Editorial

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The modern world is multidimensional. Rankings, decisions, choices, etc. are becoming ever more complex, and so is the world of finance. This Special Issue on *Vector- and Set-Valued Methods in Stochastic Finance and Related Areas* has been called for to emphasise these aspects and to attract more attention towards this challenging field.

Multiple objectives typically appear if aggregation to a single objective, for instance by a real-valued pricing functional, is not possible, or if the preference of the decision maker is incomplete and for instance represented by a finite or even infinite number of utility functions. Typical examples for such situations are market models with frictions, the presence of multiple eligible assets instead of a single numéraire, and stochastic dominance orders.

From a mathematical point of view, the major barrier which separates single- and multiple-objective models is the breakdown of all formulas involving an infimum or a supremum. This is due to the fact that only few vector orders generate lattices (for example, those generated by solvency cones in market models with proportional transaction costs usually do not), and even if infima or suprema exist, they are useless for applications because they produce infeasible or unattainable points.

This difficulty is shared by dual representations for risk measures, lower and upper quantiles, as well as value functions and duality results for constrained optimisation problems such as expected utility maximisation and risk/capital allocation. It can be

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overcome by passing to set-valued models whose objectives map into complete lattices of sets, in which infima and suprema are not only well defined, but also have a meaningful financial interpretation. An instructive example is the superhedging theorem in Kabanov [3, Theorem 3.2] which can be seen as a convex duality result for a set-valued function of multivariate contingent claims. The values of that function are the so-called upper sets $A \subseteq \mathbb{R}^d$, i.e., $A + K \subseteq A$, which means that A is stable under addition of a closed convex cone $K \subseteq \mathbb{R}^d$, which is here the solvency cone at time zero. The dual variables are consistent price processes; they correspond to the dual variables produced by recent set-valued duality theories.

This Special Issue collects six papers centering around set-valued models in finance. All submitted papers have undergone the usual rigorous reviewing process, and of course there have been more submissions than those that finally made it to acceptance.

In *Nonlinear expectations of random sets*, Molchanov and Mühlemann define nonlinear set-valued expectations generalising the scalar (real-valued) case which goes back to Peng [4]. As image space, the authors use the lattice of all closed convex subsets of $\mathbb{R}^d$ which are stable under the addition of a fixed closed convex cone. The article also gives dual representation results for sublinear expectations of random sets.

The article *Set-valued risk measures as backward stochastic difference inclusions and equations* by Ararat and Feinstein links set-valued risk measures to set-valued backward stochastic difference equations. This is done in a discrete-time setting which also may prepare for the continuous-time case; see for example Ararat et al. [1] for attempts in that direction.

In the paper *Multi-utility representations of incomplete preferences induced by set-valued risk measures*, Munari directly characterises incomplete risk preferences through a family of scalarising functions for a set-valued risk measure. The focus here is on conditions ensuring the semicontinuity and continuity of the members of that family.

Lattices of upper sets also serve as image spaces for dynamic set-valued risk measures applied to random (portfolio) sets in the contribution *Risk arbitrage and hedging to acceptability under transaction costs* by Lépinette and Molchanov. The approach here can be viewed as an extension of good-deal bounds to incomplete risk preferences modelled via set-valued risk measures.

Set-valued measures of systemic risk mapping into lattices of closed upper sets (and upper Borel sets) are the subject of the article *Elicitability and identifiability of set-valued measures of systemic risk* by Fissler, Hlavínová and Rudloff. Among other things, the paper provides strictly consistent exhaustive scoring functions for such risk measures and illustrates how to use them in a practical simulation study.

Finally, Grépat and Kabanov provide in the article *On a multi-asset version of the Kusuoka limit theorem of option superreplication under transaction costs* a Kusuoka-type limit theorem for sets of superreplicating portfolios within the framework of physical delivery. This means that a numéraire is not necessarily present.

Overall, the papers in this Special Issue show that many even technically quite demanding concepts and results can be lifted from the scalar setting to the set-valued case through the use of complete lattices of sets as image sets.

The range of applications for set-valued models in finance is very wide, and this Special Issue is far from providing a complete overview. However, we think that it

gives a good impression of some possible directions, and we hope it will stimulate more research in the future. Looking back, the Special Issue can also be seen as an intermediate milestone in a development which started in *Finance and Stochastics* with the results of Kabanov [3] on transaction costs and includes set-valued risk measures as introduced in [2] by Jouini, Meddeb and Touzi.

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Andreas H. Hamel, Guest Editor for the *Special Issue on Vector- and Set-Valued Methods in Stochastic Finance and Related Areas*

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