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RESEARCH ARTICLE

Conjunctive surface water and groundwater management in a multiple user environment

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Abstract

The conjunctive use of surface water and groundwater is practiced worldwide as a measure to address water supply uncertainty. This paper focuses on a scenario in which an aquifer is open to multiple users and examines the impact of such an open-access environment on the stabilization function of conjunctive management. We construct a non-cooperative stochastic dynamic game model where multiple users utilize groundwater intake from a common aquifer as a complement to fluctuating surface water. We also propose a simpler baseline applicable to dynamic environments to compute the benefits of the stabilization function when users are unable to adjust groundwater intake to surface water uncertainties. We then apply the model to the real-world case of the water supply environment of the Cao'e River in China. Simulation results show that open access leads to diminishing stabilization values as higher pumping costs due to declining stocks lead to a weakening of users' incentive to utilize groundwater flexibly. Furthermore, the stabilization function itself is destabilized as the number of users increases. This is because greater groundwater intake and a decline in stocks amplify variations in pumping costs caused by different patterns of surface-water fluctuation.

Keywords Groundwater · Conjunctive management · Open access · Non-cooperative game · Dynamic programing

JEL Classification D62 · C73 · Q25 · Q51 · Q54

1 Introduction

The conjunctive use of surface water and groundwater is practiced worldwide as one of the measures for tackling uncertainty in water supplies. Numerous studies on the mechanisms involved in this practice have been conducted (e.g., Burt 1964;

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Tsur 1990; Tsur & Graham-Tomasi 1991; De Wrachien and Fasso 2002; Barlow et al. 2003; Syaukat and Fox 2004; Vedula et al. 2005; Ichinose et al. 2019; Zhang 2015). Conjunctive use of surface water and groundwater can be defined as the management of these resources in a coordinated operation for the purpose of realizing greater benefits than can be obtained by using either groundwater or surface water alone (Jack 1990). One of the important functions of conjunctive use is the stabilization function, which mitigates the impact of fluctuations in surface water by adjusting the amount of groundwater used. As a rule, one increases the amount of groundwater extracted during a period of surface water shortage to stabilize the total water supply and reduces the amount extracted during a period of surface water abundance to replenish groundwater stock, with the overall aim of mitigating the vulnerability of water supply systems (Tsur 1990; Tsur and Graham-Tomasi 1991; Diao et al. 2008; Reichard and Raucher 2003; Keith and Lars 1995).

However, the analyses of the previous studies, with very few exceptions, have not discussed situations where water resources are managed by multiple users rather than a single decision-maker. But this does not capture the actual situations of urban, industrial, and agricultural areas in many countries where multiple users, even from different sectors, exploit groundwater resources under weak governance and regulations (for cases in India, see for example Garduno et al. 2011, and World Bank 2010; for cases in Kenya, Mumma et al. 2011; for cases in China, Shen 2015; for cases in Spain and Australia, Ross and Martinez-Santos 2010).

The economic characteristics of natural resources under an open access environment, where multiple users freely access and exploit the resources, have been discussed extensively in the economic literature over more than half century (e.g., Gordon 1954; Berck 1979; Berck and Perloff 1984; Wilen and Homans 1997; Wilen 2000). In most cases, open access occurs when appropriate regulations or private rights are absent or insufficient, or when the enforcements of such regulations or rights are weak. And it is widely known that open access to natural resources by multiple users can cause overexploitation of the resources.

This paper examines the impact of an open-access environment on the stabilization function of conjunctive surface water and groundwater management. Especially, unlike previous studies, we demonstrate how the stability of the stabilization function can be degraded through inefficient utilizations of the resource by multiple users. We construct a non-cooperative stochastic dynamic game model where multiple users utilize groundwater intake from a common aquifer as a complement to fluctuating surface water with the aim of maximizing their own economic benefit. Furthermore, to evaluate the impact of open access, we also construct a reference model where a single social planner distributes groundwater intake with the aim of maximizing the aggregate economic benefit of all users, and compare the simulation results of the two models by applying them to the actual conditions of the water supply environment in the Cao'e River in Shangyu District, Shaoxing City, China.

The remainder of this paper is organized as follows: Section 2 presents a review on the theoretical background and related literature. Section 3 formulates models of two regimes—multiple user and single decision-maker. Section 4 analyzes the simulation results, and Sect. 5 concludes.

2 Theoretical background

Issues in groundwater management where there is open access have been studied extensively for decades in fields such as water resource engineering and natural resource economics (e.g., Burt and Provencher 1994; Khalatbari 1977; Eswaran and Lewis 1984; Negri 1989, 1990; Reinganum and Stokey 1985; Brozovic et al. 2010). Especially, as for a contradictory argument concerning efficiency, as early as 1980, Gisser and Sánchez (1980) compared socially optimal exploitation with private exploitation. Their study shows that open access can cause pumping inefficiency underground, but if the stock of the aquifer is relatively large, the difference between the two systems is very small and for practical purposes can be ignored. Koundouri (2004) revisits the Gisser-Sánchez effect and highlights that if users' demand for groundwater is inelastic, the impact of open access cannot be ignored.

More recent studies on groundwater management under a multiple user environment are: Guilfoos et al. (2016) have explored the link between groundwater management benefits and spatial heterogeneity. They find that in an aquifer where the intensity of water demand is uneven and where each farmer faces different saturated thicknesses, the simple pricing and quantity policies are not always effective. Edwards (2016) has explored the link between groundwater management benefits and underlying aquifer characteristics, and finds that areas facing higher hydraulic conductivity and more costly common-pool problems are more likely to benefit from groundwater management. Merrill and Guilfoos (2018) examine the welfare gains from groundwater management in a multiple user environment with a stochastic recharge and spatial stock externality and show that severely depleted aquifers amplify the effects of changing rainfall. Sears et al (2020) compared groundwater use in the Beaumont Basin before and after the introduction of property rights and shows despite that imposing property rights on the previously open-access groundwater resource did not deliver significant economic benefits to groundwater users, property rights inhibited the loss of groundwater to surrounding areas.

There are only two studies that analyze the impact of overexploitation and depletion of groundwater on benefits of the conjunctive use of surface water and groundwater. Foster et al. (2017) examines the impact of aquifer depletion on the stabilization function of conjunctive use system in the case of tomato production in California's Central Valley and find that the reduction in well yields caused by overexploitation reduces the stabilizing function. They argue that groundwater should not be considered as an unconstrained substitute for surface water when aquifer depletion has led to reductions in well yields and highlight the importance of the impacts of depletion on the future economic productivity of groundwater stocks. However, Foster et al. (2017) did not explicitly consider the situation that users face the common-pool problems.

The closest work is Msangi and Hejazi (2022). They analyze the impact of suboptimal behaviors including non-cooperative extraction on the economic value of groundwater under environments with and without constraints to users' ability to abstract groundwater. They show through an empirical application to California that groundwater 'mining' tends to diminish the augmentation value of groundwater while leaving the stabilization value unaffected—as long as the resource extraction only manifests itself in terms of increased pumping costs. However, when the continued overdraft of groundwater results in restrictions on the amount of water pumped by users, the stabilization value will be attenuated. The present paper is in line with Msangi and Hejazi (2022) with respects to the general purpose of the study although we limit our attention to the case without constraints. However, we reach different conclusions by using a different approach on the evaluation of stabilization value and by shedding light, not only on levels of stabilization value but on their stability, as will be explained bellow.

In most literature, the measurement of economic values of groundwater including stabilization value is grounded on the seminal works of Tsur (Tsur 1990; Tsur and Graham-Tomas, 1991; Gemma and Tsur 2007). The basic idea is the following. To compute the stabilization value, which is the economic value generated by the stabilization function of conjunctive management, it first uses the difference between expected net economic benefits with and without conjunctive use (Tsur 1990; Reichard and Raucher 2003; Sato 2015). In other words, by taking as a baseline the case in which a user cannot utilize conjunctive use, it calculates the additional economic value generated by using groundwater and surface water in a coordinated manner. Tsur (1990) and Gemma and Tsur (2007), for example, take as a baseline a case where groundwater cannot be used and thus only surface water is used. In this case, we can write out the economic value of conjunctive use V_u as:

$$V_u \equiv E\left[F\left(w_u\right) - C\left(G_u\right) \cdot \left(w_u - S\right)\right] - E[F(S)], \tag{1}$$

where $F(\cdot)$ is a concave benefit function, w_u the benefit-maximizing total water use, $C(\cdot)$ a unit extraction cost that depends on the groundwater stock G_u, and S uncertain surface water whose known mean value is $\overline{S}$. Precisely, the unit cost should depend on the distance between water table and ground surface, but we implicitly express the mathematical transformation from the stock amount to the above distance in the form of the function $C(\cdot)$. We assume here for simplicity the user can use the surface water without any cost, and therefore the remainder $w_u - S$ represents the amount of groundwater use.

The difference in (1) however reflects not only the stabilization value of mitigating undesirable profit fluctuations due to surface water variations but also the value as augmented by the increase in average water intake through the exploitation of groundwater resources in addition to surface water (Sato 2015). The latter value is called the "augmentation value" (Gemma and Tsur 2007). To eliminate this and extract a pure stabilization value, Tsur uses, as another baseline, the difference in the benefits when there is no uncertainty in S. That is,

$$V_c \equiv F\left(w_c\right) - C\left(G_c\right) \cdot \left(w_c - \overline{S}\right) - F\left(\overline{S}\right), \tag{2}$$

where w_c represents the benefit-maximizing total water use in the case without uncertainty, and G_c is the groundwater stock. V_c represents the augmentation value of groundwater, that is, the additional economic value the user can obtain by increasing the average amount of available resource. Accordingly, the stabilization value is given as a residual:

$$SV = V_u - V_c. \tag{3}$$

In this simplified static optimization problem, if the groundwater stocks are the same, i.e., $G_u = G_c$, and so are the unit costs, the benefit-maximizing amount of the water use are also the same, that is, $w_u = w_c$. This is because the benefit-maximizing amount of the water use is determined to equate the marginal net benefit $F'(w)$ with the marginal cost (=unit cost) $C(G_u) = C(G_c)$. In other words, the user intakes the groundwater to offset the fluctuations of the surface water completely and to stabilize the entire benefit. As a result, SV can eventually be computed as the difference in the benefit with and without uncertainty when the groundwater is not available:

$$SV = V_u - V_c = F\left(\overline{S}\right) - E[F(S)]. \tag{4}$$

Several studies have applied this simplified approach to evaluate the stabilization value of actual water environments (e.g., for cases in India, Gemma and Tsur (2007) and Palanisami et al. (2012); for cases in California, Tsur (1997) and Msangi and Hejazi (2022)).

However, the simplification of (4) is not necessarily applicable to dynamic cases. This is because the sequences of groundwater stock and unit extraction cost can be affected by the extraction behavior in the preceding periods that the user determines taking such influences into consideration. Unfortunately, most literature naively apply the formulation of (4) to dynamic cases and derive incorrect results. So, we need to stay at the formulation of (3) to compute the stabilization value in dynamic cases. But, as we will see soon, the double differences with two baselines in the formulation of (3) are too complex to analyze the user's behavior and underlying mechanism of the stabilization function.

We therefore take a different approach. We focus on the informational aspect of stabilizing behavior and redefine the stabilization value as the loss that a user would incur when it could not adjust groundwater intake in line with fluctuations of surface water since it does not observe the realization of surface water before the intake decision. Specifically, we consider, as the baseline case, the situation in which the user can use both surface water and groundwater but decides on its groundwater intake considering only the known probability distribution of the surface water.

In this case, we can write out the stabilization value as:

$$SV = E\left[\sum_t F\left(g_{p,t} + S_t\right) - C\left(G_{p,t-1}\right) \cdot g_{p,t}\right] - E\left[\sum_t F\left(g_{a,t} + S_t\right) - C\left(G_{a,t-1}\right) \cdot g_{a,t}\right], \tag{5}$$

where $g_{p,t}$ is the benefit-maximizing groundwater intake at time t that is determined after the surface water realization S_t is observed, whereas $g_{a,t}$ is the benefit-maximizing groundwater intake that is determined without observing the realization.

Despite the difference in core concept, equation (5) provides the same value as the intertemporal sum of (3) (See Appendix A). But we use the latter formulation for the theoretical and empirical analyses in the following sections mainly for two reasons. First, the formulation is much simpler and more suitable to analyze the underlying behavioral mechanisms of stabilization function. In contrast, the double differences in the formulation of (3) needs to analyze each of the four cases to uncover the underlying mechanisms. Second, in empirical studies, the former formulation requires to estimate extreme situations in which the groundwater is not available at all. But in most actual locations where conjunctive use is conducted, this is an unrealistic situation and can cause some practical difficulties: for instance, the estimation of benefit function can be troublesome due to the lack of data in such extreme situations.

3 Model formulation

We consider an industrial area overlying a groundwater aquifer and having a random surface water flow. In this area, N users (firms), to be denoted $i = 1, \dots, N$, are operating. Let us denote the user set $\{1, \dots, N\}$ by N and the period set $\{1, \dots, T\}$ by T. The water use in that area is governed by a stochastic dynamic process determined by two state variables: G_t, the groundwater stock, and S_t, the surface water flow available to the users in period t. G_t belongs to an infinite set $\mathrm{G}:=\left[0, \overline{G}\right]$, where $\overline{G}$ is the maximum capacity of the aquifer. The transition equation of the groundwater stock is as follows:

$$G_t = f\left(G_{t-1}, g_{1t}, \dots, g_{Nt}\right) \triangleq G_{t-1} + R^{\mathrm{g}}_t - \sum_{i \in \mathrm{N}} g_{it}, \tag{6}$$

where $g_{it}(\geq 0)$ is the groundwater intake by user i in period t and $R^{\mathrm{g}}_t(\geq 0)$ denotes the deterministic groundwater recharge in period t. To better demonstrate users' behavior to regulate groundwater in line with fluctuations in surface water, we do not introduce uncertainty in groundwater recharge and use a fixed value, R^{g}, throughout all periods, but this simplification does not affect the main conclusions of the paper. The surface flow is generated by

$$S_t = \overline{S}_t + R^{\mathrm{s}}_t, \tag{7}$$

where $\overline{S}_t$ is the average flow amount that is expected in period t in normal years and R^{s}_t denotes the fluctuation from the average in period t, where $R^{\mathrm{s}}_t > 0$ implies a time of abundant water supply and $R^{\mathrm{s}}_t < 0$ implies a time of water scarcity. Regarding the uncertainty of water supplies, previous studies mainly use two specifications—an independent distribution (Burt 1964; Joodavi et al. 2015) and a Markov chain process (Srikanthan and McMahon 2001). Because the typical situations that this paper addresses are ones in which conjunctive water management tackles fluctuations in a relatively short period of time, such as on a monthly basis, we assume R^{s}_t

is a stationary, temporally independent random variable of known distribution with a mean of zero.

