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ORIGINALARBEITEN/ORIGINALS

A non-restrictive approach of gear geometry calculation for plastic gears

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Abstract

For the unrestricted calculation of gearboxes with a constant transmission ratio, a parameter radius ratio is developed. For this purpose, a parameter radius ratio is introduced that has a piecewise continuous and differentiable domain of definition. Thus, the parameter can be used efficiently in the algorithmic implementation for the optimisation of a contact point of two gears. Furthermore, the data processing of the surface intersection is shown. Through direct surface conjugation, two paired gears can always achieve a line contact, therefore an optimal power transmission can be achieved regarding the technical properties. The procedure is demonstrated with a practical example of a planetary gear with skewed planet.

Ein unbeschränkter Ansatz zur Berechnung der Verzahnungsgeometrie von Kunststoffzahnrädern

Zusammenfassung

Für die unbeschränkte Berechnung von Verzahnungen mit konstanter Übersetzung wird der Parameter „radius ratio" (Radienverhältnis) entwickelt. Dieser Parameter besitzt einen stückweise kontinuierlichen und differenzierbaren Definitionsbereich. Damit kann der Parameter effizient in der algorithmischen Umsetzung zur Optimierung eines Kontaktpunktes zweier Zahnräder verwendet werden. Des Weiteren wird die Datenverarbeitung der Flächenüberschneidung gezeigt. Durch die direkte Flächenkonjugation können zwei gepaarte Zahnräder immer eine Linienberührung erreichen, wodurch eine optimale Kraftübertragung hinsichtlich der technischen Eigenschaften erzielt werden kann. Das Verfahren wird an einem praktischen Beispiel eines Planetengetriebes mit windschiefem Planeten demonstriert.

1 Introduction

The continuous search for product improvement always leads to new ideas in product design. Often, a solution is found that has already been chosen and its weakness is improved in detail. Rarely is the approach to a solution questioned and, if necessary, abandoned altogether to find a new method.

To achieve a holistic approach in gear development, a basic kinematic concept was developed that can describe most of the known constant ratio gearing types. Besides the free choice of two axes and numbers of teeth, including negative numbers for internal gears, there are three more continuous parameters for the calculation of the kinematic design, which describe the outer shape of the gear. This calculation basis unfolds the most potential in connection with the freedom that manufacturing with plastic e.g., by injection moulding allows. Here, the manufacturing restrictions are usually different from those of metal cutting processes.

Mainly for metallic materials, the manufacturing processes are very advanced and highly specialised for a few types of gearboxes. Wildhaber has developed in his research [1] a fundamental theory for hypoid gears. Furthermore, Wildhaber [2], Stadtfeld [3] and Shtipelman [4] show various methods for generating the geometry that can reproduce

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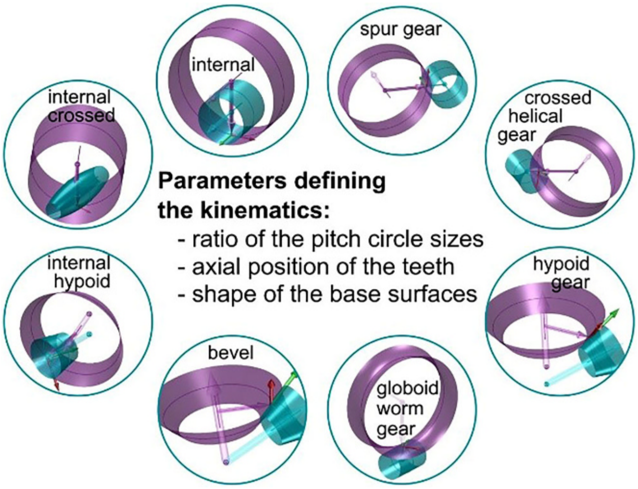


Fig. 1 Calculating several gearing types with one set of equations

machining. Other publications such as [5] deal with the optimisation of existing manufacturing systems. The aim of this work is to improve already mature processes.

Phillips [6] approaches a restriction-free path with the description of "General Spatial Involute Gearing". The starting point for gearing is an involute helicoid. This corresponds to the generalisation of common crown gear (rotation) and common rack (translational motion).

