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ORIGINAL PAPER

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The effect of a laser pulse and gravity field on a thermoelastic medium under Green–Naghdi theory

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Abstract The present work aims to investigate the effect of the gravitational field on a two-dimensional thermoelastic medium influenced by thermal loading due to a laser pulse. The bounding plane surface is heated by a non-Gaussian laser beam. The problem is discussed under Green–Naghdi theory with and without energy dissipation. The normal mode analysis method is used to get the expressions for the physical quantities. The results are illustrated graphically.

1 Introduction

The non-classical theories of thermoelasticity, so-called generalized thermoelasticity, have been developed to remove the paradox of the physically impossible phenomenon of infinite velocity of thermal signals in the conventional coupled thermoelasticity, Lord–Shulman theory [1] and Green–Lindsay theory [2]. In the 1990s, Green and Naghdi (G–N) have formulated three models (I, II, III) of thermoelasticity for homogeneous and isotropic material [3,4]. The model I of G–N theory after linearization reduced to the classical thermoelasticity theory. The model II of G–N theory [5] does not allow dissipation of the thermoelastic energy. In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among the constitutive variables. Chandrasekharaiah [6] used the Laplace method to study the one-dimensional thermal wave propagation in a half space based on the (G–N) theory of type II due to a sudden application of the temperature to the boundary. Recently, many thermoelastic problems have been discussed [7–9].

Model III of (G–N) theory confesses a dissipation of energy, where the constitutive equations are derived starting with a reduced energy equation. This model includes the thermal displacement gradient, the temperature gradient, and some independent constitutive variables.

The effect of gravity in the classical theory of elasticity is generally neglected. The effect of the gravity on the problem of propagation of waves in solids, in particular on an elastic globe, was first studied by Bromwich [10]. Ailawalia and Narah [11] depicted the effects of rotation and gravity in the generalized thermoelastic medium. Othman et al. [12] studied the influence of the gravitational field and rotation on the generalized thermoelastic medium using a dual-phase-lag model. Das et al. [13] investigated the surface waves under the influence of gravity in a non-homogeneous medium. Othman and Hilal [14] studied the rotation and gravitational field effect on two-temperature thermoelastic material with voids and temperature-dependent properties using (G–N III).

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Othman et al. [15] explained the effect of gravity on plane waves in a rotating thermomicrostretch elastic solid for a mode I crack with energy dissipation.

Very rapid thermal processes under the action of an ultra-short laser pulse are interesting from the standpoint of thermoelasticity because they require deformation fields and an analysis of the coupled temperature. This means that the laser pulse energy absorption results in a localized temperature increase, which causes thermal expansion and generates rapid movements in the structure elements, thus causing the rise in vibrations. These effects make materials susceptible to the diffusion of heat by conduction.

The ultra-short lasers are those with the pulse duration ranging from nanoseconds to femtoseconds. The high intensity, energy flux, and ultra-short duration laser beam have studied situations where very large thermal gradients or an ultra-high heating rate may exist on the boundaries, this in the case of ultra-short-pulsed laser heating [16,17]. The microscopic two-step models that are parabolic and hyperbolic are useful for modifying the material as thin films. When a metal film is heated by a laser pulse, a thermoelastic wave is generated due to thermal expansion near the surface. Wang and Xu [18] studied the stress wave induced by pico and femtosecond laser pulses in a semi infinite metal by expressing the laser pulse energy as a Fourier series. Othman et al. [19] studied the effect of rotation on a fiber-reinforced on the generalized magneto-thermoelasticity subject to thermal loading due to the laser pulse.

The present work aims to determine the distributions of the displacement components, the stresses, the temperature and the volume fraction field in a homogeneous isotropic thermoelastic medium under the influence of the laser pulse in the case of the absence and the presence of the gravity and two values of time. The model is illustrated in the context of (G–N) theory of types II and III. Expressions for the physical quantities are obtained using the normal mode analysis and are represented graphically.

2 Formulation of the problem and basic equations

Consider as a homogeneous, linear, isotropic, thermoelastic medium a half space ($x \geq 0$), the rectangular Cartesian coordinate system (x, y, z) having originated on the surface $z = 0$. In the used equations, a dot denotes differentiation with respect to time, while a comma denotes the material derivative. For two-dimensional problems, we assume the dynamic displacement vector as $\boldsymbol{u} = (u, v, 0)$, and all the considered quantities are functions of the time variable t and of the coordinates x and y.

