective.

opposite to the one obtained from the corresponding static game. Particularly open loop outcomes lead to excessive cheating, which could be somewhat mitigated by regular (even inconsequential) inspections, such that firms can adjust their policies and use the more honest Markov strategies.

The paper is organized as follows: the next section introduces the model in all its versions for the respective market structures (monopoly, cartel and duopoly). In Sect. 3 we derive the solutions for the static game. Afterwards, Sect. 4 is devoted to the dynamics, considering differing information structures. Section 5 compares the derived results. Finally, we conclude and offer directions for future research.

2 Model

What is crucial if a firm $i = 1, 2$ cheats, greenwashes or, in general, exaggerates the quality of its product? Actual quality (q_0), is normalized to zero, while $q_i > q_0 = 0$ denotes the extent to which firm i exaggerates the quality of its product. This generates a benefit on its own for the firm. However, this immediate benefit requires that cheating is not observable, at least not easily and directly, which holds in particular for experience (Nelson 1974) and credence goods (Darby and Karni 1973; Dullek and Kerschbamer 2006). Although consumers can judge the quality of an experience good once bought (e.g., the food served at an unknown restaurant or a movie in the cinema), this is not necessarily the case with a credence good. Here, consumers may also be unable to identify the properties of the good ex post, i.e., after purchase. Hence, for these types of goods it is often impossible, or at least very costly, to uncover cheating. This applies to many goods like the claims of fluoride in a toothpaste, organic food served in restaurants, to Volkswagen claiming to meet all emission standards (even by saving the costs for the more expensive SCR-catalytic converter, ‘BLUE’, that other companies installed) and to corporate social (ir)responsibility (Langevoort 2002; Lin-Hi and Müller 2013) in general.² Indeed, even claims that are known to be exaggerated matter. For example, it is common knowledge that the officially stated vehicle fuel consumption numbers will be exceeded by far in practice. Nevertheless, the relative position of claims still plays a role if car A is claimed to be more efficient than the competitor B, even though on paper the absolute numbers are implausible. However, cheating is not for free. Stated exaggerated claims can be costly because of the necessity of a camouflage (e.g., via software) or negligible if restricted to purely unverifiable technical claims. Misbehaviour may also cause moral costs for decision makers, including risk of fines and even of imprisonment. For instance, the stakes in the Dieselgate scandal are enormous. On the one hand, additional revenue streams from selling products with exaggerated quality (lemons) have been generated for many years. On the other hand, the costs range from allowing/ordering a software to fool inspectors, fines (recently, one billion dollars which Volkswagen accepted), full compensation for the cars sold in the US, costly upgrades and still pending law suits in Europe, to terms in jail (that Volkswagen’s managers face in the United States).

² Compare BP’s claims and its disasters, first in a refinery in Texas and then also ‘Deepwater Horizon’.

While cheating enhances (*ceteris paribus*) current sales and revenues this comes at the expense that a firm's future reputation is tarred. We let negative reputation, a firm's 'badwill' so to speak (B_i), evolve dynamically,

$$\dot{B}_i = q_i - \delta B_i, \quad i = 1, 2, \quad (1)$$

i.e., badwill B is an exponentially weighted (by the parameter δ , $1/\delta$ being the time constant) average of past cheating. Higher values of B_i lower firm i 's profit (π^i) but exaggerated claims (q_i) increase it. Our setup, in which truth is uncovered only slowly, seems to be particularly applicable to (imperfect) credence goods, so that the firms can hide truth only temporarily but not forever ('you can fool all of the people some of the time, you can fool some of the people all of the time, but you can't fool all of the people all of the time', Abraham Lincoln). Not only the firm's own actions and standing matter but also those of its competitors. Presumably, the relative position to the competitor j matters in both cases—the immediate gain from cheating as well as the reputational damage. Yet, instead of setting up firm i 's,

$$\text{payoff} = \text{benefit (cheating)} - \text{loss (due to badwill)} - \text{costs (cheating)},$$

