

A method for the estimation of dual transmissivities from slug tests

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Abstract Aquifer homogeneity is usually assumed when interpreting the results of pumping and slug tests, although aquifers are essentially heterogeneous. The aim of this study is to present a method of determining the transmissivities of dual-permeability water-bearing formations based on slug tests such as the pressure-induced permeability test. A bi-exponential rate-of-rise curve is typically observed during many of these tests conducted in heterogeneous formations. The work involved analyzing curves deviating from the exponential rise recorded at the Belchatow Lignite Mine in central Poland, where a significant number of permeability tests have been conducted. In most cases, bi-exponential movement was observed in piezometers with a screen installed in layered sediments, each with a different hydraulic conductivity, or in fissured rock. The possibility to identify the flow properties of these geological formations was analyzed. For each piezometer installed in such formations, a set of two transmissivity values was calculated piecewise based on the interpretation algorithm of the pressure-induced permeability test—one value for the first (steeper) part of the obtained rate-of-rise curve, and a second value for the latter part of the curve. The results of transmissivity estimation for each piezometer are shown. The discussion

presents the limitations of the interpretational method and suggests future modeling plans.

Keywords Dual-permeability · Groundwater hydraulics · Heterogeneity · Preferential flow · Poland

Introduction

The accurate determination of hydraulic conductivity (k) and transmissivity (T) of an aquifer is a crucial part of many hydrogeological studies, especially those related to groundwater control in mining areas, vulnerability mapping and the delineation of protection zones for water intakes (Rafini et al. 2017; Renard 2005). Pumping tests performed in wells can provide the average hydraulic conductivity within the cone of depression. The calculated transmissivity, i.e. the product of hydraulic conductivity and aquifer thickness, will also be an average value.

More local values of hydraulic conductivity and transmissivity can be obtained via slug tests, which require initiating and measuring water-level fluctuations in a piezometer. In order to achieve such fluctuations, water is typically added to or removed from the piezometer. Alternatively, objects heavy enough to sink into water can be inserted. Fluctuations can also be achieved by sealing off the piezometer and inducing air pressure changes inside (Marciniak 1999, 2012). Slug tests are of great significance, since they can even be conducted in cases of aquifer contamination, when water pumping is strongly inadvisable.

The most common analytic models used to interpret slug tests were developed by Bouwer and Rice (1976), Bredehoeft and Papadopoulos (1980), Cooper et al. (1965, 1967) and

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Hvorslev (1951). These models, however, usually assume homogeneity of the analyzed water-bearing formations, when many of them are, in fact, characterized by the presence of different systems of void space which may lead to preferential flow (Beckie and Harvey 2002; Coptý et al. 2011; Pechstein et al. 2016). In such cases, qualitative deviations in the return of the water level to its initial state can be observed.

The research described here is motivated by the results of many pressure-induced permeability tests which show deviations from the exponential rise (or fall) of the water level that can be approximated by bi-exponential curves (Fig. 1). It was assumed that these curves characterize heterogeneous, dual-permeability aquifers and indicate the presence of two distinct pathways for water flow. In general, a dual-permeability aquifer can be envisioned as a special type of dual-porosity reservoirs and should be defined as a medium composed of two subdomains, in which both are responsible for water flow. Fissured-porous rock—if water flow occurs through both (the fissures and the pores)—are classified as dual-permeability reservoirs, as are layered formations with layers of highly permeable sediments (Balogun et al. 2009; Leij et al. 2012). For example, two different sand layers or a layer of sand and a layer of fissured limestone or even one consolidated but fissured porous material like sandstone can fall under the definition of a dual-permeability medium.

The paper aims to present the procedure of a piecewise estimation of two transmissivity values, whenever a bi-exponential curve is observed in dual-permeability formations during a pressure-induced permeability test. However, this procedure can easily be extrapolated to any other type of slug test, since all result in recording water level movements inside a piezometer.