Let $s_{it} = \varepsilon_i S_t$ be the amount of surface water utilized by user i in period t, where $\sum_{i \in \mathrm{N}} \varepsilon_i = 1$. For the sake of simplicity, let us assume that users can take surface water within that range at no additional cost. Let w_{it} be the total amount of water utilized by user i in period t. Thus, $w_{it} = g_{it} + s_{it}$.

Now let $F_i(w_{it})$ represent the instantaneous benefit accruing to user i in period t, which is quadratic:

$$F_i(w_{it}) \triangleq a_i w_{it} - b w_{it}^2, \tag{8}$$

where a_i and b are positive constants. This implies diminishing returns to production, an assumption that accords with production experience from aquifers as reported by many water studies such as Gisser and Sánchez (1980) and by Gardner et al. (1997). We introduce the heterogeneity of users by differentiating the parameter $a_1, \ldots, a_N$. Although we do not differentiate the parameter b to obtain analytical solutions for the dynamic game, the above differentiation enables us to represent quite a broad range of heterogeneity among users in terms of both scale and production technologies, especially through heterogeneous marginal benefit functions.

Let $C_i(G_t)$ be the unit cost of user i to pump groundwater to the surface, which depends on the groundwater stock:

$$C_i(G_t) \triangleq c_i - dG_t, \tag{9}$$

where c_i and d are positive constants. This implies that cost is inversely proportional to the total stock. Note that even if groundwater is taken from the same underground aquifer, groundwater costs to individual users often vary depending on where groundwater is pumped. But since analyses of the impact of such spatial heterogeneity is beyond our scope, the present paper assumes, for the sake of simplicity, the slope parameter d of the unit cost of pumping water is common to all users.

The instantaneous net benefit, including the pumping cost, for user i in period t is given by

$$\pi_i(g_{it}, G_{t-1}, S_t) \triangleq F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1}) g_{it}. \tag{10}$$

This paper compares two dynamic models to clarify the impact of open-access conditions where multiple users freely take water from the same aquifer. One of these is a single decision-maker model where the social planner distributes groundwater intake to each user during each time period with the aim of maximizing the intertemporal sum of the aggregate net economic benefits of all users (henceforth, *single decision-maker regime*). The other is a multiple user model where each user plays a noncooperative dynamic game in choosing its amount of groundwater intake with the aim of maximizing its own intertemporal sum of net economic benefits (henceforth, *multiple user regime*). In the former model, there exists no inefficiency caused by open-access conditions; its solutions can therefore be utilized as a reference case to evaluate the impact of open access.

3.1 The single decision-maker regime

Let $\Pi_i : (\mathrm{G} \times \mathrm{S} \times \mathrm{U}_{i1} \times \mathrm{U}_{-i1}) \times \ldots \times (\mathrm{G} \times \mathrm{S} \times \mathrm{U}_{iT} \times \mathrm{U}_{-iT}) \to \mathbb{R}_{\geq 0}$ denote the discounted intertemporal sum of user i's net expected benefits:

$$\Pi_i(G_0, S_1, g_{i1}, g_{-i1}, \ldots, G_{T-1}, S_T, g_{iT}, g_{-iT}) \triangleq E\left[\sum_{t\in \mathrm{T}} \beta^{t-1}\left[F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1})g_{it}\right]\right], \tag{11}$$

where $\beta \in [0, 1]$ is a discount factor. Symbols with subscript $-i$ indicate that they are a variable or a set for users excluding user i.The social planner maximizes the discounted intertemporal sum of the aggregate net expected benefits $\Pi : (\mathrm{G} \times \mathrm{S} \times \mathrm{U}_{11} \times \ldots \times \mathrm{U}_{N1}) \times \ldots \times (\mathrm{G} \times \mathrm{S} \times \mathrm{U}_{1T} \times \ldots \times \mathrm{U}_{NT}) \to \mathbb{R}_{\geq 0}$:

$$\begin{aligned}\Pi(G_0, S_1, g_{11}, \ldots, g_{N1}, \ldots, G_{T-1}, S_T, g_{1T}, \ldots, g_{NT}) &\triangleq \sum_{i\in \mathrm{N}} \Pi_i(G_0, S_1, g_{i1}, g_{-i1}, \ldots, G_{T-1}, S_T, g_{iT}, g_{-iT}) \\ &= E\left[\sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \beta^{t-1}\left[F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1})g_{it}\right]\right],\end{aligned} \tag{12}$$

subject to (6, 7), and the initial stock level G_0, where U_{it} is the set of admissible actions of user i in period t. One of the possible requirements for admissible actions is, obviously, $\mathrm{U}_{it} := [0, G_{t-1}]$, that is, users cannot exploit the aquifer beyond its stock level. In the following, however, we omit by assumption the case in which users exploit the whole stock in a single period. This is a plausible assumption because in real-world situations it does not make sense to even discuss the stabilization function of conjunctive use in such an extreme case, which should instead be addressed as a different issue. Also, to simplify the dynamic game model and to obtain analytic solutions, we ruled out the option of using T as a control variable—that is, a scenario in which users decide on their optimal stopping time—as well as the option of introducing user heterogeneity for T.

Let $\gamma_t = (\gamma_{1t}, \ldots, \gamma_{Nt}) \in \Gamma_t = \Gamma_{1t} \times \ldots \times \Gamma_{Nt}$ denote an admissible action rule of the social planner, where Γ_{it} is the set of admissible action rules concerning user i in period t. Note that γ_{it} is dependent both on realized surface flow and current groundwater stocks, that is, $g_{it} = \gamma_{it}(S_t, G_{t-1})$. Let $V(t, G_{t-1}, S_t)$ denote the optimal value function in period $t \in \mathrm{T}$ given the current groundwater stock G_{t-1} and the realization of surface flow S_t,

$$V(t, G_{t-1}, S_t) \triangleq \max_{\gamma_t\in\Gamma_t,\ldots,\gamma_T\in\Gamma_T} E_t\left[\sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \beta^{t-1}\left[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right]\right]. \tag{13}$$

The recursive structure of the return leads to the following Bellman optimality equation (Bellman 1952; Basar 2012):

$$V(t, G_{t-1}, S_t) = \max_{\gamma_t\in\Gamma_t} \sum_{i\in \mathrm{N}} \left[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right] + \beta E_{t+1}\left[V(t+1, G_t, S_{t+1})\right], \tag{14}$$

and

$$V(T+1, G_T, S_{T+1}) = 0, \forall G_T \in \mathrm{G} \text{and } S_{T+1} \in \mathrm{S}. \tag{15}$$

Proposition 1 *For the stochastic control problem described by (14 and 15), there exists a unique solution* $\{\gamma^*_{it}(S_t, G_{t-1}), S_t \in \mathrm{S}, G_{t-1} \in \mathrm{G}\}_{i\in\mathrm{N},t\in\mathrm{T}}$ *such that*

$$\gamma^*_{iT}(S_T, G_{T-1}) = \frac{1}{2b}\left[\Theta_i(S_T) - Nd^2\beta G_{T-1}\right],$$

$$\gamma^*_{it}(S_t, G_{t-1}) = \frac{1}{v_t}\left[\frac{v_t}{2b}\Theta_i(S_t) + \frac{Nd^2\beta\rho_{t+1}}{2b}\Theta(S_t) - d\beta\rho_{t+1}\Theta(\overline{S}) - Nd^2\beta\eta_t R^g + d(v_{t+1} - Nd\beta\rho_{t+1})G_{t-1}\right], t \le T-1. \tag{16}$$

For the definitions of v_t, ρ_t, η_t, $\Theta_i(\cdot)$, $\Theta(\cdot)$, *and the proof of Proposition 1, see Appendix B.*

3.2 The multiple user regime

User i maximizes the discounted intertemporal sum of the net expected benefits (11) subject to (7), the initial stock level G_0, and the transition equations of the groundwater stock:

$$G_t = f_i(G_{t-1}, g_{it}, g_{-it}) \triangleq G_{t-1} + R^g - g_{it} - \sum_{\substack{j\in N\\ j\neq i}} g_{jt}, t \in T. \tag{17}$$

Let γ_{it} denote an admissible strategy of user i for $S_t \in \mathrm{S}, G_{t-1} \in \mathrm{G}, t \in \mathrm{T}$ and let Γ_{it} denote the set of admissible strategies. We can then describe the dynamic process as an N-user T-stage discrete-time stochastic dynamic noncooperative game of a finite horizon defined by $\left\{\mathrm{N}, \mathrm{T}, \mathrm{G}, \mathrm{S}, \{U_{it}\}_{i\in\mathrm{N},t\in\mathrm{T}}, \{f_{it}\}_{i\in\mathrm{N},t\in\mathrm{T}}, \{\Gamma_{it}\}_{i\in\mathrm{N},t\in\mathrm{T}}, \{\Pi_i\}_{i\in\mathrm{N}}\right\}$.

Let $V^i(t, G_{t-1}, S_t)$ denote the optimal value function of user i at time $t \in \mathrm{T}$ given the groundwater stock G_{t-1} and the realization of surface flow S_t,

$$V^i(t, G_{t-1}, S_t) \triangleq \max_{\gamma_{it}\in\Gamma_{it},\ldots,\gamma_{iT}\in\Gamma_{iT}} E_t\left[\sum_{\tau=t}^{T}\beta^{t-1}\left[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right]\right]. \tag{18}$$

This leads to the Bellman optimality equation

$$V^i(t, G_{t-1}, S_t) = \max_{\gamma_{it}\in\Gamma_{it}} F_i(\gamma_{it} + \varepsilon_i S_t) - C_i(G_{t-1})\gamma_{it} + \beta E_t\left[V^i(t+1, f_i(G_{t-1}, \gamma_{it}, \gamma_{-it}), S_{t+1})\right], \tag{19}$$

and

$$V^i(T+1, G_T, S_{T+1}) = 0, \forall G_T \in \mathrm{G} \text{ and } S_{T+1} \in \mathrm{S}. \tag{20}$$

Proposition 2 *For the* N*-user dynamic noncooperative game* $\{\mathrm{N}, \mathrm{T}, \mathrm{G}, \mathrm{S}, \{U_{it}\}_{i\in\mathrm{N},t\in\mathrm{T}}, \{f_{it}\}_{i\in\mathrm{N},t\in\mathrm{T}}, \{\Gamma_{it}\}_{i\in\mathrm{N},t\in\mathrm{T}}, \{\Pi_i\}_{i\in\mathrm{N}}\}$, *the set of strategies* $\{\gamma^{**}_{it}(G_{t-1}, S_t); t \in \mathrm{T}, i \in \mathrm{N}\}$ *provides a feedback Nash equilibrium solution if, and* *only if, there exist functions* $V^i(t,\cdot):\mathrm{G}\times\mathrm{S}\to\mathbb{R}_{\geq 0}, t\in\mathrm{T}, i\in\mathrm{N}$ *such that the following recursive relations are satisfied*:

$$\begin{gathered} V^i(t,G,S)=\max_{\gamma_{it}\in\Gamma_{it}}\left\{F_i\left(\gamma_{it}+\varepsilon_i S\right)-C(G)\gamma_{it}+\beta E_t\left[V^i\left(t+1,f_i^*\left(G,\gamma_{it},\gamma_{-it}^{**}(G,S)\right)\right)\right]\right\} \\ =F_i\left(\gamma_{it}^{**}(G,S)+\varepsilon_i S\right)-C(G)\gamma_{it}^{**}(G,S)+\beta E_t\left[V^i\left(t+1,f_i^{**}\left(G,\gamma_{it}^{**}(G,S),\gamma_{-it}^{**}(G,S)\right)\right)\right], \\ V^i\left(T+1,G_T,S_{T+1}\right)=0, i\in N, \# \end{gathered} \tag{21}$$

where

$$f_i^{**}\left(G,\gamma_i,\gamma_{-i}^{**}(G,S)\right)\triangleq G+R^{\mathrm{g}}-\gamma_i-\sum_{\substack{j\in N\\ j\neq i}}\gamma_j^{**}(G,S). \tag{22}$$

For the proof of Proposition 2, see Appendix C. For the definition of feedback Nash equilibrium and its extended theory, see Basar (2012).

Proposition 3 *The N-user dynamic noncooperative game* $\left\{\mathrm{N},\ \mathrm{T},\mathrm{G},\ \mathrm{S},\left\{U_{it}\right\}_{i\in\mathrm{N},t\in\mathrm{T}},\left\{f_{it}\right\}_{i\in\mathrm{N},t\in\mathrm{T}},\left\{\Gamma_{it}\right\}_{i\in\mathrm{N},t\in\mathrm{T}},\left\{\Pi_i\right\}_{i\in\mathrm{N}}\right\}$ *admits a unique feedback Nash equilibrium solution* $\left\{\gamma_{it}^{**}\left(G_{t-1},S_t\right),t\in T\right\}$, *which amounts to the total groundwater intake of*:

$$\begin{gathered} \gamma_{iT}^{**}\left(G_T,S_T\right)=\frac{1}{2b}\left[\Theta_i\left(S_T\right)+dG_{T-1}\right], \\ \gamma_{it}^{**}\left(G_{t-1},S_t\right)=\frac{1}{\tilde{v}_t}\left[\frac{\tilde{v}_t}{2b}\Theta_i\left(S_t\right)-\frac{d\beta\tilde{v}_t\left(\tilde{\rho}_{t+1}+N\tilde{\varphi}_{t+1}\right)}{2b\tilde{v}_{t+1}}\Theta_i\left(\overline{S}\right)+\frac{d^2\beta\tilde{\rho}_{t+1}}{2b}\Theta\left(S_t\right)\right. \\ \left.-\frac{d\beta\left(d^2\beta\tilde{\rho}_{t+1}^2-\tilde{v}_t\tilde{\varphi}_{t+1}\right)}{2b\tilde{v}_{t+1}}\Theta\left(\overline{S}\right)-d^2\beta\tilde{\eta}_t R^{\mathrm{g}}+d\left(\tilde{v}_{t+1}-d\beta\tilde{\rho}_{t+1}\right)G_{t-1}\right], \\ t\leq T-1. \end{gathered} \tag{23}$$

For the definitions of $\tilde{v}_t$, $\tilde{\rho}_t$, $\tilde{\eta}_t$, $\tilde{\varphi}_t$, and the proof of Proposition 3, see Appendix D.