According to Green and Naghdi [5], the field equations and the constitutive relations of a linear homogenous, isotropic generalized thermoelastic medium for body forces, heat sources and extrinsic equilibrated body force in the context of (G–N) theory of type III for can be written as

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta T_{,i} + G_i = \rho \ddot{u}_i, \tag{1}$$

$$k T_{,ii} + k^* \dot{T}_{,ii} = \rho C_e \ddot{T} + \beta T_0 \ddot{e} - \rho \dot{Q}, \tag{2}$$

$$\sigma_{ij} = [\lambda u_{k,k} - \beta T] \delta_{ij} + 2\mu e_{ij}, \quad i, j, k = 1, 2, 3, \tag{3}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad i, j = 1, 2, 3$$

where λ, μ are the Lamé's constants, T is the temperature distribution, $\beta = (3\lambda + 2\mu)\alpha_t$ such that α_t is the coefficient of thermal expansion, ρ is the density, C_e is the specific heat, k is the thermal conductivity, k^* is the material constant characteristic of the theory, T_0 is the reference temperature chosen so that $|(T - T_0)/T_0| << 1$, e is the dilation, e_{ij} are the strain tensor components, σ_{ij} are the stress tensor components, δ_{ij} is the Kronecker delta, G_i is the gravity force, and Q is the heat input of the laser pulse.

As $k^* \to 0$, Eq. (2) will be reduced to the heat condition equation in (G–N) theory (of type II).

The plate surface is illuminated by the laser pulse given by the heat input

$$Q = \frac{I_0 \gamma}{2\pi r^2} \exp\left(-\frac{y^2}{r^2} - \gamma x\right) f(t) \tag{4}$$

where I_0 is the absorbed energy, r is the beam radius, and γ is constant.

The temporal profile $f(t)$ can be defined as

$$f(t) = \frac{t}{t_0^2} \exp\left(-\frac{t}{t_0}\right)$$

where t_0 is the pulse rising time.

The basic governing equations of a linear, homogenous thermoelastic medium under the influence of a laser pulse and the gravitational field will be in the forms:

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial e}{\partial x} - \beta\frac{\partial T}{\partial x} + \rho g\frac{\partial v}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$\mu\nabla^2 v + (\lambda+\mu)\frac{\partial e}{\partial y} - \beta\frac{\partial T}{\partial y} - \rho g\frac{\partial u}{\partial x} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{6}$$

$$k\nabla^2 T + k^*\frac{\partial}{\partial t}\nabla^2 T = \rho C_e\frac{\partial^2 T}{\partial t^2} + \beta T_0\frac{\partial^2 e}{\partial t^2} - \rho\frac{\partial}{\partial t}Q. \tag{7}$$

Introducing the following dimensionless variables:

$$x' = \frac{\omega_1^*}{c_1}x,\quad x' = \frac{\omega_1^*}{c_1}x,\quad u' = \frac{\omega_1^*}{c_1}u,\quad v' = \frac{\omega_1^*}{c_1}v,\quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu},\quad \theta' = \frac{T}{T_0},$$
$$g' = \frac{g}{c_1\omega_1^*},\quad t' = \omega_1^* t,\quad Q' = \frac{Q}{\omega_1^* T_0 C_e},\quad c_1^2 = \left(\frac{\lambda+2\mu}{\rho}\right)\ \text{and}\ \omega_1^* = \frac{\rho C_e c_1^2}{k}, \tag{8}$$

Eqs. (5)–(7) will be rewritten into the non-dimensional forms with dropping primes for convenience:

$$\nabla^2 u + b_1\frac{\partial e}{\partial x} - b_2\frac{\partial\theta}{\partial x} + b_3\frac{\partial v}{\partial x} = b_4\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\nabla^2 v + b_1\frac{\partial e}{\partial y} - b_2\frac{\partial\theta}{\partial z} - b_3\frac{\partial u}{\partial x} = b_4\frac{\partial^2 v}{\partial t^2}, \tag{10}$$

$$\varepsilon_3\nabla^2\theta + \varepsilon_2\frac{\partial}{\partial t}\nabla^2\theta = \frac{\partial^2\theta}{\partial t^2} + \varepsilon_1\frac{\partial^2 e}{\partial t^2} - \frac{\partial}{\partial t}Q \tag{11}$$

where

$$b_1 = \frac{\lambda+\mu}{\mu},\quad b_2 = \frac{\beta T_0}{\mu},\quad b_3 = \frac{\rho g c_1^2}{\mu},\quad b_4 = \frac{\rho c_1^2}{\mu},\quad \varepsilon_1 = \frac{\beta}{\rho C_e},\quad \varepsilon_2 = \frac{k^*\omega_1^*}{\rho C_e c_1^2},\quad \varepsilon_3 = \frac{k}{\rho C_e c_1^2}.$$

Here ε_1, ε_2 and ε_3 are the coupling constants.

Using the expressions relating the displacement components $u(x, y, t)$ and $v(x, y, t)$ to each of the potential functions $\psi_1(x, y, t)$ and $\psi_{\mathbf{2}}(x, y, t)$ in the dimensionless forms:

$$u = \frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y},\quad v = \frac{\partial\psi_1}{\partial y} - \frac{\partial\psi_2}{\partial x} \tag{12}$$

gives

$$e = \nabla^2\psi_1\ \text{and}\ \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \nabla^2\psi_2. \tag{13}$$

