



Sliding on wet sand

Rinse W. Liefferink¹ · Mojgan Aliasgari² · Nahid Maleki-Jirsaraei² · Shahin Rouhani³ · Daniel Bonn¹

Received: 10 October 2019 / Published online: 23 May 2020
© The Author(s) 2020

Abstract

We present sliding experiments of a sledge on wetted sand and describe that the frictional response is controlled by the penetration hardness of the granular medium. Adding a small amount of water to sand increases the hardness which results in a decrease of the sliding friction. Pouring even more water to sand results in a decrease of the hardness and a subsequent increase of the friction. This inverse correlation between hardness of a wetted granular material and its frictional response to sliding is found to be due to ploughing of the sledge. When the load of the sledge exceeds the penetration hardness of the water-sand mixture the granular material is irreversibly deformed, which is evident by a trace of the slider left after its passage. The penetration hardness sets how deep the trace of the slider is which, in turn, controls the ploughing force. Consequently, increasing the hardness of the water-sand mixtures makes pulling a sledge over it easier. In addition, we quantify the critical shear strain which sets the transition of an elastic to plastic response of (wet) granular materials which enables us to directly relate the shear modulus, in the elastic regime, to the hardness, in the plastic regime.

Keywords Wet granular material · Friction · Hardness

1 Introduction

The mechanical response of sand is largely dominated by the amount of water added to it. With a small amount of water, a pile of sand can be made into a sandcastle where, however, too much water results in a muddy puddle [1–4]. Pouring water in sand enables the formation of capillary liquid bridges between the grains where the curvature of the liquid interface leads to a capillary pressure [5–8]. The capillary bridges act as a cohesive force between the grains which, for example, enables building a sandcastle [1, 2]. The elastic response on deformation can be quantified by the increase of the elastic shear modulus G' with increasing water volume

fraction [10–12]. However, with continuously increasing the water fraction the capillary bridges starts to coalesce which decreases the shear modulus again. The consequence of the nonmonotonic behaviour of the shear modulus for increasing water fraction is evident when walking over a beach, but also interesting because it governs for instance the force necessary for sliding a sledge over sand.

It was reported recently that when a sledge is pulled over a water-sand mixture, the sliding friction follows a roughly opposite nonmonotonic behaviour: adding a small amount of water greatly reduces the friction where, however, increasing the water fraction even more results in a muddy mixture that is difficult to slide over [13, 14]. This seemed already to be known by the ancient Egyptians. A celebrated tomb drawing, see Fig. 1, suggests that, for the transportation of a large statue through the desert, one person was pouring water in front of the sledge to decrease the friction [9]. It was previously shown that the friction decreases roughly linearly with the increase in shear modulus, which suggests that when the sand is ‘stiffer’, the friction force decreases [13].

However, if the slider leaves a trace in the sand after its passage, the granular medium is responding plastically rather than elastically and the (linear) elastic modulus is not the pertinent quantity to consider. Sand-grains irreversibly move when a critical pressure, the penetration hardness P_h ,

Rinse W. Liefferink and Mojgan Aliasgari have contributed equally to this work.

✉ Rinse W. Liefferink
r.w.liefferink@uva.nl

¹ Institute of Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands

² Complex Systems Laboratory, Physics and Chemistry Department, Alzahra University, P.O. Box 1993893973, Tehran, Iran

³ Department of Physics, Sharif University of Technology, P.O. Box 11365-9161, Tehran, Iran

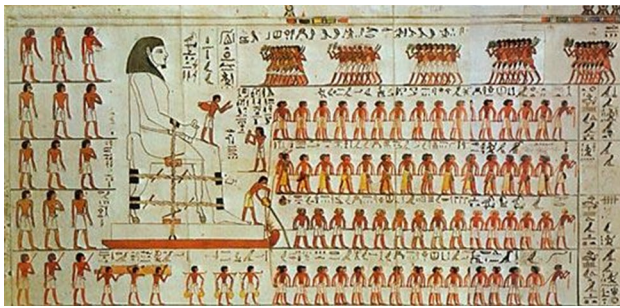


Fig. 1 A wall painting from 1880 B.C. on the tomb of Djehutihotep [9]. A sledge with a big statue is pulled through the desert, where a person is painted who pours water onto the sand

is reached. Therefore, the sliding friction is controlled by the hardness of the material instead of the linear shear modulus. In this paper, we present sliding experiments coupled with hardness measurements for increasing water volume fraction and show that the sliding friction is a result of the plastic response of the granular material. Furthermore, the transition from the elastic to the plastic regime is unraveled which enables us to explain the link between hardness and stiffness.

