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CORRECTION

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Correction to: Analytical and numerical techniques for damage localization of rectangular plates using higher-order moments of inertia

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In the original publication, few errors were noted and are corrected in this correction.

In Section 6, under heading "Concluding remarks," the last two paragraphs should be correctly read as:

Future works can include the damage detection of the rectangular plate with holes of arbitrary shapes with smooth boundaries. Also, some examples of computing the higher-order moments of inertia for the plates weakened by some holes can be presented using the experimental data. The results can be validated by available elasticity solutions.

To identify human faces in images or videos, or search for a face among an extensive collection of existing images, some facial features are identified and measured. Similarly, as we saw in this paper, there are some damage detector features, namely higher-order moments of inertia which are used for identifying the actual shape and location of a damage or a set of damages. Machine learning is a rapidly growing field and has been used for various tasks, such as facial recognition. Therefore, using machine learning algorithms to identify the shape and locations of damage or a set of damages in a plate via the measured higher-order moments of inertial is of particular interest for future work.

Equation 15 should be read as:

$$
\begin{aligned}
M_{1,0}^{t} &= \frac{x_0 + x_1 + x_2}{3} M_{0,0}^{1} + \frac{x_2' + x_1' + x_0'}{3} M_{0,0}^{2} \\
M_{0,1}^{t} &= \frac{y_0 + y_1 + y_2}{3} M_{0,0}^{1} + \frac{y_2' + y_1' + y_0'}{3} M_{0,0}^{2} \\
M_{2,0}^{t} &= \frac{x_0^2 + x_1^2 + x_2^2 + x_1 x_2 + x_0 x_2 + x_0 x_1}{6} M_{0,0}^{1} \\
&\quad + \frac{x_0'^2 + x_1'^2 + x_2'^2 + x_1' x_2' + x_0' x_2' + x_0' x_1'}{6} M_{0,0}^{2} \\
M_{0,2}^{t} &= \frac{y_0^2 + y_1^2 + y_2^2 + y_1 y_2 + y_0 y_2 + y_0 y_1}{6} M_{0,0}^{1}
\end{aligned}
$$