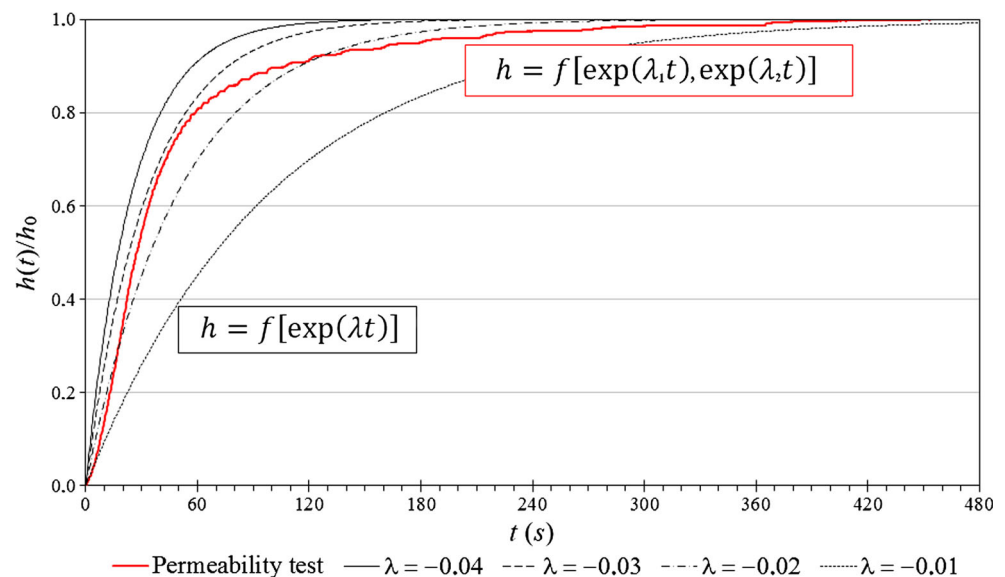
Materials and methods

The pressure-induced permeability test

The pressure-induced permeability (slug) test conducted in field conditions determines the in situ hydraulic conductivity of an aquifer in the area around a piezometer (Kaczmarek et al. 2016; Marciniak 2001; Marciniak et al. 2013). The test is performed in the following manner. First, the upper part of the piezometer is sealed off using a sealing device. Simultaneously, a probe is placed inside the piezometer in the area of expected water level fluctuations. Next, air is pumped into the sealed piezometer to decrease the water level. After pumping is stopped and the water level has stabilized, the air valve of the sealing device is opened and the air inside the piezometer decompresses, leading to the free rise of the water level to its initial state. An alternate way of conducting the test is to apply suction and successively increase the water level in the piezometer. In such case, a decrease of the water level is observed after the air valve is opened. The rate at which the water returns to its initial level determines the transmissivity of the investigated aquifer.

The mathematical model of water level changes within the piezometer characterizes two possible cases: damped oscillations or aperiodic movement (Krauss 1974, 1977; Marciniak 2001, 2012). By using the model in the identification algorithms of the pressure-induced permeability test (PARAMEX-type), it is possible to calculate the transmissivity of an aquifer based on the free rise or fall of the water level that is recorded within a piezometer (Marciniak 2001). Moreover, hydraulic conductivity of the investigated aquifer can be easily determined by dividing the obtained transmissivity value by the thickness of the aquifer.

Fig. 1 Bi-exponential water level rise recorded during a pressure-induced permeability test (bold red line) and attempts to fit theoretical exponential curves (thin black lines) to field data



In a vast majority of cases, an aperiodic movement of the water level can be observed. The water level’s rate-of-rise or rate-of-fall as a function of time, $h(t)$, can be described by the following exponential function:

$$h(t) = h_0[1 - \exp(\lambda t)] \tag{1}$$

where h_0 is the initial water level (cm) and $\lambda < 0$ (1/s) is an exponent dependent on the transmissivity.

Unusual bi-exponential water-level movement has been observed during ca. 12% of these tests. The possible causes of such movement have been analyzed thoroughly. Initially, piezometer malfunctions, i.e. leaky isolating clay seals or faulty sand packs around the piezometers, were taken into consideration. Such types of damage have been observed during many pressure-induced permeability tests but, in such cases, the initial stage of water level rise is rapid and short and the next stage is exponential.

In order to find the hydraulic connections that result in the bi-exponential behavior of the water level, several laboratory models of water flow have been investigated by Marciniak et al. (2013). Only one model called the “W-tube” allowed such behavior to be observed. This laboratory system of water flow indicates that dual-permeability conditions are required to record curves deviating from the exponential rise. Such conditions are observed in aquifers characterized by the presence of two subdomains (e.g. two layers) with different transmissivities and hydraulic conductivities, as well as a weak hydraulic connection between them.

When conducting any type of slug test in a piezometer installed in a dual-permeability aquifer, and if the screen section covers both subdomains, faster water flow should be present in the subdomain with a higher transmissivity compared to the subdomain with a lower transmissivity. Generally, higher contrast between transmissivity and hydraulic conductivity values will result in a more apparent bi-exponential water level movement.

The laboratory W-tube model of a permeability test conducted in heterogeneous aquifers

The “W-tube” is a laboratory model that was thoroughly discussed in two earlier publications (Kaczmarek et al. 2016; Marciniak et al. 2013). The model is considered to reenact the hydraulic conditions of a field pressure-induced permeability test conducted in heterogeneous, dual-permeability aquifers. It consists of three hydraulically connected columns. The two outer columns are equipped with cylindrical chambers where samples of different sands are placed. The central column, which represents a piezometer, contains a measuring probe that records the water level in this column as a function of time (Fig. 2).

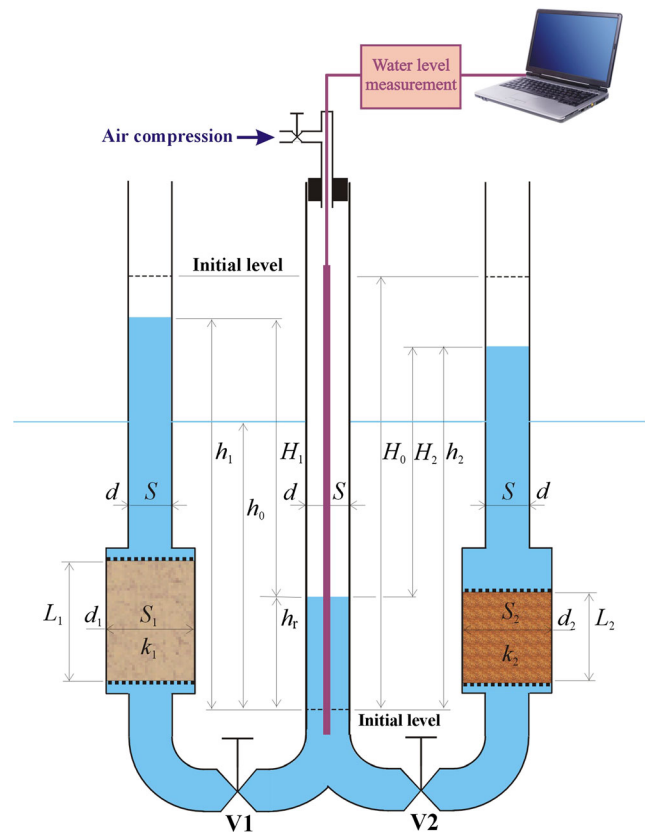


Fig. 2 The W-tube used in laboratory experiments and modeling

The preparation of a test requires filling all three columns with water to the same level. Afterwards, by pumping air into the central column using an air compressor, the water level in the central column is displaced down and the water level in the outer columns is displaced up. Over time, the water level stabilizes, then a test is initiated when the air valve at the top of the central column is opened, causing water to return to the initial level due to its filtration through the sand samples in the outer columns.