To examine how the stabilizing behaviors of the multiple user regime deviate from those of the single decision-maker regime, let us compare two analytic solutions shown in Proposition 1 and 3. We evaluate the gaps in the total groundwater intakes between two cases of surface water realization, S_H and S_L, where $S_H > S_L$. Namely, we compare the followings:

$$\begin{gathered} \Delta\gamma_t^*\triangleq\sum_{i\in\mathrm{N}}\left[\gamma_{it}^*\left(S_H,G_{t-1}\right)-\gamma_{it}^*\left(S_L,G_{t-1}\right)\right]=\begin{cases}-\left(S_H-S_L\right),t=T\\ -\frac{2bv_{t+1}}{v_t}\left(S_H-S_L\right),t\leq T-1\end{cases} \\ \Delta\gamma_t^{**}\triangleq\sum_{i\in\mathrm{N}}\left[\gamma_{it}^{**}\left(S_H,G_{t-1}\right)-\gamma_{it}^{**}\left(S_L,G_{t-1}\right)\right]=\begin{cases}-\left(S_H-S_L\right),t=T\\ -\frac{2b\tilde{v}_{t+1}}{\tilde{v}_t}\left(S_H-S_L\right),t\leq T-1.\end{cases}\# \end{gathered} \tag{24}$$

As we can see above, if a decision is one-shot, just like at $t = T$, a groundwater intake responds to a fluctuation of surface water on one-to-one basis in the opposite direction in both regimes. That is, the groundwater completely offsets the surface-water fluctuations to stabilize the total water use.

To analyze the movements for the periods before T, we use a numerical illustration. First, just for the simplification, let us suppose $S_H - S_L = 1\left(m^3\right)$. Also, we set the values of b, d and T, which are used explicitly, and implicitly through v_t and $\tilde{v}_t$, in the above equations, as the ones used in the numerical experiments in the next section. But the essence of the following analyses is completely independent on the choices of the above values.

Figure 1a shows the temporal transition of $\Delta\gamma_t^*$ and $\Delta\gamma_t^{**}$ in the case of $N = 5$. In both regimes, the responses of groundwater intake are more than offsetting the surface-water fluctuations but gradually converge to − 1 as it approaches to T. However, the extent to which the multiple user regime responds to the fluctuations is less than the single decision-maker regime throughout the periods before T. Figure 1b shows such differences in responses, i.e., $\Delta\gamma_t^* - \Delta\gamma_t^{**}$, in the case of $N = 2$, 4, 6, and 10. As the number of users increases, the deviations of the multiple user regime from the single decision-maker regime broadens. We can therefore anticipate that overexploitation under open-access conditions leads to weaker stabilizing behaviors of users and that it is worsen as the number of users increases.

3.3 Evaluation of the stabilization function

As stated in the previous section, we consider, as the baseline to compute the stabilization value, the situation in which users decide on groundwater intake without observing the surface water realization but considering only its known probability distribution. We can easily show, using the calculations in the proofs of Proposition 1 and Proposition 3, that the unique solution (strategy) of the baseline single decision-maker regime is given by replacing $\gamma_{it}^*\left(S_t, G_{t-1}\right)$ with

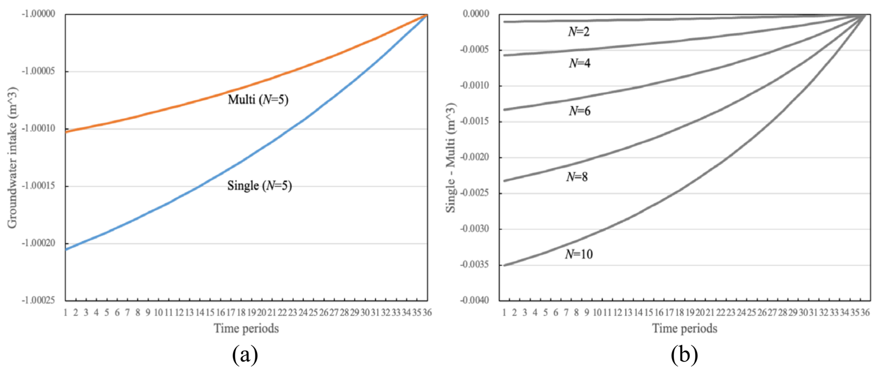


Fig. 1 Gaps in groundwater intake between the case of S_H and S_L ($S_H - S_L = 1$)

$\gamma_{it}^{*}\left(\overline{S}_t, G_{t-1}\right)$, and the unique solution (strategy) of the baseline multiple user regime is given by replacing $\gamma_{it}^{**}\left(S_t, G_{t-1}\right)$ with $\gamma_{it}^{**}\left(\overline{S}_t, G_{t-1}\right)$. Then, the stabilization value of the single decision-maker regime, v_{single}, is given by

$$\begin{aligned}\mathcal{V}_{\text{single}} \triangleq E\Bigg[&\sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{N}}\left[F_i\left(\gamma_{it}^{*}\left(S_t, G_{t-1}^{*}\right)+\varepsilon_i S_t\right)-C_i\left(G_{t-1}^{*}\right)\gamma_{it}^{*}\left(S_t, G_{t-1}^{*}\right)\right]\\ &-\sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{N}}\left[F_i\left(\gamma_{it}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right)+\varepsilon_i S_t\right)-C_i\left(\overline{G}_{t-1}^{*}\right)\gamma_{it}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right)\right]\Bigg], \#\end{aligned} \tag{25}$$

where $G_0^{*} \triangleq G_0$, $\overline{G}_0^{*} \triangleq G_0$ and, for $t \in T/\{1\}$,

$$G_t^{*} \triangleq f\left(G_{t-1}^{*}, \gamma_{i1}^{*}\left(S_t, G_{t-1}^{*}\right), \dots, \gamma_{Nt}^{*}\left(S_t, G_{t-1}^{*}\right)\right) \tag{26}$$

and

$$\overline{G}_t^{*} \triangleq f\left(\overline{G}_{t-1}^{*}, \gamma_{i1}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right), \dots, \gamma_{Nt}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right)\right). \tag{27}$$

Similarly, the stabilization value of the multiple user regime, v_{multi}, is given by

$$\begin{aligned}\mathcal{V}_{\text{multi}} \triangleq E\Bigg[&\sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{N}}\left[F_i\left(\gamma_{it}^{**}\left(S_t, G_{t-1}^{**}\right)+\varepsilon_i S_t\right)-C_i\left(G_{t-1}^{**}\right)\gamma_{it}^{**}\left(S_t, G_{t-1}^{**}\right)\right]\\ &-\sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{N}}\left[F_i\left(\gamma_{it}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right)+\varepsilon_i S_t\right)-C_i\left(\overline{G}_{t-1}^{**}\right)\gamma_{it}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right)\right]\Bigg],\end{aligned} \tag{28}$$

where $G_0^{**} \triangleq G_0$, $\overline{G}_0^{**} \triangleq G_0$ and, for $t \in T/\{1\}$,

$$G_t^{**} \triangleq f\left(G_{t-1}^{**}, \gamma_{i1}^{**}\left(S_t, G_{t-1}^{**}\right), \dots, \gamma_{Nt}^{**}\left(S_t, G_{t-1}^{**}\right)\right) \tag{29}$$

and

$$\overline{G}_t^{**} \triangleq \left(\overline{G}_{t-1}^{**}, \gamma_{i1}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right), \dots, \gamma_{Nt}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right)\right). \tag{30}$$

Note that we don't have a discount factor in the above formulation of $\mathcal{V}_{\text{single}}$ and $\mathcal{V}_{\text{multi}}$, although the solutions used here, i.e., $\gamma_{it}^{*}(\cdot,\cdot)$ and $\gamma_{it}^{**}(\cdot,\cdot)$, are the results of users' decisions with discounting. We thereby evaluate the stabilization values of each period equally across periods. Of course, summing up the discounted net benefits is another option to evaluate the stabilization values in dynamic context, and it might sometimes be more appropriate for resource management practices. But we, as researchers, take the former approach for our analytic purpose and deal with the users' behaviors equally throughout the periods.

4 Experimental analysis

4.1 Case study

This section numerically examines the stabilizing behaviors of a single decision-maker and multiple users and corresponding stabilization values by applying the analytic solutions presented in the previous section to the water supply environment in the Cao'e River in Shaoxing City, China. Shaoxing City is located in East China, in the north-central part of Zhejiang Province. It is in a subtropical monsoon climate with abundant precipitation, and the average annual rainfall from 2010 to 2020 was 1,795 mm. The Cao'e River flows through the Shangyu District of Shaoxing City and is adjacent to the Shangyu Economic Development Zone. This is an industrial park mainly focused on textile and manufacturing industries. In particular, the district exhibits, within a small land area, a considerable concentration of small factories exploiting local groundwater. We chose this area mainly because it exhibits the practical conditions in developing countries to which our model can be applied, that is, where multiple groundwater users utilize the same aquifer intensively without regulation.

We performed experimental simulations of a scenario where companies take water from the Cao'e River and use the groundwater in Shangyu District, Shaoxing City. The unit period, t, is one month, and the total length of the simulation period, T, is 36—or, in other words, three years.

To overcome the limited availability of water supply data of the district, we make some assumptions about both surface water supply and groundwater resources. For surface water supply, we dispense with the approach of using different means and variances for each month and instead use fixed values throughout the simulation period. This is because the source data—records of the daily surface water level—are only available for the three years from 2018 to 2020

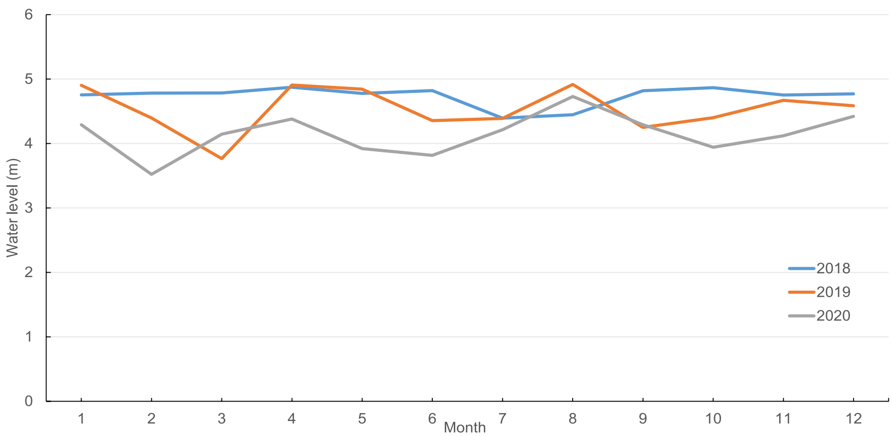


Fig. 2 Monthly temporal evolution of the surface water level of the Cao'e River (Water and Rain Regime Service System in Shangyu 2021)

(Water and Rain Regime Service System in Shangyu 2021), and these data are insufficient to establish credible values for different means and variances for each month. But at least judging from the monthly temporal evolution over the three years, they show no apparent seasonal variations (Figure 2). We obtain the mean surface water withdrawal $\overline{S}$ for Shangyu District by first calculating the eleven-year average (2010–2020) withdrawal for Zhejiang Province as a whole and then multiplying this by the ratio of the two populations—that of Shangyu District and that of Zhejiang Province (Water Resources Bulletin 2020; Demographic Statistics of Zhejiang Province 2022). We assume that the fluctuation from the average, R_t^s, independently follows a zero-mean normal distribution with a standard deviation σ_s, which we calculate by multiplying $\overline{S}$ by the coefficient of variation of the surface water level from 2018 to 2020.

The groundwater stock data is, basically, not available for the district. We therefore use an arbitrary initial stock value G_0 that is sufficient to avoid triggering a groundwater depletion throughout the simulation period. Again, this is plausible since it is meaningless to discuss the stabilization function of conjunctive use in an extreme case of depletion. We found in the simulations, moreover, that adding to the initial stock level does not change the overall conclusions presented below. Regarding groundwater recharge, we use a fixed rate in order to limit our attention to companies' reactions to surface water fluctuations. We assume that groundwater recharge is solely due to precipitation. We calculate the precipitation volume from the average monthly rainfall for the period 2018 to 2020 and use it as the fixed groundwater recharge R^g.

Concerning companies utilizing water resources, we divide our analyses in two parts. We first assumed homogeneity in technology but performed simulations for different numbers of companies (Sects. 4.1 and 4.2). That is, we fixed the parameter a_i of the production function as $a = 10,000$ and the parameter c_i of the unit cost function as $c = 5000$ for all companies, but we varied the number of companies N from 2 to 10. We thereby focused on examining the effects of open access and number of users. In the second part, we fixed the number of companies but

Table 1 Parameter values used in simulations

Parameter	Description	Value
a_i	First-order coefficient of instantaneous benefit function	10,000
b	Second-order coefficient of instantaneous benefit function	500
c	Pumping cost intercept	5000
d	Pumping cost slope	1.5
G_0	Initial groundwater stock	$3000(10^7 m^3)$
$\overline{S}$	Average surface water withdrawal	$6.7(10^7 m^3)$
σ_s	Standard deviation of surface water withdrawal	$0.54(10^7 m^3)$
R^g	Natural groundwater recharge	$0.02\left(10^7 m^3\right)$
ε_i	Share of water rights	$\varepsilon_i = 1/N$
β	Discount rate	0.98

introduced heterogeneity in technology so that we can focus on examining the effects of open access and heterogeneous users (Sect. 4.3). As for other parameter d of the pumping cost, we set $d = 1.5$ so that the effect of water withdrawal behavior on the stock level would be relatively small. Again, we did so to avoid triggering a depletion throughout the simulation period.

For the first part, we performed 10,000 simulations for each of the above nine cases ($N = 2, \ldots, 10$). All parameter values required for these simulations are organized in Table 1.

4.2 Results

The estimated stabilization values of the single decision-maker regime and the multiple user regime for the nine cases are listed in Table 2. There are three major findings here. First, the stabilization values of the single decision-maker regime are higher than those of the multiple user regime for all cases, and the extent to which the former outperforms the latter increases as the number of companies increases (A/B in the table). Second, the stabilization value in both regimes decreases as the number of companies increases. Third, the standard error for the multiple user regime increases with the number of companies to much more extent than the single decision-maker regime, or, in other words, the multiple user regime destabilizes the stabilization function as more companies become involved. This means that in multiple user regime, users get smaller and more unstable benefit from the flexible use of groundwater.