Park [7] describes the general connection of hypoid gearing with a common crown rack. Besides other aspects, he shows the continuous transition from bevel gear to crossed helical gear and from crossed axis gear to parallel axis gear.

For further separation within the concept of gearing, this paper refers exclusively to gears that correspond to a linear transmission function from gear 1 to gear 2. Another boundary condition is the integer number of teeth of both gears, whereby zero teeth are not permissible and negative numbers of teeth represent an internal toothed gear.

Fig. 1 gives an overview of the continuous transition of all known gears. This paper is intended to describe the calculation method of the hyperboloids and to introduce a new parameter to control the properties.

2 Generalisation of the kinematics calculation for gears using radius ratio (*RR*)

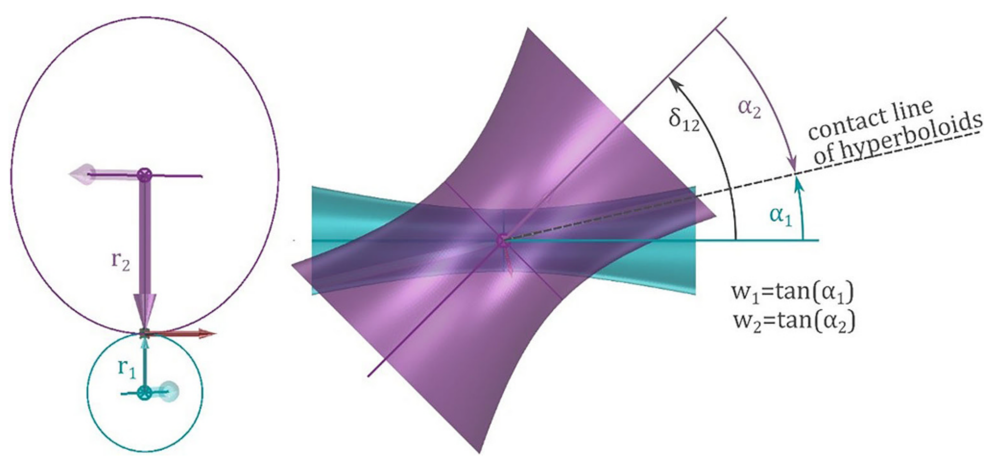
The calculation of the contact point is based on the calculation of two hyperboloids that always touch along a contact line. For this condition to be fulfilled, the two hyperboloids must satisfy the following Eq. 1, where r_1, r_2, w_1, and w_2 are taken from Fig. 2. The both radii r_1 and r_2 dividing the distance between the axis into two directed distances. The two calculation variables α_1 and α_2 divide the sum angle δ_{12} into two directional angles. The values w_1 and w_2 are calculated as the tangent function of the angles α_1 and α_2. If the axes of the two gears are skewed, the tangential velocities along these contact lines are no longer collinear. A third velocity component, the sliding velocity, must compensate for this velocity difference so that the velocity behaviour at the contact point is solved.

Furthermore, the direction of this sliding velocity must be determined with a control parameter. This control parameter should have the following properties so that it can be implemented effectively and as generally valid as possible in the programmatic implementation.

First there should be a reference to the number of teeth ratio so that a conceptual parallel to the gear development is created. From a mathematical algorithmic point of view, a continuous definition range is important, which maps both externally-externally toothed gear pairs and enables externally-internally and internally-externally toothed gears.

The control parameter radius ratio *RR* is introduced, which influences the outer shape of the base surfaces of the gear pair and defines the two hyperboloids using the following equation.

Fig. 2 Parameters for kinematic calculation model



$$RR = -\frac{r_2}{r_1} = -\frac{w_2}{w_1} \tag{1}$$

In this context, the radius is signed and is given the same sign as the number of teeth. An internal-toothed gear thus has a negative radius. With this definition, an external-external toothed gear pair has a negative *RR* and an external-internal or internal-external toothed gear pair has a positive *RR*.