The mathematical model of the W-tube test was formulated using a balance of forces for water flowing through the sand samples. It was assumed that the inner cross-sections of all three columns, denoted as S , are equal. S_1 and S_2 are the cross-sections of the sand samples, L_1 and L_2 denote the heights of these samples, and their hydraulic conductivities are k_1 and k_2 . Initially, the water level in the central column is lower than that in the adjacent columns and serves as the reference level for the measurements of the water level h_r in the central column and the levels h_1 and h_2 in the adjacent columns. The mathematical model neglects all inertial forces related to the motion of water in the columns and in the pore space of the sand samples. The interaction forces between water and the porous material are approximated according to the linear Darcy’s law. Taking into account the hydrostatic forces caused by the water columns as well as the gravitational forces and the interaction forces between water and the porous material,

the following balance equations for pore water can be written (Kaczmarek 2009; Reh binder 1992):

$$\begin{cases} \rho g(h_1 - h_r)S_1 + \frac{\rho g}{k_1} L_1 \frac{dh_1}{dt} S = 0 \\ \rho g(h_2 - h_r)S_2 + \frac{\rho g}{k_2} L_2 \frac{dh_2}{dt} S = 0 \end{cases} \quad (2)$$

where ρ is the density of water, g is the acceleration due to gravity, and the dh_1/dt and dh_2/dt derivatives denote the rate at which the water height changes. This rate is equal to the filtration velocities in the appropriate samples.

Due to the conservation of mass in the system, the changes of water heights in the columns for any time increment dt are given by the following relation:

$$dh_1 + dh_2 + dh_r = 0 \quad (3)$$

It is assumed that the h_1 and h_2 values are as follows:

$$\begin{cases} h_1(t=0) = H_0 - h_r \\ h_2(t=0) = H_0 - h_r \end{cases} \quad (4)$$

After some basic transformations presented by Marciniak et al. (2013), the analytical form of the solution, height h_r , is obtained:

$$h_r = \frac{H_0}{3} [2 + (B-1)\exp(\lambda_1 t) - (B+1)\exp(\lambda_2 t)] \quad (5)$$

where the constant B is derived from the following formula:

$$B = \frac{A_2 + A_1}{2\sqrt{A_1^2 + A_2^2 - A_1 A_2}} \quad (6)$$

and $\lambda_{1,2}$ equals:

$$\lambda_{1,2} = -A_1 - A_2 \pm \sqrt{A_1^2 \pm A_2^2 - A_1 A_2} \quad (7)$$

Constants A_1 and A_2 are related to the geometric parameters of the analyzed sand samples: S_1 , S_2 , L_1 , L_2 and to their hydraulic conductivities k_1 and k_2 :

$$A_1 = \frac{k_1 S_1}{L_1 S}, \quad A_2 = \frac{k_2 S_2}{L_2 S} \quad (8)$$

After making use of Eq. (8), constants B and $\lambda_{1,2}$ can be written in the following form:

$$B = \frac{k_1 L_2 S_1 + k_2 L_1 S_2}{2\sqrt{k_1^2 L_2^2 S_1^2 + k_2^2 L_1^2 S_2^2 - k_1 k_2 L_1 L_2 S_1 S_2}} \quad (9)$$

$$\lambda_{1,2} = -\frac{k_1 S_1}{L_1 S} - \frac{k_2 S_2}{L_2 S} \pm \sqrt{\frac{k_1^2 S_1^2}{L_1^2 S^2} + \frac{k_2^2 S_2^2}{L_2^2 S^2} - \frac{k_1 S_1 k_2 S_2}{L_1 L_2 S^2}} \quad (10)$$

Hence, with the solution of Eq. (5), it is possible to determine the water height in the central column as a function of time.

The results of laboratory tests performed on the W-tube were also presented in the authors' previous publications. In Marciniak et al. (2013), the so-called direct problem is solved, i.e. bi-exponential rate-of-rise curves experimentally obtained in the W-tube for various combinations of sand samples are compared with the predictions of the mathematical model (generated in MATLAB). A root-mean-square error (RMSE) was calculated for every rate-of-rise curve and the corresponding prediction, good convergence was obtained. In Kaczmarek et al. (2016) the inverse problem is solved—a numerical optimization procedure is applied in order to jointly estimate a pair of hydraulic conductivity values based on a bi-exponential rate-of-rise curve recorded in the W-tube. In this case, the predictions of the mathematical model varied in accuracy. Relatively small percent errors between the numerically obtained hydraulic conductivity values and the values derived from constant head measurements were usually obtained for highly permeable sands. Higher errors were obtained for sands with a low permeability or in cases when the contrast in permeability between the two analyzed sand samples in a pair was small.

