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Even imprudent risk lovers may engage in precautionary saving

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Abstract
Recent developments in dynamic consumption theory have shown that risk-loving agents, much like their risk-averse analogues, can exhibit downside risk aversion (prudence) and thus demand precautionary savings. I complement this finding by showing that risk-seeking preferences also magnify the role of natural borrowing limits in shaping consumers' behavior, causing risk lovers to increase savings against income uncertainty in cases where risk averters would not: even imprudent risk lovers may engage in precautionary saving.

Keywords Risk lovers · Precautionary saving · Prudence

JEL Classification D11 · D81 · E21

1 Introduction

Since the seminal work of Leland (1968) and Sandmo (1970), the theory of precautionary saving in response to income uncertainty has gained central stage in the economic analysis of choices involving risk. A well-known feature of precautionary saving behavior lies in its characterization in terms of the convexity of the marginal utility function or *prudence*, in the context of the conventional expected utility framework – see Rothschild and Stiglitz (1971) and Kimball (1990).[1]

[1] The theoretical literature on precautionary saving is definitely too broad to be adequately reviewed here. The relationship between the source(s) of risk, the risk attitudes of expected utility maximizers and their incentive to engage in precautionary saving has been extensively framed in dynamic models to examine, among other issues, the role of multiple risks (e.g. Li 2012; Baiardi et al. 2014), relative prudence and interest rate risk (e.g. Magnani 2017), higher-order changes in interest rate risk (e.g. Wong 2019), nonlinear risk effects (e.g. Bonilla and Vergara 2022). To gain further insights on the many lines of research on the topic under scrutiny, I would like to refer the reader to the excellent survey by Baiardi et al. (2020).

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In a basic model setup, Menegatti (2007) establishes an equivalence result between the existence of excess (precautionary) saving and the desire of *risk-averse* agents to reduce the disutility (*pain*) from income uncertainty. Addressing the issue of linking the precautionary saving motive to the saver's attitude toward risk without advocating conditions on the third derivative of the utility function, Menegatti (2007) argues that precautionary saving behavior emerges because risk aversion entails a loss in expected utility due to uncertainty – measured by the so-called *utility premium* first introduced in Friedman and Savage (1948) – and risk-averse agents strongly prefer to curtail this loss by saving more relative to the certainty case. Eeckhoudt and Schlesinger (2009) further explore the relevance of the utility premium for decision-making under multiplicative (e.g. interest rate) risk.

In this note, I first complement the analysis in Menegatti (2007) and Eeckhoudt and Schlesinger (2009) by showing that the very same incentive operates for prudent *risk lovers*, whose preference for combining good with good (e.g. Crainich et al. 2013) can induce them to increase savings *vis-à-vis* the certainty scenario, in order to feed the utility premium associated with risk exposure. Intuitively, since risk-lovers seek to gather consumption opportunities in a single period rather than smoothing them over time, they will save against income risk in order to achieve the largest possible utility premium that enters positively their utility function. I then prove, and this is all the more remarkable, that the equivalent characterizations of risk preferences at third order by means of prudence or decreasing utility premium do not qualify as a universal trait of saving behavior across risk attitudes: risk-seeking in fact magnifies the role of natural borrowing limits in shaping consumers' behavior, causing risk lovers to demand precautionary saving in cases where risk averters would not.

The key mechanism at work here is the effect of risk on borrowing opportunities. The literature on consumption theory under income uncertainty has long established that liquidity constraints can produce a positive demand for precautionary saving under very general circumstances. Deaton (1991) was the first to emphasize the possibility that risk-averse and prudent consumers face explicit liquidity constraints. In his relatively simple model, limited borrowing opportunities interact with the standard precautionary motive for saving – i.e. the desire to self-insure against income risk – because even impatient consumers may find it optimal to accumulate assets when times are good, a buffer stock that shields consumption from future income downturns. Remarkably, liquidity constraints can cause precautionary savings to occur even if preferences exhibit linear marginal utility from consumption (i.e. in the absence of prudence), see e.g. Besley (1995).

