CORRECTION

A. Arbind · J. N. Reddy

Correction to: A one-dimensional model of 3-D structure for large deformation: a general higher-order rod theory

Received: 16 May 2018 / Published online: 31 August 2018
 © Springer-Verlag GmbH Austria, part of Springer Nature 2018

Correction to: *Acta Mech* 229, 1803–1831 (2018)
<https://doi.org/10.1007/s00707-017-2048-4>

Correction in Appendix B

The correction in Eq. (B.1)

$$\mathbf{e}_s = \frac{\partial \mathbf{R}}{\partial s} = \xi \mathbf{T} + \tau \mathbf{e}_\theta, \quad \text{with } \xi = 1 - r\kappa \cos \theta \quad (1)$$

Because of this change, the follow-up corrections in the equations are as follows:
 In Eq. (B.2)

$$g_{ss} = \xi^2 + r^2 \tau^2, \quad g_{s\theta} = g_{\theta s} = r^2 \tau, \quad g_{rr} = 1, \quad g_{\theta\theta} = r^2 \quad (2)$$

In Eq. (B.3)

$$\mathbf{e}^s = g^{sk} \mathbf{e}_k = \frac{1}{\xi} \mathbf{T}, \quad \mathbf{e}^r = \mathbf{e}_r, \quad \mathbf{e}^\theta = -\frac{\tau}{\xi} \mathbf{T} + \frac{1}{r^2} \mathbf{e}_\theta \quad (3)$$

In Eq. (B.4)

$$\mathbf{e}_s = \xi \mathbf{T} + \tau \mathbf{e}_\theta \quad (4)$$

In Eq. (B.9)

$$\begin{aligned} \frac{\partial \mathbf{e}_\theta}{\partial s} &= \frac{r}{\xi} \hat{\kappa}_2 \mathbf{e}_s - \frac{r}{\xi} \hat{\kappa}_2 \tau \mathbf{e}_\theta - r \tau \mathbf{e}_r \\ \frac{\partial \mathbf{e}_r}{\partial s} &= -\frac{1}{\xi} \hat{\kappa}_1 \mathbf{e}_s + \left(\frac{1}{\xi} \hat{\kappa}_1 \tau + \frac{\tau}{r} \right) \mathbf{e}_\theta \\ \frac{\partial \mathbf{e}_s}{\partial s} &= \frac{1}{\xi} \left(\frac{\partial \xi}{\partial s} + r \hat{\kappa}_2 \tau \right) \mathbf{e}_s + (\xi \hat{\kappa}_1 - r \tau^2) \mathbf{e}_r + \left(\frac{\partial \tau}{\partial s} - \frac{1}{\xi} \frac{\partial \xi}{\partial s} \tau - \frac{\xi}{r} \hat{\kappa}_2 - \frac{r}{\xi} \hat{\kappa}_2 \tau^2 \right) \mathbf{e}_\theta \end{aligned} \quad (5)$$

where

$$\hat{\kappa}_1 = \kappa \cos \theta, \quad \hat{\kappa}_2 = \kappa \sin \theta$$

The original article can be found online at <https://doi.org/10.1007/s00707-017-2048-4>.

A. Arbind · J. N. Reddy (✉)
 Advanced Computational Mechanics Laboratory, Department of Mechanical Engineering, Texas A&M University, College Station, TX, USA
 E-mail: jnreddy@tamu.edu

In Eq. (B.10),

$$\begin{aligned}\Gamma_{ss}^s &= \frac{1}{\xi} \left(\frac{\partial \xi}{\partial s} + r \hat{\kappa}_2 \tau \right), \quad \Gamma_{ss}^r = (\xi \hat{\kappa}_1 - r \tau^2), \quad \Gamma_{ss}^\theta = \left(\frac{\partial \tau}{\partial s} - \frac{1}{\xi} \frac{\partial \xi}{\partial s} \tau - \frac{\xi}{r} \hat{\kappa}_2 - \frac{r}{\xi} \hat{\kappa}_2 \tau^2 \right) \\ \Gamma_{rs}^s &= \Gamma_{sr}^s = -\frac{1}{\xi} \hat{\kappa}_1, \quad \Gamma_{rs}^\theta = \Gamma_{sr}^\theta = \left(\frac{1}{\xi} \hat{\kappa}_1 \tau + \frac{\tau}{r} \right), \quad \Gamma_{\theta s}^s = \Gamma_{s\theta}^s = \frac{r}{\xi} \hat{\kappa}_2, \quad \Gamma_{\theta s}^\theta = \Gamma_{s\theta}^\theta = -\frac{r}{\xi} \hat{\kappa}_2 \tau\end{aligned}\quad (6)$$