Here, we solve the puzzle for the linear relation between the measured friction coefficient and the linear modulus raised by Fall et al. [13]. In spite of the relatively large attention the paper by Fall et al. has attracted, it has remained without a quantitative explanation so far. Liefferink et al. showed recently that the penetration hardness is related to the shear modulus, making the plausible argument that this related the friction with the modulus [15]. However quantitatively, this remained to be demonstrated, and that is exactly what we do here. The results can then be used to finally understand the results of Fall et al. in a quantitative fashion.

2 Method

The sliding experiment is done by pulling a wooden sledge (10.3 cm by 6.8 cm with a total mass 273 g) horizontally over a well-mixed water-sand mixture of known composition with use of a step motor (pulling velocity of 2 cm/s) coupled to a load cell, see Fig. 2a. ‘Iranian sand’, which contains mainly grains in the 212–500 μm range, is first dried in an oven (150°C) and cooled down to room temperature. De-ionized water is gradually added up to a given water volume fraction and the thoroughly mixed water-sand mixture is instantly used. The sledge is pulled over a distance of 10 cm; after 2 cm a stable friction force is reached. The friction force is monitored and the friction coefficient, defined as the ratio of the friction to the normal force (given by the total weight of the sledge), is calculated. The error bars

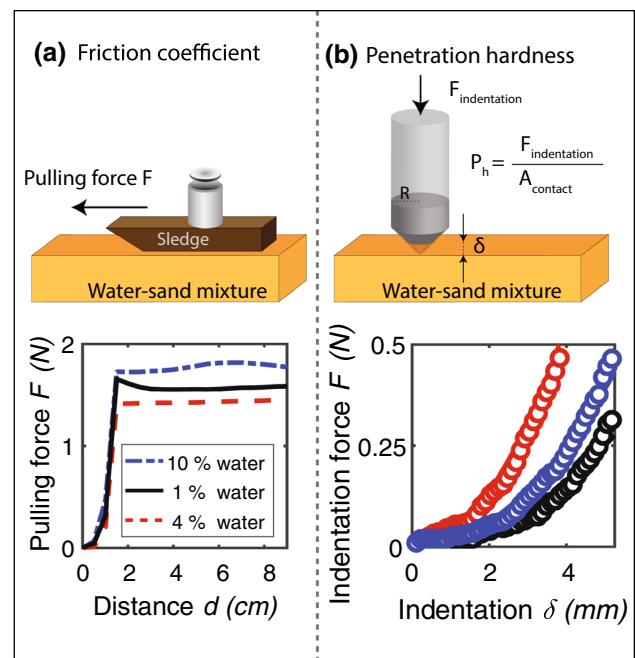


Fig. 2 Schematic representation and corresponding curves of the sliding experiment and hardness test. **a** Friction coefficient μ , the ratio of friction to normal force, is measured by sliding a sledge including a weight (total mass of 273 g) over a water-sand mixture. The friction force is quantified by monitoring the pulling force over a distance d and is systematically measured for different water volume fraction ϕ_w . **b** Penetration hardness P_h of the water-sand mixture is measured with use of a vertical indenter. A cone with apex angle $\alpha = 75^\circ$ and base-radius $R = 5.05$ mm is pushed into the water-sand mixture with use of a step motor. The force and indentation depth δ is followed in time which enables us to calculate an average penetration hardness for every water-sand mixture with use of dividing the force by the contact area A_c , where the latter can be calculated based on $A_c = \pi \tan^2(\frac{1}{2}\alpha) \delta^2$

reflect the fluctuations in the friction force over time giving the standard deviation of approximately 15 data points.

After sliding, the penetration hardness P_h of the water-sand mixture is measured on the same sample with use of an indentation-experiment (see Fig. 2b). A cone (with apex angle $\alpha = 75^\circ$ and base-radius $R = 5.05$ mm) is pushed vertically in the water-sand mixture and the indentation force and depth are monitored. The penetration hardness is defined as the ratio of indentation force F and the contact area A_c on which it acts ($A_c = \pi \tan^2(\frac{1}{2}\alpha) \delta^2$). The cone enables us to calculate the average penetration hardness for increasing contact area and confirm its independence on indentation depth.