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$$
\begin{aligned}
&+ \frac{y_0'^2 + y_1'^2 + y_2'^2 + y_1'y_2' + y_0'y_2' + y_0'y_1'}{6} M_{0,0}^2 \\
M_{3,0}^t &= \frac{1}{10}\Big[x_0^3 + x_1^3 + x_2^3 + x_0^2(x_1 + x_2) + x_1^2(x_0 + x_2) + x_2^2(x_0 + x_1) \\
&\quad + x_0x_1x_2\Big]M_{00}^1 + \frac{1}{10}\Big[x_0'^3 + x_1'^3 + x_2'^3 + x_0'^2\big(x_1' + x_2'\big) + x_1'^2\big(x_0' + x_2'\big) \\
&\quad + x_2'^2\big(x_0' + x_1'\big) + x_0'x_1'x_2'\Big]M_{0,0}^2 \\
M_{0,3}^t &= \frac{1}{10}\Big[y_0^3 + y_1^3 + y_2^3 + y_0^2(y_1 + y_2) + y_1^2(y_0 + y_2) + y_2^2(y_0 + y_1) \\
&\quad + y_0y_1y_2\Big]M_{00}^1 + \frac{1}{10}\Big[y_0'^3 + y_1'^3 + y_2'^3 + y_0'^2\big(y_1' + y_2'\big) + y_1'^2\big(y_0' + y_2'\big) \\
&\quad + y_2'^2\big(y_0' + y_1'\big) + y_0'y_1'y_2'\Big]M_{0,0}^2 \\
M_{4,0}^t &= \frac{1}{15}\Big[x_0^4 + x_1^4 + x_2^4 + x_0^3(x_1 + x_2) + x_1^3(x_0 + x_2) + x_2^3(x_0 + x_1) \\
&\quad + x_0x_1x_2(x_0 + x_1 + x_2) + x_0^2x_1^2 + x_1^2x_2^2 + x_0^2x_2^2\Big]M_{0,0}^1 \\
&\quad + \frac{1}{15}\Big[x_0'^4 + x_1'^4 + x_2'^4 + x_0'^3\big(x_1' + x_2'\big) + x_1'^3\big(x_0' + x_2'\big) + x_2'^3\big(x_0' + x_1'\big) \\
&\quad + x_0'x_1'x_2'\big(x_0' + x_1' + x_2'\big) + x_0'^2x_1'^2 + x_1'^2x_2'^2 + x_0'^2x_2'^2\Big]M_{0,0}^2 \\
M_{0,4}^t &= \frac{1}{15}\Big[y_0^4 + y_1^4 + y_2^4 + y_0^3(y_1 + y_2) + y_1^3(y_0 + y_2) + y_2^3(y_0 + y_1) \\
&\quad + y_0y_1y_2(y_0 + y_1 + y_2) + y_0^2y_1^2 + y_1^2y_2^2 + y_0^2y_2^2\Big]M_{0,0}^1 \\
&\quad + \frac{1}{15}\Big[y_0'^4 + y_1'^4 + y_2'^4 + y_0'^3\big(y_1' + y_2'\big) + y_1'^3\big(y_0' + y_2'\big) + y_2'^3\big(y_0' + y_1'\big) \\
&\quad + y_0'y_1'y_2'\big(y_0' + y_1' + y_2'\big) + y_0'^2y_1'^2 + y_1'^2y_2'^2 + y_0'^2y_2'^2\Big]M_{0,0}^2 \\
M_{5,0}^t &= \frac{1}{21}\Big[x_0^5 + x_1^5 + x_2^5 + x_0^4(x_1 + x_2) + x_1^4(x_0 + x_2) + x_2^4(x_0 + x_1) \\
&\quad + x_0^3\big(x_1^2 + x_1x_2 + x_2^2\big) + x_1^3\big(x_0^2 + x_0x_2 + x_2^2\big) + x_2^3\big(x_0^2 + x_0x_1 + x_1^2\big) \\
&\quad + x_0x_1x_2(x_1x_2 + x_0x_2 + x_0x_1)\Big]M_{0,0}^1 + \frac{1}{21}\Big[x_0'^5 + x_1'^5 + x_2'^5 + x_0'^4\big(x_1' + x_2'\big) \\
&\quad + x_1'^4\big(x_0' + x_2'\big) + x_2'^4\big(x_0' + x_1'\big) + x_0'^3\big(x_1'^2 + x_1'x_2' + x_2'^2\big) \\
&\quad + x_1'^3\big(x_0'^2 + x_0'x_2' + x_2'^2\big) + x_2'^3\big(x_0'^2 + x_0'x_1' + x_1'^2\big) \\
&\quad + x_0'x_1'x_2'\big(x_1'x_2' + x_0'x_2' + x_0'x_1'\big)\Big]M_{0,0}^2 \\
M_{0,5}^t &= \frac{1}{21}\Big[y_0^5 + y_1^5 + y_2^5 + y_0^4(y_1 + y_2) + y_1^4(y_0 + y_2) + y_2^4(y_0 + y_1) \\
&\quad + y_0^3\big(y_1^2 + y_1y_2 + y_2^2\big) + y_1^3\big(y_0^2 + y_0y_2 + y_2^2\big) + y_2^3\big(y_0^2 + y_0y_1 + y_1^2\big) \\
&\quad + y_0y_1y_2(y_1y_2 + y_0y_2 + y_0y_1)\Big]M_{0,0}^1 + \frac{1}{21}\Big[y_0'^5 + y_1'^5 + y_2'^5 + y_0'^4\big(y_1' + y_2'\big) \\
&\quad + y_1'^4\big(y_0' + y_2'\big) + y_2'^4\big(y_0' + y_1'\big) + y_0'^3\big(y_1'^2 + y_1'y_2' + y_2'^2\big) \\
&\quad + y_1'^3\big(y_0'^2 + y_0'y_2' + y_2'^2\big) + y_2'^3\big(y_0'^2 + y_0'y_1' + y_1'^2\big) \\
&\quad + y_0'y_1'y_2'\big(y_1'y_2' + y_0'y_2' + y_0'y_1'\big)\Big]M_{0,0}^2 \\
M_{6,0}^t &= \frac{1}{28}\Big[x_0^6 + x_1^6 + x_2^6 + x_0^5(x_1 + x_2) + x_1^5(x_0 + x_2) + x_2^5(x_0 + x_1)
\end{aligned}
$$