Equation (B.11) becomes

$$\begin{aligned}\nabla \mathbf{u} &= \left(\frac{\partial u^j}{\partial z^i} + \Gamma_{ik}^j u^k \right) \mathbf{e}^i \mathbf{e}_j \\ &= \left(\frac{\partial u^s}{\partial s} + \frac{1}{\xi} \left(\frac{\partial \xi}{\partial s} + r \hat{\kappa}_2 \tau \right) u^s - \frac{1}{\xi} \hat{\kappa}_1 u^r + \frac{r}{\xi} \hat{\kappa}_2 u^\theta \right) \mathbf{e}^s \mathbf{e}_s \\ &\quad + \left(\frac{\partial u^r}{\partial s} + (\xi \hat{\kappa}_1 - r \tau^2) u^s - \tau r u^\theta \right) \mathbf{e}^s \mathbf{e}_r \\ &\quad + \left(\frac{\partial u^\theta}{\partial s} + \left(\frac{\partial \tau}{\partial s} - \frac{1}{\xi} \frac{\partial \xi}{\partial s} \tau - \frac{\xi}{r} \hat{\kappa}_2 - \frac{r}{\xi} \hat{\kappa}_2 \tau^2 \right) u^s + \left(\frac{1}{\xi} \hat{\kappa}_1 \tau + \frac{\tau}{r} \right) u^r - \frac{r}{\xi} \hat{\kappa}_2 \tau u^\theta \right) \mathbf{e}^s \mathbf{e}_\theta \\ &\quad + \left(\frac{\partial u^s}{\partial r} - \frac{1}{\xi} \hat{\kappa}_1 u^s \right) \mathbf{e}^r \mathbf{e}_s + \frac{\partial u^r}{\partial r} \mathbf{e}^r \mathbf{e}_r + \left(\frac{\partial u^\theta}{\partial r} + \left(\frac{1}{\xi} \hat{\kappa}_1 \tau + \frac{\tau}{r} \right) u^s + \frac{u^\theta}{r} \right) \mathbf{e}^r \mathbf{e}_\theta \\ &\quad + \left(\frac{\partial u^s}{\partial \theta} + \frac{r}{\xi} \hat{\kappa}_2 u^s \right) \mathbf{e}^\theta \mathbf{e}_s + \left(\frac{\partial u^r}{\partial \theta} - \tau r u^s - r u^\theta \right) \mathbf{e}^\theta \mathbf{e}_r \\ &\quad + \left(\frac{\partial u^\theta}{\partial \theta} - \frac{r}{\xi} \hat{\kappa}_2 \tau u^s + \frac{u^r}{r} \right) \mathbf{e}^\theta \mathbf{e}_\theta\end{aligned}\quad (7)$$

In Eq. (B.12)

$$\hat{\mathbf{e}}_s = \mathbf{T}.\quad (8)$$

In Eq. (B.13)

$$u^s = \frac{\hat{u}^s}{\xi}, \quad u^\theta = \frac{\hat{u}^\theta}{r} - \frac{\tau}{\xi} \hat{u}^s\quad (9)$$

In Eq. (B.14)

$$\begin{aligned}\nabla \mathbf{u} &= \left(\frac{1}{\xi} \frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_1 \hat{u}^r + \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^\theta \right) \hat{\mathbf{e}}_s \hat{\mathbf{e}}_s + \frac{1}{\xi} \left(\frac{\partial \hat{u}^r}{\partial s} - \tau \frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1 \hat{u}^s \right) \hat{\mathbf{e}}_s \hat{\mathbf{e}}_r \\ &\quad + \left(\frac{1}{\xi} \frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^s \right) \hat{\mathbf{e}}_s \hat{\mathbf{e}}_\theta + \frac{\partial \hat{u}^s}{\partial r} \hat{\mathbf{e}}_r \hat{\mathbf{e}}_s + \frac{\partial \hat{u}^r}{\partial r} \hat{\mathbf{e}}_r \hat{\mathbf{e}}_r + \frac{\partial \hat{u}^\theta}{\partial r} \hat{\mathbf{e}}_r \hat{\mathbf{e}}_\theta \\ &\quad + \frac{1}{r} \frac{\partial \hat{u}^s}{\partial \theta} \hat{\mathbf{e}}_\theta \hat{\mathbf{e}}_s + \left(\frac{1}{r} \frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} \right) \hat{\mathbf{e}}_\theta \hat{\mathbf{e}}_r + \left(\frac{1}{r} \frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r} \right) \hat{\mathbf{e}}_\theta \hat{\mathbf{e}}_\theta\end{aligned}\quad (10)$$