The calculation of the parameters w_1 and w_2 is done with the help of the quadratic solution formula and the following correlations for p and q:

$$p = \frac{1 - RR}{\tan(\delta_{12})} \tag{2}$$

$$q = RR \tag{3}$$

$$w_{2_1} = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q} \tag{4}$$

$$w_{2_2} = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} \tag{5}$$

The following separation is to be made.

$$w_2 = \begin{cases} w_{2_1}, & if \quad RR > 1 \\ w_{2_2}, & else \end{cases} \tag{6}$$

The calculation of the remaining parameters r_1, r_2 and w_1 is done by substituting and solving Eq. 1.

According to Fig. 3, there are three continuously differentiable domains for *RR* depending on the number of teeth of the two gears. If two numbers of teeth are specified, it is no longer possible to switch between these domains, which means that for the algorithmic evaluation of a gear pair with a specified transmission ratio, there is always one domain to which a continuously differentiable range of values is assigned.

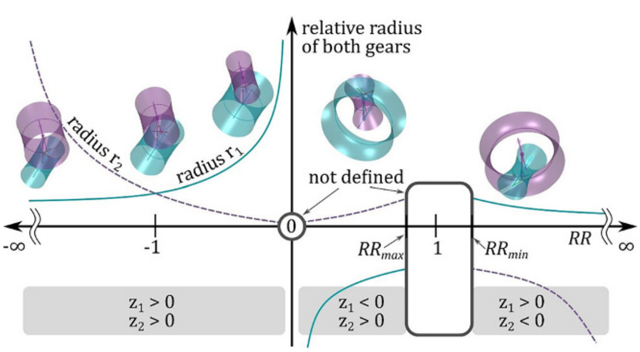


Fig. 3 Influence of radius ratio (*RR*) to calculate the base surface and contact point

For externally toothed gear pairs, $-\infty < RR < 0$ is the domain that can be selected independently of the selected transmission ratio i_{12} to set the corresponding gear characteristics.

Since the parameter *RR* generally has no technical relevance, *RR* is used in the algorithmic implementation to optimise for technically relevant parameters using optimisation methods. For example, the helix angle of both gears or the pitch p and the normal modulus m_n can be used as optimisation targets.

The selection of a certain section of the hyperboloid determines whether the gear is a bevel, hypoid, worm, crown, spur, helical, crossed helical gear according to conventional nomenclature.

2.1 Restrictions of the domain

In general, the parameter *RR* must have the same sign as the associated ratio i_{12}. Therefore:

$$\frac{i_{12}}{RR} > 0 \tag{7}$$

All *RR* that belong to a certain transmission ratio i_{12} and do not fulfil this condition are not permitted. Furthermore, there are restrictions for internally toothed gears. According to Fig. 3, a RR_{min} and RR_{max} is defined that limits the domain of definition for internally toothed gears. Here RR_{min} applies as the lower limit of the definition for gear pairs with $z_2 < 0$. RR_{max} applies as the upper limit of the definition range for gear pairs with $z_1 < 0$.

$$RR_{\min} = \frac{\tan\left(\frac{\pi}{4} + \frac{\delta_{12}}{2}\right)}{\tan\left(\frac{\pi}{4} - \frac{\delta_{12}}{2}\right)} \tag{8}$$

$$RR_{\max} = \frac{\tan\left(\frac{\pi}{4} - \frac{\delta_{12}}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{\delta_{12}}{2}\right)} \tag{9}$$

For the special cases of parallel axes and crossing axes, the parameter *RR* is not arbitrary, otherwise the tangential speeds of both gears cannot be compensated with the sliding movement. In this case, the parameter *RR* must always fulfil Eq. 10, which was developed based on Eq. 11 from Klingelnberg [8, p. 41].

$$RR = -\frac{\cos(\delta_{12}) - i_{12}}{\cos(\delta_{12}) - \frac{1}{i_{12}}} \tag{10}$$

$$\tan(\alpha_1) = \frac{\sin(\delta_{12})}{\cos(\delta_{12})} - i_{12} \quad (11)$$

3 Surface calculation

The surface calculation is performed on a discrete point grid. For this purpose, a common rack is defined by analytically describing surface points and their normals and transferring them into a local coordinate system. Based on this geometry description, the surface points can be conjugated between the surfaces of the toothed objects using various algorithms.