$$
\begin{aligned}
&+ x_0x_1x_2(x_0+x_1+x_2)\big(x_0^2+x_1^2+x_2^2\big) + \big(x_0^2+x_1^2\big)x_0^2x_1^2\\
&+ x_0^2x_2^2\big(x_0^2+x_2^2\big) + x_1^2x_2^2\big(x_1^2+x_2^2\big) + x_0^3x_1^3 + x_1^3x_2^3 + x_0^3x_2^3 + x_0^2x_1^2x_2^2\Big]M_{0,0}^1\\
&+ \frac{1}{28}\Big[x_0'^6 + x_1'^6 + x_2'^6 + x_0'^5\big(x_1'+x_2'\big) + x_1'^5\big(x_0'+x_2'\big) + x_2'^5\big(x_0'+x_1'\big)\\
&+ x_0'x_1'x_2'\big(x_2'+x_1'+x_0'\big)\big(x_0'^2+x_1'^2+x_2'^2\big) + \big(x_0'^2+x_1'^2\big)x_0'^2x_1'^2\\
&+ x_0'^2x_2'^2\big(x_0'^2+x_2'^2\big) + x_1'^2x_2'^2\big(x_1'^2+x_2'^2\big) + x_0'^3x_1'^3 + x_1'^3x_2'^3 + x_0'^3x_2'^3\\
&+ x_0'^2x_1'^2x_2'^2\Big]M_{0,0}^2\\
M_{0,6}^t = &\frac{1}{28}\Big[y_0^6 + y_1^6 + y_2^6 + y_0^5(y_1+y_2) + y_1^5(y_0+y_2) + y_2^5(y_0+y_1)\\
&+ y_0y_1y_2(y_0+y_1+y_2)\big(y_0^2+y_1^2+y_2^2\big) + \big(y_0^2+y_1^2\big)y_0^2y_1^2\\
&+ y_0^2y_2^2\big(y_0^2+y_2^2\big) + y_1^2y_2^2\big(y_1^2+y_2^2\big) + y_0^3y_1^3 + y_1^3y_2^3 + y_0^3y_2^3\\
&+ y_0^2y_1^2y_2^2\Big]M_{0,0}^1 + \frac{1}{28}\Big[y_0'^6 + y_1'^6 + y_2'^6 + y_0'^5\big(y_1'+y_2'\big) + y_1'^5\big(y_0'+y_2'\big) + y_2'^5\big(y_0'+y_1'\big)\\
&+ y_0'y_1'y_2'\big(y_2'+y_0'+y_1'\big)\big(y_0'^2+y_1'^2+y_2'^2\big) + \big(y_0'^2+y_1'^2\big)y_0'^2y_1'^2\\
&+ y_0'^2y_2'^2\big(y_0'^2+y_2'^2\big) + y_1'^2y_2'^2\big(y_1'^2+y_2'^2\big) + y_0'^3y_1'^3 + y_1'^3y_2'^3 + y_0'^3y_2'^3\\
&+ y_0'^2y_1'^2y_2'^2\Big]M_{0,0}^2
\end{aligned}
\tag{15}
$$

Equation 40 should be read as:

$$
\begin{aligned}
&\sum_{i=0}^{m}\sum_{j=0}^{m-i}\{a_{ij}(r+2)(i+2)M_{i+r+2,j+s}\\
&\quad + \nu(j+2)\big[(r+2)M_{i+r+1,j+s+1} - a(r+1)M_{i+r,j+s+1}\big]b_{ij}\}\\
&\quad - \sum_{i=0}^{m}\sum_{j=0}^{m-i}\{a[(r+1)(i+2)+(r+2)(i+1)]a_{ij}\\
&\quad + (r+2)\nu bb_{ij}(j+1)\}M_{i+r+1,j+s}\\
&\quad + a(r+1)\sum_{i=0}^{m}\sum_{j=0}^{m-i}\big[aa_{ij}(i+1) + \nu bb_{ij}(j+1)\big]M_{i+r,j+s}\\
&\quad - \frac{(\nu-1)}{2}s\left\{\sum_{i=0}^{m}\sum_{j=1}^{m-i}a_{ij}j\big(M_{i+r+4,j+s-2} - 2aM_{i+r+3,j+s-2} + a^2M_{i+r+2,j+s-2}\big)\right.\\
&\quad \left. + \sum_{i=0}^{m}\sum_{j=1}^{m-i}b_{ij}i\big(M_{i+r+1,j+s+1} - aM_{i+r,j+s+1} - bM_{i+r+1,j+s} + abM_{i+r+1,j+s}\big)\right\}\\
&\quad = W_{rs}, 0 \le r \le m, 0 \le s \le m-r\\
&\sum_{i=0}^{m}\sum_{j=0}^{m-i}\{\nu a_{ij}(i+2)\big[(s+2)M_{i+r+1,j+s+1} - b(s+1)M_{i+r+1,j+s}\big]\\
&\quad + (s+2)(j+2)b_{ij}M_{i+r,j+s+2}\} - \sum_{i=0}^{m}\sum_{j=0}^{m-i}\{\nu a(s+2)(i+1)a_{ij}\\
&\quad + [(s+2)(j+1)+(s+1)(j+2)]bb_{ij}\}M_{i+r,j+s+1}\\
&\quad + b(s+1)\sum_{i=0}^{m}\sum_{j=0}^{m-i}\big[\nu aa_{ij}(i+1) + bb_{ij}(j+1)\big]M_{i+r,j+s}
\end{aligned}
$$