Xu (1995) studies the effects of liquidity constraints on saving behavior in lifecycle models by disentangling *precautionary savings* driven by liquidity constraints from the conventional *precautionary savings* as a self-insurance device. Conditions for existence of liquidity constrained-driven demand for precautionary savings are then identified and their implications for the evaluation of the effects of policy reform discussed. In a rather general setting, Carroll et al. (2021) formally establish that liquidity constraints and income risk both entail a concavification of the optimal consumption policy of risk-averse agents facing uncertainty, which in turn intensifies the prudence of the value function. As a result, liquidity constraints and risk reinforce the precautionary saving motive and have similar effects on intertemporal consumption/saving choices. More generally, constraints on borrowing opportunities may complement standard precautionary motives for saving, and thereby affect optimal consumption behavior in models of intertemporal choice involving risk – see e.g. Jappelli and Pistaferri (2017).

I contribute to this literature – in which risk aversion is the fundamental working assumption – by considering model environments where (i) agents exhibit *risk-loving preferences*, and where (ii) the standard precautionary motive for saving is not operative (*imprudence*). I formally show that, even in cases where explicit liquidity constraints are absent, the occurrence of natural borrowing limits (which stem from non-negativity constraints on consumption, see e.g. Aiyagari 1994) can stimulate additional savings *vis-à-vis* the certainty case when agents are *imprudent risk lovers*, while failing to do so when agents are *prudent risk averters*. Intuitively, a natural borrowing limit distorts the saving incentives of agents who wish to borrow against future income, while net savers are not affected. To achieve their consumption gathering goal, risk-seeking agents may then find it optimal to borrow against their future income in the certainty scenario, and to carry instead all of their current resources onto the future under income uncertainty.[2]

The remainder of the paper proceeds as follows. Section 2 presents a simple two-period framework with income uncertainty and a few preliminary definitions, that will jointly allow me to analyze the optimal saving behavior of risk lovers. Section 3 offers some concluding remarks.

2 Risk loving and the (dis)utility from uncertainty

2.1 A simple model

As a vehicle for comparing optimal saving choices with and without income uncertainty, I consider a simple two-period model in which an expected utility maximizer (or consumer), whose preferences are described by a thrice differentiable, strictly increasing utility function $U(\cdot)$, engages in intertemporal consumption choices given her income in period 0 ($y_0 > 0$) and in period 1 ($y_1 > 0$). To narrow down the effects of income uncertainty on saving behavior, I posit that the market interest rate and the subjective rate of time preference are both equal to zero.[3] When income levels y_0 and y_1 are deterministic and known to the consumer, the problem is in the form

[2] Carroll (1997) emphasizes the possibility that future (uncertain) income can fall to zero with positive probability to generate a natural borrowing limit, which fully discourages consumers from borrowing; as a result, the optimal saving strategy of risk-averse agents resembles that of a model with explicit liquidity constraints.

[3] While these simplifying assumptions restrict the set of models to which my analysis applies, they are conventionally adopted in the literature dealing with precautionary saving behavior (see e.g. Kimball 1990; Gollier 2001), and can definitely be relaxed.

$$\max_s \; U(y_0 - s) + U(y_1 + s) \tag{1}$$

provided consumption in both periods is non-negative, i.e. $c_0 = y_0 - s \geq 0$ and $c_1 = y_1 + s \geq 0$.

By contrast, when the second period income $\tilde{y}_1$ is uncertain, e.g. it is a random variable with finite first moment $\mathbb{E}[\tilde{y}_1] = y_1$ and finite variance, then the problem reads as

$$\max_s \; U(y_0 - s) + \mathbb{E}\big[U(\tilde{y}_1 + s)\big] \tag{2}$$

provided $c_0 \geq 0, c_1 \geq 0$. Here $\mathbb{E}[\cdot]$ denotes the (statistical) expectation operator. Without loss of generality, I henceforth set $\tilde{y}_1 = y_1 + \tilde{\epsilon}$, where $\tilde{\epsilon}$ is a zero mean random variable whose compact support $[\underline{\epsilon}, \overline{\epsilon}]$ is defined so that $y_1 + \underline{\epsilon} \geq 0$ and the realized future income $y_1 + \epsilon$ lies in the domain of $U(\cdot)$.[4]

2.2 Risk attitudes and borrowing constraints

Before turning to the conditions for precautionary savings, I will use the simple model under certainty (1) in order to shed light on the role of borrowing constraints in shaping optimal saving behavior across risk attitudes. In this respect, notice that the requirement of non-negative consumption entails a natural borrowing limit $s \geq -y_1$, whereas positive saving cannot exceed the current income y_0. For the purpose of the analysis, assume further that $y_0 > y_1$, and that the consumer is bound to face a more stringent *liquidity constraint* of the form $s \geq 0$ (i.e. borrowing is not allowed).