For the case of transferring gear to gear (two rotating objects), the algorithm of Johann and Scheuerle [9] is used. For the transfer of the conjugate points between a rack and a gear, another analytical algorithm is used, implementing the general law of gearing for translational-rotational motion transformation.

3.1 Filtering the surface model

An especially important aspect is the post-processing of the conjugated surfaces. During the conjugation process according to Johann and Scheuerle [9], valid and invalid points are generated. Invalid points occur, for example, when the tip off the tooth is cut off and when undercutting occurs on gears with a small number of teeth. In general, these invalid points can occur at all regions of the geometry, so that an algorithm is used to recognise the corresponding invalid points and then determine the intersection points and the intersection edges between the valid neighbouring partial surfaces. Zimmer uses a filter method in [10] that exclusively uses linear interpolation to find the intersection points in the regular grid. This usually leads to inaccurate intersection points. Furthermore, the discrete point grids have strongly varying distances to each other after conjugation, so that a secant effect occurs.

The filtering process is realised within this paper in five steps:

1. Calculate the distorted grid through the conjugation process.
2. Calculate a regular grid.
3. Find the invalid points.
4. Calculate the intersection points and introduce cut lines.
5. Recalculate the individual parts and merge to a continuous surface.

The result of these five steps is a surface model of one tooth of the gear. There is a continuous transition between the individual partial surfaces as well as to the neighbouring teeth. Figure 4 shows the merged gear surface on the right and corresponding data base on the left. The common rack introduces a uniform point grid, which gets distorted by the conjugation.

Due to this process, gears with highly variable cross sections can be calculated and designed. In particular, the peripheral areas of gears, such as this spherical planet gear, can be involved in the power transmission.

The disadvantage of gears that deviate from the involute shape is that they usually have a transmission error when the position changes due to tolerances and manufacturing errors.

4 Advantages for plastic gears

Detaching gears from the restrictions caused by the manufacturing method leads to technical advantages, especially

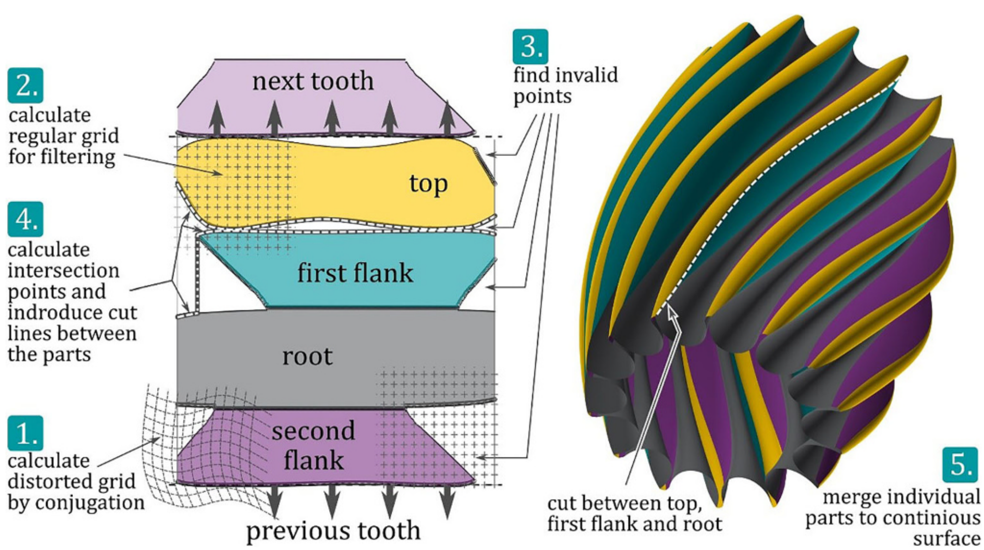


Fig. 4 Steps of the filter process with the gear surface and the different data models

in the development of special gears, as the example in the next section shows. It is initially irrelevant from which material the gears are made.