$$-\frac{(\nu-1)}{2}r\left\{\sum_{i=1}^{m}\sum_{j=0}^{m-i}b_{ij}i\left(M_{i+r-2,j+s+4}-2bM_{i+r-2,j+s+3}+b^2M_{i+r-2,j+s+2}\right)\right.$$
$$\left.+\sum_{i=0}^{m}\sum_{j=1}^{m-i}a_{ij}j\left(M_{i+r+1,j+s+1}-aM_{i+r,j+s+1}-bM_{i+r+1,j+s}+abM_{i+r,j+s}\right)\right\}$$
$$=V_{rs}, 0\le r\le m, 0\le s\le m-r \tag{40}$$

In Appendix C, C_{13} should be read as:

$$C_{30}=2\left(m_{0,0}^1\right)^3\left(m_{0,0}^1+1\right)^2,$$
$$C_{22}=-3\left(m_{0,0}^1\right)^2\left(m_{0,0}^1-1\right)\left(m_{0,0}^1+1\right)^2,$$
$$C_{21}=-6\left(m_{0,0}^1\right)^2m_{1,0}^t\left[\left(m_{0,0}^1\right)^2+4m_{0,0}^1+3\right],$$
$$C_{20}=-9\left(m_{0,0}^1\right)^2\left[4m_{0,0}^1\left(m_{0,0}^1+1\right)m_{2,0}^t-5m_{0,0}^1\left(m_{1,0}^t\right)^2-3\left(m_{1,0}^t\right)^2\right],$$
$$C_{14}=3m_{0,0}^1\left(m_{0,0}^1+1\right)^2\left(\left(m_{0,0}^1\right)^2+1\right),$$
$$C_{13}=-12m_{0,0}^1m_{1,0}^t[\left(m_{0,0}^1\right)^3+3\left(m_{0,0}^1\right)^2+5m_{0,0}^1+3]$$
$$C_{12}=6m_{0,0}^1\left(m_{0,0}^1+1\right)\left[2m_{0,0}^1\left(2m_{0,0}^1-3\right)m_{2,0}^t+9\left(m_{0,0}^1+3\right)\left(m_{1,0}^t\right)^2\right],$$
$$C_{11}=36m_{0,0}^1m_{1,0}^t\left[2m_{0,0}^1\left(2m_{0,0}^1+3\right)m_{2,0}^t-9\left(m_{0,0}^1+1\right)\left(m_{1,0}^t\right)^2\right],$$
$$C_{10}=3m_{0,0}^1\left\{-10m_{4,0}^t\left(m_{0,0}^1\right)^3\left(m_{0,0}^1+1\right)+40\left(m_{0,0}^1\right)^3m_{3,0}^tm_{1,0}^t\right.$$
$$\left.+24\left(m_{0,0}^1+3\right)\left(m_{0,0}^1\right)^2\left(m_{2,0}^t\right)^2-108m_{0,0}^1\left(m_{0,0}^1+1\right)\left(m_{1,0}^t\right)^2m_{2,0}^t+27\left(2m_{0,0}^1+3\right)\left(m_{1,0}^t\right)^4\right\},$$
$$C_{06}=-\left[\left(m_{0,0}^1\right)^3-1\right]\left(m_{0,0}^1+1\right)^2,$$
$$C_{05}=6m_{1,0}^t\left[\left(m_{0,0}^1\right)^4+\left(m_{0,0}^1\right)^3-2\left(m_{0,0}^1\right)^2-5m_{0,0}^1-3\right],$$
$$C_{04}=18\left(m_{1,0}^t\right)^2-3\left[\left(m_{0,0}^1\right)^3+1\right]\left[3\left(m_{1,0}^t\right)^2+4m_{0,0}^1m_{2,0}^t\right]$$
$$-18\left[\left(m_{0,0}^1\right)^2-1\right]\left[m_{0,0}^1m_{2,0}^t-3\left(m_{1,0}^t\right)^2\right]-12\left(m_{0,0}^1+1\right)\left[2m_{0,0}^1m_{2,0}^t-15\left(m_{1,0}^t\right)^2\right],$$
$$C_{03}=4\left(m_{0,0}^1+1\right)\left[-5m_{3,0}^t\left(m_{0,0}^1\right)^3+54m_{0,0}^1m_{2,0}^tm_{1,0}^t-135\left(m_{1,0}^t\right)^3\right]$$
$$+108\left(m_{0,0}^1\right)^2\left[m_{0,0}^1m_{2,0}^tm_{1,0}^t-\left(m_{1,0}^t\right)^3\right],$$
$$C_{02}=3\left\{5\left(m_{0,0}^1\right)^3\left[1-\left(m_{0,0}^1\right)^2\right]m_{4,0}^t+20\left(m_{0,0}^1\right)^3\left(m_{0,0}^1+2\right)m_{3,0}^tm_{1,0}^t\right.$$
$$+12\left(m_{0,0}^1\right)^2\left[\left(m_{0,0}^1\right)^2-2m_{0,0}^1+3\right]\left(m_{2,0}^t\right)^2-54m_{0,0}^1\left[\left(m_{0,0}^1\right)^2+4m_{0,0}^1+6\right]\left(m_{1,0}^t\right)^2m_{2,0}^t$$
$$\left.+27\left[\left(m_{0,0}^1\right)^2+10m_{0,0}^1+15\right]\left(m_{1,0}^t\right)^4\right\},$$
$$C_{01}=6\left\{\left(m_{0,0}^1\right)^4[7\left(m_{0,0}^1+1\right)m_{5,0}^t-40m_{3,0}^tm_{2,0}^t]\right.$$
$$-15\left(m_{0,0}^1\right)^3\left[m_{4,0}^tm_{1,0}^t+2\left(m_{1,0}^t\right)^2m_{3,0}^t\right]+36\left(m_{0,0}^1\right)^2\left(m_{0,0}^1-3\right)m_{1,0}^t\left(m_{2,0}^t\right)^2$$
$$\left.+108m_{0,0}^1\left(m_{0,0}^1+3\right)\left(m_{1,0}^t\right)^3m_{2,0}^t-81\left(m_{0,0}^1+3\right)\left(m_{1,0}^t\right)^5\right\},$$
$$C_{00}=-9\left\{2\left(m_{0,0}^1\right)^4\left(7m_{5,0}^tm_{1,0}^t-10m_{4,0}^tm_{2,0}^t\right)+3\left(m_{0,0}^1\right)^3\left[-5m_{4,0}^t\left(m_{1,0}^t\right)^2+16\left(m_{2,0}^t\right)^3\right]\right.$$
$$\left.-108\left(m_{0,0}^1\right)^2\left(m_{1,0}^t\right)^2\left(m_{2,0}^t\right)^2+162m_{0,0}^1\left(m_{1,0}^t\right)^4m_{2,0}^t-81\left(m_{1,0}^t\right)^6\right\},$$

