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ERRATUM

Erratum to: Asset price bubbles from heterogeneous beliefs about mean reversion rates

Xi Chen · Robert V. Kohn

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Theorem 5.1 of [1] draws correct conclusions, however the proof is incomplete. Indeed, the final paragraph appeals to a "basic comparison theorem for viscosity super- and subsolutions, see e.g. Theorem 5.1 of [2]." Alas, the cited result from [2] concerns equations of the form $u(x) + F(Du, D^2u) - f(x) = 0$, whereas the equation under consideration in [1] does not have this form.

The purpose of that final paragraph was to conclude that $\Phi(D) \le P_*(D)$, where P_* is the minimal equilibrium price and Φ is the unique C^2 solution of

$$-\max\{\kappa_1(\theta - D), \kappa_2(\theta - D)\}\Phi' - \frac{1}{2}\sigma^2\Phi'' + \lambda\Phi - D = 0 \tag{1.1}$$

with linear growth at infinity. Actually, appeal to a general comparison result is unnecessary. The desired conclusion follows easily from the fact that $P_*(D)$ is a viscosity supersolution, using the asymptotic properties of $P_*(D)$ and $\Phi(D)$ as $|D| \to \infty$. Thus Theorem 5.1 of [1] can be replaced with the following:

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X. Chen · R.V. Kohn (✉)
Courant Institute of Mathematical Sciences, New York University, New York, USA
e-mail: kohn@cims.nyu.edu

Present address:
X. Chen
Royal Bank of Canada, Toronto, Ontario, Canada
e-mail: roger.chen@rbccm.com

Theorem 5.1 *The equilibrium price $\Phi(D)$ identified in Sect.* 4 *and the minimal equilibrium price $P_*(D)$ discussed in Sect.* 3 *have the following properties*:

(i) $P_*(D) \le \Phi(D)$, *and* $\Phi(D) - P_*(D) \to 0$ *as* $|D| \to \infty$.
(ii) $P_*(D)$ *is a lower semicontinuous function*.
(iii) $P_*(D)$ *is a viscosity supersolution of* (1.1).

Furthermore, *assertions* (i)–(iii) *imply*

$$\Phi(D) \le P_*(D), \tag{1.2}$$

so $\Phi = P_*$. Thus, *the unique classical solution of the differential equation* (1.1) *with linear growth at infinity is in fact the minimal equilibrium price*.

Proof The assertion $P_*(D) \le \Phi(D)$ is obvious, since Φ is an equilibrium price and P_* is the minimal equilibrium price. We also know $P_*(D) \ge I(D)$, where

$$I(D) = \begin{cases} \frac{D}{\lambda+\kappa_1} + \frac{\theta\kappa_1}{\lambda(\lambda+\kappa_1)}, & \text{if } D \le \theta, \\ \frac{D}{\lambda+\kappa_2} + \frac{\theta\kappa_2}{\lambda(\lambda+\kappa_2)}, & \text{if } D \ge \theta \end{cases}$$

is the intrinsic value (cf. (2.3) of [1]), since the definition of an equilibrium price (Definition 2.1 of [1]) includes this inequality. Theorem 4.1(b) of [1] shows that $\Phi(D) - I(D) \to 0$ as $|D| \to \infty$. This gives (i), since $\Phi(D) - P_*(D) \le \Phi(D) - I(D)$.

Assertions (ii) and (iii) are stated and proved in Theorem 5.1 of [1].

For the final conclusion (1.2), consider the variational problem

$$\inf_{D\in\mathbb{R}} \{P_*(D) - \Phi(D)\}.$$

If a minimizing sequence tends to $\pm\infty$ then the minimum value is 0 by (i), and (1.2) is true. If on the other hand a minimizing sequence stays bounded, then the minimum is achieved at some D_1, by (ii). Since P_* is a viscosity supersolution we have

$$-\max\{\kappa_1(\theta - D_1), \kappa_2(\theta - D_1)\}\Phi'(D_1) - \frac{1}{2}\sigma^2\Phi''(D_1) + \lambda P_*(D_1) - D_1 \ge 0.$$

It follows since Φ solves (1.1) that

$$-\lambda\Phi(D_1) + \lambda P_*(D_1) \ge 0.$$

Since the discount rate λ is positive, we conclude that $P_*(D_1) - \Phi(D_1) \ge 0$. Thus $P_*(D) - \Phi(D) \ge P_*(D_1) - \Phi(D_1) \ge 0$, completing the proof of (1.2). □

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References

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