In Eq. (B.15)

$$\begin{aligned}(\nabla \mathbf{u})^T &= \left(\frac{1}{\xi} \frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_1 \hat{u}^r + \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^\theta \right) \hat{\mathbf{e}}_s \hat{\mathbf{e}}_s + \frac{\partial \hat{u}^s}{\partial r} \hat{\mathbf{e}}_s \hat{\mathbf{e}}_r \\ &\quad + \frac{1}{r} \frac{\partial \hat{u}^s}{\partial \theta} \hat{\mathbf{e}}_s \hat{\mathbf{e}}_\theta + \frac{1}{\xi} \left(\frac{\partial \hat{u}^r}{\partial s} - \tau \frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1 \hat{u}^s \right) \hat{\mathbf{e}}_r \hat{\mathbf{e}}_s + \frac{\partial \hat{u}^r}{\partial r} \hat{\mathbf{e}}_r \hat{\mathbf{e}}_r + \left(\frac{1}{r} \frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} \right) \hat{\mathbf{e}}_r \hat{\mathbf{e}}_\theta \\ &\quad + \left(\frac{1}{\xi} \frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^s \right) \hat{\mathbf{e}}_\theta \hat{\mathbf{e}}_s + \frac{\partial \hat{u}^\theta}{\partial r} \hat{\mathbf{e}}_\theta \hat{\mathbf{e}}_r + \left(\frac{1}{r} \frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r} \right) \hat{\mathbf{e}}_\theta \hat{\mathbf{e}}_\theta\end{aligned}\quad (11)$$

In Eq. (B.17), the strain components in terms of the components of displacement field are:

$$\begin{aligned}
E_{ss} &= \left(\frac{1}{\xi} \frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_1 \hat{u}^r + \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^\theta \right) + \frac{1}{2} \left[\left(\frac{1}{\xi} \frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_1 \hat{u}^r + \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^\theta \right)^2 \right. \\
&\quad \left. + \frac{1}{\xi^2} \left(\frac{\partial \hat{u}^r}{\partial s} - \tau \frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1 \hat{u}^s \right)^2 + \left(\frac{1}{\xi} \frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^s \right)^2 \right] \\
E_{rr} &= \frac{\partial \hat{u}^r}{\partial r} + \frac{1}{2} \left[\left(\frac{\partial \hat{u}^s}{\partial r} \right)^2 + \left(\frac{\partial \hat{u}^r}{\partial r} \right)^2 + \left(\frac{\partial \hat{u}^\theta}{\partial r} \right)^2 \right] \\
E_{\theta\theta} &= \left(\frac{1}{r} \frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r} \right) + \frac{1}{2} \left[\left(\frac{1}{r} \frac{\partial \hat{u}^s}{\partial \theta} \right)^2 + \left(\frac{1}{r} \frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} \right)^2 + \left(\frac{1}{r} \frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r} \right)^2 \right] \\
2E_{r\theta} &= \frac{1}{r} \frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} + \frac{\partial \hat{u}^\theta}{\partial r} + \frac{1}{r} \frac{\partial \hat{u}^s}{\partial r} \frac{\partial \hat{u}^s}{\partial \theta} + \frac{\partial \hat{u}^r}{\partial r} \left(\frac{1}{r} \frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} \right) + \frac{\partial \hat{u}^\theta}{\partial r} \left(\frac{1}{r} \frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r} \right) \\
2E_{\theta s} &= \frac{1}{\xi} \frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^s + \frac{1}{r} \frac{\partial \hat{u}^s}{\partial \theta} + \frac{1}{r} \frac{\partial \hat{u}^s}{\partial r} \left(\frac{1}{\xi} \frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_1 \hat{u}^r + \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^\theta \right) \\
&\quad + \frac{1}{\xi} \left(\frac{1}{r} \frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} \right) \left(\frac{\partial \hat{u}^r}{\partial s} - \tau \frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1 \hat{u}^s \right) + \left(\frac{1}{r} \frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r} \right) \left(\frac{1}{\xi} \frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^s \right) \\
2E_{sr} &= \frac{1}{\xi} \left(\frac{\partial \hat{u}^r}{\partial s} - \tau \frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1 \hat{u}^s \right) + \frac{\partial \hat{u}^s}{\partial r} + \frac{\partial \hat{u}^s}{\partial r} \left(\frac{1}{\xi} \frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_1 \hat{u}^r + \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^\theta \right) \\
&\quad + \frac{1}{\xi} \frac{\partial \hat{u}^r}{\partial r} \left(\frac{\partial \hat{u}^r}{\partial s} - \tau \frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1 \hat{u}^s \right) + \frac{\partial \hat{u}^\theta}{\partial r} \left(\frac{1}{\xi} \frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi} \frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi} \hat{\kappa}_2 \hat{u}^s \right)
\end{aligned} \tag{12}$$