in a more or less ad hoc manner, we motivate the firms' payoffs from an underlying (linear) demand system. As a consequence, firms compete in cheating and production. We consider two market segments of identical size and their inverse demands, or respectively, their marginal willingness to pay, p_i , $i = 1, 2$,

$$p_1 = 1 + aq_1 - bq_2 - \alpha B_1 + \beta B_2 - x_1 - x_2, \quad (2)$$

$$p_2 = 1 + aq_2 - bq_1 - \alpha B_2 + \beta B_1 - x_1 - x_2, \quad (3)$$

where x_i denotes the quantity of firm i 's product and a, b, α and β are positive constants. Thus, willingness to pay for an additional unit decreases with respect to total supply ($x_1 + x_2$). Absent cheating, the maximum willingness to pay is normalized to unity. Technically, the two goods are homogeneous, however, from the consumers' point of view they differ according to their respective perceived quality. More precisely, the difference between the two segments is that consumers from segment 1 are more susceptible to the claims and the reputation of firm 1 while consumers from segment 2 have the same symmetric feelings towards firm 2. Exaggerated (false) claims of the 'own' good are beneficial, yet similar claims of the competitor are harmful. It seems plausible to assume that the own effect dominates,

$$a > b. \quad (4)$$

A firm's negative reputation, $B_i > 0$, lowers consumers willingness to pay as the firm's claims are mistrusted, while a negative reputation of the competitor is beneficial; again own effects dominate, $\alpha > \beta$. Of course, one may allow for $\beta < 0$, as increased badwill of a firm can have a negative spillover on the competing firm since the entire industry faces increasing distrust. In order to render cheating demand enhancing and thus at least potentially profitable, the short run gain must exceed the associated

(marginal) long run damage. More precisely, since cheating by one (infinitesimal) unit creates the benefit a but implies the (stationary) damage α/δ , or respectively after accounting for discounting (r), we require

$$a > \frac{\alpha}{r + \delta}, \quad (5)$$

and similarly for the cross effects. In particular, for a monopoly (or cartel) selling two brands we need,

$$b > \frac{\beta}{r + \delta} \wedge a - b > \frac{\alpha - \beta}{r + \delta} > 0. \quad (6)$$

These inequalities must hold strictly as the costs of cheating, for simplicity quadratic,

$$\frac{c}{2} q_i^2, \quad c > 0, \quad (7)$$

must be covered too.

Summarizing, if firms maximize their net present value of profits over an infinite horizon,

$$\max_{q_i, x_i} \int_0^\infty e^{-rt} \pi^i dt, \quad i = 1, 2, \quad (8)$$

with

$$\pi^i = p_i x_i - \frac{c}{2} q_i^2, \quad (9)$$

then they must solve the above optimizations subject to the evolution of the (two) states, B_i , $i = 1, 2$, accounting for the competitor's strategies; production costs are normalized to zero.

Remark In this setup production and cheating are chosen simultaneously by both players and both move simultaneously in each period t . An alternative and also plausible assumption is that production (x_i) takes place after reputations (the states B_i) and the degrees of cheating (actions, controls, q_i) are determined. Computing the Nash equilibrium of the intraperiod levels of production for given cheating and badwill would result in,

$$x_i = \frac{1 + a(2q_i - q_j) - b(2q_j - q_i) - \alpha(2B_i - B_j) - \beta(B_i - 2B_j)}{3},$$

which implies the reduced form profit,

$$\pi^i = \frac{(1 + (2a + b)q_i - (a + 2b)q_j - (2\alpha + \beta)B_i + (\alpha + 2\beta)B_j)^2}{9} - \frac{c}{2} q_i^2,$$

of firm i , with cheating as then the only control. This objective includes all linear, mixed and quadratic terms between firms' actions and states. Of course, in this case

a monopoly would keep the flexibility over both controls, cheating and production, since a sequential optimization would be suboptimal.