For the feasibility of a certain toothing, the shape of the base surface can be examined first. Cylindrical and conical gears can probably be easily produced by injection moulding with a single-shell tool. Hyperbolic, globoid, and spherical gears require more effort to manufacture and can usually not be produced with single-shell tools.

The design freedom of the described procedure regarding kinematics and surface definition is further extended in its variety. By implementing a tooth definition, which may have any three-dimensional shape, a design is possible that ensures a constant transmission ratio combined with optimised material utilisation.

Another advantage of plastic moulding is the ability to describe flank modifications generically. Flank modifications are typically set via the tool kinematics or considered in the tool contour when machining with a profiled tool. In general, this leads to the fact that flank modifications in both dimensions of the surface are dependent on each other. Through a generic implementation, arbitrary deviations to the conjugate surface are possible.

5 Example planetary gear (WPK29) with skewed planet

A planetary gear with a skewed planet serves as a practical example, see Fig. 5. The starting point for the development is a two-stage planetary gearbox that was to be replaced due to its non-optimal NVH behaviour. The static transmission ratio i_0 of this new planetary gear is calculated to the value 17 according to Eq. 12.

$$i_0 = -\frac{z_{\text{ring}}}{z_{\text{sun}}} = -\frac{-34}{2} = 17 \tag{12}$$

According to the installation restriction for the number of teeth of the planet gear z_{Planet} for planetary gearboxes with parallel axes, Eq. 13 applies. A deviation from this equation in the case of parallel axes is only possible with the use of a profile offset.

$$z_{\text{planet}} = \frac{-z_{\text{ring}} - z_{\text{sun}}}{2} = 16 \tag{13}$$

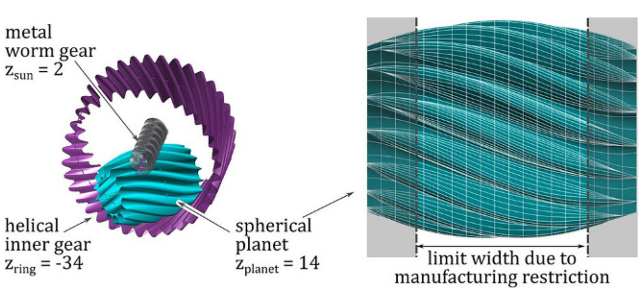


Fig. 5 Example gearing of the skewed axis planetary gear WPK29

5.1 Kinematic calculation

Due to the skewed position of the planet gear, this installation restriction is not necessarily fulfilled any more, as the following description shows. The model from Sect. 2 is used to calculate the kinematic relations. By introducing the parameter *RR*, the radius and the number of teeth no longer must correlate and can be selected independently of each other by means of the parameter *RR*.

Initially, two gear pairs are created that are independent of each other. One gear pair represents the combination of worm gear and planet gear, and the second gear pair represents the combination of planet gear and internal toothed gear. For both gear pairs, the parameter *RR* can be freely selected at first.

Since the planet gear is the same gear, there is another condition that must be considered in the calculation. The planet gear must have an identical helix angle in both gear pairs as well as an identical pitch and radius.

For this purpose, an optimisation is used that merges the two parameters *RR* of the two gear pairs and optimises numerically using the Newton method so that the helix angle of the planet gear is identical in both calculation models.

This eliminates the installation restriction from Eq. 13 due to the skewed position of the planets. In the example shown in this paper, a planet gear with $z_{\text{planet}} = 14$ is used.

5.2 Manufacturing

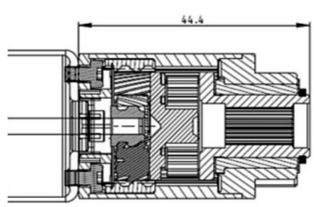
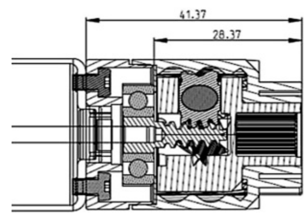
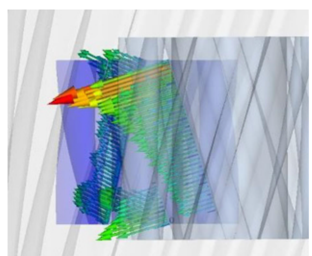
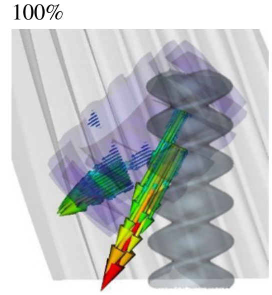
The gearbox is manufactured using materials and processes that are suitable for series production. This paper refers to the development and manufacturing process of the planet gear, but it should be mentioned that the worm gear is made of metal and the internal toothed gear is injection moulded.