Corrections in Sects. 2 and 4

In Eq. (8)

$$\begin{aligned}
E_{ss} &= \left(\frac{1}{\xi} \mathbf{A}_s \Phi_{s,s} - \frac{\tau}{\xi} \mathbf{A}_{s,\theta} \Phi_s - \frac{1}{\xi} \hat{\kappa}_1 \mathbf{A}_r \Phi_r + \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_\theta \Phi_\theta \right) \\
&\quad + \frac{1}{2} \left[\left(\frac{1}{\xi} \mathbf{A}_s \Phi_{s,s} - \frac{\tau}{\xi} \mathbf{A}_{s,\theta} \Phi_s - \frac{1}{\xi} \hat{\kappa}_1 \mathbf{A}_r \Phi_r + \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_\theta \Phi_\theta \right)^2 \right. \\
&\quad \left. + \frac{1}{\xi^2} (\mathbf{A}_r \Phi_{r,s} - \tau \mathbf{A}_{r,\theta} \Phi_r + \hat{\kappa}_1 \mathbf{A}_s \Phi_s)^2 + \left(\frac{1}{\xi} \mathbf{A}_\theta \Phi_{\theta,s} - \frac{\tau}{\xi} \mathbf{A}_{\theta,\theta} \Phi_\theta - \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_s \Phi_s \right)^2 \right] \\
E_{rr} &= \mathbf{A}_{r,r} \Phi_r + \frac{1}{2} \left[(\mathbf{A}_{s,r} \Phi_s)^2 + (\mathbf{A}_{r,r} \Phi_r)^2 + (\mathbf{A}_{\theta,r} \Phi_\theta)^2 \right] \\
E_{\theta\theta} &= \left(\frac{1}{r} \mathbf{A}_{\theta,\theta} \Phi_\theta + \frac{\mathbf{A}_r \Phi_r}{r} \right) + \frac{1}{2} \left[\left(\frac{1}{r} \mathbf{A}_{s,\theta} \Phi_s \right)^2 + \left(\frac{1}{r} \mathbf{A}_{r,\theta} \Phi_r - \frac{\mathbf{A}_\theta \Phi_\theta}{r} \right)^2 \right. \\
&\quad \left. + \left(\frac{1}{r} \mathbf{A}_{\theta,\theta} \Phi_\theta + \frac{\mathbf{A}_r \Phi_r}{r} \right)^2 \right] \\
2E_{r\theta} &= \frac{1}{r} \mathbf{A}_{r,\theta} \Phi_r - \frac{\hat{u}^\theta}{r} + \mathbf{A}_{\theta,r} \Phi_\theta + \frac{1}{r} \mathbf{A}_{s,r} \Phi_s \mathbf{A}_{s,\theta} \Phi_s + \mathbf{A}_{r,r} \Phi_r \left(\frac{1}{r} \mathbf{A}_{r,\theta} \Phi_r - \frac{\hat{u}^\theta}{r} \right) \\
&\quad + \mathbf{A}_{\theta,r} \Phi_\theta \left(\frac{1}{r} \mathbf{A}_{\theta,\theta} \Phi_\theta + \frac{\hat{u}^r}{r} \right) \\
2E_{\theta s} &= \frac{1}{\xi} \mathbf{A}_\theta \Phi_{\theta,s} - \frac{\tau}{\xi} \mathbf{A}_{\theta,\theta} \Phi_\theta - \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_s \Phi_s + \frac{1}{r} \mathbf{A}_{s,\theta} \Phi_s \\
&\quad + \frac{1}{r} \mathbf{A}_{s,\theta} \Phi_s \left(\frac{1}{\xi} \mathbf{A}_s \Phi_{s,s} - \frac{\tau}{\xi} \mathbf{A}_{s,\theta} \Phi_s - \frac{1}{\xi} \hat{\kappa}_1 \mathbf{A}_r \Phi_r + \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_\theta \Phi_\theta \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\xi} \left(\frac{1}{r} \mathbf{A}_{r,\theta} \Phi_r - \frac{\mathbf{A}_\theta \Phi_\theta}{r} \right) (\mathbf{A}_r \Phi_{r,s} - \tau \mathbf{A}_{r,\theta} \Phi_r + \hat{\kappa}_1 \mathbf{A}_s \Phi_s) \\
& + \left(\frac{1}{r} \mathbf{A}_{\theta,\theta} \Phi_\theta + \frac{\mathbf{A}_r \Phi_r}{r} \right) \left(\frac{1}{\xi} \mathbf{A}_\theta \Phi_{\theta,s} - \frac{\tau}{\xi} \mathbf{A}_{\theta,\theta} \Phi_\theta - \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_s \Phi_s \right) \\
2E_{sr} = & \frac{1}{\xi} (\mathbf{A}_r \Phi_{r,s} - \tau \mathbf{A}_{r,\theta} \Phi_r + \hat{\kappa}_1 \mathbf{A}_s \Phi_s) \\
& + \mathbf{A}_{s,r} \Phi_s + \mathbf{A}_{s,r} \Phi_s \left(\frac{1}{\xi} \mathbf{A}_s \Phi_{s,s} - \frac{\tau}{\xi} \mathbf{A}_{s,\theta} \Phi_s - \frac{1}{\xi} \hat{\kappa}_1 \mathbf{A}_r \Phi_r + \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_\theta \Phi_\theta \right) \\
& + \frac{1}{\xi} \mathbf{A}_{r,r} \Phi_r (\mathbf{A}_r \Phi_{r,s} - \tau \mathbf{A}_{r,\theta} \Phi_r + \hat{\kappa}_1 \mathbf{A}_s \Phi_s) \\
& + \mathbf{A}_{\theta,r} \Phi_\theta \left(\frac{1}{\xi} \mathbf{A}_\theta \Phi_{\theta,s} - \frac{\tau}{\xi} \mathbf{A}_{\theta,\theta} \Phi_\theta - \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_s \Phi_s \right)
\end{aligned} \tag{13}$$