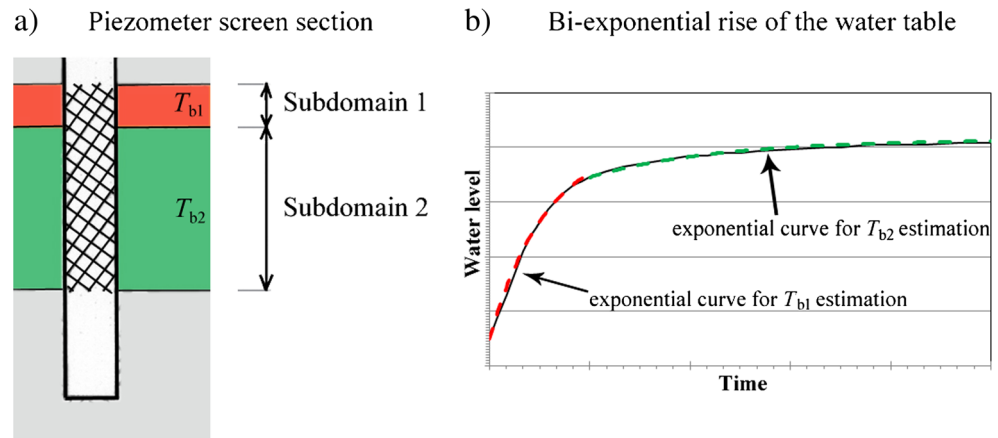
A piecewise exponential model for dual transmissivities in field conditions

When interpreting field pressure-induced permeability tests that result in recording bi-exponential curves, it should be assumed that the piezometer screen is in contact with two subdomains, both characterized by a distinct system of water flow, even though dual permeability is not always apparent when analyzing hydrogeological data (Fig. 3). Thin layers of sediment or zones of weathered material are frequently omitted in the lithological description of boreholes. Since it is impossible to calculate a pair of hydraulic conductivity values for the two screened subdomains if one of them is unnoticed or if their thickness is unknown, the authors propose a simplified analytic method of estimating dual transmissivity values piecewise (in two intervals).

The bi-exponential movement of the water level is approximated using two theoretical exponential curves:

$$h = \begin{cases} h_1 = h_{01} [1 - \exp(\lambda_1 t)^{01}] & \text{for } t_{01} \leq t < t_b \\ h_2 = h_{02} [1 - \exp(\lambda_2 t)] & \text{for } t_b \leq t < t_f \end{cases} \quad (11)$$

Fig. 3 **a** Piezometer screening two subdomains with different conductivities and **b** the bi-exponential rise of the water level observed during permeability tests



and a pair of transmissivity values, T_{b1} and T_{b2} , is calculated based on the interpretation algorithm of the slug test. The transmissivity of the subdomain with a higher conductivity (T_{b1}) is calculated from the exponential curve that approximates the first (steeper) part of the bi-exponential rise of the water level – from the initial time t_0 to “breakthrough” time t_b . Transmissivity of the subdomain with lower conductivity (T_{b2}) is calculated from the exponential curve that approximates the second part of the bi-exponential movement observed during the test—from time t_b to time t_f , i.e. the time when water reaches its initial level. Slightly shifting time t_b (altering the intervals) does not cause any significant change in the estimated T values.

Furthermore, a goodness-of-fit analysis can be performed to estimate how well the observed water level corresponds to the predictions of the model. The authors usually calculate root-mean-square errors (RMSE) and normalized root-mean-square errors (NRMSE) to assess the differences between observed and predicted data:

$$\text{RMSE} = \frac{1}{n} \sqrt{\sum_{i=1}^n (h_{\text{pred}} - h_{\text{obs}})^2} \quad (12)$$

$$\text{NRMSE} = \frac{\text{RMSE}}{h_{\text{max}} - h_{\text{min}}} \quad (13)$$

where h_{obs} is the water level observed during the slug test in a piezometer, h_{pred} is the water level estimated using the analytic method, n is the number of measurements, h_{max} and h_{min} are the maximum and minimum water levels achieved during the test, respectively. When estimating the goodness-of-fit, the two theoretical exponential curves are analyzed jointly to obtain one RMSE and one NRMSE value for the entire duration of a slug test (time t_0 to t_f).

Study area

Pressure-induced permeability tests (PARAMEX-type) have been performed in over 1,400 piezometers in various regions

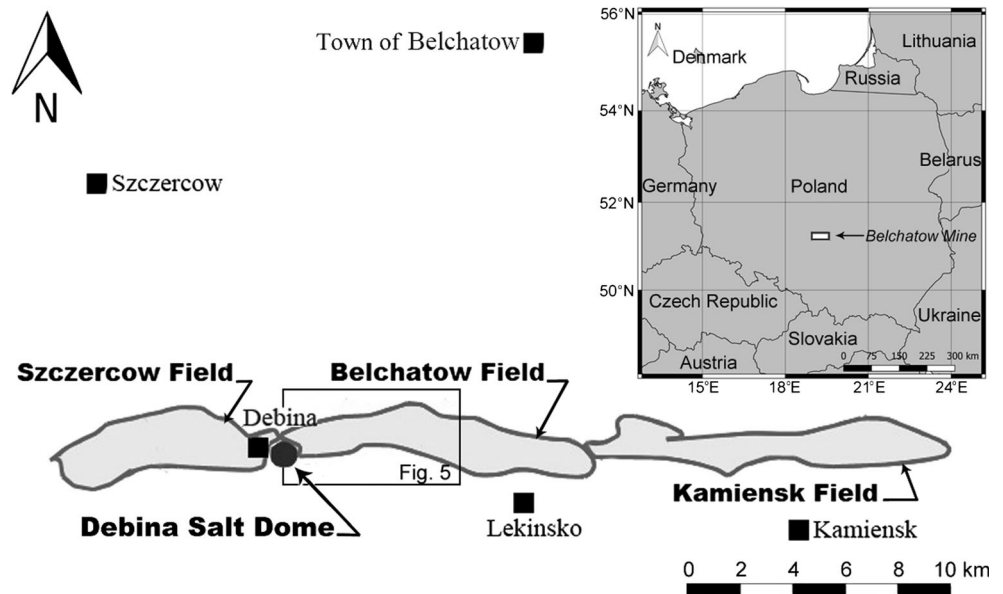
of Poland. Usually, single tests are performed in a small number of piezometers, e.g. in water-intake-protection zones. The only region with a significant number of performed tests is the Belchatow Lignite Mine. In this mine, a crew of hydrogeologists routinely performs PARAMEX tests to determine the technical condition of the piezometer network (Marciniak 2001). So far, over 200 piezometers have been analyzed at the mine. Most of them (154) are located within the Belchatow mining field, one of the two fields where opencast mining takes place; therefore, this field was chosen as the study area (Fig. 4).