$$D_{40} = \left(m^1_{0,0}\right)^4\left(m^1_{0,0} + 1\right)^3,$$
$$D_{32} = -6\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)^2\left[\left(m^1_{0,0}\right)^2 - 1\right],$$
$$D_{31} = -12\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)^2\left(m^1_{0,0} + 3\right)m^1_{1,0},$$
$$D_{30} = -6\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)\left[4\left(\left(m^1_{0,0}\right)^2 + m^1_{0,0}\right)m^t_{2,0} - 3\left(5m^1_{0,0} + 3\right)\left(m^t_{1,0}\right)^2\right],$$
$$D_{24} = 3\left(m^1_{0,0}\right)^2\left(m^1_{0,0} + 1\right)^3\left[\left(m^1_{0,0}\right)^2 - 8m^1_{0,0} + 1\right],$$
$$D_{23} = 12\left(m^1_{0,0}\right)^2 m^t_{1,0}\left(m^1_{0,0} + 1\right)^2\left[7\left(m^1_{0.0}\right)^2 + 14m^1_{0,0} - 3\right],$$
$$D_{22} = -6\left(m^1_{0,0}\right)^2\left(m^1_{0,0} + 1\right)\left\{3\left[25\left(m^1_{0,0}\right)^2 + 12m^1_{0,0} - 9\right]\left(m^t_{1,0}\right)^2 \right.$$
$$\left. +2m^1_{0,0}\left[-10\left(m^1_{0,0}\right)^2 - m^1_{0,0} + 9\right]m^t_{2,0}\right\},$$
$$D_{21} = 36\left(m^1_{0,0}\right)^2 m^t_{1,0}\left\{3\left[\left(m^1_{0,0}\right)^2 - 6m^1_{0,0} - 3\right]\left(m^t_{1,0}\right)^2 \right.$$
$$\left. +2m^1_{0.0}\left[2\left(m^1_{0.0}\right)^2 + 11m^1_{0,0} + 9\right]m^t_{2,0}\right\},$$
$$D_{20} = 3\left(m^1_{0,0}\right)^2\left\{9\left[6\left(m^1_{0,0}\right)^2 + 39m^1_{0,0} + 9\right]\left(m^t_{1,0}\right)^4 + 24\left(m^1_{0,0}\right)^2[\left(m^1_{0,0}\right)^2 + 4m^1_{0,0} \right.$$
$$+ 3]\left(m^t_{2,0}\right)^2 - 36m^1_{0,0}\left[3\left(m^1_{0,0}\right)^2 + 14m^1_{0,0} + 9\right]\left(m^t_{1,0}\right)^2 m^t_{2,0} - 10\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)^2 m^t_{4,0}$$
$$\left. +40\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)m^t_{3,0}m^t_{1,0}\right\},$$
$$D_{16} = -2m^1_{0,0}\left(m^1_{0,0} + 1\right)^3\left(m^1_{0,0} - 1\right)\left[\left(m^1_{0,0}\right)^2 - 5m^1_{0,0} + 1\right],$$
$$D_{15} = -12m^1_{0,0}m^t_{1,0}\left(m^1_{00} + 1\right)^2\left[3\left(m^1_{0,0}\right)^3 + 2\left(m^1_{0,0}\right)^2 - 12m^1_{0,0} + 3\right],$$