Equations (10) and (11) should be replaced by:

$$\begin{aligned}
\mathbf{A}_1 &= \begin{bmatrix} -\frac{\tau}{\xi} \mathbf{A}_{s,\theta} & -\frac{1}{\xi} \hat{\kappa}_1 \mathbf{A}_r & \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_\theta \\ \mathbf{0} & \mathbf{A}_{r,r} & \mathbf{0} \\ \mathbf{0} & \frac{1}{r} \mathbf{A}_r & \frac{1}{r} \mathbf{A}_{\theta,\theta} \\ +\frac{1}{r} \mathbf{A}_{s,\theta} - \frac{1}{\xi} \hat{\kappa}_2 \mathbf{A}_s & \mathbf{0} & -\frac{1}{r} \mathbf{A}_\theta + \mathbf{A}_{\theta,r} \\ \frac{\hat{\kappa}_1}{\xi} \mathbf{A}_s + \mathbf{A}_{s,r} & -\frac{\tau}{\xi} \mathbf{A}_{r,\theta} & \mathbf{0} \end{bmatrix}, \quad \mathbf{A}_2 = \frac{1}{\xi} \begin{bmatrix} \mathbf{A}_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_\theta \\ \mathbf{0} & \mathbf{A}_r & \mathbf{0} \end{bmatrix} \\
\mathbf{A}_{nl_s} &= \begin{bmatrix} \frac{1}{\xi^2} (u_{s,s} - \tau u_{s,\theta} - \hat{\kappa}_1 u_r + \hat{\kappa}_2 u_\theta) \mathbf{A}_s & \frac{1}{\xi^2} (u_{r,s} - \tau u_{r,\theta} + \hat{\kappa}_1 u_s) \mathbf{A}_r & \frac{1}{\xi^2} (u_{\theta,s} - \tau u_{\theta,\theta} - \hat{\kappa}_2 u_s) \mathbf{A}_\theta \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \frac{1}{r\xi} u_{s,\theta} \mathbf{A}_s & \frac{1}{r\xi} (u_{r,\theta} - u_\theta) \mathbf{A}_r & \frac{1}{r\xi} (u_{\theta,\theta} + u_r) \mathbf{A}_\theta \\ \frac{1}{\xi} u_{s,r} \mathbf{A}_s & \frac{1}{\xi} u_{r,r} \mathbf{A}_r & \frac{1}{\xi} u_{\theta,r} \mathbf{A}_\theta \end{bmatrix} \\
\mathbf{A}_{nl} &= \begin{bmatrix} -\frac{\tau}{\xi} \left(\frac{1}{\xi} u_{s,s} - \frac{\tau}{\xi} u_{s,\theta} - \frac{1}{\xi} \hat{\kappa}_1 u_r + \frac{1}{\xi} \hat{\kappa}_2 u_\theta \right) \mathbf{A}_{s,\theta} & -\frac{\hat{\kappa}_1}{\xi} \left(\frac{1}{\xi} u_{s,s} - \frac{\tau}{\xi} u_{s,\theta} - \frac{1}{\xi} \hat{\kappa}_1 u_r + \frac{1}{\xi} \hat{\kappa}_2 u_\theta \right) \mathbf{A}_r & \frac{\hat{\kappa}_2}{\xi} \left(\frac{1}{\xi} u_{s,s} - \frac{\tau}{\xi} u_{s,\theta} - \frac{1}{\xi} \hat{\kappa}_1 u_r + \frac{1}{\xi} \hat{\kappa}_2 u_\theta \right) \mathbf{A}_\theta \\ +\frac{\hat{\kappa}_1}{\xi^2} (u_{r,s} - \tau u_{r,\theta} + \hat{\kappa}_1 u_s) \mathbf{A}_s & -\frac{1}{\xi} \hat{\kappa}_1 u_r + \frac{1}{\xi} \hat{\kappa}_2 u_\theta \mathbf{A}_r & -\frac{1}{\xi} \hat{\kappa}_1 u_r + \frac{1}{\xi} \hat{\kappa}_2 u_\theta \mathbf{A}_\theta \\ -\frac{\hat{\kappa}_2}{\xi^2} (u_{\theta,s} - \tau u_{\theta,\theta} - \hat{\kappa}_2 u_s) \mathbf{A}_s & -\frac{\tau}{\xi^2} (u_{r,s} - \tau u_{r,\theta} + \hat{\kappa}_1 u_s) \mathbf{A}_{r,\theta} & -\frac{\tau}{\xi^2} (u_{\theta,s} - \tau