An injection moulding process is also used to manufacture the planet gear. The difficulty with this part is the demould ability. To achieve a high surface quality of the tooth flanks, the tooth flank must be produced with a single-shell tool in this area. Due to the spherical shape, there is no demould ability from a purely geometrical point of view.

Two effects support the production with a single-shell tool:

1. Shrinkage of the material.
2. Elastic and damage tolerant behaviour of the material.

Table 1 Properties of WPK29 in comparison to the reference 2 stage PK28LN

Property	PK28LN (2 stage, reference)	WPK29 (1 stage)
Transmission ratio *i*	18.37:1	18:1
Number of planets	3/4 (1st stage/2nd stage)	3
Gear types	Planetary gear with helical gears (1st stage)/ planetary gear (2nd stage)	Skewed axis planetary gear
Efficiency (nominal)	85 ± 3%	75 ± 5%
Efficiency (–40 °C to 80 °C)	72–88%	65–85%
Characteristic frequencies at input frequency 50 Hz	11.67 Hz (output 1st stage) 536.7 Hz (meshing 1st stage) 2.72 Hz (output 2nd stage) 125.2 Hz (meshing 2nd stage)	2.77 Hz (output 1st stage) 94.44 Hz (meshing 1st stage)
Package	100%	93%
		
Number of parts	24 (100%)	16 (67%)
Load capacity (experimental)	150–200%	100%
Tooth contact forces		
Contact ratio	Sun → planet 2nd stage: 1.1	Worm (sun) → planet: 2.0

Furthermore, geometric adjustments of the planet gear are necessary. On the one hand, a lateral cut is applied to an area in which the flank is still sufficiently pronounced. Figure 5 shows that the supporting flank at the edge almost no longer exists. Secondly, the thickness of the teeth is reduced by means of flank modification so that demoulding is improved.

The demoulding itself is realised by axial deforming.

5.3 Properties of WPK29 in comparison to the reference 2 stage PK28LN

The design of WPK29 has both advantages and disadvantages compared to the replaced product. Due to the skewed position of the planetary gear, there is inevitably a sliding component in the contact point. This leads to increased power loss and thus lower efficiency. Due to the use in short-time operation S2 regarding to IEC 60034-1 as a small actuator in highly changing load conditions, this is not critical. Within the specific application the lower efficiency is more of an advantage, as it increases the resistance to backward driving.

Furthermore, axial forces and tilting moments occur, which result in higher stress on the bearing areas. For this reason, a ball bearing was used in the WPK, while in the reference gearbox the axial load can be compensated by the motor bearing. The lower load capacity is not decisive in this application (Table 1).

By replacing two gear stages with a single gear stages, the required packaging is reduced, and fewer parts are needed. The planet carrier is more complex in design, but it is injection moulded and there is only one planet carrier needed, instead of two.

Especially regarding the contact behaviour, a flank modification is used so that the line of contact, which typically occurs in the conjugation process, is changed in such a way that tilting of the planet gear under load does not lead to edge contact. This allows the material to be used in an optimal way.