3 Results and discussion

The friction coefficient of a sledge sliding over a water-sand mixture defined as the friction force over the normal force is then measured for increasing water volume fractions ϕ_w . Adding some water initially decreases the friction coefficient, as can be observed in Fig. 3 (blue squares). However, adding more water than 4 vol% results in an increase of the friction. This nonmonotonic behaviour is qualitatively in agreement with earlier measurements where the optimum volume fraction, here 4%, is set by the grain size distribution of the used granular material [13, 14]. After each sliding experiment, the slider leaves a trace in the water-sand mixture marking the width and sliding distance of the sledge. This so-called ploughing track indicates that, indeed, during sliding, the water-sand mixture is plastically deformed.

The deformation of the water-sand mixture is a result of the pressure exerted by the sledge along the normal direction which exceeds the critical pressure of the granular material. Interestingly, the penetration hardness of the water-sand mixture also varies nonmonotonically with the water volume fraction (red circles in Fig. 3) in a way that suggests an inverse correlation with the friction coefficient. Initially, adding water increases the hardness of the granular material due to formation of liquid bridges between neighboring

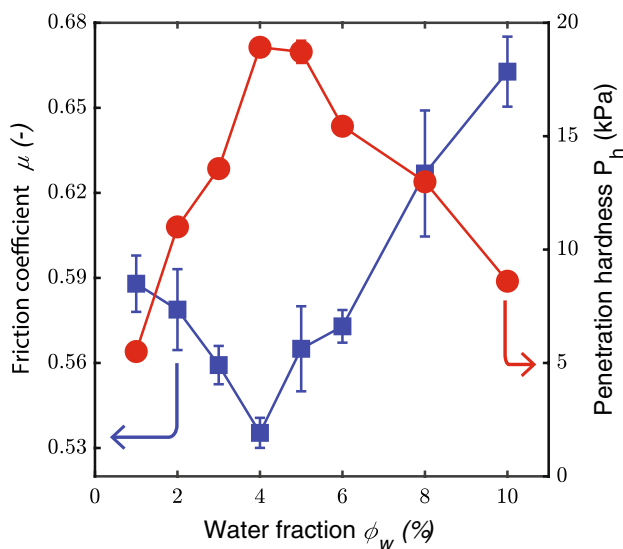


Fig. 3 Evolution of the friction coefficient μ (blue squares) and the penetration hardness P_h (red circles) for the water volume fraction ϕ_w . The friction coefficient obtained from the measured friction force (for a fixed normal force of 2.7N) displays a nonmonotonic behaviour for increasing water fraction. A similar nonmonotonic behaviour for increasing water fraction is found for the obtained penetration hardness where the minimum in friction corresponds to the maximum in hardness at the water volume fraction of $\phi_w = 4\%$. The error bars represent the standard deviation (color figure online)

grains, this is the so-called ‘pendular regime’ [7, 16]. The water-sand mixture can resist a larger load before the critical penetration hardness is reached whereafter the material will irreversibly deform. Deformation of the granular material results in reorientation of the grains including breaking and rebuilding of capillary liquid bonds. Pouring even more water to sand results initially in larger liquid bridges which decreases the Laplace pressure due to the smaller curvature; the ‘funicular regime’ is reached. Furthermore, a high water volume fraction eventually results in coalescence of the liquid bridges, the ‘capillary regime’, which decreases the plastic response of the granular material even more.

The nonmonotonic behaviour of the sliding friction and the penetration hardness for increasing water volume fraction indicate a direct relation; for increasing hardness of the water-sand mixture, pulling a sledge over it becomes easier. This is quantified in Fig. 4 where the friction coefficient roughly linearly decreases as a function of the penetration hardness.

The sliding friction coefficient therefore decreases linearly with increasing shear modulus G' [13] and with increasing penetration hardness P_h of the water sand-mixture. However, the question then remains how the elastic and plastic response relate to each other and is the found friction force of the slider a result of the elastic or the plastic response?