Returning to our simultaneous setup, we consider two different monopolistic solutions: a single-brand monopoly and a monopoly with different brands, or equivalently, a cartel of different firms. Selling different brands allows to benefit from the cross effects, even if symmetric strategies, to which the analysis is limited, are applied. Therefore, a monopoly or cartel can be modelled in two ways, even if assuming a symmetric allocation, i.e.,

$$q = q_1 = q_2, \quad x = x_1 = x_2,$$

and thus,

$$B = B_1 = B_2.$$

1. A single-brand monopoly cannot benefit/lose from cross effects, thus,

$$b = \beta = 0.$$

2. A multi-brand monopoly or cartel, such that the cross effects matter, i.e., $b \neq 0$ and $\beta \neq 0$ and both are presumably positive.

Therefore, a higher willingness to pay results if the monopoly commits to a single brand if the current loss induced on the other brand ($b > 0$), is larger than the marginal gain due to the implied stationary badwill (β/δ), and again ($\beta/(r + \delta)$) on a net present value basis; this is analogue to assumption (6). Combining this with the assumption that own effects dominate, inequality (4), introducing different brands will lower the willingness to pay and will lower cheating due to the overall negative cross effects. Therefore, more brands are, if at all, only preferable for the opposite inequalities, and will, surprisingly, result in less cheating.

3 Solution of the static game

Unfortunately, an analytical solution of the dynamic setup is either too cumbersome to draw analytical conclusions (in the case of a monopoly and a cartel) or not existing (at least, we could not obtain a closed-form solution). Therefore, we first consider the outcome using the steady state relationships and then use carefully calibrated relations to draw some inferences how competition, actual or inter-brand, (as in the case of different brands sold by a monopolist) affects the honesty of the firms, transiently and in the long run. Computing the symmetric equilibrium choices of x_i and B_i , substituting for $q_i = \delta B_i$, i.e., the steady states of (1), yields the following equilibria.

Result 1 *For the single-brand monopoly (superscript m),*

$$B^m = \frac{a\delta - \alpha}{4c\delta^2 - (a\delta - \alpha)^2}, \quad (10)$$

$$x^m = \frac{c\delta^2}{4c\delta^2 - (a\delta - \alpha)^2}, \quad (11)$$

for the cartel, or equivalently a two-brands monopoly (superscript c),

$$B^c = \frac{(a\delta - \alpha) - (b\delta - \beta)}{4c\delta^2 - ((a\delta - \alpha) - (b\delta - \beta))^2}, \quad (12)$$

$$x^c = \frac{c\delta^2}{4c\delta^2 - ((a\delta - \alpha) - (b\delta - \beta))^2}, \quad (13)$$

and for a duopoly (superscript d),

$$B^d = \frac{a\delta - \alpha}{3c\delta^2 - (a\delta - \alpha)((a\delta - \alpha) - (b\delta - \beta))^2}, \quad (14)$$

$$x^d = \frac{c\delta^2}{3c\delta^2 - (a\delta - \alpha)((a\delta - \alpha) - (b\delta - \beta))^2}. \quad (15)$$

A single brand monopoly engages in more cheating than a monopoly selling two brands,

$$B^m > B^c, \quad (16)$$

and a duopoly cheats even more than a single brand monopoly,

$$B^d > B^m \iff a < \frac{c\delta^2 + \alpha(b\delta - \beta)}{\delta(b\delta - \beta)}, \quad (17)$$

i.e., except high immediate gains from cheating (large a).