u_{\theta,\theta} - \hat{\kappa}_2 u_s) \mathbf{A}_{\theta,\theta} \\ u_{s,r} \mathbf{A}_{s,r} & u_{r,r} \mathbf{A}_{r,r} & u_{\theta,r} \mathbf{A}_{\theta,r} \\ \frac{1}{r^2} u_{s,\theta} \mathbf{A}_{s,\theta} & \frac{1}{r^2} (u_{r,\theta} - u_\theta) \mathbf{A}_{r,\theta} & -\frac{1}{r^2} (u_{r,\theta} - u_\theta) \mathbf{A}_\theta \\ & +\frac{1}{r^2} (u_{\theta,\theta} + u_r) \mathbf{A}_r & +\frac{1}{r^2} (u_{\theta,\theta} + u_r) \mathbf{A}_{\theta,\theta} \\ \frac{1}{r} (u_{s,r} \mathbf{A}_{s,\theta} + u_{s,\theta} \mathbf{A}_{s,r}) & \frac{1}{r} (u_{r,\theta} - u_\theta) \mathbf{A}_{r,r} & -\frac{1}{r} u_{r,r} \mathbf{A}_\theta + \frac{1}{r} u_{\theta,r} \mathbf{A}_{\theta,\theta} \\ & +\frac{1}{r} u_{r,r} \mathbf{A}_{r,\theta} + \frac{1}{r} u_{\theta,r} \mathbf{A}_r & +\frac{1}{r} (u_{\theta,\theta} + u_r) \mathbf{A}_{\theta,r} \\ \frac{1}{r\xi} (u_{s,s} - \tau u_{s,\theta} - \hat{\kappa}_1 u_r + \hat{\kappa}_2 u_\theta) \mathbf{A}_{s,\theta} - \frac{\tau}{r\xi} u_{s,\theta} \mathbf{A}_{s,\theta} & -\frac{\hat{\kappa}_1}{r\xi} u_{s,\theta} \mathbf{A}_r - \frac{\tau}{r\xi} (u_{r,\theta} - u_\theta) \mathbf{A}_{r,\theta} & -\frac{1}{r\xi} (u_{r,s} - \tau u_{r,\theta} + \hat{\kappa}_1 u_s) \mathbf{A}_\theta \\ +\frac{\hat{\kappa}_1}{r\xi} (u_{r,\theta} - u_\theta) \mathbf{A}_s & +\frac{1}{r\xi} (u_{r,s} - \tau u_{r,\theta} + \hat{\kappa}_1 u_s) \mathbf{A}_{r,\theta} & +\frac{\hat{\kappa}_2}{r\xi} u_{s,\theta} \mathbf{A}_\theta \\ -\frac{\hat{\kappa}_2}{r\xi} (u_{\theta,\theta} + u_r) \mathbf{A}_s & +\frac{1}{r\xi} (u_{\theta,s} - \tau u_{\theta,\theta} - \hat{\kappa}_2 u_s) \mathbf{A}_r & +\frac{1}{r\xi} (u_{\theta,s} - \tau u_{\theta,\theta} - \hat{\kappa}_2 u_s) \mathbf{A}_{\theta,\theta} \\ & & -\frac{\tau}{r\xi} (u_{\theta,\theta} + u_r) \mathbf{A}_{\theta,\theta} \\ \frac{1}{\xi} (u_{s,s} - \tau u_{s,\theta} - \hat{\kappa}_1 u_r + \hat{\kappa}_2 u_\theta) \mathbf{A}_{s,r} - \frac{\tau}{\xi} u_{s,r} \mathbf{A}_{s,\theta} & \frac{1}{\xi} (u_{r,s} - \tau u_{r,\theta} + \hat{\kappa}_1 u_s) \mathbf{A}_{r,r} & \frac{\hat{\kappa}_2}{\xi} u_{s,r} \mathbf{A}_\theta - \frac{\tau}{\xi} u_{\theta,r} \mathbf{A}_{\theta,\theta} \\ +\frac{\hat{\kappa}_1}{\xi} u_{r,r} \mathbf{A}_s - \frac{\hat{\kappa}_2}{\xi} u_{\theta,r} \mathbf{A}_s & -\frac{\hat{\kappa}_1}{\xi} u_{s,r} \mathbf{A}_r - \frac{\tau}{\xi} u_{r,r} \mathbf{A}_{r,\theta} & +\frac{1}{\xi} (u_{\theta,s} - \tau u_{\theta,\theta} - \hat{\kappa}_2 u_s) \mathbf{A}_{\theta,r} \end{bmatrix}
\end{aligned}$$

In Eqs. (12), (14), (15), (19), (20), (27), (34), (37) and (38), dA should be replaced by ξdA .