As is well-known, if $U''(\cdot) < 0$ (risk aversion) optimal savings s^* satisfy the Kuhn-Tucker conditions

$$U'(y_0 - s^*) + \mu = U'(y_1 + s^*) + \lambda \tag{3}$$

$$s^* \geq 0, \quad y_0 - s^* \geq 0 \tag{4}$$

$$\lambda \cdot s^* = 0, \quad \mu \cdot [y_0 - s^*] = 0 \tag{5}$$

where $\lambda \geq 0$ and $\mu \geq 0$ are the multipliers associated with the relevant constraints. The above conditions clearly deliver the unique interior solution $s^* = (y_0 - y_1)/2 \in (0, y_0)$: the inability of risk-averse consumers to borrow against future income – as enforced by the liquidity constraint $s \geq 0$ – does not affect their optimal saving plan, who reflects a desire for consumption smoothing.

Let now $U(\cdot)'' > 0$ (risk loving). By continuity and strict convexity of the objective function, and its domain being a compact and convex set (whether or not the liquidity constraint is imposed), Bauer's maximum principle (Bauer 1958)

[4] Notice that $\underline{\epsilon} < 0$ for $\mathbb{E}[\tilde{\epsilon}] = 0$ to hold true.

guarantees that the maximum is attained at the extreme points of such set. Notice that, absent the liquidity constraint, both $s^* = y_0$ and $s^* = -y_1$ would deliver the same total utility $U(0) + U(y_0 + y_1)$, implying non-uniqueness of the solution of the utility maximization problem. The presence of the liquidity constraint $s \geq 0$ can change the picture: zero current consumption and positive savings ($s^* = y_0$) may be strictly preferred by risk-loving consumers who seek to concentrate consumption opportunities in a single period rather than smooth them over time. This is the case, e.g., when $U(0) = 0$ (and the maintained assumption that $y_0 > y_1$), since the convexity of the utility function $U(\cdot)$ implies the latter is super-additive, i.e. $U(y_0 + y_1) > U(y_0) + U(y_1)$. Under these circumstances, the inability of risk-seeking consumers to borrow against future income overturns their saving incentives by stimulating them to save all of their current income so as to favor future consumption (consumption gathering).

When future income is uncertain, the presence of the *natural borrowing limit*, rather than explicit liquidity constraint, is able to induce excess sensitivity of consumption to the precautionary effect of uncertain income for risk lovers but not for risk averters, as discussed next.

2.3 Prudence, utility premium and precautionary saving

Let s^* solve the utility maximization problem (1), and $\hat{s}$ solve the expected utility maximization problem (2). Without a priori constraining the sign of $U'''(\cdot)$, Menegatti (2007) shows that, if $U''(\cdot) < 0$ – i.e. if the consumer is risk-averse – then positive precautionary savings are obtained ($\hat{s} > s^*$) if and only if the function

$$\pi\left(y_1 + \hat{s};\tilde{\epsilon}\right) = U(y_1 + \hat{s}) - \mathbb{E}\left[U(y_1 + \hat{s} + \tilde{\epsilon})\right] \tag{6}$$

is monotonically decreasing in its domain. This function, known as *utility premium*, measures the pain from risk induced by the loss in expected future utility incurred by the consumer when her future income y_1 is uncertain.[5]

This result, whose proof (Menegatti, 2007, p. 278) explicitly uses the strict concavity of $U(\cdot)$, appears to suggest that precautionary savings are associated with a specific feature of the consumer's attitude toward risk, namely the desire to curtail the disutility from uncertainty, which is in turn generated by risk aversion. However, as pointed out in Crainich et al. (2013), even risk-loving consumers can be prudent – and thus engage in precautionary behavior – for they can express the same preference at third order (downside risk aversion) as risk averters'.[6]

[5] Notice that Menegatti (2007, p. 277) inadvertently defines $\pi(\cdot)$ as being equal to $\mathbb{E}\left[U(\tilde{y}_1 + \hat{s})\right] - U(\mathbb{E}[\tilde{y}_1] + \hat{s})$, claiming that the strict concavity of $U(\cdot)$ ensures that this latter quantity is positive by virtue of Jensen's inequality. The opposite is actually true: $\mathbb{E}\left[U(\tilde{y}_1 + \hat{s})\right] - U(\mathbb{E}[\tilde{y}_1] + \hat{s})$ would be negative if $U''(\cdot) < 0$; a similar remark applies when showing that $\pi'(\cdot) < 0$ is satisfied if and only if $U'''(\cdot) > 0$ (Menegatti, 2007, p. 280). Using the correct definition of utility premium given in (6), the analytical results in Menegatti (2007) retain full validity.