$$D_{14} = -6m^1_{00}\left(m^1_{00} + 1\right)\left\{3\left[-7\left(m^1_{0,0}\right)^3 + 32\left(m^1_{0,0}\right)^2 + 30m^1_{0,0} - 15\right]\left(m^t_{1,0}\right)^2 \right.$$
$$\left. +2m^1_{0,0}\left[2\left(m^1_{0,0}\right)^3 - 21\left(m^1_{0,0}\right)^2 - 20m^1_{0,0} + 3\right]m^t_{2,0}\right\},$$
$$D_{13} = 8m^1_{0,0}\left\{10\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)^2 m^t_{3,0} + 27\left[5\left(m^1_{0,0}\right)^3 + 12\left(m^1_{0,0}\right)^2 - 5\right]\left(m^t_{1,0}\right)^3 \right.$$
$$\left. -18m^1_{0,0}\left[9\left(m^1_{0,0}\right)^3 + 22\left(m^1_{0,0}\right)^2 + 10m^1_{0,0} - 3\right]m^t_{2,0}m^t_{1,0}\right\},$$
$$D_{12} = -6m^1_{0,0}\left\{25\left(m^1_{0,0}\right)^3\left(m^1_{0,0} + 1\right)\left[1 - \left(m^1_{0,0}\right)^2\right]m^t_{4,0} \right.$$
$$+ 20\left(m^1_{0,0}\right)^3\left(5m^1_{0,0} + 1\right)\left(m^1_{0,0} + 1\right)m^t_{3,0}m^t_{1,0}$$
$$+ 12\left(m^1_{0,0}\right)^2\left[11\left(m^1_{0,0}\right)^2 + 5\left(m^1_{0,0}\right)^3 - 3m^1_{0,0} - 9\right]\left(m^t_{2,0}\right)^2$$
$$+ 18m^1_{0,0}\left[-53\left(m^1_{0,0}\right)^2 - 15\left(m^1_{0,0}\right)^3 - 18m^1_{0,0} + 18\right]\left(m^t_{1,0}\right)^2 m^t_{2,0}$$
$$\left. + 27\left[5\left(m^1_{0,0}\right)^3 + 17\left(m^1_{0,0}\right)^2 - 15m^1_{0,0} - 15\right]\left(m^t_{1,0}\right)^4\right\},$$
$$D_{11} = -36m^1_{0,0}m^t_{1,0}\{5\left(m^1_{0,0}\right)^3\left[\left(m^1_{0,0}\right)^2 - 4m^1_{0,0} - 5\right]m^t_{4,0} + 20\left(m^1_{0,0}\right)^3\left(2 - m^1_{0,0}\right)m^t_{3,0}m^t_{1,0}$$
$$+ 12\left(m^1_{0,0}\right)^2\left[-\left(m^1_{0,0}\right)^2 + 6m^1_{0,0} + 9\right]\left(m^t_{2,0}\right)^2 + 18m^1_{0,0}\left[3\left(m^1_{0,0}\right)^2 - 8m^1_{0,0} - 6\right]\left(m^t_{1,0}\right)^2 m^t_{2,0}$$
$$+ 9\left[-3\left(m^1_{0,0}\right)^2 + 18m^1_{0,0} + 9\right]\left(m^t_{1,0}\right)^4\},$$
$$D_{10} = 18m^1_{0,0}\left\{20\left(m^1_{0,0}\right)^4[\left(m^1_{0,0} + 1\right)m^t_{4,0}m^t_{2,0} - 4m^t_{1,0}m^t_{2,0}m^t_{3,0}] \right.$$
$$+ 324\left(m^1_{0,0}\right)^2\left(m^1_{0,0} + 1\right)\left(m^t_{1,0}\right)^2\left(m^t_{2,0}\right)^2$$