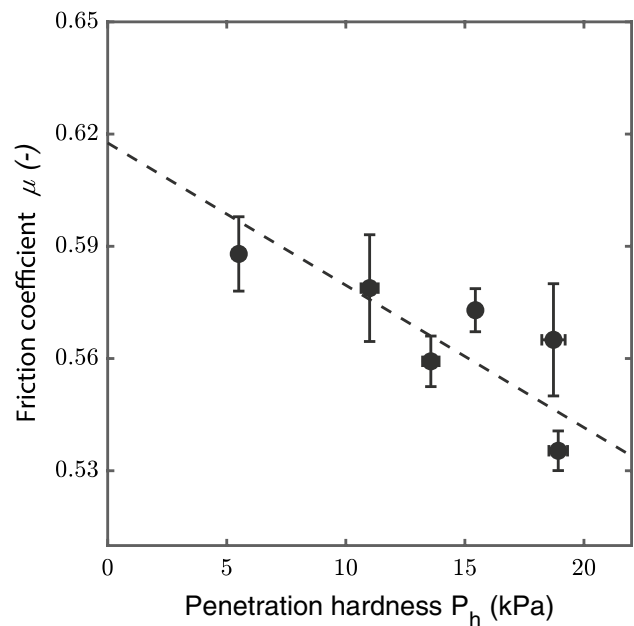


Fig. 4 Parametric plot of the friction coefficient μ as a function of the penetration hardness P_h , both obtained as a function of the water volume fraction ϕ_w as given in Fig. 3. A roughly linear decrease of the friction coefficient for increasing hardness is found; it becomes easier to slider over a hard water-sand mixture. Note that water volume fractions higher than 7% are excluded from the analysis, high water fractions (‘slurry sand’) results in a very heterogeneous water and air distribution in the sand packing

The water-sand mixture can hold a finite deformation in the elastic regime before it fails whereafter a plastic irreversible deformation occurs. This finite deformation, the critical shear strain γ_c , defines the transition between elastic and plastic behaviour. To be able to quantify this critical shear strain, we can make use of the measured penetration hardness and shear modulus as a function of the water fraction. The latter was published previously for ‘Iranian’ sand for the same range of water volume fractions, see Fig. 4 from Fall et al. [13]. The penetration hardness increases linearly with the shear modulus, see Fig. 5a, as $P_h \sim \gamma_c G'$ with $\gamma_c = 3\%$ the critical shear strain. The critical strain of yielding is a constant for the given system and does not depend notably on the water volume fraction. For soft solids in general it has been shown that indeed the critical shear strain for a given system (emulsion, microgel suspensions, foams, gels) are reasonably constant [17].

The puzzle from the data of Fall et al. is that the friction coefficient decreases linearly for increasing shear modulus for different types of sand. Now, using of the relationship between penetration hardness and shear modulus, we can calculate the penetration hardness for various types of sand as shown in Fig. 5b. The observed linear dependence of the friction coefficient on the hardness is a result of the geometry, in this case a flat slider over a flat sand-surface. Recently, for a sphere-on-flat geometry a quantitative relation is found where the the friction coefficient depends on the penetration

hardness as $\mu \sim P_h^{-1/2}$ [15]. Due to the simple geometry, the relevant contact areas in the normal- and tangential direction can be calculated based on the measured ploughing track which, in the end, controls the friction coefficient for sliding. A peculiar pressure dependent friction was shown before by Crassous et al. [18] for sliding along a sandy slope. Objects on an inclined granular surface close to the avalanche threshold only slide for a narrow range of applied pressures. Sliding occur only when the granular surface may be slightly deformed by the slider weight, but not enough to create a rim able to stop the object.

These results therefore show that the relation between the sliding friction and the (linear) shear modulus shown in Fall et al. [13] can be understood by considering the (non-linear) penetration hardness that turns out to be roughly proportional to the shear modulus (Fig. 5a). The penetration hardness then allows for a quantitative explanation of ploughing friction. Modelling of the plasticity of (wet) granular materials could qualitatively explain the ploughing friction. However, due its complexity, this is beyond the scope if this paper.

Ploughing is a very general type of wear, which occurs generally when a hard material slide over a softer one. Tabor et al. described it for the first time for metal-metal systems where the hardness and geometries of the slider were varied [19]. However, ploughing friction, as can be quantified