[6] Menegatti (2001) shows that, provided $U'''(x)$ does not change sign over the domain $[0, \infty)$, then local non-satiation ($U'(x) > 0$) and risk-aversion ($U''(x) < 0$) for every $x \geq 0$ are jointly sufficient for prudence ($U'''(x) > 0$) to obtain. By its very nature, this condition does not prevent risk-lover consumers

A natural question is then whether prudence – or, equivalently, a decreasing utility premium – stands as a *necessary condition* for precautionary saving behavior to emerge in the risk-loving scenario. Notice that the potential for multiple (corner) solutions requires caution in defining precautionary savings in dynamic choice problems involving risk-seeking consumers. Within this setting, the question whether income uncertainty stimulates precautionary saving behavior is to be framed as follows: under what conditions, if any, risk lovers facing future income uncertainty *never choose to curtail* their optimal savings relative to the certainty case? In the context of the model of Sect. 2, I complement Menegatti (2007)'s and Crainich et al. (2013)'s insights by showing that prudence is not required, even when risk-seeking consumers face no explicit liquidity constraints. Formally

Proposition 1 *Consider problems* (1) *and* (2), *and assume* $U'(\cdot) > 0$ (*non-satiation*) *and* $U''(\cdot) > 0$ (*risk-loving*). *Then* $\pi'(\cdot) < 0$ *is not necessary for* $\hat{s} \geq s^*$.

Proof Consider first the certainty case (1). As mentioned in the previous Section, a non-unique corner solution will emerge, i.e. $s^* \in \{-y_1, y_0\}$.

Consider now the income uncertainty case (2). Since there is a positive probability that the consumer is hit by her worst possible income shock in the second period, the natural borrowing limit is $s \geq -(y_1 + \underline{\epsilon})$. Again by virtue of Bauer's maximum principle, either $\hat{s} = -(y_1 + \underline{\epsilon})$ or $\hat{s} = y_0$. The latter occurs if and only if

$$U(0) + \mathbb{E}\left[U(y_0 + y_1 + \tilde{\epsilon})\right] > U(y_0 + y_1 + \underline{\epsilon}) + \mathbb{E}\left[U(\tilde{\epsilon} - \underline{\epsilon})\right] \tag{7}$$

or equivalently

$$\mathbb{E}\left[U\left(y_0 + y_1 + \underline{\epsilon} + \tilde{\theta}\right)\right] - \mathbb{E}\left[U\left(\tilde{\theta}\right)\right] > U\left(y_0 + y_1 + \underline{\epsilon}\right) - U(0) \tag{8}$$

where $\tilde{\theta} = \tilde{\epsilon} - \underline{\epsilon}$. Using the definition of the utility premium from equation (6), the latter inequality can be rewritten as

$$U(y_0 + y_1) - \pi\left(y_0 + y_1 + \underline{\epsilon};\tilde{\theta}\right) - U(-\underline{\epsilon}) + \pi\left(0;\tilde{\theta}\right) > U\left(y_0 + y_1 + \underline{\epsilon}\right) - U(0) \tag{9}$$

Since $U(\cdot)$ is strictly convex, we have

$$U(y_0 + y_1) - U\left(y_0 + y_1 + \underline{\epsilon}\right) > U(-\underline{\epsilon}) - U(0) \tag{10}$$

and thus we can have $\hat{s} = y_0$ even in cases where $\pi\left(0;\tilde{\theta}\right) \leq \pi\left(y_0 + y_1 + \underline{\epsilon};\tilde{\theta}\right)$, i.e. when $\pi'(\cdot) < 0$ does not hold everywhere in its domain.[7] □

Footnote 6 (continued)
($U''(x) > 0$) to also exhibit prudence ($U'''(x) > 0$). However, when $U(\cdot)'$ is strictly positive and bounded, then risk aversion is a necessary condition for prudence to occur, i.e. risk lovers must be imprudent (Menegatti 2014).