The main advantage of this type of design is the NVH behaviour, which benefits from the use of plastic as well

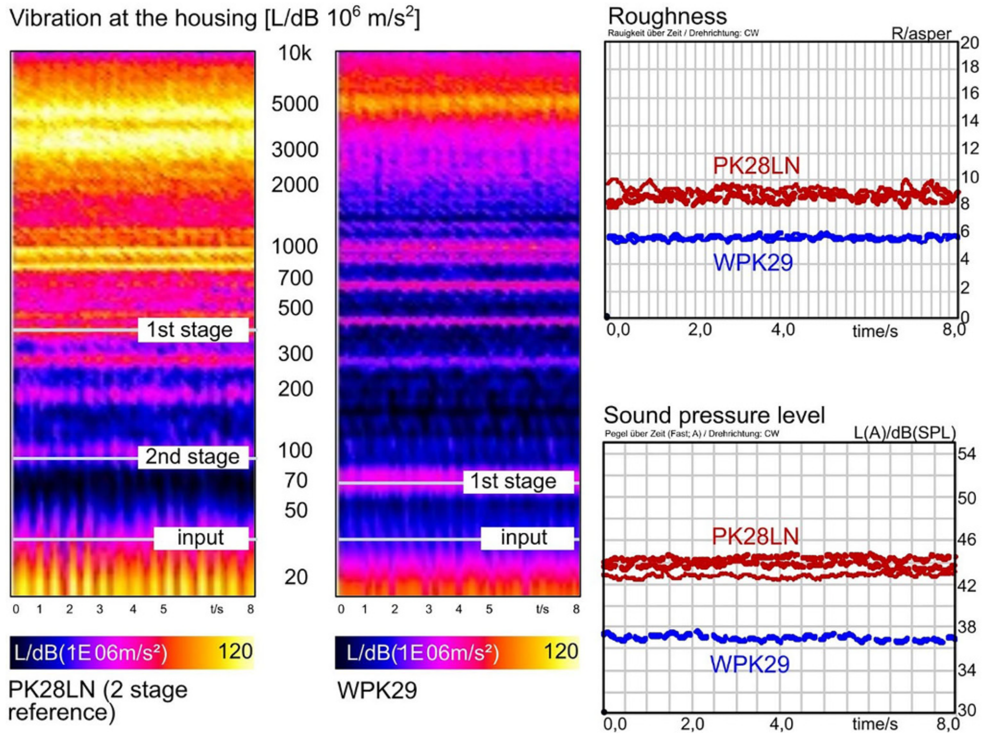


Fig. 6 Comparison of acoustic behaviour of PK28LN and WPK29 in clockwise operation

as from the kinematic and kinetic conditions at the contact points, regarding to Fig. 6. It is shown that the roughness as well as the noise emission and the vibration of the housing can be significantly reduced compared to the reference gearbox PK28LN. The drive frequency of 50 Hz was identical for both gearboxes.

6 Conclusion

The calculation method of conjugate surfaces for the computation of gears is not generally new. By introducing the control parameter radius ratio RR, it is possible to describe a basic model for the calculation of sliding rolling bodies, which can calculate the continuous transition of the conventionally known gear types into each other. Based on two hyperboloids that have a common contact line, the basic bodies of the two gears can be derived. Due to the piecewise differentiable characteristic of relevant gear parameters in dependency of the parameter RR, it is suitable for the algorithmic implementation of the approach.

Besides the description of a kinematic model, the surface calculation of the gearing is important in the context of this paper. By providing a common rack or crown gear, the gear surfaces can be calculated by suitable conjugation methods. With the help of the intersection of the partial surfaces to a continuous tooth contour of arbitrary shape, the actual potential in manufacturing with plastics and the associated manufacturing processes is realised.

The example shown of a planetary gear with a skewed planet gear makes it apparent that special gearing can not only lead to technically more suitable products but can also save resources. Breaking away from entrenched development patterns leads to better and sustainable products for its own application. The conventional manufacturing processes remain but can be replaced at certain points by other processes and thus by more suitable gearings. In a new product development, it is important to define the main properties to be improved, since the example shows that other properties may degrade.

Manufacturing processes such as injection moulding or 3D printing make more complex gear geometries possible, which can avoid possible weak points of conventionally manufactured gears. The calculation method described above can not only be used for plastic gears. Other materials and manufacturing processes are also suitable for realising technically better gears compared to gears produced by machining.

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Author Contribution All authors made substantial contributions to the conception of the work and the creation of the new software used in this work.

Conflict of interest F. Eigner, S. Oberle and M. Berger declare that they have no competing interests.

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