$$+3\left(m_{0,0}^1\right)^3\left[60m_{3,0}^t\left(m_{1,0}^t\right)^3-16\left(m_{0,0}^1+1\right)\left(m_{2,0}^t\right)^3-5\left(3m_{0,0}^1+5\right)m_{4,0}^t\left(m_{1,0}^t\right)^2\right]$$
$$-54m_{0,0}^1\left(11m_{0,0}^1+3\right)\left(m_{1,0}^t\right)^4m_{2,0}^t+81\left(3m_{0,0}^1+1\right)\left(m_{1,0}^t\right)^6\Big\},$$

$$D_{08}=\left(m_{0,0}^1+1\right)^3\left[\left(m_{0,0}^1\right)^2-m_{0,0}^1+1\right]^2,$$

$$D_{07}=-24m_{1,0}^t\left(m_{0,0}^1+1\right)^2\left(\left(m_{0,0}^1\right)^2-m_{0,0}^1+1\right),$$

$$D_{06}=12\left(m_{0,0}^1+1\right)\left\{\left[9\left(m_{0,0}^1\right)^3+21\right]\left(m_{1,0}^t\right)^2+m_{0,0}^1\left[-5\left(m_{0,0}^1\right)^3+4m_{0,0}^1-1\right]m_{2,0}^t\right\},$$

$$D_{05}=72m_{1,0}^t\left(m_{0,0}^1+1\right)\{m_{0,0}^1\left[4\left(m_{0,0}^1\right)^3+\left(m_{0,0}^1\right)^2-9m_{0,0}^1+3\right]m_{2,0}^t$$
$$-3\left[\left(m_{0,0}^1\right)^3+7\right]\left(m_{1,0}^t\right)^2\Big\},$$

$$D_{04}=\Big\{120\left(m_{0,0}^1\right)^3\left[\left(m_{0,0}^1\right)^3+1\right]m_{3,0}^tm_{1,0}^t-30\left(m_{0,0}^1\right)^3\left(m_{0,0}^1+1\right)\left[\left(m_{0,0}^1\right)^3+1\right]m_{4,0}^t$$
$$+162\left(m_{0,0}^1+1\right)\left[\left(m_{0,0}^1\right)^3+35\right]\left(m_{1,0}^t\right)^3+36\left(m_{0,0}^1\right)^2\left[2\left(m_{0,0}^1\right)^4+2\left(m_{0,0}^1\right)^3-20\left(m_{0,0}^1\right)^2\right.$$
$$\left.-17m_{0,0}^1+3\right]\left(m_{2,0}^t\right)^2+108m_{0,0}^1\left[24\left(m_{0,0}^1\right)^2-5\left(m_{0,0}^1\right)^3-15+15m_{0,0}^1\right.$$
$$\left.-3\left(m_{0,0}^1\right)^4\right]\left(m_{1,0}^t\right)^2m_{2,0}^t\Big\},$$

$$D_{03}=24\Big\{-20\left(m_{0,0}^1\right)^5m_{3,0}^tm_{2,0}^t-20\left(m_{0,0}^1\right)^4m_{3,0}^tm_{2,0}^t-60\left(m_{0,0}^1\right)^3m_{3,0}^t\left(m_{1,0}^t\right)^2$$
$$+15\left(m_{0,0}^1\right)^3\left(m_{0,0}^1+1\right)m_{4,0}^tm_{1,0}^t+18\left(m_{0,0}^1\right)^2\left[10\left(m_{0,0}^1\right)^2+10m_{0,0}^1-3\right]m_{1,0}^t\left(m_{2,0}^t\right)^2$$
$$+54m_{0,0}^1\left[-4\left(m_{0,0}^1\right)^2+5\right]\left(m_{1,0}^t\right)^3m_{2,0}^t-567\left(m_{0,0}^1+1\right)\left(m_{1,0}^t\right)^5\Big\},$$