[7] Alternatively, and keeping in mind that $\pi'(\cdot) < 0$ if and only if $U'''(\cdot) > 0$, the inequality (8) is satisfied if and only if $\mathbb{E}[U'(x + \tilde{\theta})] > U'(x)$ for all x; by virtue of $U''(\cdot) > 0$ one has $U'(\mathbb{E}[x + \tilde{\theta}]) > U'(x)$. By Jensen's inequality, $U'''(\cdot) > 0$ implies $\mathbb{E}[U'(x + \tilde{\theta})] > U'(\mathbb{E}[x + \tilde{\theta}])$, showing again that $\pi'(\cdot) < 0$ is not required for non-negative precautionary savings.

As pointed out by Ebert (2013), *not all* risk lovers are prudent. The result above shows that *imprudent risk lovers* may still decide to accumulate precautionary savings as long as risk seeking embodies a preference for gathering consumption opportunities in a single period of time. Different from the case of risk aversion, a strictly decreasing utility premium (or equivalently, a strictly positive third derivative of the utility function) therefore fails to characterize the saving behavior of risk lovers even in the absence of other frictions (e.g. liquidity constraints) that would entail a rise in savings in the face of future income uncertainty. Of course, there exist situations where such a condition proves necessary for precautionary saving to arise in economies populated by risk lovers; the simple setting studied in Crainich et al. (2013), where $y_1 = 0$ (so that $\mathbb{E}[\tilde{y}_1] = 0$) and $s \geq 0$ (ad hoc liquidity constraint), is a case in point.

More generally, given the structure of the underlying problem, the emergence of precautionary saving behavior on the part of risk-loving consumers can be fully characterized in terms of *bounded average rates of change* – rather than signed marginal ones – of the utility premium. Formally, for the model under scrutiny, define $h := y_0 + y_1 + \underline{\epsilon}$ and let

$$\Delta_h[\pi](y_1 + \hat{s};\tilde{\theta}) = \pi(h;\tilde{\theta}) - \pi(0;\tilde{\theta}) \tag{11}$$

denote the forward difference of the function $\pi(\cdot)$, where the spacing h is strictly positive. I can then state the following

Proposition 2 *Consider problems* (1) *and* (2), *and assume* $U'(\cdot) > 0$ (*non-satiation*) *and* $U''(\cdot) > 0$ (*risk-loving*). *Then there exists a positive threshold* $\bar{\Delta}$ *such that* $\hat{s} \geq s^*$ *if and only if*

$$\frac{\Delta_h[\pi](y_1 + \hat{s};\tilde{\theta})}{h} < \bar{\Delta} \tag{12}$$

Proof Define $\bar{\Delta} := h^{-1}\left[U(h - \underline{\epsilon}) + U(0) - U(h) - U(-\underline{\epsilon})\right]$, which is strictly positive due to $U''(\cdot) > 0$. Then the inequality in (12) is equivalent to the characterization in (9), and the assertion follows.

Clearly, the necessary and sufficient condition in (12) allows for non-monotonic behavior of the utility premium $\pi(\cdot)$, making prudence ($\pi'(\cdot) < 0$) a relatively stronger requirement. Notice however that the threshold $\bar{\Delta}$ is model-specific – it depends on the actual degree of convexity of the utility function and its behavior at zero, the current income level, the average future income and the worst future income draw. This suggests that the existence of non-negative precautionary savings for risk-loving consumers cannot be exclusively framed in terms of their attitude toward downside risk.

3 Concluding remarks

Crainich et al. (2013) establish that, under risk loving, prudence entails non-negative precautionary savings – that is, savings in the presence of certainty are not larger than in the presence of (income) uncertainty. The present manuscript shows that a further mechanism stimulating precautionary saving behavior, based on natural restrictions on borrowing opportunities, is particularly significant under risk seeking preferences, since it makes prudence (or equivalently a decreasing utility premium) not necessary for saving to increase under uncertainty. This conclusion complements that obtained by Menegatti (2007) in the traditional case of risk aversion, showing the differences obtained in the case of risk seeking, analyzed by Crainich et al. (2013).

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