The mine is located in the Kleszczow Graben (central Poland). This arc-shaped graben, developed on the Epi-Variscan platform, is 40–50 km long, 4–5 km wide, and stretches in a E–W direction (Gruszka and van Loon 2007). Its depth reaches 350 m, which makes it the deepest neotectonic graben in the Polish Lowland. The Kleszczow Graben was formed due to the movement of faults in the substratum which were reactivated during the Alpine orogeny (Gruszka 2001). The basement is represented by Jurassic and Cretaceous sediments, mainly sandstones, limestones, shales and marls. Permian rocks are solely present in a small area of the “Debina” salt dome (Gotowala and Haluszczak 2002; Haluszczak 2007). The graben is filled with Neogene (early Miocene–Pliocene) and Quaternary deposits, predominantly lignite, sands and clays. Their displacements are caused by tectonic deformations of the basement, especially near the fault zones and the salt dome. Moreover, the exploitation of lignite involves relocating vast amounts of gangue to surrounding areas, which evokes tectonic activities in the region such as earthquakes, rock displacement and rock fracturing (Widera 2016).

Results

As already mentioned, pressure-induced permeability tests were conducted in 154 piezometers located in the Belchatow mining field; however, water movement could not be achieved in all of them. In some piezometers, the water level was

Fig. 4 The Belchatow lignite mine, central Poland



located at the height of the screen, preventing air compression and decompression, whereas in several other cases, the analyzed piezometers were damaged. Ultimately, rate-of-rise curves were successfully recorded in 92 piezometers. Bi-exponential water level movement was observed in 11 of these piezometers—6 of them screened Mesozoic formations (Upper Jurassic or Cretaceous), 4 others screened Paleogene sediments underlying the lignite deposit, and 1 piezometer was installed in Quaternary material (Table 1).

In all piezometers where a bi-exponential curve was recorded (Fig. 5), the screen was 5 m long. Furthermore, in most cases the lithology of the water-bearing formations surrounding the screen suggests the presence of at least two distinct pathways for water flow (Table 2). Most piezometers either screen weathered/fissured sedimentary rock layers (Mesozoic) or layers of two different unconsolidated sediments (Paleogene or Quaternary). Only in two cases—piezometers PW-424-2 and PW-354—the lithology suggests the presence of one single water-bearing formation around the screen. However, it should be noted that these piezometers and most of the other analyzed piezometers in the Belchatow field are

over 200 m deep. It cannot be ruled out that thin layers and the presence of weathered or fissured zones were omitted in the lithological description of the boreholes; moreover, the Rock Quality Designation (RQD) index was not always established. Such information is available for the screen section of piezometers KT-89, KT-117-BIS and PW-420. RQD values calculated for these piezometers, ranging from 10 to 50%, indicate the presence of a weathered or completely weathered rock mass around the screen (Deere 1964; Deere et al. 1967).

The results of transmissivity estimation for each piezometer are presented in Table 3. Calculations of dual-transmissivities (T_{b1} and T_{b2}) were performed in accordance with the methodology presented in section ‘A piecewise exponential model for dual transmissivities in field conditions’. Table 3 also includes “standard” transmissivity (T_s) estimates from the mathematical model of the pressure-induced permeability test, i.e. from a single exponential function that treats the geological medium to be homogeneous. A comparison of transmissivity values T_{b1} and T_{b2} is also presented in Fig. 6. The distance from the diagonal line illustrates the heterogeneity of the analyzed dual-permeability formation (high T_{b1}/T_{b2} ratio). Moreover, graphs showing the bi-exponential rise of the water level in four piezometers are depicted in Fig. 7, along with the best-fit exponential curves for dual and single transmissivity estimation.