We begin here with an explanation of the second finding. The decrease in the stabilization value occurs because the average proportion of surface water in the total water use per company decreases as the number of companies increases—a consequence of the total amount of surface water following the same probability distribution irrespective of the number of companies. This reduces the impact of surface water fluctuations on a company's profit and thereby reduces the value of the stabilization function.

Table 2 Stabilization values ($)

Number of companies	Single decision-maker regime (A)	Multiple user regime (B)	A/B
2	2617 (6.6)	2614 (9.1)	1.001,148
3	1744 (6.6)	1738 (14.4)	1.003,452
4	1307 (6.6)	1298 (20.1)	1.006,934
5	1045 (7.6)	1030 (26.8)	1.014,563
6	870 (8.7)	856 (32.4)	1.016,355
7	746 (9.7)	728 (37.4)	1.024,725
8	651 (10.1)	632 (41.9)	1.030,063
9	579 (11.5)	558 (46.0)	1.037,634
10	521 (12.4)	498 (49.6)	1.046,185

The values in parentheses are standard errors

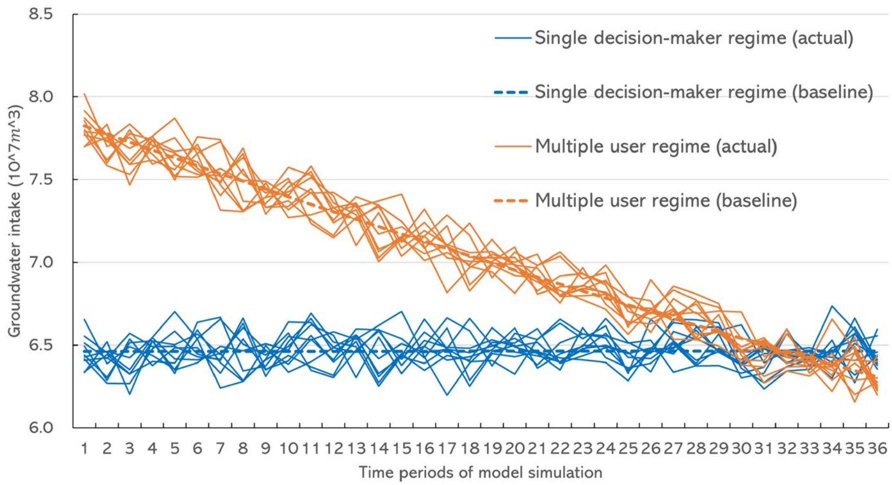


Fig. 3 10 samples of temporal evolution of groundwater intake per user

Below, we examine the underlying mechanisms of the first and third findings.

4.2.1 The difference in stabilization value

As mentioned above, our first finding was the smaller stabilization values in the case of the multiple user regime. Two factors contribute to this result. The first factor is reduced flexibility in groundwater use due to higher pumping costs. Figure 3 shows a comparison of 10 samples of temporal evolution of groundwater intake per user, in the case of $N = 5$, between the single decision-maker regime (blue lines) and the multiple user regime (orange lines). The dotted lines represent the baseline case where users cannot observe the realization of surface water before making their intake decisions, and the solid lines represent the actual intake where users are able to make this observation. Note that groundwater intake in the baseline case does not fluctuate from one simulation to another in either of the regimes. The major difference between the two regimes is that groundwater intake in the multiple user regime is much higher than that in the single decision-maker regime in the earlier periods but then decreases over time to levels that are slightly lower than those in the single decision-maker regime. This difference obviously comes from open access. While a social planner optimizes the intertemporal allocation of intake to smooth the instantaneous gross benefits throughout the time horizon, users in an open-access situation overexploit groundwater such that the pumping cost keeps rising due to the declining stock level, which in turn leads to a decline in water intake over time. As the number of companies increases, the overexploitation gets worse, and water intake exhibits a sharper drop from its initially higher level. The higher pumping cost hinders users' incentive to utilize groundwater flexibly in response to surface water fluctuations. Figure 4 shows the average standard deviation and the error range (black lines) for the 36 time periods in the differences between baseline and actual groundwater

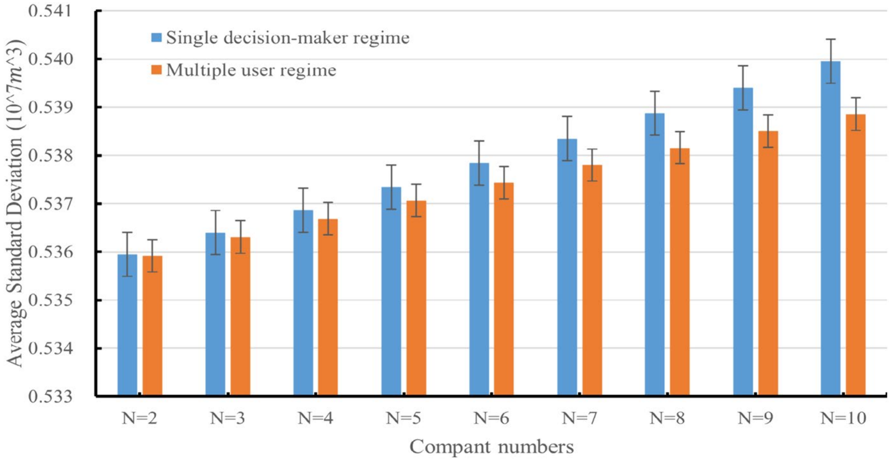


Fig. 4 The average standard deviation and the error range (black lines) in the differences between baseline and actual groundwater intake of all users in each period

intake of all users in each period, for both the single decision-maker regime (blue lines) and the multiple user regime (orange lines), with varying numbers of users. These values represent how flexibly groundwater is utilized in response to surface water fluctuations. The single decision-maker regime exhibits higher flexibility than the multiple user regime, and the difference between the two regimes gets larger

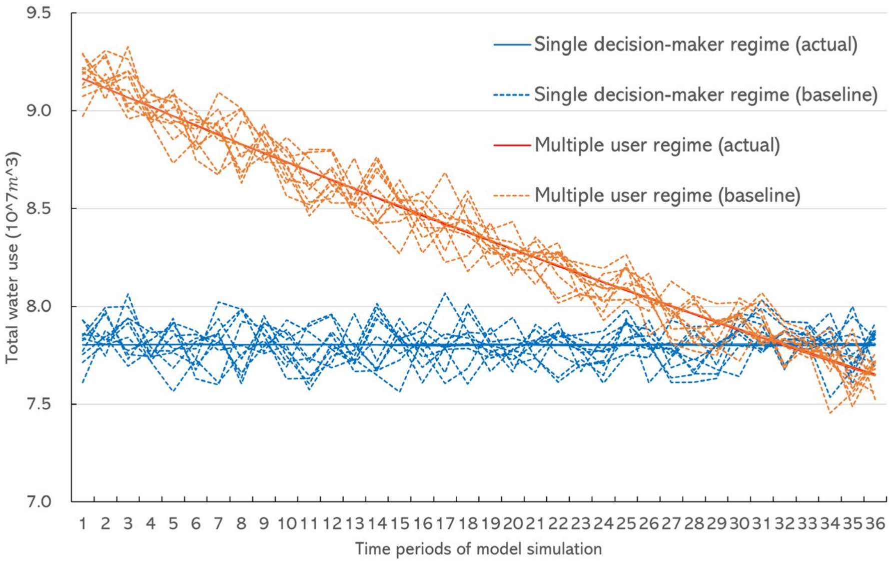


Fig. 5 10 samples of temporal evolution of total water use per user

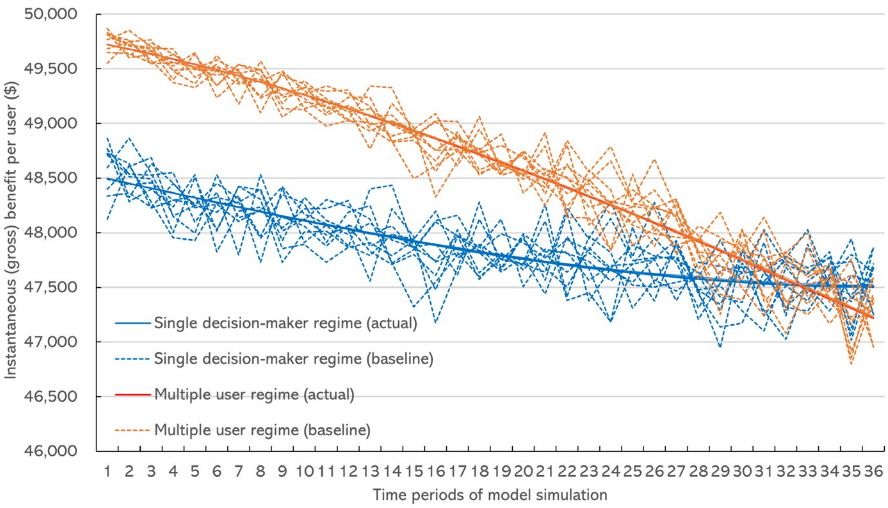


Fig. 6 10 samples of temporal evolution of instantaneous (gross) benefit per user

as the number of companies increases. The reduced flexibility of groundwater use leads directly to a decline in the stabilization function.

Although the flexibility gap is clear from the results shown above, its actual impact on the stabilization function is not that large. In fact, the differences in the values between the two regimes in Figure 4 are extremely small. We therefore need to look at another factor that contributes to a smaller stabilization value in multiple user regimes. But it requires a stepwise explanation. Figure 5 shows a comparison of 10 samples of temporal evolution of total water use per user, in the case of $N = 5$, between the two regimes. Figure 6 shows this same comparison with respect to instantaneous (gross) benefit per user before deducting pumping costs. For both figures, again, the blue lines represent the single decision-maker regime and the orange lines the multiple user regime; the dotted lines indicate the baseline case, and the solid lines show the actual intake. In contrast with groundwater intake, both the total water use and the instantaneous benefit in the actual case show hardly any fluctuation for either regime, owing to the stabilization function of conjunctive water management. But if we compare Fig. 4 with Fig. 5, we notice that the instantaneous benefit in the baseline multiple user regime (orange dotted lines) exhibits smaller fluctuations in the earlier time periods even though the extent of fluctuation in total water use looks almost the same for the two regimes.

Two factors generate this difference: overexploitation due to open access and the diminishing marginal productivity of the instantaneous benefit function. To describe how these factors work, let us look at a simpler static case (Fig. 7). We first consider the single decision-maker regime (Fig. 7a). Suppose the amount of surface water available for a company is S_L (=0 for simplicity) or S_H, with a probability of $1/2$ for each and with a mean value of $\overline{S}\left(= S_H/2\right)$. The unit pumping cost is given by C. In the actual case, the amount of total water use w is determined at point E—the

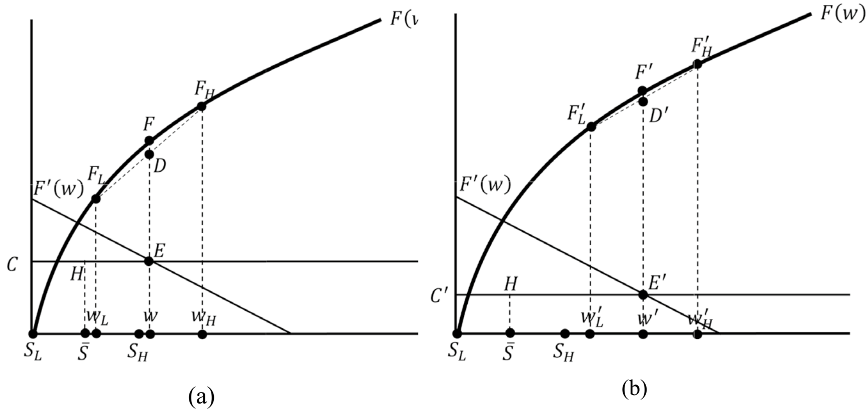


Fig. 7 Static-case illustration of the stabilization value

intersection of the marginal benefit curve $F\prime(w)$ and the marginal cost curve (unit pumping cost) C—and groundwater intake is given by the difference between w and the realized amount of surface water S_L or S_H. The benefit is therefore given by the line segment wF, and the expected pumping cost is given by the rectangle $\overline{S}HEw$. In the baseline case, the groundwater intake is fixed by the line segment $\overline{S}w$; total water use fluctuates between w_L and w_H; and the benefit fluctuates between w_LF_L and w_HF_H. Accordingly, the expected benefit is given by the line segment wD and the pumping cost by $\overline{S}HEw$. Thus, the stabilization value is given by the line segment FD. What happens if the groundwater is pumped under open-access conditions (Fig. 7b)? Suppose the private pumping cost for individual users drops to C', since they do not take external costs incurred by other users into consideration. In that case, groundwater intake in the reference case fluctuates within larger quantities, between w'_L and w'_H, and the benefit fluctuates to a higher level but within a smaller range, between $w'_LF'_L$ and $w'_HF'_H$, due to the strict concavity of the benefit function $F(W)$.

This coincides with the fact that the instantaneous benefit in the baseline multiple user regime exhibits smaller fluctuations in the earlier time periods, as shown in Fig. 6. And, importantly, the stabilization value in the multiple user regime is given by the line segment $F'D'$, which is smaller than that for the single decision-maker regime FD. The concavity of the benefit function reflects, needless to say, diminishing marginal productivity with respect to water use, which is one of the major features of typical production technologies not restricted to quadratic cases like ours.

In sum, overexploitation under open-access conditions leads to a decline in the stabilization value because of less flexible groundwater intake due to higher pumping costs and because of decreased potential variations in the instantaneous benefit due to increased groundwater use and the diminishing marginal productivity of the users.

4.2.2 Destabilization of the stabilization function

Identifying the mechanism underlying the increasing instability of the stabilization function in the multiple user regime requires a more complicated examination. But to begin with we can at least suppose that the source of the instability is variation between simulations in the sequence of surface water flows and that these variations are amplified in some way through users' behavior under open-access conditions and thus have a greater magnitude than under a single decision-maker regime. On this premise, we have examined two extreme patterns—a sequence of surface water flows leading to a maximum sample stabilization value and another sequence leading to a minimum sample stabilization value—as examples to help identify factors that foster instability in the stabilization function.