Proof The first-order-conditions (FOC's) of the cartel (assuming symmetry) result from equating the marginal costs of cheating to the induced marginal revenue, and production results from equating marginal revenues to the marginal production costs (assumed to be zero),

$$\begin{aligned} c\delta^2 B &= ((a\delta - \alpha) - (b\delta - \beta))x, \\ 1 + ((a\delta - \alpha) - (b\delta - \beta))B - 2x &= 0. \end{aligned}$$

Solving these two equations for (B, x) yields the claim (B^c, x^c) in (12) and (13). These conditions simplify for the monopoly, $b = \beta = 0$, to

$$\begin{aligned} c\delta^2 B &= (a\delta - \alpha)x, \\ 1 + (a\delta - \alpha)B - 2x &= 0, \end{aligned}$$

and thus imply (B^m, x^m) as reported in (10) and (11). The FOC's of the duopoly (again after imposing symmetry),

$$c\delta^2 B = (a\delta - \alpha)x,$$

$$1 + ((a\delta - \alpha) - (b\delta - \beta))B - 3x = 0,$$

imply the same *marginal revenue equals marginal costs condition* for cheating as the monopoly. Solving those two equations verifies the claim (B^d, x^d) in (14) and (15).

Compared to the cartel, the monopoly equates its marginal costs to higher marginal revenues from cheating (keeping output the same) and, similarly, from production (now keeping cheating constant). Directly, the numerator in the monopoly's state, (10), is larger than the one of the cartel, (12), yet the denominator of (10) is smaller since a larger number is subtracted from the term $4c\delta^2$. This is the case under (6), hence, $B^m > B^c$, as reported in (16). A comparison of the numerators of (10) and (14) yields

$$a\delta - \alpha > (a - b)\delta - (\alpha - \beta) \iff b\delta > \beta,$$

i.e., if the immediate effect is dominating the intertemporal one, which is in accordance to our assumptions in (6). Now, comparing the denominators of (10) and (14) yields,

$$4c\delta^2 - (\alpha - a\delta)^2 < 4c\delta^2 - (\alpha - \beta - (a - b)\delta)^2,$$

$$\iff (\beta - b\delta)(\beta - b\delta + 2a\delta - 2\alpha) < 0,$$

and the claim is equivalent to that this difference is negative, as B^d must have the smaller denominator. Solving this inequality for a verifies that $B^d > B^m$ and yields the critical value reported in (17). $\square$

Cheating, $B > 0$, results for a duopoly and a single-brand monopoly only if the intuitively derived condition in assumption (5) holds. For a monopoly selling two brands, the analogous assumption (6) is necessary for cheating. The monopoly cheats more than the cartel (or a multi-brand monopoly), because the associated marginal revenue from cheating is diminished for a cartel due the cannibalization by the different brands (the cross effect). The duopoly's high rate of cheating is a consequence of the larger production, because cheating is determined by a condition identical to the monopoly's and the marginal revenues are scaled by the output.

4 Intertemporal equilibria

4.1 Monopoly and cartel

A symmetric cooperative equilibrium eliminates the gain from cheating that results from relative positions. This mitigates but does not eliminate the incentive for cheating. In spite of the two cases, we can restrict the analysis to the more general case of the cartel by solving the Hamilton–Jacobi–Bellman (HJB) equation for the value function J ,

$$rJ(B) = \max_{q_i, x_i} \left\{ \sum_{i=1}^2 \left[p_i x_i - \frac{c}{2} q_i^2 + J_{B_i} (q_i - \delta B_i) \right] \right\}, \quad (18)$$

and obtain the monopoly solution by setting $\beta = b = 0$. The first-order-conditions of the maximization on the right side of (18),

$$\begin{aligned} ax_1 - bx_2 + J_{B_1} - cq_1 &= 0, \\ 1 + aq_1 - bq_2 - 2(x_1 + x_2) - \alpha B_1 + \beta B_2 &= 0, \end{aligned}$$

imply for the strategies of the monopoly (symmetry allows to confine the analysis to $q = q_1 = q_2$ and $x = x_1 = x_2$),

$$q^m = \frac{a(\alpha B - 1) - 4J'}{a^2 - 4c}, \quad (19)$$

$$x^m = \frac{c(\alpha B - 1) - aJ'}{a^2 - 4c}. \quad (20)$$

as well as the cartel,

$$q^c = \frac{(a - b)[(\alpha - \beta)B - 1] - 4J'}{(a - b)^2 - 4c}, \quad (21)$$