Table 1 Breakdown of all pressure-induced permeability tests conducted in the Belchatow field

Age of screened formations	Number of piezometers where a pressure-induced permeability test was conducted	Number of piezometers where water level movement was achieved	Number of piezometers where a bi-exponential rate-of-rise curve was recorded
Quaternary	18	10	1
Paleogene	56	36	4
Mesozoic	80	46	6
Total	154	92	11

Discussion

A goodness-of-fit analysis was performed for all investigated slug tests which resulted in observing bi-exponential water-level movement. Root-mean-square errors (RMSE) and normalized root-mean-square errors (NRMSE) were calculated to measure the difference between bi-exponential rate-of-rise

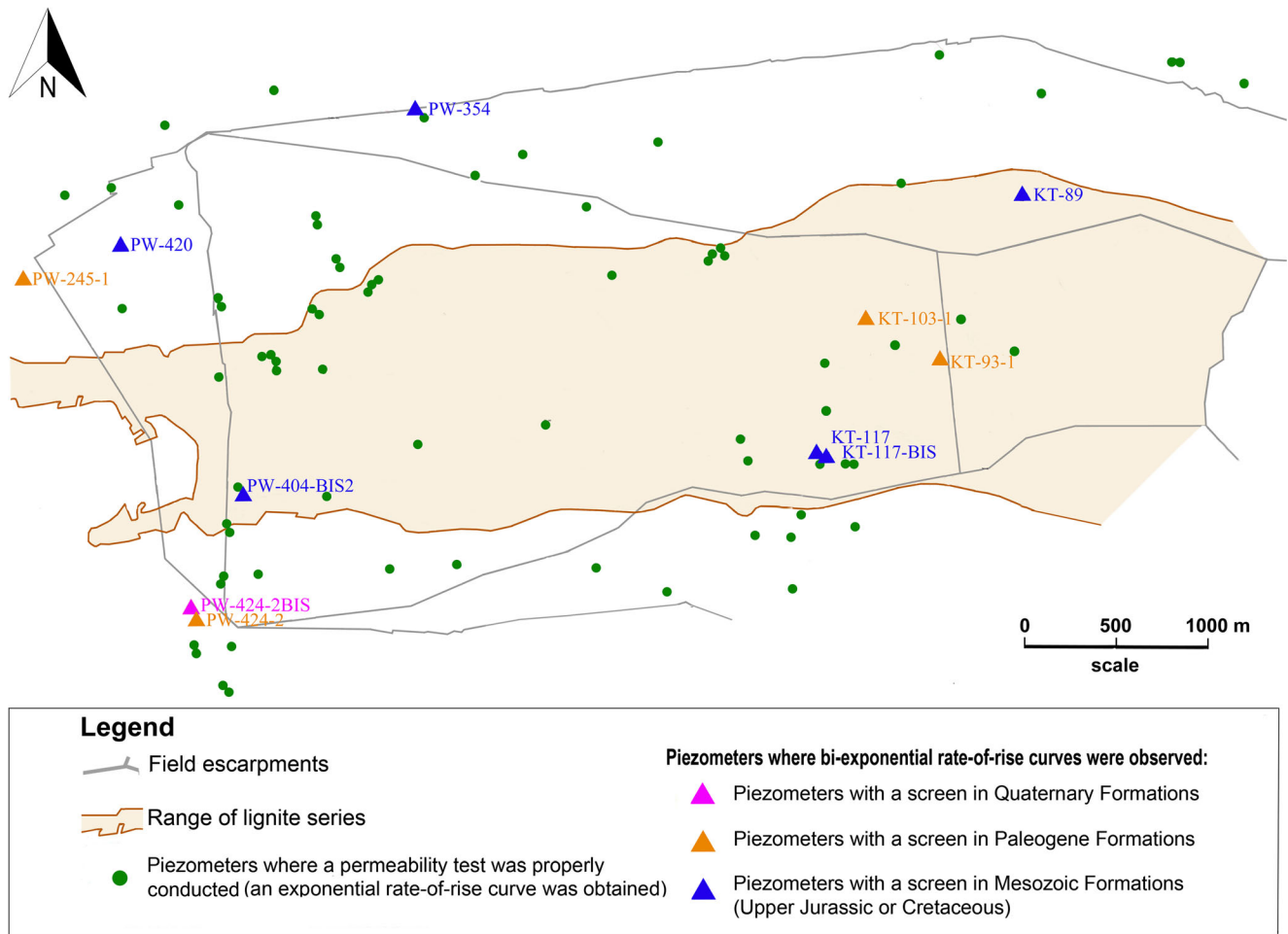


Fig. 5 A map of the analyzed piezometers within the Belchatow mining field

curves recorded in the field and the adjusted exponential curves. Error values are presented in Table 4.

In all cases, relatively small RMSE and NRMSE errors are obtained for the dual-transmissivity approach, i.e.

when the bi-exponential rate-of-rise curve is approximated piecewise by two exponential curves. When using one single exponential function the errors are roughly 3–8 times larger.

Table 2 List of piezometers characterized by a bi-exponential movement of the water level during permeability tests

Age of screened formations	Piezometer symbol	Lithology of geological formations around the screen
Quaternary	PW-424-2BIS	Two different layers of sand: fine sand and medium sand
Paleogene	KT-93-1	Two different layers of sand: fine sand and medium sand
	KT-103-1	Fine sand with several layers of sandstone rubble
	PW-424-2	Fine sand
	PW-425-1	Fine and silty sand with smaller layers of medium sand
Mesozoic	KT-89	Oncolitic limestone (RQD of 40%)
	KT-117	Limestone and gaize-type sediments, highly disintegrated (debris pieces with a diameter of 5 cm)
	KT-117-BIS	Fine-grained, glauconitic and marly sandstone, locally as a breccia with numerous vertical fissures (RQD of 10%)
	PW-354	Gaize-type limestone with many silicate zones
	PW-404-BIS2	Oncolitic limestone breccia, disintegrated
	PW-420	Sandy, calcareous marlstone, the number of sandy zones increases with depth (RQD of 50%)

Table 3 Dual transmissivity (T_{b1} and T_{b2}) and single transmissivity (T_s) estimates for the investigated piezometers