Let us first define the term "sample stabilization value." Let v_{single} and v_{multi} denote realized samples of the difference between the intertemporal sum of the actual aggregate net benefits and the intertemporal sum of the baseline net benefit, respectively, in the single decision-maker regime and the multiple user regime:

$$\begin{aligned} v_{\text{single}} \triangleq & \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[F_i\left(\gamma_{it}^{*}\left(S_t, G_{t-1}^{*}\right) + \varepsilon_i S_t\right) - C_i\left(G_{t-1}^{*}\right)\gamma_{it}^{*}\left(S_t, G_{t-1}^{*}\right)\right] \\ & - \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[F_i\left(\gamma_{it}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right) + \varepsilon_i S_t\right) - C_i\left(\overline{G}_{t-1}^{*}\right)\gamma_{it}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right)\right], \end{aligned} \tag{31}$$

$$\begin{aligned} v_{\text{multi}} \triangleq & \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[F_i\left(\gamma_{it}^{**}\left(S_t, G_{t-1}^{**}\right) + \varepsilon_i S_t\right) - C_i\left(G_{t-1}^{**}\right)\gamma_{it}^{**}\left(S_t, G_{t-1}^{**}\right)\right] \\ & - \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[F_i\left(\gamma_{it}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right) + \varepsilon_i S_t\right) - C_i\left(\overline{G}_{t-1}^{**}\right)\gamma_{it}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right)\right]. \end{aligned} \tag{32}$$

For the sake of convenience, we call v_{single} and v_{multi} "sample stabilization values," although they are in fact not samples of stabilization values, which are defined by the expectations in (25 and 28), but rather are realized sample values that are obtained in experimental simulations designed to estimate stabilization values.

As the two extreme patterns, we take two sequences of surface water flows that give the maximum and minimum values of the 10,000 sample stabilization values of the multiple user regime ($N = 10$). Then we see how, under these two sequences, the social planner and individual users behave in the cases of $N = 2$ and $N = 10$ and in the actual and baseline cases. We therefore have, for each of the $N = 2$ and $N = 10$ cases, four scenarios: the maximum sample stabilization value scenario in the single decision-maker regime (hereafter, "MaxS"), the maximum sample stabilization value scenario in the multiple user regime ("MaxM"), the minimum sample stabilization value scenario in the single decision-maker regime ("MinS"), and the minimum sample stabilization value scenario in the multiple user regime ("MinM"). Figure 8 shows the monthly temporal evolution of surface water disturbance (R_t^s) over 36 periods, i.e., three years, in the scenarios of MaxS and MaxM and of MinS and MinM. It is easy to see that surface water is relatively abundant in most months in the Max scenarios, while it is relatively scarce in the Min scenarios.

We decompose the sample stabilization values (31 and 32)—into two parts, the benefit gap and the (negative of) the cost gap, as follows:

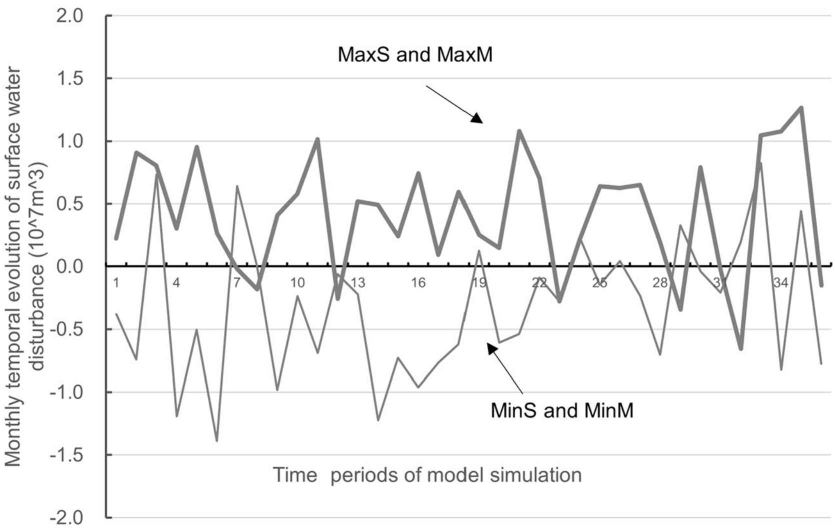
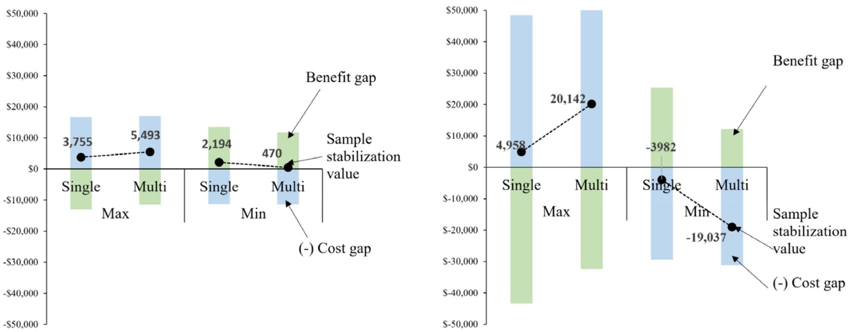


Fig. 8 Monthly temporal evolution of surface water disturbance in the Max and Min scenarios



(a) $N = 2$ (b) $N = 10$

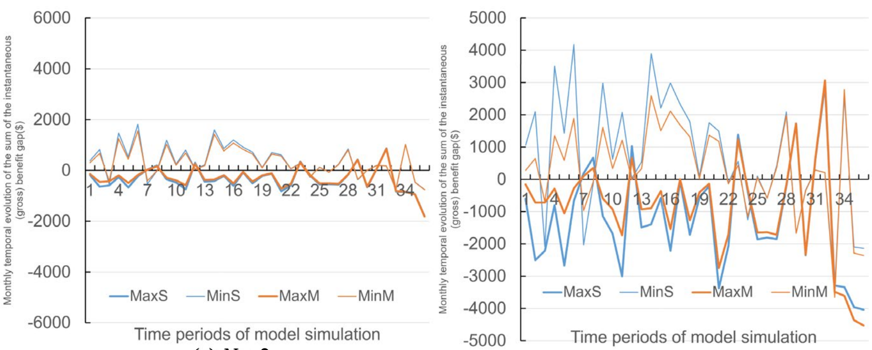
Fig. 9 Decomposition of sample stabilization values

$$v_{\text{single}} = \left\{ \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[F_i\left(\gamma_{it}^{*}\left(S_t, G_{t-1}^{*}\right) + \varepsilon_i S_t\right) - F_i\left(\gamma_{it}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right) + \varepsilon_i S_t\right) \right] \right\} + \left\{ - \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[C_i\left(G_{t-1}^{*}\right)\gamma_{it}^{*}\left(S_t, G_{t-1}^{*}\right) - C_i\left(\overline{G}_{t-1}^{*}\right)\gamma_{it}^{*}\left(\overline{S}_t, \overline{G}_{t-1}^{*}\right) \right] \right\}, \tag{33}$$

$$v_{\text{multi}} = \left\{ \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[F_i\left(\gamma_{it}^{**}\left(S_t, G_{t-1}^{**}\right) + \varepsilon_i S_t\right) - F_i\left(\gamma_{it}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right) + \varepsilon_i S_t\right) \right] \right\} + \left\{ - \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{N}} \left[C\left(G_{t-1}^{**}\right)\gamma_{it}^{**}\left(S_t, G_{t-1}^{**}\right) - C\left(\overline{G}_{t-1}^{**}\right)\gamma_{it}^{**}\left(\overline{S}_t, \overline{G}_{t-1}^{**}\right) \right] \right\}. \tag{34}$$

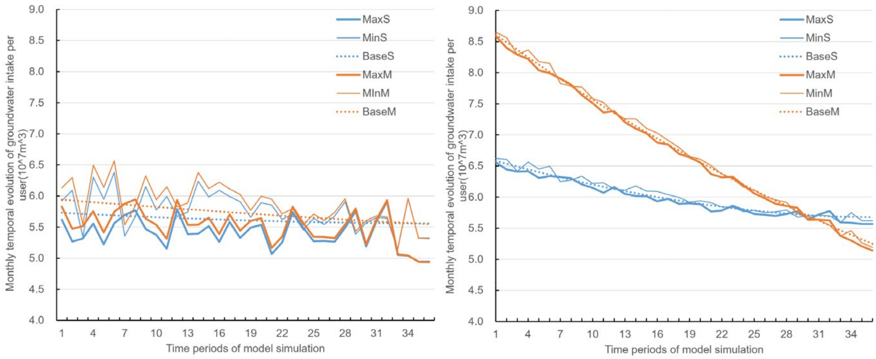
Figure 9 shows the decompositions in the MaxS, MinS, MaxM, and MinM scenarios. The stabilization value are represented by the black dots in the figure. Note that in the Max scenarios, the benefit gaps are negative and the (negative of) the cost gaps are positive, while in the Min scenarios, the signs are the opposite. This is because users pump less (more) groundwater than in the baseline case when the surface water is abundant (scarce) compared to the average. Also, in both $N = 2$ and $N = 10$, the sample stabilization values of MaxM are higher than those of MaxS, while those of MinM are lower than those of MinS, but these differences between the two regimes are much larger in the case of $N = 10$ than in the case of $N = 2$. And this increase in the degree of difference comes both from the benefit gap and from the cost gap: The extent to which the benefit gap for the multiple user regime falls below that for the single decision-maker regime (negative under the Max situation and positive under the Min situation) is much greater in the case of $N = 10$ than in the case of $N = 2$. Likewise, the extent to which the cost gap for the former regime exceeds that for the latter regime (positive under the Max situation and negative under the Min situation) is much greater in the case of $N = 10$ than in the case of $N = 2$.

Below, we examine the benefit gap and the cost gap individually. Figure 10 shows the monthly temporal evolution of the sum of the instantaneous (gross) benefit gap for all users over 36 periods. The bold lines represent the Max scenarios, and the thin ones the Min scenarios. The blue lines represent the single decision-maker regime, and the orange ones the multiple user regime. Note that when surface water is abundant, the groundwater intake and the benefit become smaller than in the baseline case, and the gap swings to negative. When surface water is scarce, the intake and the benefit become larger, and the gap swings to positive. We can see that, especially in the early periods, the multiple user regime exhibits smaller variations in the cases of both $N = 2$ and $N = 10$. However, the magnitude of the difference between the two regimes is much greater when $N = 10$, and in the latter periods the multiple user regime even outperforms the single decision-maker regime.



(a) $N = 2$ (b) $N = 10$

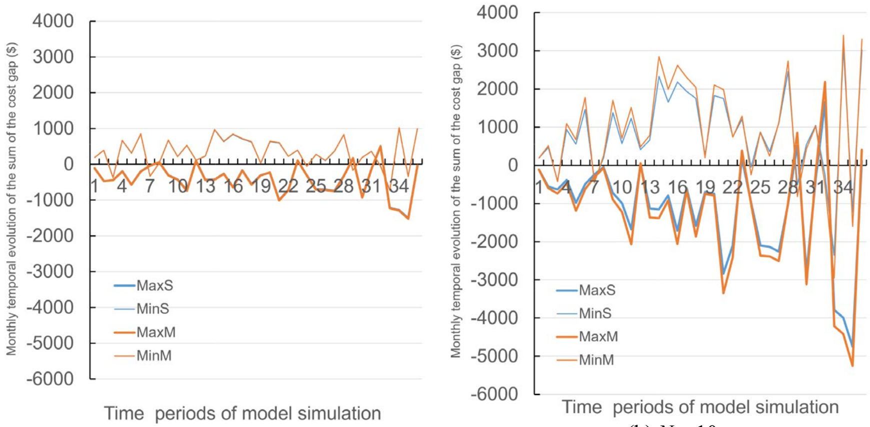
Fig. 10 Monthly temporal evolution of the sum of the instantaneous (gross) benefit gap



(a) $N = 2$ (b) $N = 10$

Fig. 11 Monthly temporal evolution of groundwater intake per user

What causes this difference? Figure 11 shows the monthly temporal evolution of groundwater intake per user over 36 periods. The lines representing the Max and Min scenarios and the two regimes are as in Fig. 10; the dotted lines represent the baseline case. Note that the fluctuations in actual intake around the baseline amount are smaller in the case of $N = 10$ than in the case of $N = 2$. This is because the amount of surface water (and its variation) per user gets smaller as the number of companies increases. Also, as we've already seen (in subsection 4.2.1), due to open access, groundwater intake in the multiple user regime starts from a much higher level than in the single decision-maker regime, but then it decreases over time. And, more importantly, as the number of companies increases, overexploitation gets worse, and water intake exhibits sharper drops from the higher level. Greater intake in earlier periods, coupled with the (strict)



(a) $N = 2$ (b) $N = 10$

Fig. 12 Monthly temporal evolution of the sum of the cost gap concavity of the benefit function, leads to smaller variations in the baseline instantaneous benefit and, therefore, to a smaller benefit gap for the multiple user regime when $N = 10$ in Fig. 10. This reduction of the benefit gap is eventually reflected in the phenomenon shown in Fig. 9, where the extent to which the benefit gap for the multiple user regime falls below that for the single decision-maker regime is much greater when $N = 10$. In sum, overexploitation due to open access and the diminishing marginal productivity of users are the driving forces, with respect to the benefit gap, in the instability of the stabilization function.

Next, let us examine the cost gap. Fig. 12 shows the monthly temporal evolution of the sum of the cost gap for all users over 36 periods. Note that when surface water is abundant, both groundwater intake and costs drop below their respective levels in the baseline case, and the gap swings to negative. When surface water is scarce, intake and costs become larger, and the gap swings to positive. We can see that the multiple user regime exhibits larger variations in the cases of both $N = 2$ and $N = 10$. However, the magnitude of the difference between the two regimes is much greater when $N = 10$.