$$x^c = \frac{(\alpha - \beta)cB - c - (a - b)J'}{(a - b)^2 - 4c}. \quad (22)$$

i.e., optimal cheating from a myopic perspective is adjusted for the intertemporal effect (J') of how cheating affects the future badwill. Although a closed-form solution exists, after all only a quadratic polynomial must be solved after proper substitutions and rearrangements when comparing coefficients of the Hamilton–Jacobi–Bellman equation for a quadratic guess of J , the result is very cumbersome and thus suppressed. Instead, we will present a numerical comparison in Sect. 5.

4.2 Duopoly

Open loop

The open loop equilibrium results if both firms solve their intertemporal optimization of (8) subject to (1), where the strategy $\{q_i(t), t \in [0, \infty)\}$ is a best response to the strategy of the opponent $\{q_j(t), t \in [0, \infty)\}$ and vice versa. Moreover, due to symmetry we have $q_i = q_j$ in any Nash equilibrium. This setup ignores how the state, in particular of the competitor, evolves and is thus not subgame perfect if the opponent deviates, for whatever reason, from his optimal policy. However, the open loop strategy can make sense in this case, at least temporarily, since the badwill as well as cheating of the competitor may not be observable—or only occasionally, such as at the beginning of the game. Indeed, Volkswagen and presumably others, were able to hide their policy of cheating for quite some time.

Setting up the Hamiltonian of player i ,

$$H^i = \pi^i + \lambda^i (q_i - \delta B_i) + \mu^i (q_j - \delta B_j), \quad i \neq j,$$

the first-order-conditions for interior solutions are,

$$\begin{aligned} ax_i - cq_i + \lambda^i &= 0, \\ 1 + aq_i - bq_j - 2x_i - x_j - \alpha B_i + \beta B_j &= 0, \\ \dot{\lambda}^i &= (r + \delta) \lambda^i + \alpha x_i, \quad \lim_{t \rightarrow \infty} e^{-rt} \lambda^i B^i = 0, \\ \dot{\mu}^i &= (r + \delta) \mu^i - \beta x_i, \quad \lim_{t \rightarrow \infty} e^{-rt} \mu^i B^j = 0. \end{aligned}$$

The maximization of the Hamiltonian is independent of the competitor's action and state and moreover, the second costate vanishes, $\mu^i = 0$ for $i = 1, 2$ due to the corresponding transversality condition. Assuming a symmetric equilibrium (thus dropping the index), the open loop strategies are given by,

$$q^o = \frac{(\alpha - \beta) a B - a - 3\lambda}{a^2 - ab - 3c}, \quad (23)$$

$$x^o = \frac{(\alpha - \beta) c B - c - (a + b) \lambda}{a^2 - ab - 3c}. \quad (24)$$

The symmetric, intertemporal open loop Nash equilibrium is characterized by the following pair of differential equations,

$$\begin{aligned} \dot{B} &= \frac{(\alpha - \beta) a B - a - 3\lambda}{a^2 - ab - 3c} - \delta B, \\ \dot{\lambda} &= (r + \delta) \lambda + \frac{(\alpha - \beta) \alpha c B - \alpha c - (a + b) \alpha \lambda}{a^2 - ab - 3c}, \end{aligned}$$

which implies the steady state,

$$B_\infty^o = \frac{\alpha - a(r + \delta)}{A}, \quad \lambda_\infty^o = \frac{\alpha c \delta}{A}, \quad (25)$$

with

$$A = (\alpha - \beta + b\delta)(\alpha - a(r + \delta)) - \alpha a \delta + \delta(r + \delta)(a^2 - 3c).$$