Age of screened formations	Piezometer	Transmissivity T_{b1} [m ² /s]	Transmissivity T_{b2} [m ² /s]	T_{b1}/T_{b2}	Transmissivity T_s [m ² /s]
Quaternary Paleogene	PW-424-2BIS	4.10×10^{-4}	1.45×10^{-4}	2.83	3.06×10^{-4}
	KT-93-1	4.63×10^{-4}	1.19×10^{-4}	3.90	3.27×10^{-4}
	KT-103-1	9.90×10^{-5}	3.08×10^{-5}	3.21	5.10×10^{-5}
	PW-424-2	1.08×10^{-4}	2.40×10^{-5}	4.50	5.70×10^{-5}
Mesozoic	PW-425-1	2.70×10^{-4}	7.72×10^{-5}	3.50	1.72×10^{-4}
	KT-89	3.09×10^{-4}	4.30×10^{-5}	7.18	1.86×10^{-4}
	KT-117	2.58×10^{-4}	1.05×10^{-4}	2.45	1.94×10^{-4}
	KT-117BIS	5.82×10^{-4}	5.53×10^{-5}	10.53	2.25×10^{-4}
	PW-354	6.14×10^{-5}	1.77×10^{-5}	3.47	3.30×10^{-5}
	PW-404-BIS-2	6.40×10^{-5}	3.03×10^{-5}	2.11	4.60×10^{-5}
	PW-420	7.00×10^{-5}	3.65×10^{-5}	1.92	5.90×10^{-5}

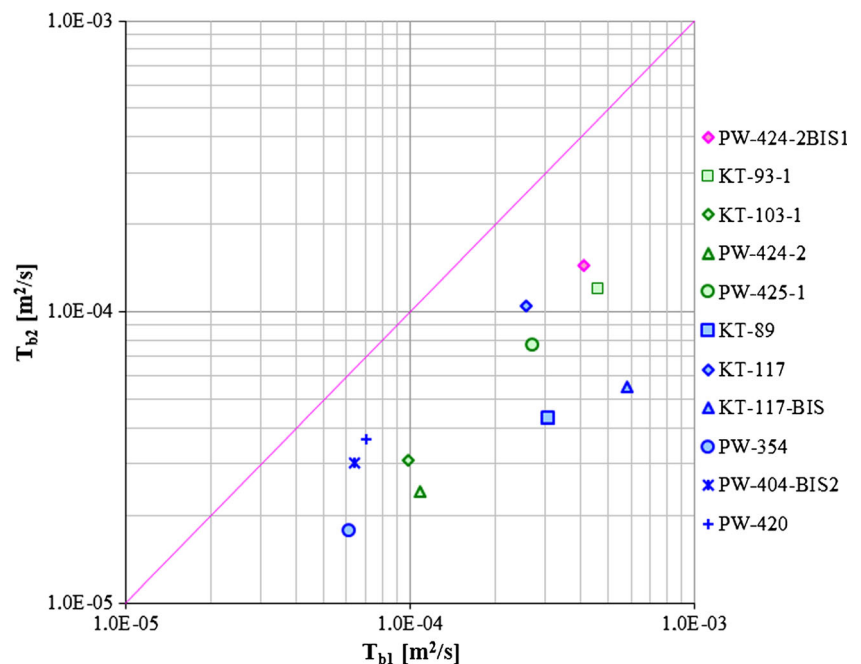
It should also be noticed that single transmissivity (T_s) estimates fall between the two dual transmissivity values (T_{b1} and T_{b2}). This is understandable, considering the fact that the T_s curve is merely an “averaged” approximation of the bi-exponential movement.

Since the area of contact between each of the two subdomains and the piezometer screen is unknown, the authors could only determine dual transmissivities accurately, without determining the exact hydraulic conductivities of both subdomains near the piezometer screen. However, knowing the transmissivity values, it is possible to calculate several potential pairs of hydraulic conductivities assuming different subdomain thickness ratios (e.g. assuming that the thickness of one subdomain is in contact with 10% of the screen, whereas the other covers the remaining 90%). In such case, the k values should be calculated for each subdomain by dividing the obtained transmissivity values by the appropriate thickness (i.e. the length of the screen covered by the subdomain).

Eventually, the authors plan to expand the W-tube model and make it applicable in field conditions where the thickness of both subdomains is known. It would therefore be possible to calculate a set of hydraulic conductivities using optimization techniques, whenever a slug test results in observing bi-exponential movement of the water table.

Conclusions

During some pressure-induced permeability tests performed in the Belchatow Lignite Mine, an unusual bi-exponential type of water-level movement was observed in several piezometers. The movement clearly differs from the exponential rise usually recorded in such experiments. The location of these piezometers and the lithology of the screened formations suggest the possible presence of a dual-permeability system of water flow that may be responsible for such deviations.