What causes this difference? Let us denote the baseline case cost by $\theta_{i,t}^{\mathrm{base}} \triangleq C_t^{\mathrm{base}} g_{i,t}^{\mathrm{base}}$, where C_t^{base} and $g_{i,t}^{\mathrm{base}}$ are the unit cost and user i's intake, respectively, and the actual cost by $\theta_{i,t} \triangleq C_t g_{i,t} = \left(C_t^{\mathrm{base}} + \Delta_t^C\right)\left(g_{i,t}^{\mathrm{base}} + \Delta_{i,t}^g\right)$, where C_t and $g_{i,t}$ are the unit cost and user i's intake, while Δ_t^C and $\Delta_{i,t}^g$ are the differences from the baseline case. $\Delta_{i,t}^g$ s are positive when surface water is abundant and negative when it is scarce, and, as a result of the accumulation of these increments and decrements, Δ_t^C s are positive in the Min scenarios and negative in the Max scenarios. The cost gap $\Delta_{i,t} \triangleq \theta_{i,t} - \theta_{i,t}^{\mathrm{base}}$ can be decomposed into three parts:

$$\Delta_{i,t} = \Delta_t^C \Delta_{i,t}^g + \Delta_{i,t}^g C_t^{\mathrm{base}} + \Delta_t^C g_{i,t}^{\mathrm{base}}. \tag{35}$$

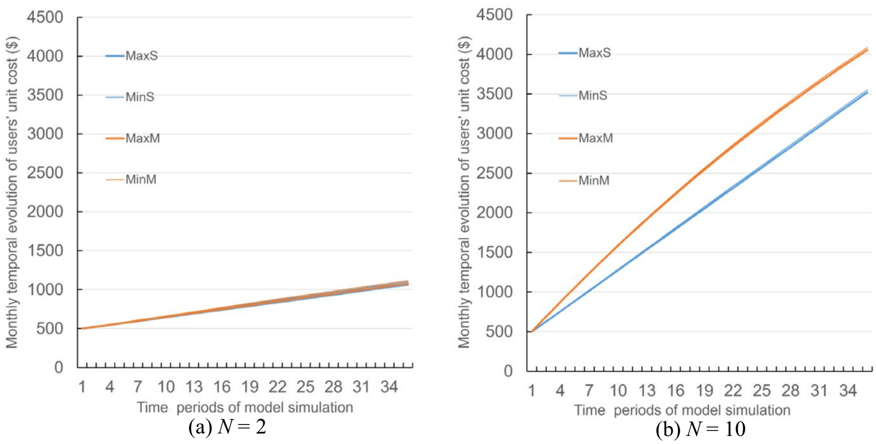


Fig. 13 Monthly temporal evolution of users' unit cost

The first term on the right side of the equation represents the cost gap generated by the increment (decrement) of the unit cost and the intake, the second term represents the gap generated by the increment of the intake through the existing part of the unit cost, and the third term represents the gap generated by the increment of the unit cost through the existing part of the intake. The cost gap, therefore, depends not only on the increment (decrement), Δ_t^C and $\Delta_{i,t}^g$, but also on the base amounts of the baseline case, C_t^{base} and $g_{i,t}^{\text{base}}$. Figure 13 shows the monthly temporal evolution of users' unit cost over 36 periods. As can be seen in Figs. 13 and 11, for most of the time horizon the multiuser regime outperforms the single decision-maker regime in the base amount of unit cost and groundwater intake as a result of open access and the corresponding decline in groundwater stock. And, more importantly, the magnitude of the outperformance is much greater when $N = 10$. This amplifies the impact of the increments (decrement)s Δ_t^C s and $\Delta_{i,t}^g$ s through the second and the third terms of (35) and thereby amplifies the variation in the total cost gap $\Delta_{i,t}$ compared to the case of $N = 2$. As a matter of fact, the differences in the increments (decrements) Δ_t^C s and $\Delta_{i,t}^g$ between the two scenarios are the only sources of variation in the sample stabilization values with respect to pumping cost, since the unit cost and the groundwater intake in the baseline case do not differ across simulations. But the impact of these differences is amplified by greater groundwater intake and a decline in stocks caused by open access. This amplification is eventually reflected in the phenomenon shown in Fig. 9, where the extent to which the cost gap for the multiple user regime exceeds that for the single decision-maker regime is much greater when $N = 10$.

In summary, open access and the resulting overexploitation of groundwater resources contribute to the instability of the stabilization function through the dual pathways of benefits and costs. In the first pathway, the increase in the number of companies enlarges the extent to which the multiple user regime falls below the single decision-maker regime with respect to their benefit gaps vis-à-vis the reference case. This is because greater intake coupled with the diminishing marginal productivity of users leads to smaller variations in baseline instantaneous benefit and, therefore, to a smaller benefit gap. In the second pathway, the increase in the number of companies enlarges the extent to which the multiple user regime exceeds the single decision-maker regime with respect to their gaps in pumping costs vis-à-vis the reference case. This is because greater intake and the resulting stock decline amplify variations in pumping cost caused by different patterns of surface-water fluctuation.

4.3 Heterogeneous users

In this section we extend our analyses to the case of heterogeneous users. We examine the simplest hypothetical case of $N = 2$ as an example. In our models, there are two sources of heterogeneity: a_i in the benefit function and c_i in the unit cost function. But they only take a form of $a_i - c_i$ (contained in $\Theta_i(\cdot)$) in the solution equations of (16 and 23) in each regime. So what only matters here is the relative levels between the two coefficients of the benefit and cost functions, not individual absolute levels. As shown in Table 3, we start from the homogeneous case and then

Table 3 Stabilization values of heterogeneous users ($)

$a_2 - c_2$	Single decision-maker regime			Multiple user regime		
	User 1	User 2	Total	User 1	User 2	Total
10,000	1309	1309	2617 (6.59)	1308	1308	2614 (9.09)
12,000	1309	1308	2617 (6.77)	1273	1350	2623 (9.94)
14,000	1310	1307	2617 (6.96)	1243	1404	2648 (10.85)
16,000	1310	1307	2617 (7.17)	1218	1467	2685 (11.80)
18,000	1311	1306	2617 (7.40)	1196	1537	2733 (12.81)
20,000	1311	1306	2617 (7.64)	1176	1614	2790 (13.86)
22,000	1312	1305	2617 (7.90)	1159	1695	2854 (14.93)
24,000	1312	1304	2616 (8.17)	1144	1780	2925 (16.02)
26,000	1312	1304	2616 (8.46)	1131	1870	3001 (17.12)
28,000	1313	1303	2616 (8.75)	1120	1962	3081 (18.23)
30,000	1314	1303	2616 (9.06)	1109	2056	3166 (19.35)

The values in parentheses are standard errors

broaden $a_i - c_i$ of user 2 from 10,000 to 30,000 keeping that of user 1 constant at 10,000.

The results show that while the total stabilization value of the single decision-maker regime only slightly decreases, that of the multiple user regime significantly increases as the difference between the two users broaden. As a result, the latter regime overperforms the former, which is an opposite result from the homogeneous case, although the multiple user regime still suffers from the greater instability of the stabilization value. Also, while the social planner assigns the stabilization value to the two users relatively equally, user 2 in the multiple user regime achieves greater stabilization values in expense of smaller values of user 1.

What causes these results? Table 4 shows the groundwater intake of each user. In both cases in the single decision-maker regime, the intake of user 1 decreases and that of user 2 increases as the difference between the two users broadens. In other words, the social planner concentrates the groundwater intake to the user with higher productivity and/or lower cost. Note also that the extent to which user 2 increases its intake is much more than the extent to which user 1 decreases its intake. The multiple user regime also exhibits similar behaviors. However, while the gaps from the baseline ((A)-(B)) in the former regime stay almost stable, in the latter regime, the gaps of user 1 turn negative and keep decreasing and those of user 2 keep increasing. In other words, user 1 decreases its intake more than the average of the baseline and user 2 increase it more than the average of the baseline.

To interpret this, let us use the static case simplification that we used in Fig. 7. Figure 14a shows the homogeneous case in the single decision-maker regime, where the stabilization values can be measured by the differences shown in the figure (SV_1, SV_2). (b) is its heterogeneous case, where the dotted curve represents the benefit function of user 2. User 1 decreases and user 2 increases the groundwater intake and thereby the total water use compared to the homogeneous

Table 4 Groundwater intake of heterogeneous users ($10^7 m^3$)

$a_2 - c_2$	Single decision-maker regime					
	Actual (A)		Baseline (B)		(A) – (B)	
	User 1	User 2	User 1	User 2	User 1	User 2
10,000	202	202	202	202	0.01	0.01
12,000	199	271	199	271	0.01	0.01
14,000	196	340	196	340	0.01	0.01
16,000	193	409	193	409	0.01	0.01
18,000	190	478	190	478	0.01	0.01
20,000	187	547	187	547	0.01	0.01
22,000	184	616	184	616	0.01	0.01
24,000	181	685	181	685	0.01	0.01
26,000	178	754	178	754	0.01	0.01
28,000	175	823	175	823	0.01	0.01
30,000	171	891	171	891	0.01	0.01
$a_2 - c_2$	Multiple user regime					
	Actual (A)		Baseline (B)		(A) – (B)	
	User 1	User 2	User 1	User 2	User 1	User 2
10,000	206	206	206	206	0.01	0.01
12,000	205	275	205	275	– 0.21	0.23
14,000	203	344	203	343	– 0.39	0.42
16,000	201	413	202	412	– 0.55	0.57
18,000	200	481	200	481	– 0.68	0.71
20,000	198	550	199	549	– 0.80	0.82
22,000	196	619	197	618	– 0.90	0.92
24,000	195	688	196	687	– 0.99	1.01
26,000	193	756	194	755	– 1.07	1.09
28,000	191	825	192	824	– 1.14	1.16
30,000	189	894	191	893	– 1.20	1.23

case. Note that the new stabilization value of user 2 ($\widetilde{SV_2}$) is not shrunk so much compared to the old one in the homogeneous case (SV_2) even though the user increases the water use. This is because the new benefit function of user 2 is given by shifting the original curve to the upper right, and the user can enjoy the stabilization value at the similar level of curvature as before. Also, as we saw in Table 4, the extent to which user 2 increases its water use is larger than the extent to which user 1 decreases it. (c) and (d) are the multiple user regime. In the heterogeneous user case (d), user 1 decreases and user 2 increases the total water use more than the baseline case, as we saw in Table 4. As a result, even though the stabilization value of user 1 slightly decreases ($\widetilde{SV_1\prime}$), that of user 2 significantly increases ($\widetilde{SV}_1^{\prime}$). This eventually leads to the predominance of the multiple user

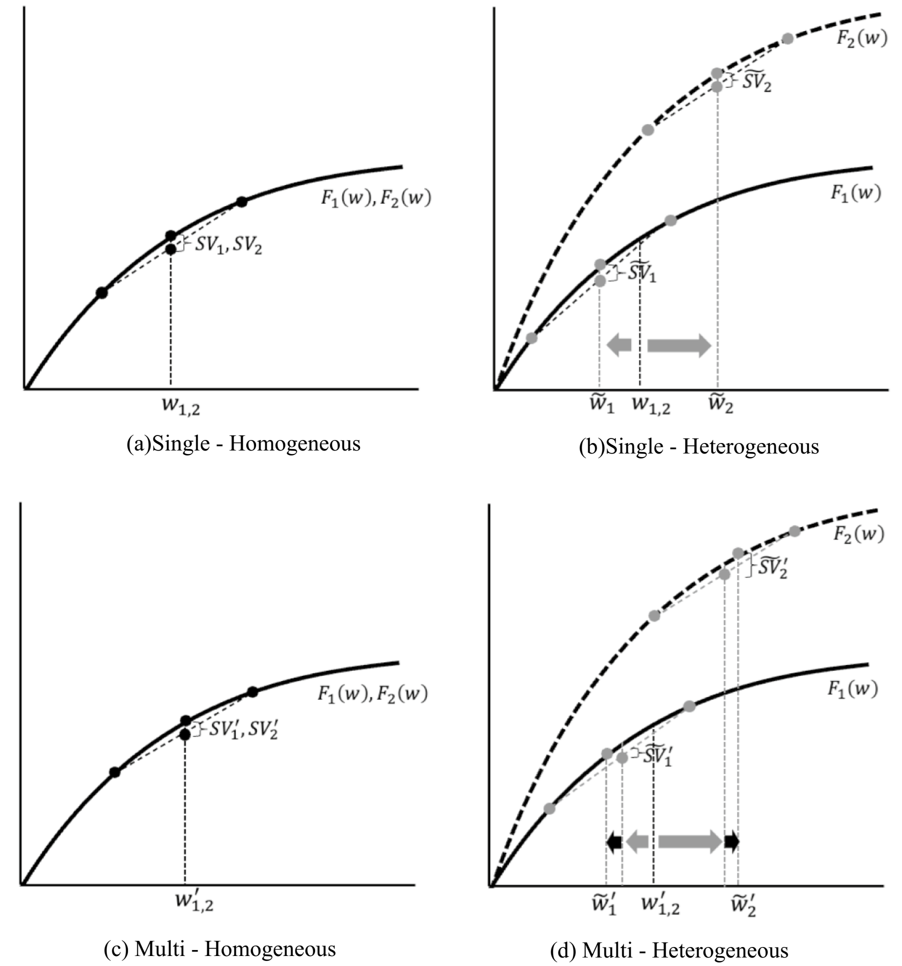


Fig. 14 Static-case illustration of the stabilization value in homogeneous and heterogeneous user cases

regime we saw in Table 3. But its interpretation requires further considerations. It is obvious from the figure that the range of the surface water fluctuation that the groundwater stabilizes are more than $\widetilde{SV}'_1$ but much less than $\widetilde{SV}'_2$. We will be back to this point in the discussion section.

5 Discussion and conclusions

In this study, we have examined the impact of an open-access environment on the stabilization function of conjunctive surface water and groundwater management. We constructed a noncooperative stochastic dynamic game model where multiple users utilize groundwater intake from a common aquifer as a complement to fluctuating surface water with the aim of maximizing their own economic benefits. We applied this model to the actual water supply environment in the Cao'e River in Shangyu District, Shaoxing City, China, and compared the simulated stabilization values with those of a reference model in which a single social planner distributes groundwater intake with the aim of maximizing the aggregate benefit of all users. In assessing stabilization values, we used a baseline case in which users could observe the realization of surface water only after making their groundwater intake decisions for each period, meaning that they could not flexibly use groundwater in line with fluctuations in surface water.

The major findings are, first, that stabilization values are reduced in an open-access environment, and the extent of the reduction increases as the number of users increases. The excessive use of groundwater brings this result through two underlying mechanisms: one is that higher pumping costs due to declining stock diminish users' incentive to use groundwater flexibly in a way that would effectively reflect fluctuations in surface water. At the same time, potential benefits from regulating groundwater use decline since the larger quantities of water that are input into production technologies with diminishing marginal productivity mitigate the potential impact of water fluctuations on economic benefits.