Markov perfection

Finally, we consider the possibility of state-contingent strategies as well. In contrast to the time-dependent strategies in the open loop setup, the duopolistic outcome is now characterized by linear Markov strategies. The resulting equilibrium is strongly time-consistent, more precisely, subgame perfect such that the strategies remain optimal even if a player took an erroneous move. The Markov strategies are the natural outcome if applying (continuous time) dynamic programming. Therefore, we set up the Hamilton–Jacobi–Bellman equation (see Dockner et al. 2003) for the value function of player $i = 1$,

$$r V^1(B_1, B_2) = \max_{q_1, x_1} \left\{ \pi^i + V_{B_1}^1(q_1 - \delta B_1) + V_{B_2}^1(q_2 - \delta B_2) \right\}, \quad (26)$$

and analogously for player $i = 2$. The first-order-condition of the maximization of the right hand side for the instrument of cheating is,

$$cq_i = ax_i + V_{B_i}^i, \quad i = 1, 2, \quad (27)$$

i.e., the marginal cost of cheating must equal the marginal benefit, which consists of the induced additional revenues (ax_i) minus the intertemporal loss from a damaged reputation, captured by $V_{B_i}^i < 0$. The production choice follows from the familiar condition that marginal revenue must equal marginal cost (zero in our case),

$$1 + aq_i - bq_j - \alpha B_i + \beta B_j - 2x_i - x_j = 0.$$

As usual for LQ dynamic games, we guess a quadratic solution of the value functions,

$$V^1 = v_0 + v_1 B_1 + v_2 B_2 + \frac{1}{2} \left(v_{11} B_1^2 + 2v_{12} B_1 B_2 + v_{22} B_2^2 \right), \quad (28)$$

$$V^2 = w_0 + w_1 B_1 + w_2 B_2 + \frac{1}{2} \left(w_{11} B_1^2 + 2w_{12} B_1 B_2 + w_{22} B_2^2 \right) \quad (29)$$

with coefficients $v_i, w_i \in \mathbb{R}$ and then assume symmetry,

$$w_0 = v_0, w_1 = v_2, w_2 = v_1, w_{11} = v_{22}, w_{12} = v_{12}, w_{22} = v_{11},$$

such that ultimately only the coefficients of V^1 must be determined. Yet, even exploiting all these conditions does not allow for a meaningful analytical representation of the Markov perfect equilibrium. Therefore, the following numerical examples have to substitute for this shortcoming.

5 Comparison

We base our analysis on the following reference case:

$$a = 0.2, b = 0.1, \alpha = 0.02, \beta = 0.01, c = 0.1, r = 0.05, \delta = 0.2. \quad (30)$$

We let firms discount at 5% per annum and the consumers' rate of forgetting/forgiving be given by 20%, which assumes a rather weak memory. Turning to the demand parameters, cheating by one unit increases a consumer's willingness to pay by $a = 0.2$ (for the preferred good/brand), i.e., by 20% relative to the otherwise maximum willingness to pay (which has been normalized to unity). This increase in willingness to pay is reduced by $b = 0.1$ if also the competitor exaggerates his claims by one unit, which is half of the own effect and in line with assumption (4). The long run effect due to the firm's bad reputation amounts to $\alpha = 0.02$, which satisfies assumption (5), and the effect from the competitor's badwill as half of that, $\beta = 0.01$. More precisely, the immediate effects of cheating are twice the long run effects, own

Fig. 2 Strategies, cheating in **a** and **c** and production in **b** and **d**, in state space

and cross: cheating by one incremental unit triggers the gain $a = 0.2$ (and respectively the loss $b = 0.1$ from the competitor's cheating) but also the long run reputational loss $\alpha/\delta = 0.1$ and $\beta/\delta = 0.05$ if ignoring discounting and transient behavior. That is, substantial incentives for cheating are assumed. The parameters also satisfy inequality (6). Finally, the cost parameter implies for hypothetical cheating levels of $1/2$, 1 and 2 , marginal cheating costs of 0.05 , 0.1 , and 0.2 .