Correction in Appendix D

The matrices $\tilde{\mathbf{P}}_i$ for $i = 1, 2, 3, 4$ in Eq. (38) can be given as follows:

$$\begin{aligned}
 \tilde{\mathbf{P}}_1 &= \frac{1}{\xi^2} \begin{bmatrix} \sigma_{ss}(\hat{\kappa}_1^2 + \hat{\kappa}_2^2) \mathbf{A}_s^T \mathbf{A}_s \\ + \xi^2 \sigma_{rr} \mathbf{A}_{s,r}^T \mathbf{A}_{s,r} \\ + \left(\frac{\xi^2 \sigma_{r\theta}}{r} - \xi \tau \sigma_{sr} \right) (\mathbf{A}_{s,r}^T \mathbf{A}_{s,\theta} \\ + \mathbf{A}_{s,\theta}^T \mathbf{A}_{s,r}) + \left(\frac{\xi^2 \sigma_{\theta\theta}}{r^2} \right. \\ \left. + \sigma_{ss} \tau^2 - 2 \tau \sigma_{\theta s} \frac{\xi}{r} \right) \mathbf{A}_{s,\theta}^T \mathbf{A}_{s,\theta} \end{bmatrix} \\
 &\quad \begin{bmatrix} \left(\sigma_{ss} \hat{\kappa}_1 \tau - \frac{\xi}{r} \sigma_{\theta s} \hat{\kappa}_1 \right) (\mathbf{A}_{s,\theta}^T \mathbf{A}_r \\ - \mathbf{A}_s^T \mathbf{A}_{r,\theta}) - \frac{\xi}{r} \sigma_{\theta s} \hat{\kappa}_2 \mathbf{A}_s^T \mathbf{A}_r \\ + \xi \sigma_{sr} \hat{\kappa}_1 (\mathbf{A}_s^T \mathbf{A}_{r,r} - \mathbf{A}_{s,r}^T \mathbf{A}_r) \\ + \xi \sigma_{sr} \hat{\kappa}_2 (\mathbf{A}_{s,r}^T \mathbf{A}_\theta - \mathbf{A}_s^T \mathbf{A}_{\theta,r}) \end{bmatrix} \\
 &\quad \begin{bmatrix} \left(\sigma_{ss} \hat{\kappa}_1^2 + \frac{\xi^2}{r^2} \sigma_{\theta\theta} \right) \mathbf{A}_r^T \mathbf{A}_r \\ + \left(\sigma_{ss} \tau^2 + \frac{\xi^2}{r^2} \sigma_{\theta\theta} \right. \\ \left. - \sigma_{ss} \hat{\kappa}_1 \hat{\kappa}_2 \mathbf{A}_r^T \mathbf{A}_\theta + \left(\frac{\xi}{r} \tau \sigma_{\theta s} \right. \right. \\ \left. \left. - \mathbf{A}_{r,\theta}^T \mathbf{A}_s \right) - \frac{\xi}{r} \sigma_{\theta s} \hat{\kappa}_2 \mathbf{A}_r^T \mathbf{A}_s \right. \\ \left. - \frac{2\xi}{r} \tau \sigma_{\theta s} \right) \mathbf{A}_{r,\theta}^T \mathbf{A}_{r,\theta} + \left(\frac{\xi^2}{r} \sigma_{r\theta} \right. \\ \left. - \frac{\xi^2}{r^2} \sigma_{\theta\theta} \right) (\mathbf{A}_{r,\theta}^T \mathbf{A}_\theta - \mathbf{A}_r^T \mathbf{A}_{\theta,\theta}) \\ \left. + \xi \sigma_{sr} \hat{\kappa}_1 (\mathbf{A}_{r,r}^T \mathbf{A}_s - \mathbf{A}_r^T \mathbf{A}_{s,r}) \right. \\ \left. - \xi \sigma_{sr} \tau \right) (\mathbf{A}_{r,r}^T \mathbf{A}_{r,\theta} + \mathbf{A}_{r,\theta}^T \mathbf{A}_{r,r}) \\ \left. + \frac{\xi^2}{r} \sigma_{r\theta} (\mathbf{A}_r^T \mathbf{A}_{\theta,r} - \mathbf{A}_{r,\theta}^T \mathbf{A}_r) \right. \\ \left. + \xi^2 \sigma_{rr} \mathbf{A}_{r,r}^T \mathbf{A}_{r,r} \right. \\ \left. + \frac{\xi^2}{r} \sigma_{r\theta} (\mathbf{A}_{\theta,r}^T \mathbf{A}_r - \mathbf{A}_{\theta,\theta}^T \mathbf{A}_{r,r}) \right. \\ \left. + \left(\frac{\xi^2}{r} \sigma_{r\theta} - \xi \sigma_{sr} \tau \right) (\mathbf{A}_{\theta,\theta}^T \mathbf{A}_{\theta,r} \right. \\ \left. + \mathbf{A}_{\theta,r}^T \mathbf{A}_{\theta,\theta}) \end{bmatrix} \\