Second, the stabilization function itself is "destabilized" in an open-access environment as the number of users increases and stable stabilization values are no longer attainable. Overexploitation contributes to this instability through the dual pathways of benefits and costs. The stabilization value can be decomposed into two gaps vis-à-vis the baseline (which is characterized by inflexibility in pumping): a gap in benefits and a gap in (the negative of) costs. And because users pump less (more) groundwater than the baseline amount when surface water is abundant (scarce) compared to the average, the two gaps take opposite signs: the former is negative (positive) and the latter is positive (negative) when surface water is abundant (scarce). Thus, the smaller the absolute value of the benefit gap and the larger that of the cost gap, the wider the discrepancy in the sum of the two gaps between water-abundant and water-scarce cases, with the result that one cannot expect a stable stabilization value. With regard to benefits, again, the larger water input with diminishing marginal productivity mitigates the potential impact of the water fluctuation and thereby lessens the gap from the baseline. With regard to costs, the greater groundwater intake and the resulting stock decline increase the gap from the baseline by amplifying variations in pumping costs caused by different patterns of surface-water fluctuation.

In both findings, the diminishing marginal productivity of users' production technologies plays a certain role, as described above. But this requires further consideration when we interpret the results. On the one hand, this feature of production technologies is not so unusual, i.e., it is not restricted to quadratic cases like our model. In particular, if for some reason other inputs are fixed or relatively inflexible, it appears quite commonly in production processes in the short run. But on the other hand, the reason this feature decreases stabilization values and their stability is that it reduces the potential benefits of conjunctive use by mitigating the potential impact of water fluctuations that the baseline case of inflexible pumping would have incurred. In other words, the benefit that one can obtain by conjunctive use is itself inherently low. Therefore, we should ignore the impact of benefits and focus instead on the cost pathway when we interpret results and derive policy implications.

The third major finding is that when there exists heterogeneity regarding users' benefit functions and/or unit cost functions, stabilization values can increase in an open-access environment, and the extent of the increase grows as the differences between users increases. Such increases can contain more than the value of surface water fluctuations that groundwater actually stabilizes. The extra values contained here are not the results of stabilizing behavior but those of non-cooperative behaviors of users. Especially, under an open-access condition, users with higher efficiency can take advantage of observing surface water realizations better than those with lower efficiency. This gives the former users seemingly higher stabilization values in expense of the latter ones. Also, we found the "destabilization" issue in the heterogeneous user case too. Considering such issues, the true finding should be that when there exists heterogeneity in users in an open-access environment, the measurement of the stabilization value further reduces its reliability.

As for policy implications, first, our findings augment the importance of mitigating open-access conditions of groundwater. It is essential not just for avoiding overexploitation of resources but also for fully utilizing the stabilization function of groundwater and thereby strengthening the resilience and sustainability of water resource management. Concrete measures to mitigate open-access conditions are proposed in some literature: Provencher and Burt (1993, 1994) argued that managing groundwater by adopting the regime of private tradeable water permits, may generate considerable welfare in a stochastic framework by providing opportunities for risk management. Rouillard et al. (2021) emphasizes that under co-management of groundwater, attention should not only be paid to the construction of institutions but also to the process of building trust and social learning between state actors and users. Secondly, our findings require recalculations or reevaluations of the economic value of groundwater in actual water resource management. As explained in the introduction, the measurement of economic values in most empirical studies are grounded on the simplified approach that uses the economic benefit where the groundwater is not available (equation (4)). But this is simply incorrect in dynamic cases, especially in environments with multiple users. Such incorrect valuations can mislead policy planning and resource management in actual situations.

The main contributions of the present paper can be summarized as follows: first, it provides an original theoretical framework for evaluating the dynamic impact of a multiuser environment with open-access conditions on the conjunctive management of surface water and groundwater. Specifically, we propose a new baseline to redefine the benefits of the stabilization function when users are unable to adjust groundwater intake to surface water uncertainties. Second, by applying this framework to an actual water supply environment, we have identified the mechanisms through which open access restrains the stabilization value of conjunctive management. Third, we have found that one characteristic of open access is its potential to "destabilize" the stabilization function of conjunctive use. Finally, through this framework we have presented a theoretical foundation for policy considerations against issues related to conjunctive use and open access environments in developing countries.

On the other hand, our framework does have a few methodological limitations. First, the benefit function (production function) of our model is quadratic, and the heterogeneity we introduced is limited to two parameters in the benefit function and the unit cost function. This formulation enables us to derive analytic, and even reduced-form, solutions for noncooperative dynamic games while still allowing for a considerable range of applications and heterogeneities. But obviously, its generalizability is not unlimited. Second, our model excluded possibilities of complex hydrological interaction between surface water flows and groundwater stock. Several types of interactions are possible between groundwater and surface water (Sophocleous 2002). For example, on flatland with uniform precipitation and permeability, groundwater flow velocity and groundwater flow direction are highly dependent on surface water flow (Hubbert 1940). Also, surface water seeping into aquifers is an essential source of groundwater recharge (Hantush 2005; Fleckenstein et al. 2006). In many studies of conjunctive use management, such hydrological interactions are reflected in the modeling (Danskin and Gorelick 1985; Reichard and Bredehoeft 1984). By bringing in richer data—for example on permeability of surface water obtained from measurements of parameters such as precipitation, temperature, and relative humidity (Ntona et al. 2022)—it can be expected that our model will enable more reliable simulations.

Appendix A

We take the two-period model with a single user as an example to show the equivalency of equation (3) and equation (5). Let us first calculate (3). For the second period, we obtain a unique solution $g_2\left(G_1, S_2\right)$ for given G_1 and S_2,

$$g_2\left(G_1, S_2\right) = \frac{a-c+dG_1}{2b} - S_2. \tag{36}$$

With the solution (36) substituted into (14), we obtain a unique solution for the first period $g_1\left(G_0, S_1\right)$ and $\overline{g}_1\left(G_0, \overline{S}\right)$ for the uncertain case and the certain case respectively:

$$\begin{aligned} g_1\left(G_0, S_1\right) &= \frac{1}{4b^2-d^2}\left[(2b-d)\left(a-c+dG_0\right) - 2b\left(2bS_1 - d\overline{S}\right) - d^2R^g\right], \\ \overline{g}_1\left(G_0, \overline{S}\right) &= \frac{1}{4b^2-d^2}\left[\left(a-2b\overline{S}-c+dG_0\right)(2b-d) - d^2R^g\right].\# \end{aligned} \tag{37}$$

With the solution (36 and 37), we obtain $V_u, V_c andSV$:

$$V_u = \left(a + c - dG_0\right)\overline{S} - b\overline{S}^2 + \tfrac{2b-d}{4b(2b+d)}\left(X - 2b\overline{S}\right)^2 + \tfrac{1}{4b}X^2 + \tfrac{bd^2}{4b^2-d^2}\sigma_s^2 - 2\left[a\overline{S} - b\left(\overline{S}^2 + \sigma_s^2\right)\right],$$
$$V_c = \left(a + c - dG_0\right)\overline{S} - b\overline{S}^2 + \tfrac{2b-d}{4b(2b+d)}\left(X - 2b\overline{S}\right)^2 + \tfrac{1}{4b}X^2 - 2\left[a\overline{S} - b\overline{S}^2\right],$$
$$SV = \tfrac{bd^2}{4b^2-d^2}\sigma_s^2 + 2b\sigma_s^2. \tag{38}$$

where σ_s is standard deviation of surface water and $X \triangleq a - c + dG_0$.

Now we verify the result of (5). Using (36 and 37), we can easily obtain the expected net benefit in the case where the user determines the groundwater intake after the surface water realization:

$$E\left[\sum_{t=1}^{2} F\left(g_{p,t} + S_t\right) - C\left(G_{p,t-1}\right) \cdot g_{p,t}\right] = \left(a + c - dG_0\right)\overline{S} - b\overline{S}^2 + \tfrac{2b-d}{4b(2b+d)}\left(X - 2b\overline{S}\right)^2 + \tfrac{1}{4b}X^2 + \tfrac{bd^2}{4b^2-d^2}\sigma_s^2. \tag{39}$$

In the case where the user does not observe the surface water supply, for the second period, we obtain a unique solution $g_{a,2}\left(G_1, \overline{S}\right)$:

$$g_{a,2}\left(G_1, \overline{S}\right) = \tfrac{a-c+dG_1}{2b} - \overline{S}. \tag{40}$$

With the solution (40) substituted into (14), we obtain a unique solution $g_{a,1}\left(G_0, \overline{S}\right)$:

$$g_{a,1}\left(G_0, \overline{S}\right) = \tfrac{1}{4b^2-d^2}\left[(2b - d)\left(a - c + dG_0 - 2b\overline{S}\right) - d^2R^g\right]. \tag{41}$$

With the solution (40 and 41), we obtain:

$$E\left[\sum_{t=1}^{2} F\left(g_{a,t} + S_t\right) - C\left(G_{a,t-1}\right) \cdot g_{a,t}\right] = \left(a + c - dG_0\right)\overline{S} - b\overline{S}^2 + \tfrac{2b-d}{4b(2b+d)}\left(X - 2b\overline{S}\right)^2 + \tfrac{1}{4b}X^2 - 2b\sigma_s^2. \tag{42}$$

From(41 and 42), we can easily show:

$$SV = E\left[\sum_{t=1}^{2} F\left(g_{p,t} + S_t\right) - C\left(G_{p,t-1}\right) \cdot g_{p,t}\right] - E\left[\sum_{t=1}^{2} F\left(g_{a,t} + S_t\right) - C\left(G_{a,t-1}\right) \cdot g_{a,t}\right] = \tfrac{bd^2}{4b^2-d^2}\sigma_s^2 + 2b\sigma_s^2,$$

which is the same value as Eq. 43. Note that the above derivation can easily be applied to a model with a longer horizon and multiple users.

Appendix B

In addition to equation (16) in Proposition 1, we prove the following:

$$E_{t-1}\left[\frac{\partial V(t,G_{t-1},S_t)}{\partial G_{t-1}}\right] = \frac{d}{\upsilon_t}\left[\rho_t\Theta\left(\overline{S}\right) + Nd\beta\eta_t(2b-Nd)R^{g} + Nd\rho_t G_{t-1}\right], \quad (43)$$

where

$$\Theta_i(S_t) \triangleq a_i - c_i - 2b\varepsilon_i S_t, \Theta(S_t) \triangleq \sum_{i=1}^{N}(a_i - c_i) - 2bS_t,$$

$$\rho_t \triangleq \begin{cases} 1, & t = T \\ \upsilon_{t+1} - 2\beta\rho_{t+1}(Nd-b), & t \le T-1 \end{cases}$$

$$\upsilon_t \triangleq \begin{cases} 2b, & t = T \\ 2b\upsilon_{t+1} - N^2d^2\beta\rho_{t+1}, & t \le T-1 \end{cases}$$

$$\eta_t \triangleq \begin{cases} 0, & t = T \\ \beta\eta_{t+1}(2b-Nd) + \rho_{t+1}, & t \le T-1. \end{cases}$$

For $t = T$ and $T-1$, solving backward from T, we can easily show equation (16 and 43) hold. Assume they also holds for some $t = k+1 (1 \le k \le T-2)$, that is:

$$\begin{aligned} \gamma^*_{ik+1}(G_k, S_{k+1}) = \frac{1}{\upsilon_{k+1}}\Big[&\frac{\upsilon_{k+1}}{2b}\Theta_i(S_{k+1}) + \frac{Nd^2\beta\rho_{k+2}}{2b}\Theta(S_{k+1}) - d\beta\rho_{k+2}\Theta\left(\overline{S}\right) \\ &-Nd^2\beta\eta_{k+1}R^{g} + d\left(\upsilon_{k+2} - Nd\beta\rho_{k+2}\right)G_k\Big], \end{aligned}$$

$$E_k\left[\frac{\partial V(k+1,G_k,S_{k+1})}{\partial G_k}\right] = \frac{d}{\upsilon_{k+1}}\left[\rho_{k+1}\Theta\left(\overline{S}\right) + Nd\beta\eta_{k+1}(2b-Nd)R^{g} + Nd\rho_{k+1}G_k\right]. \quad (44)$$

Consider the problem for $t = k$:

$$\max_{g_{1,k},\ldots,g_{N,k}} \Omega(S_k) + \sum_{i=1}^{N}\left[\Theta_i(S_k) + dG_{k-1}\right]g_{ik} - b\sum_{i=1}^{N}g_{ik}^2 + \beta E_k\left[V(k+1, G_k, S_{k+1})|S_{k-1}\right],$$

where

$$\Omega(S_k) \triangleq \left(\sum_{i=1}^{N}a_i\varepsilon_i\right)S_k - b\left(\sum_{i=1}^{N}\varepsilon_i^2\right)S_k^2.$$

By using (44), we obtain the solution:

$$\gamma_{ik}^{*}\left(G_{k-1},S_{k}\right)=\frac{1}{\upsilon_{k}}\left[\frac{\upsilon_{k}}{2b}\Theta_{i}\left(S_{k}\right)+\frac{Nd^{2}\beta\rho_{k+1}}{2b}\Theta\left(S_{k}\right)-d\beta\rho_{k+1}\Theta\left(\overline{S}\right)\right. \\ \left.-Nd^{2}\beta\eta_{k}R^{g}+d\left(\upsilon_{k+1}-Nd\beta\rho_{k+1}\right)G_{k-1}\right]. \tag{45}$$

By using (45), we can show:

$$E_{k-1}\left[\frac{\partial V\left(k,G_{k-1},S_{k}\right)}{\partial G_{k-1}}\right]=\frac{d}{v_{k}}\left[\rho_{k}\Theta\left(\overline{S}\right)+Nd\beta\eta_{k}(2b-Nd)R^{g}+Nd\rho_{k}G_{k-1}\right]. \tag{46}$$

From equation (45 and 46) above, equation (16 and 43) also holds for $t=k$. By a mathematical induction, they are true for all $t\leq T-1$.