Starting with the monopoly and the reference case, Fig. 2a and b compare the monopolistic strategies of a single brand with a multi-brand monopoly. This figure highlights two things: first, how much more cheating is employed by the single-brand monopoly because countereffects are missing. Second, both monopolistic strategies are increasing in their own badwill, i.e., the worse the reputation the more the monopoly tries to compensate by exaggerating claims and surprisingly, declining with respect to the other brand's state of badwill. This is clearly counterintuitive and we may just quote as an explanation a proverb (which rhymes in German) that one can proceed unabashedly once one's reputation is ruined. The same signs apply to the production strategy, and we note that both strategies (cheating and production) are fairly insensitive to changes in the state.

Figure 2c and d compare the cartel (multi-brand) and the oligopolistic Markov perfect strategies also in state space. In contrast to the monopoly, the oligopolistic

Table 1 Steady state comparison of the controls and states including total discounted utility

	Monopoly		Cartel		Duopoly		
	Stat.	Dyn.	Stat.	Dyn.	Stat.	Dyn.	MP
q_{∞}	0.26	0.31	0.13	0.15	0.34	0.41	0.13
x_{∞}	0.26	0.26	0.25	0.25	0.34	0.34	0.34
p_{∞}	0.51	0.52	0.50	0.50	0.34	0.34	0.34
B_{∞}	1.28	1.55	0.63	0.76	1.69	2.04	0.64
NPV_i	0.13	5.19	0.13	5.05	0.11	2.16	2.24

strategy has the expected sign: cheating decreases with respect to the own badwill but increases with respect to the opponent's. Table 1 compares the steady states of the reference case across the different scenarios, including the outcomes of the static game. The insensitivity of all strategies with respect to the actual state of badwill is confirmed by how little production changes due to the dynamic analysis. However, compared to the cartel, the monopoly's badwill increases and almost doubles, which rather shamefully exploits its monopoly position—not only with respect to output but also with respect to cheating.

In contrast, the dynamic analysis of reputational effects in duopolistic competition shows a dramatic reduction—stationary badwill is cut to below one half of the static game, which is the opposite for a single-brand monopoly. In fact, the duopoly ends up in less cheating than the monopoly (at least stationary and weighted over the evolution as the lowest level of badwill results). However, this positive characterization of competition is due to strong intertemporal strategic interactions (i.e., the linear Markov perfect equilibrium), as the open loop equilibrium of the dynamic game ends up at the highest level of badwill. The reason is that playing in open loop strategies ignores the evolution of the opponent's badwill, which in this case leads to excessive cheating.

Result 2 *Accounting for dynamics results in an ambiguous effect on the long run behaviour of the firm, depending on the market and information structure. For the monopoly as well as the cartel (marginally) more cheating results, whereas a more nuanced picture emerges for actual competition. To summarize:*

$$q_{dyn}^m > q_{stat}^m, q_{dyn}^c > q_{stat}^c \text{ and } q_{OL}^d > q_{stat}^d > q_{MP}^d.$$

In particular, Markov strategies reduce cheating substantially compared to the open loop outcome.

That is, a dynamic consideration can turn the static results upside down if the firms play in the usually more aggressive Markov strategies. These findings are robust as the sensitivity analyses in Fig. 3, which compares stationary levels with respect to the model parameters around the reference case, document. Figure 3a displays the evolution of the long run state for all market structures with respect to δ and shows that the relation is non-monotonic. However, increasing δ (i.e., consumers' memories

Fig. 3 a–d Sensitivity of stationary values with respect to model parameters around the reference case (vertical line) in (30). Production of the monopoly (constant at 0.24) and cartel (increasing but rather flat, from 0.25 to 0.26) are much lower and excluded in d in order to highlight the difference between the open loop and the Markov perfect equilibrium

become shorter and shorter), the dynamic solutions, including the Markov perfect one, approach the static ones. Therefore, the relation between monopoly and duopoly can be reversed at very high values of δ .³ While long run cheating is declining in the marginal effect of one's own state in all models (Fig. 3b), this is not true for the opponent's state (Fig. 3c). In the Markov perfect equilibrium cheating (as well as the state) is declining in β , which then also results in slightly less production due to a smaller inflated market size (Fig. 3d). These figures corroborate the conclusions from Fig. 2 and the previous