Fig. 6 A comparison of the transmissivity values T_{b1} and T_{b2} for each of the analyzed piezometers

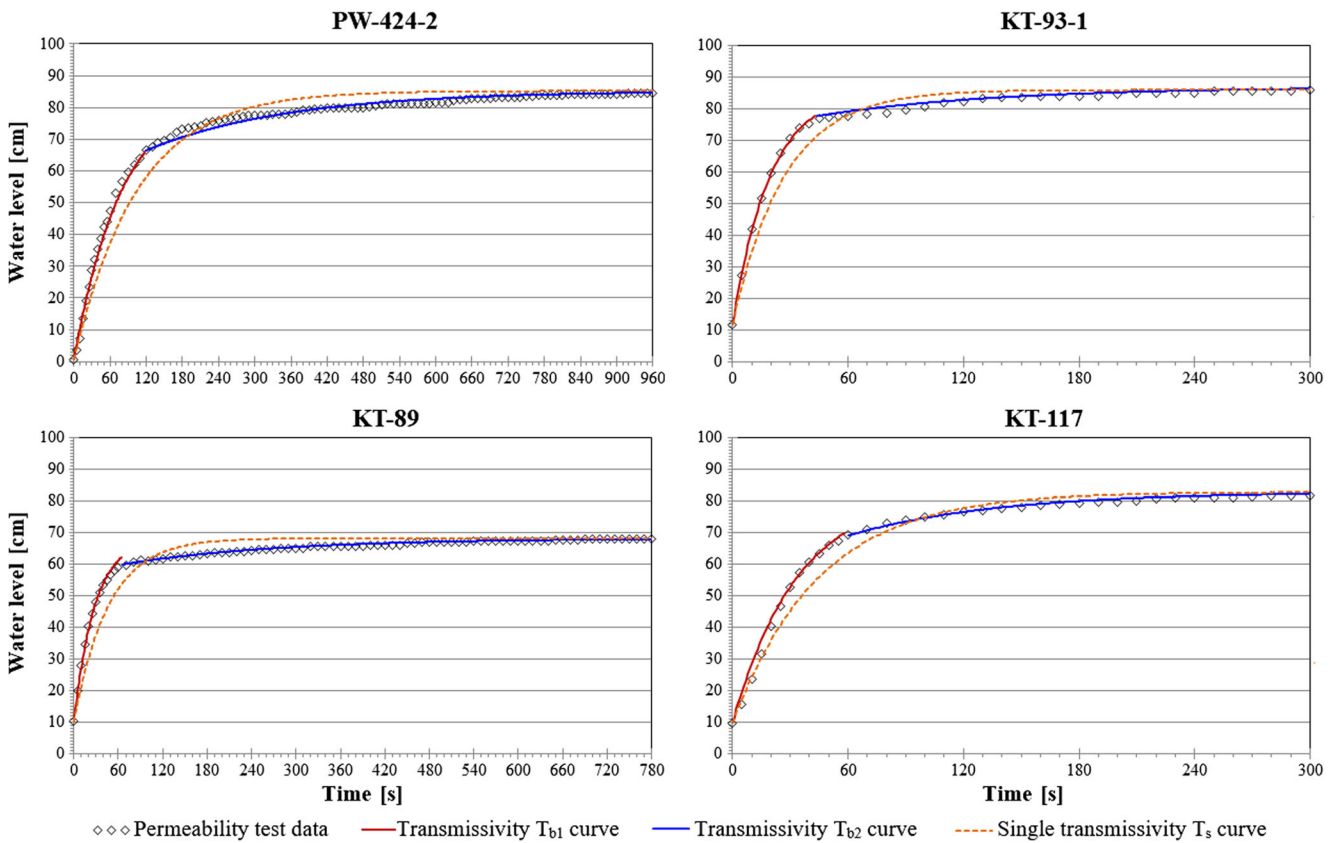


Fig. 7 Attempts to approximate bi-exponential water level movement in four piezometers with exponential curves (for T estimation)

Some analogies are apparent between slug tests and other methods such as the pumping test. Hydrogeological parameters of heterogeneous formations are determined during a slug test based on the shape of the experimentally recorded rate-of-rise or rate-of-fall curve, whereas during a pumping test this shape indicates a possible leakage of water from adjacent layers. When analyzing dual-permeability formations, slug tests (e.g. pressure-induced permeability tests) can be used

to determine a pair of transmissivity values, even if the thickness of both subdomains responsible for water flow is unknown.

Determining hydraulic conductivities of a water-bearing formations composed of two layers of porous material is possible if the exact thickness of these layers around the piezometer screen, or the ratio of two flow paths, are known. Determining hydraulic conductivities in fissured or fissured-

Table 4 RMSE and NRMSE values showing differences between observed and theoretical water levels (for the dual and single transmissivity approach)

Age of screened formations	Piezometer	Dual transmissivity approach		Single transmissivity approach		
		RMSE [cm]	NRMSE [%]	RMSE [cm]	NRMSE [%]	
Quaternary	PW-424-2BIS	0.853	1.09	3.324	4.25	
	Paleogene	KT-93-1	0.574	0.77	3.336	4.48
		KT-103-1	1.056	1.48	4.951	6.96
		PW-424-2	1.600	1.89	5.472	6.46
		PW-425-1	0.767	1.44	3.179	5.99
Mesozoic	KT-89	0.428	0.74	3.374	5.86	
	KT-117	0.751	1.06	2.690	3.79	
	KT-117BIS	0.249	1.04	2.035	8.51	
	PW-354	0.362	1.84	1.666	8.46	
	PW-404-BIS-2	0.876	1.25	3.395	4.83	
	Mesozoic	PW-420	0.646	0.84	2.171	2.79

porous rock is more difficult, since the ratio of different flow paths in such rock is rarely examined.

The proposed piecewise method of determining transmissivity values of heterogeneous water-bearing formations is an approximate one. The limitations of the method may serve as a motivation to elaborate new mathematical models of water flow and interpretation algorithms based on slug tests. These algorithms could be implemented in field conditions, whenever a bi-exponential water level rise is observed. Such algorithms would increase the reliability of slug tests conducted in heterogeneous formations.

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