Appendix C

Let $\dot{\Pi}_{i}\left(\gamma_{11},\ldots,\gamma_{1T};\gamma_{21},\ldots,\gamma_{2T};\ldots,;\gamma_{N1}\ldots,\gamma_{NT},G_{0},S_{1},\ldots,S_{T}\right)$ denote the intertemporal sum of user i's net benefit when the strategies $\left\{\gamma_{11},\ldots,\gamma_{1T}\right\},\ldots,\left\{\gamma_{N1},\ldots,\gamma_{NT}\right\}$ are adopted by the users. Note that for period T the set of strategies $\gamma_{iT}^{**}\left(G_{T-1},S_{T}\right)$ is uniquely determined as a solution to the problem of maximizing the net benefit $\pi_{i}\left(\gamma_{iT},G_{T-1},S_{T}\right)$ because $V^{i}\left(T+1,G_{T},S_{T+1}\right)=0$. Now, consider period $T-1$. When $\left\{\gamma_{it}^{**}\left(G_{t-1},S_{t}\right);t\in\mathrm{T},i\in\mathrm{N}\right\}$ is a feedback Nash equilibrium solution, for all $\gamma_{it}\in\Gamma_{t}^{i},i\in\mathrm{N},t\leq T-2$ the following set of inequalities are satisfied:

$$\begin{gathered}
E_{T-1}\left[\dot{\Pi}_{1}\left(\gamma_{11},\ldots,\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\ldots,\gamma_{2T-1}^{**},\gamma_{2T}^{**};\ldots;\gamma_{N1},\ldots,\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_{0},S_{1},\ldots,S_{T}\right)\right]\\
\geq E_{T-1}\left[\dot{\Pi}_{1}\left(\gamma_{11},\ldots,\gamma_{1T-1},\gamma_{1T}^{**};\gamma_{21},\ldots,\gamma_{2T-1}^{**},\gamma_{2T}^{**};\ldots;\gamma_{N1},\ldots,\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_{0},S_{1},\ldots,S_{T}\right)\right]\\
E_{T-1}\left[\dot{\Pi}_{2}\left(\gamma_{11},\ldots,\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\ldots,\gamma_{2T-1}^{**},\gamma_{2T}^{**};\ldots;\gamma_{N1},\ldots,\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_{0},S_{1},\ldots,S_{T}\right)\right]\\
\geq E_{T-1}\left[\dot{\Pi}_{2}\left(\gamma_{11},\ldots,\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\ldots,\gamma_{2T-1},\gamma_{2T}^{**};\ldots;\gamma_{N1},\ldots,\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_{0},S_{1},\ldots,S_{T}\right)\right]\\
\cdots\\
E_{T-1}\left[\dot{\Pi}_{N}\left(\gamma_{11},\ldots,\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\ldots,\gamma_{2T-1}^{**},\gamma_{2T}^{**};\ldots;\gamma_{N1},\ldots,\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_{0},S_{1},\ldots,S_{T}\right)\right]\\
\geq E_{T-1}\left[\dot{\Pi}_{N}\left(\gamma_{11},\ldots,\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\ldots,\gamma_{2T-1}^{**},\gamma_{2T}^{**};\ldots;\gamma_{N1},\ldots,\gamma_{NT-1},\gamma_{NT}^{**},G_{0},S_{1},\ldots,S_{T}\right)\right].\#
\end{gathered} \tag{47}$$

Note that this condition holds for all values of G_{T-2} that are reachable by utilization of these strategies. And this condition is the same as solving a static game with the following profits:

$$\pi_{iT-1}\left(\gamma_{iT-1},G_{T-2},S_{T-1}\right)+\beta E_{T-1}\left[\pi_{iT}\left(\gamma_{iT}^{**},f_{i}\left(G_{T-2},\gamma_{iT-1},\gamma_{-iT-1}\right),S_{T}\right)\right]. \tag{48}$$

This is what equation (19) expresses for $T-1$, and it leads to a set of Nash equilibrium strategies $\left\{\gamma_{iT-1}^{**}\left(G_{T-2},S_{T-1}\right);i\in\mathrm{N}\right\}$ for all reachable states of G_{T-2}. Now, consider period $T-2$. Again, from the definition of the feedback Nash equilibrium solution, for all $\gamma_{it} \in \Gamma_t^i, i \in \mathrm{N}, t \le T-3$ the following set of inequalities are satisfied:

$$\begin{gathered}
E_{T-2}\left[\dot{\Pi}_1\left(\gamma_{11},\dots,\gamma_{1T-2}^{**},\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\dots,\gamma_{2T-2}^{**},\gamma_{2T-1}^{**},\gamma_{2T}^{**};\dots;\gamma_{N1},\dots,\gamma_{NT-2}^{**},\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_0,S_1,\dots,S_T\right)\right] \\
\ge E_{T-2}\left[\dot{\Pi}_1\left(\gamma_{11},\dots,\gamma_{1T-2},\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\dots,\gamma_{2T-2}^{**},\gamma_{2T-1}^{**},\gamma_{2T}^{**};\dots;\gamma_{N1},\dots,\gamma_{NT-2}^{**},\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_0,S_1,\dots,S_T\right)\right] \\
E_{T-2}\left[\dot{\Pi}_2\left(\gamma_{11},\dots,\gamma_{1T-2}^{**},\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\dots,\gamma_{2T-2}^{**},\gamma_{2T-1}^{**},\gamma_{2T}^{**};\dots;\gamma_{N1},\dots,\gamma_{NT-2}^{**},\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_0,S_1,\dots,S_T\right)\right] \\
\ge E_{T-2}\left[\dot{\Pi}_2\left(\gamma_{11},\dots,\gamma_{1T-2}^{**},\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\dots,\gamma_{2T-2},\gamma_{2T-1}^{**},\gamma_{2T}^{**};\dots;\gamma_{N1},\dots,\gamma_{NT-2}^{*},\gamma_{NT-1}^{**},\gamma_{NT}^{*},G_0,S_1,\dots,S_T\right)\right] \\
\dots \\
E_{T-2}\left[\dot{\Pi}_N\left(\gamma_{11},\dots,\gamma_{1T-2}^{**},\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\dots,\gamma_{2T-2}^{**},\gamma_{2T-1}^{**},\gamma_{2T}^{**};\dots;\gamma_{N1},\dots,\gamma_{NT-2}^{**},\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_0,S_1,\dots,S_T\right)\right] \\
\ge E_{T-2}\left[\dot{\Pi}_N\left(\gamma_{11},\dots,\gamma_{1T-2}^{**},\gamma_{1T-1}^{**},\gamma_{1T}^{**};\gamma_{21},\dots,\gamma_{2T-2}^{**},\gamma_{2T-1}^{**},\gamma_{2T}^{**};\dots;\gamma_{N1},\dots,\gamma_{NT-2},\gamma_{NT-1}^{**},\gamma_{NT}^{**},G_0,S_1,\dots,S_T\right)\right]\#.
\end{gathered}$$

Again, this condition holds for all values of G_{T-3} that are reachable by these strategies, and it is equivalent to solving a static game with the following profits:

$$\begin{aligned}
&\pi_{iT-2}\left(\gamma_{iT-2},G_{T-3},S_{T-2}\right)+\beta E_{T-2}\left[\pi_{iT-1}\left(\gamma_{iT-1}^{**},f\left(G_{T-3},\gamma_{iT-2},\gamma_{-iT-2}\right),S_{T-1}\right)\right. \\
&\left.+\beta V^i\left(T,f\left(G_{T-2},\gamma_{1T-1}^{**},\dots,\gamma_{-iT-1}^{**}\right),S_T\right)\right)\Big],i\in\mathrm{N},
\end{aligned}$$

which is what (19) expresses for $T-2$, and it leads to a set of Nash equilibrium strategies $\left\{\gamma_{iT-2}^{**}\left(G_{T-3},S_{T-2}\right);i\in\mathrm{N}\right\}$ for all reachable states of G_{T-3}. Proposition 2 can be proved by repeating the above process.

Appendix D

In addition to equation (23) in Proposition 3, we prove the following:

$$\begin{aligned}
E_{t-1}\left[\frac{\partial V^i\left(t,G_{t-1},S_t\right)}{\partial G_{t-1}}\right]=\frac{d}{\widetilde{\upsilon}_t}\Big[\left(\widetilde{\rho}_t+N\widetilde{\varphi}_t\right)\Theta_i\left(\overline{S}\right)-\widetilde{\varphi}_t\Theta\left(\overline{S}\right) \\
+d\beta\widetilde{\eta}_t\widetilde{\mu}_t(2b-Nd)R^{\mathrm{g}}+d\widetilde{\rho}_tG_{t-1}\Big],
\end{aligned}\tag{49}$$

where

$$\widetilde{\rho}_t \triangleq \begin{cases} 1, & t=T \\ \widetilde{\upsilon}_{t+1}-\beta\widetilde{\rho}_{t+1}(Nd+d-2b)-\frac{(N-1)d\beta\widetilde{\rho}_{t+1}(2b-Nd)\left(\widetilde{\upsilon}_{t+1}-d\beta\widetilde{\rho}_{t+1}\right)}{\widetilde{\upsilon}_t}, & t\le T-1 \end{cases}$$

$$\widetilde{\upsilon}_t \triangleq \begin{cases} 2b, & t=T \\ 2b\widetilde{\upsilon}_{t+1}-Nd^2\beta\widetilde{\rho}_{t+1}, & t\le T-1 \end{cases}$$

$$\widetilde{\eta}_t \triangleq \begin{cases} 0, & t=T \\ \beta\widetilde{\eta}_{t+1}\widetilde{\mu}_{t+1}(2b-Nd)+\widetilde{\rho}_{t+1}, & t\le T-1 \end{cases}$$

$$\widetilde{\mu}_t \triangleq \begin{cases} 1, & t = T \\ \frac{2b\widetilde{\upsilon}_{t+1} - d^2\beta\widetilde{\rho}_{t+1}}{\widetilde{\upsilon}_t}, & t \le T-1 \end{cases}$$

$$\widetilde{\varphi}_t \triangleq \begin{cases} 0, & t = T \\ \frac{\beta\widetilde{\mu}_t(2b-Nd)\left[d\widetilde{\rho}_{t+1}\left(\widetilde{\upsilon}_{t+1} - d\beta\widetilde{\rho}_{t+1}\right) + \widetilde{\upsilon}_t\widetilde{\varphi}_{t+1}\right]}{2b\widetilde{\upsilon}_{t+1}}, & t \le T-1. \end{cases}$$

For $t = T$ and $T - 1$, solving backward from T, we can easily show equation (21 and 49) hold. Assume they also holds for some $t = k + 1(1 \le k \le T - 2)$, that is:

$$\begin{aligned} \gamma^{**}_{ik+1}\left(G_k, S_{k+1}\right) = \frac{1}{\widetilde{\upsilon}_{k+1}}\Bigg[\frac{\widetilde{\upsilon}_{k+1}}{2b}\Theta_i\left(S_{k+1}\right) - \frac{d\beta\widetilde{\upsilon}_{k+1}\left(\widetilde{\rho}_{k+2} + N\widetilde{\varphi}_{k+2}\right)}{2b\widetilde{\upsilon}_{k+2}}\Theta_i\left(\overline{S}\right) + \frac{d^2\beta\widetilde{\rho}_{k+2}}{2b}\Theta\left(S_{k+1}\right) \\ - \frac{d\beta\left(d^2\beta\widetilde{\rho}^2_{k+2} - \widetilde{\upsilon}_{k+1}\widetilde{\varphi}_{k+2}\right)}{2b\widetilde{\upsilon}_{k+2}}\Theta\left(\overline{S}\right) - d^2\beta\widetilde{\eta}_{k+1}R^g + d\left(\widetilde{\upsilon}_{k+2} - d\beta\widetilde{\rho}_{k+2}\right)G_k\Bigg], \end{aligned}$$

$$\begin{aligned} E_k\left[\frac{\partial V^i\left(k+1, G_k, S_{k+1}\right)}{\partial G_k}\right] = \frac{d}{\widetilde{\upsilon}_{k+1}}\Big[\left(\widetilde{\rho}_{k+1} + N\widetilde{\varphi}_{k+1}\right)\Theta_i\left(\overline{S}\right) - \widetilde{\varphi}_{k+1}\Theta\left(\overline{S}\right) \\ + d\beta\widetilde{\eta}_{k+1}\widetilde{\mu}_{k+1}(2b - Nd)R^g + d\widetilde{\rho}_{k+1}G_k\Big].\# \end{aligned} \tag{50}$$

Consider the problem for $t = k$:
$\max_{g_{i,k}} \Omega\left(S_k\right) + \left[\Theta_i\left(S_k\right) + dG_{k-1}\right]g_{ik} - bg^2_{ik} + \beta E_k\left[V^i\left(k+1, G_k, S_{k+1}\right)|S_{k-1}\right].$
By using (D.2), we obtain the solution:

$$\begin{aligned} \gamma^{**}_{ik}\left(G_{k-1}, S_k\right) = \frac{1}{\tilde{\upsilon}_k}\Bigg[\frac{\tilde{\upsilon}_k}{2b}\Theta_i\left(S_k\right) - \frac{d\beta\tilde{\upsilon}_k\left(\tilde{\rho}_{k+1} + N\tilde{\varphi}_{k+1}\right)}{2b\tilde{\upsilon}_{k+1}}\Theta_i\left(\overline{S}\right) + \frac{d^2\beta\tilde{\rho}_{k+1}}{2b}\Theta\left(S_k\right) \\ - \frac{d\beta\left(d^2\beta\tilde{\rho}^2_{k+1} - \tilde{\upsilon}_k\tilde{\varphi}_{k+1}\right)}{2b\tilde{\upsilon}_{k+1}}\Theta\left(\overline{S}\right) - d^2\beta\tilde{\eta}_k R^g + d\left(\tilde{\upsilon}_{k+1} - d\beta\tilde{\rho}_{k+1}\right)G_{k-1}\Bigg]. \end{aligned} \tag{51}$$

By using (D.3), we can show:

$$\begin{aligned} E_{k-1}\left[\frac{\partial V^i\left(k, G_{k-1}, S_k\right)}{\partial G_{k-1}}\right] = \frac{d}{\tilde{\upsilon}_k}\Big[\left(\tilde{\rho}_k + N\tilde{\varphi}_k\right)\Theta_i\left(\overline{S}\right) - \tilde{\varphi}_k\Theta\left(\overline{S}\right) \\ + d\beta\tilde{\eta}_k\tilde{\mu}_k(2b - Nd)R^g + d\tilde{\rho}_k G_{k-1}\Big]. \end{aligned} \tag{52}$$

From equation (51 and 52) above, equation (23 and 49) also holds for $t = k$. By a mathematical induction, they are true for all $t \le T - 1$.

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Data availability The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Conflict of interest The authors declare no conflicts of interest relevant to this study.

Ethical approval The research involved no human participants or animals; thus, compliance with the ethical standard is unnecessary.

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