³ The rankings described in Result 1 and 2 hold not only for changes in δ but also for large intervals around the reference case for all other parameters not shown here due to brevity.

results and further emphasize that Markovian strategies mitigate cheating. We note further that the monopoly's cheating (and also of the cartel over certain domains of low values of δ and over significant spillovers β) exceeds that of Markov perfect (but not open loop) competition.⁴ This result—the cheating of German carmakers is due to their implicit cartelization—seems to fit hypotheses expressed in various media, including Domen and Hawranek (2017). As a consequence, further investigations by the EU commission in the direction of suspected long-lasting collusion among German carmakers have also been initiated recently.

Hence, information structure is crucial, i.e., the ability to monitor the state of the opponent (and the fact that this also holds for the opponent) leads to a dramatic reduction in cheating and therefore also to less reputational loss. On the other hand, lacking this knowledge or lacking the ability to condition one's strategy on the evolution of the states leads to overshooting and therefore to the worst outcome with respect to consumers (i.e., by how much they are cheated, see Table 1). A trivial way of revealing the true underlying state is by inspections. These would make information public and thus would enable firms to credibly adjust their strategies in time. We note that even inconsequential audits, that is, audits devoid of any material consequences like a punishment fine, are sufficient in mitigating misbehaviour of firms. Indeed, this tightening of standards and regulations has followed after Dieselgate, but probably did not go far enough.

6 Final remarks

In this paper we introduced a simple dynamic game theoretic representation of cheating. Both aspects—dynamics as well as strategic interactions—are crucial. Indeed, a major result is that the analysis of the dynamic game turns the counterpart from the static game upside down. More precisely, a Markov perfect oligopoly equilibrium leads to the lowest level of badwill while a duopoly leads to the highest level of cheating (and thus also of badwill) in the static game and in the dynamic game in open loop strategies. Therefore, not only the consideration of dynamics is important but also the degree of strategic interaction. It is the interaction in the Markov perfect equilibrium that lowers cheating, which is in contrast to the resulting outcome if the duopolists compete in open loop strategies. The reason seems to be the countervailing effect of the opponent's strategy, such that the Markov perfect strategies are fairly flat in the symmetric equilibrium.

Our model is general enough so that it may be applied to many examples. In regards to the game the German carmakers played, one may conjecture that those firms have not acted completely non-cooperatively. Or they colluded tacitly which was enabled by the reliance on common suppliers (e.g., Bosch). We observe that a cartel but even more so, a game played in commitment strategies, increases the incentives to cheat (i.e., the cheating need not be due to cartelization but due to the commitment, e.g., to certain technical solutions). This suggests that establishing competition (as is the

⁴ That is, for most pairs of parameters governing firm internal, as well as external benefits (α, β).

case in most markets anyway) but coupled with sufficient public verification of claims should have the highest priority.

These observations are based to some extent on numerical examples and a theoretical and more robust verification of our claims could be part of future research. Of course, this is a tough task given a dynamic game with multiple states and their interaction in the objectives. Also, the analysis of possible multiple nonlinear equilibria is ignored, because in this setting, a partial differential equation implied by the Hamilton–Jacobi–Bellman equation rules out graphical phase plane techniques in order to derive and characterize nonlinear strategies. Therefore, this is also left for future research (and as the one above, hard to solve too). Other possible extensions would be to include spillovers in the dynamics of the competitor’s badwill and to include stochastic processes. Considering the discrete event of ‘Dieselgate’, Poisson jumps could capture the uncovering of cheating and its subsequent effect on the consumers’ perception (see Hirschmann 2014).

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