 \tilde{\mathbf{P}}_2 &= \frac{1}{\xi^2} \begin{bmatrix} \left(\frac{\xi}{r} \sigma_{\theta s} - \tau \sigma_{ss} \right) \mathbf{A}_{s,\theta}^T \mathbf{A}_s \\ + \xi \sigma_{sr} \mathbf{A}_{s,r}^T \mathbf{A}_s \\ - \hat{\kappa}_1 \sigma_{ss} \mathbf{A}_r^T \mathbf{A}_s \\ \hat{\kappa}_2 \sigma_{ss} \mathbf{A}_\theta^T \mathbf{A}_s \\ - \hat{\kappa}_1 \sigma_{ss} \mathbf{A}_r^T \mathbf{A}_s \\ - \hat{\kappa}_2 \sigma_{ss} \mathbf{A}_\theta^T \mathbf{A}_s \end{bmatrix} \\
 &\quad \begin{bmatrix} \hat{\kappa}_1 \sigma_{ss} \mathbf{A}_s^T \mathbf{A}_r \\ \left(\frac{\xi}{r} \sigma_{\theta s} - \tau \sigma_{ss} \right) \mathbf{A}_{r,\theta}^T \mathbf{A}_r \\ + \xi \sigma_{sr} \mathbf{A}_{r,r}^T \mathbf{A}_r \\ - \frac{\xi}{r} \sigma_{\theta s} \mathbf{A}_\theta^T \mathbf{A}_r \\ - \hat{\kappa}_1 \sigma_{ss} \mathbf{A}_s^T \mathbf{A}_r \\ - \hat{\kappa}_2 \sigma_{ss} \mathbf{A}_s^T \mathbf{A}_r \end{bmatrix} \\
 &\quad \begin{bmatrix} -\hat{\kappa}_2 \sigma_{ss} \mathbf{A}_s^T \mathbf{A}_\theta \\ \frac{\xi}{r} \sigma_{\theta s} \mathbf{A}_r^T \mathbf{A}_\theta \\ \left(\frac{\xi}{r} \sigma_{\theta s} - \tau \sigma_{ss} \right) \mathbf{A}_{\theta,\theta}^T \mathbf{A}_\theta \\ + \xi \sigma_{sr} \mathbf{A}_{\theta,r}^T \mathbf{A}_\theta \end{bmatrix} \\
 \tilde{\mathbf{P}}_3 &= \frac{1}{\xi^2} \begin{bmatrix} \left(\frac{\xi}{r} \sigma_{\theta s} - \tau \sigma_{ss} \right) \mathbf{A}_s^T \mathbf{A}_{s,\theta} \\ + \xi \sigma_{sr} \mathbf{A}_s^T \mathbf{A}_{s,r} \\ \hat{\kappa}_1 \sigma_{ss} \mathbf{A}_r^T \mathbf{A}_s \\ - \hat{\kappa}_2 \sigma_{ss} \mathbf{A}_\theta^T \mathbf{A}_s \end{bmatrix} \\
 &\quad \begin{bmatrix} -\hat{\kappa}_1 \sigma_{ss} \mathbf{A}_s^T \mathbf{A}_r \\ -\tau \sigma_{ss} \mathbf{A}_r^T \mathbf{A}_{r,\theta} \\ + \frac{\xi}{r} \sigma_{\theta s} \mathbf{A}_r^T \mathbf{A}_{r,\theta} + \xi \sigma_{sr} \mathbf{A}_r^T \mathbf{A}_{r,r} \\ \frac{\xi}{r} \sigma_{\theta s} \mathbf{A}_\theta^T \mathbf{A}_r \end{bmatrix} \\
 &\quad \begin{bmatrix} \hat{\kappa}_2 \sigma_{ss} \mathbf{A}_s^T \mathbf{A}_\theta \\ -\frac{\xi}{r} \sigma_{\theta s} \mathbf{A}_r^T \mathbf{A}_\theta \\ \left(\frac{\xi}{r} \sigma_{\theta s} - \tau \sigma_{ss} \right) \mathbf{A}_\theta^T \mathbf{A}_{\theta,\theta} \\ + \xi \sigma_{sr} \mathbf{A}_\theta^T \mathbf{A}_{\theta,r} \end{bmatrix} \\
 \tilde{\mathbf{P}}_4 &= \frac{\sigma_{ss}}{\xi^2} \begin{bmatrix} \mathbf{A}_s^T \mathbf{A}_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_r^T \mathbf{A}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_\theta^T \mathbf{A}_\theta \end{bmatrix}
 \end{aligned}$$

Acknowledgements The authors acknowledge with thanks the helpful remarks of Professor A. Srinivasa.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.