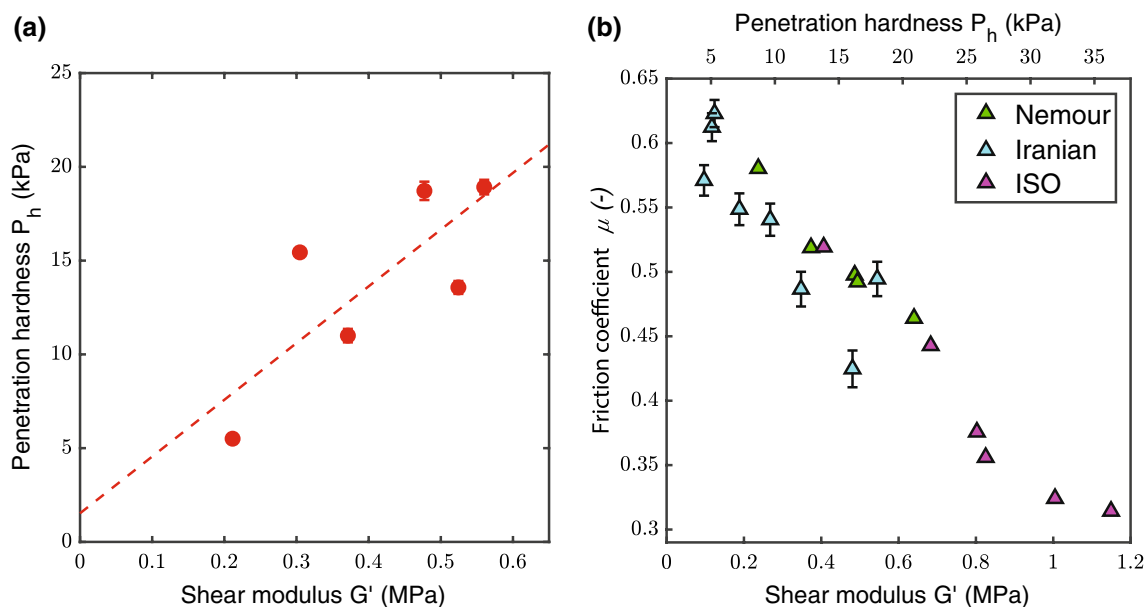


Fig. 5 **a** Penetration hardness P_h as a function of the shear modulus G' for different water fractions in sand. The shear modulus, reused from Fig. 4b in Ref. [13], and the penetration hardness are both obtained with so-called ‘Iranian sand’ (grain sizes of 212–500 μm). A roughly linear dependence is found: $P_h = \gamma_c G' + 1528$ with $\gamma_c = 0.030$ the critical shear strain. **b** Friction coefficient μ as a function the shear modulus G' for three sand types, reused from Fig. 5 in Ref. [13].

Furthermore, on the upper x-axis the penetration hardness P_h is given derived from the linear dependency with the shear modulus. The three sand-types ‘Nemour’, ‘Iranian’ and ‘ISO’, with respectively grain sizes in the ranges of 150–300 μm , 212–500 μm and 100–1000 μm , were mixed with varying amounts of water

by the material hardness and sliding geometry, can for instance also be found on ice and snow, which can have a major impact on ice skating and skiing. The hardness of ice decreases for increasing temperature which, close to the melting point of ice, results in a large ploughing contribution to the friction of ice-skating [20]. Therefore, the friction contribution sharply increases close to the melting point and large traces of the skate can be found on this ‘soft’ ice.

Sliding friction is therefore based on two contributions independent on the system: the friction due to ploughing (wear) and the surface contribution. The former can be quantified based on the geometry of the system and hardness of the materials, where the latter gives the friction when no ploughing occurs.

4 Conclusions

In conclusion, we have performed sliding experiments of a sledge on wetted sand and measured the penetration hardness of this granular material. A roughly inverse correlation between the penetration hardness and sliding friction coefficient is obtained, which is due to ploughing. During sliding, the sledge irreversibly moves the grains, as is evident by a trace in the sand after the passage of the sledge. This trace is less pronounced if the hardness is increased by adding a small amount of water to the sand which, subsequently, results in less friction. Therefore, increasing the hardness of the water-sand mixtures makes pulling a sledge over it easier. This seems already be experienced by the ancient Egyptians, where by pouring water in front of a sledge the sliding friction is decreased.

Pulling a sledge over sand results in irreversible movement of the grains and therefore the water-sand mixture is responding plastically. However, the mixture can hold a finite deformation elastically where the ‘stiffness’ can be quantified by the shear modulus G' . We found that the penetration hardness increases linearly with increasing shear modulus as $P_h \sim \gamma_c G'$ with $\gamma_c = 3\%$ the critical shear strain which sets the transition from elastic to plastic response. Therefore, after the critical shear strain, the granular material will respond plastically and ploughing will occur. The transition from elastic to plastic deformation and the amount of ploughing during sliding is of interest in many applications, where sliding of a sledge over sand is one of them.