$$D_{02}=4\Big\{\left(m_{0,0}^1+1\right)[5103\left(m_{1,0}^t\right)^6-3645m_{0,0}^1\left(m_{1,0}^t\right)^4m_{2,0}^t-405\left(m_{0,0}^1\right)^3m_{4,0}^t\left(m_{1,0}^t\right)^2$$
$$+225\left(m_{0,0}^1\right)^4m_{4,0}^tm_{2,0}^t-28\left(m_{0,0}^1\right)^5\left(m_{0,0}^1+1\right)m_{6,0}^t+100\left(m_{0,0}^1\right)^5\left(m_{3,0}^t\right)^2\Big]$$
$$+972\left(m_{0,0}^1\right)^3\left(m_{1,0}^t\right)^4m_{2,0}^t+1620\left(m_{0,0}^1\right)^3\left(m_{1,0}^t\right)^3m_{3,0}^t$$
$$+162\left(m_{0,0}^1\right)^2\left[-10\left(m_{0,0}^1\right)^2-9m_{0,0}^1+9\right]\left(m_{1,0}^tm_{2,0}^t\right)^2$$
$$+108\left(m_{0,0}^1\right)^3\left[2\left(m_{0,0}^1\right)^2-m_{0,0}^1-3\right]\left(m_{2,0}^t\right)^3+180\left(m_{0,0}^1\right)^4\left(1+2m_{0,0}^1\right)m_{1,0}^tm_{2,0}^tm_{3,0}^t\Big\},$$

$$D_{01}=24m_{1,0}^t\Big\{\left(m_{0,0}^1+1\right)\left[28\left(m_{0,0}^1\right)^5m_{6,0}^t-729\left(m_{1,0}^t\right)^6+135\left(m_{0,0}^1\right)^3m_{4,0}^t\left(m_{1,0}^t\right)^2\right]$$
$$-162\left(4m_{0,0}^1+3\right)\left(m_{0,0}^1\right)^2\left(m_{1,0}^tm_{2,0}^t\right)^2-45\left(m_{0,0}^1\right)^4\left(4m_{0,0}^1+5\right)m_{4,0}^tm_{2,0}^t$$
$$+108\left(2m_{0,0}^1+3\right)\left(m_{0,0}^1\right)^3\left(m_{2,0}^t\right)^3+729m_{0,0}^1\left(2m_{0,0}^1+1\right)\left(m_{1,0}^t\right)^4m_{2,0}^t$$
$$-100\left(m_{0,0}^1\right)^5\left(m_{3,0}^t\right)^2+360\left(m_{0,0}^1\right)^4m_{1,0}^tm_{2,0}^tm_{3,0}^t-540\left(m_{0,0}^1\right)^3\left(m_{1,0}^t\right)^3m_{3,0}^t\Big\},$$

$$D_{00}=9\Big\{m_{0,0}^1[-27\left(m_{1,0}^t\right)^4+5\left(m_{0,0}^1\right)^3m_{4,0}^t-4\left(m_{0,0}^1\right)^2\left[3\left(m_{2,0}^t\right)^2+5m_{1,0}^tm_{3,0}^t\right]$$
$$+54m_{0,0}^1\left(m_{1,0}^t\right)^2m_{2,0}^t]^2+[5\left(m_{0,0}^1\right)^3m_{4,0}^t-12\left(m_{0,0}^1\right)^2\left(m_{2,0}^t\right)^2+54m_{0,0}^1\left(m_{1,0}^t\right)^2m_{2,0}^t$$
$$-27\left(m_{1,0}^t\right)^4]\left[5\left(m_{0,0}^1\right)^3m_{4,0}^t-12\left(m_{0,0}^1\right)^2\left(m_{2,0}^t\right)^2+126m_{0,0}^1\left(m_{1,0}^t\right)^2m_{2,0}^t-27\left(m_{1,0}^t\right)^4\right]$$
$$+16m_{0,0}^1\left(m_{1,0}^t\right)^2\{27m_{2,0}^t\left[2\left(m_{00}^1\right)^2\left(m_{2,0}^t\right)^2-15m_{0,0}^1\left(m_{1,0}^t\right)^2m_{2,0}^t+9\left(m_{1,0}^t\right)^4\right]-7\left(m_{0,0}^1\right)^4m_{6,0}^t\Big\} \tag{C.1}$$

The original article has been corrected.