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Unfortunately, in the original publication, the conditions Q_1 , Q_2 and Q_3 given in Sect. 7 intended as convergence criteria for the algorithm (Kulański 2015, Listing 1) are incorrect. Their values calculated under the assumptions of the example (Kulański 2015, Sect. 5) are also erroneous. Hence, the conditions and the values are corrected in this erratum.

The equation system [Kulański 2015, (42)] describing the iterative updates of priorities carried out by the algorithm (Kulański 2015, Listing 1) can be written as:

$$\begin{aligned} &\mu(c_1) - \frac{1}{n-1}M(c_2, c_1)\mu(c_2) + \cdots - \frac{1}{n-1}M(c_k, c_1)\mu(c_k) = b_1 \\ &-\frac{1}{n-1}M(c_1, c_2)\mu(c_1) + \mu(c_2) - \frac{1}{n-1}M(c_3, c_2)\mu(c_3) - \cdots - \frac{1}{n-1}M(c_k, c_2)\mu(c_k) = b_2 \\ &\dots\dots\dots \\ &-\frac{1}{n-1}M(c_1, c_k)\mu(c_1) + \cdots - \frac{1}{n-1}M(c_{k-1}, c_k)\mu(c_{k-1}) + \mu(c_k) = b_k \end{aligned} \quad (1)$$

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It is easy to see that, during the second and subsequent iterations, the procedure (Kułakowski 2015, Listing 1) follows the *Jacobi* iterative method for solving a linear equation system:

$$A\mu = b \quad (2)$$

where

$$A = \begin{pmatrix} 1 & -a_{12} & \cdots & -a_{1k} \\ -a_{21} & 1 & \cdots & -a_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ -a_{k1} & -a_{k2} & \cdots & 1 \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu(c_1) \\ \mu(c_2) \\ \vdots \\ \mu(c_k) \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{pmatrix} \quad (3)$$

and $a_{ij} \stackrel{\text{df}}{=} \frac{1}{n-1} M(c_j, c_i)$ for $i \neq j$. In other words, the same equations as (1) are used by the *Jacobi* method (Quarteroni et al. 2000) to the iterative improving $\mu(c_1), \dots, \mu(c_k)$. Since every b_i is a sum of strictly positive components [Kułakowski 2015, (41)], thus also $b_i > 0$, and, accordingly, every $\mu(c_1), \dots, \mu(c_k)$ is strictly positive (1). Hence, if the procedure (Kułakowski 2015, p. 191, Listing 1) converges, the resulting vector μ must be strictly positive.

According to Quarteroni et al. (2000, p. 129), the *Jacobi* method is convergent if A is strictly diagonally dominant by rows, i.e. $|a_{ii}| > \sum_{j=1, j \neq i}^k |a_{ij}|$. Thus, the procedure (Kułakowski 2015, p. 191, Listing 1) leads to an admissible (i.e. real and positive) solution if the conditions $Q^{(1)}, \dots, Q^{(k)}$ are less than 1, where:

$$Q^{(i)} \stackrel{\text{df}}{=} \sum_{j=1, j \neq i}^k \left| \frac{1}{n-1} M(c_j, c_i) \right| \quad (4)$$

Remark 1 Like the conditions Q_2 and Q_3 presented in Kułakowski (2015), also $Q^{(i)}$ has the form of the sum of entries of A . Due to this fact, the conclusions formulated in the paragraph following [Kułakowski 2015, (48)] still remain valid.

The values of Q_1 , Q_2 and Q_3 given in the last paragraph of (Kułakowski 2015, Sect. 7) computed under the assumptions of the numerical example (Kułakowski 2015, Sect. 5) are incorrect. The values of the new correct criteria are $Q^{(1)} = 0.0844$, $Q^{(2)} = 0.182$ and $Q^{(3)} = 0.133$. Since all of them are below 1, the procedure (Kułakowski 2015, Listing 1) applied to the example (Kułakowski 2015, Sect. 5) leads to an admissible solution. Further information on the *Heuristic Rating Estimation* approach, including the criteria given above and a discussion on the convergence, can be found in Kułakowski (2014).

Also in the formulas (2), (25) and (33), the summation should start from $i = 1$ instead of $i = 0$.

References

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