Acknowledgements R.W.L. thanks Shell for financial support (PT 67354).

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

1. Pakpour, M., Habibi, M., Møller, P., Bonn, D.: How to construct the perfect sandcastle. *Sci. Rep.* **2**, 549 (2012)
2. Hornbaker, D., Albert, R., Albert, I., Barabási, A.L., Schiffer, P.: What keeps sandcastles standing? *Nature* **387**(6635), 765 (1997)
3. Schiffer, P.: Granular physics: a bridge to sandpile stability. *Nat. Phys.* **1**(1), 21 (2005)
4. Yuliza, E., Amalia, N., Rahmayanti, H.D., Munir, R., Munir, M.M., Khairurrijal, K., Abdullah, M.: Stability of granular tunnel. *Granul. Matter* **20**(4), 75 (2018)
5. Brutin, D.: *Droplet Wetting and Evaporation: From Pure to Complex Fluids*. Academic Press, Amsterdam (2015)
6. Fournier, Z., Geromichalos, D., Herminghaus, S., Kohonen, M., Mugele, F., Scheel, M., Schulz, M., Schulz, B., Schier, C., Seemann, R., et al.: Mechanical properties of wet granular materials. *J. Phys. Condens. Matter* **17**(9), S477 (2005)
7. Scheel, M., Seemann, R., Brinkmann, M., Di Michiel, M., Sheppard, A., Breidenbach, B., Herminghaus, S.: Morphological clues to wet granular pile stability. *Nat. Mater.* **7**(3), 189 (2008)
8. Kudrolli, A.: Granular matter: sticky sand. *Nat. Mater.* **7**(3), 174 (2008)
9. Newberry, P.E.: El Bersheh: The tomb of Tehuti-Hetep. *Egypt Exploration Fund*, vol. 1 (1895)
10. Møller, P.C., Bonn, D.: The shear modulus of wet granular matter. *EPL* **80**(3), 38002 (2007)
11. Richefeu, V., El Youssoufi, M.S., Radjai, F.: Shear strength properties of wet granular materials. *Phys. Rev. E* **73**(5), 051304 (2006)
12. Badetti, M., Fall, A., Chevoir, F., Roux, J.N.: Shear strength of wet granular materials: macroscopic cohesion and effective stress. *Eur. Phys. J. E* **41**(5), 68 (2018)
13. Fall, A., Weber, B., Pakpour, M., Lenoir, N., Shahidzadeh, N., Fiscina, J., Wagner, C., Bonn, D.: Sliding friction on wet and dry sand. *Phys. Rev. Lett.* **112**(17), 175502 (2014)
14. Aliasgari, M., Maleki-Jirsaraei, N., Rouhani, S.: The effect of grain size and shape on sliding friction of wet granular media. *Zeitschrift für Physikalische Chemie* **234**(1), 107–116 (2020). <https://doi.org/10.1515/zpch-2018-1267>
15. Liefverink, R., Weber, B., Bonn, D.: Ploughing friction on wet and dry sand. *Phys. Rev. E* **98**(5), 052903 (2018)
16. Badetti, M., Fall, A., Hautemayou, D., Chevoir, F., Aïmedieu, P., Rodts, S., Roux, J.N.: Rheology and microstructure of unsaturated wet granular materials: experiments and simulations. *J. Rheol.* **62**(5), 1175 (2018)
17. Dinkgreve, M., Paredes, J., Denn, M.M., Bonn, D.: On different ways of measuring “the” yield stress. *J. Non Newtonian Fluid-mech.* **238**, 233 (2016)
18. Crassous, J., Humeau, A., Boury, S., Casas, J.: Pressure-dependent friction on granular slopes close to avalanche. *Phys. Rev. Lett.* **119**(5), 058003 (2017)

19. Bowden, F., Moore, A., Tabor, D.: The ploughing and adhesion of sliding metals. *J. Appl. Phys.* **14**(2), 80 (1943)
20. Weber, B., Nagata, Y., Ketzetzi, S., Tang, F., Smit, W.J., Bakker, H.J., Backus, E.H., Bonn, M., Bonn, D.: Molecular insight into the slipperiness of ice. *J. Phys. Chem. Lett.* **9**(11), 2838 (2018)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.