Using (12) and (13) into (9)–(11) yields

$$\left[(1+b_1)\nabla^2 - b_4\frac{\partial^2}{\partial t^2}\right]\psi_1 - b_3\frac{\partial}{\partial x}\psi_2 - b_2\theta = 0, \tag{14}$$

$$b_3\frac{\partial}{\partial x}\psi_1 + \left[\nabla^2 - b_4\frac{\partial^2}{\partial t^2}\right]\psi_2 = 0, \tag{15}$$

$$-\varepsilon_1\frac{\partial^2}{\partial t^2}\nabla^2\psi_1 + \left(\varepsilon_3 + \varepsilon_2\frac{\partial}{\partial t}\right)\nabla^2\theta - \frac{\partial^2\theta}{\partial t^2} = -\frac{\partial}{\partial t}Q. \tag{16}$$

The constitutive relations will be

$$\sigma_{xx} = b_4 \frac{\partial u}{\partial x} + (b_1 - 1)\frac{\partial v}{\partial y} - b_2\theta, \tag{17}$$

$$\sigma_{yy} = b_4 \frac{\partial v}{\partial y} + (b_1 - 1)\frac{\partial u}{\partial x} - b_2\theta, \tag{18}$$

$$\sigma_{zz} = (b_1 - 1)e - b_2\theta, \tag{19}$$

$$\sigma_{xy} = \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \quad \text{and} \quad \sigma_{xz} = \sigma_{zy} = 0. \tag{20}$$

3 The normal mode analysis

We can decompose the solution of the physical quantities in terms of the normal mode as follows:

$$[\psi_1, \psi_2, \theta](x, y, t) = [\psi_1^*, \psi_2^*, \theta^*](x) \exp[i(\omega t + ay)] \tag{21}$$

where $[\psi_1^*, \psi_2^*, \theta^*](x)$ are the amplitudes of the physical quantities, ω is the angular frequency, $i = \sqrt{-1}$, and a is the wave number.

Using (21), Eqs. (14)–(16) will be

$$[D^2 - B_1]\psi_1^* - B_2 D\psi_2^* - B_3\theta^* = 0, \tag{22}$$

$$b_3 D\psi_1^* + [D^2 - B_4]\psi_2^* = 0, \tag{23}$$

$$B_5[D^2 - a^2]\psi_1^* + [D^2 - a^2]\theta^* = B_6 \frac{\partial}{\partial t} Q \tag{24}$$

where $B_1 = a^2 - \frac{b_4\omega^2}{1+b_1}$, $B_2 = \frac{b_3}{1+b_1}$, $B_3 = \frac{b_2}{1+b_1}$, $b_6 = \frac{a_3}{b_1}$, $B_4 = a^2 - b_4\omega^2$, $b_9 = \varepsilon_1\omega^2$, $B_5 = \frac{\varepsilon_1\omega^2}{\varepsilon_3 + i\varepsilon_2\omega + \omega^2}$, $B_6 = \frac{-1}{\varepsilon_3 + i\varepsilon_2\omega + \omega^2}$, and $D = \frac{\mathrm{d}}{\mathrm{d}x}$.

Eliminating ψ_1^*, ψ_2^*, and θ^* from Eqs. (22)–(24) gives the differential equations

$$[D^6 - B_7 D^4 + B_8 D^2 - B_9]\psi_1^* = B_{10}\left(1 - \frac{t}{t_0}\right)\exp\left[-\left(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay\right)\right], \tag{25}$$

$$[D^6 - B_7 D^4 + B_8 D^2 - B_9]T^* = B_{11}\left(1 - \frac{t}{t_0}\right)\exp\left[-\left(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay\right)\right], \tag{26}$$

$$[D^6 - B_7 D^4 + B_8 D^2 - B_9]\psi_2^* = B_{12}\left(1 - \frac{t}{t_0}\right)\exp\left[-\left(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay\right)\right] \tag{27}$$

where

$$B_7 = B_1 + B_4 - B_2 b_3 - B_3 B_5 + a^2, \quad B_8 = a^2 B_1 + a^2 B_4 - a^2 B_2 b_3 - a^2 B_3 B_5 - B_3 B_5 B_4 + B_1 B_4,$$

$$B_9 = a^2 B_1 B_4 - a^2 B_2 b_3 - a^2 B_3 B_5 B_4, \quad B_{10} = B_3 B_6(\gamma^2 - B_4)\frac{I_o\gamma}{2\pi r^2 t_o^2},$$

$$B_{11} = B_6\left[(\gamma^2 - B_1)(\gamma^2 - B_4) + (\gamma^2 B_2 b_3)\right]\frac{I_o\gamma}{2\pi r^2 t_o^2}, \quad B_{12} = -B_3 B_6 b_3 \frac{I_o\gamma^2}{2\pi r^2 t_o^2}.$$

Equation (25) can be factored as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)\psi_1^* = B_{10}\left(1 - \frac{t}{t_0}\right)\exp\left[-\left(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay\right)\right] \tag{28}$$

where $k_n^2 (n = 1, 2, 3)$ are the roots of the characteristic equation of the homogeneous equations of Eqs. (25)–(27).

The general solutions of (25)–(27) bound as $x \to \infty$ are given by

$$\psi_1(x,y,t) = \sum_{n=1}^{3} R_n \exp(-k_n x + i\omega t + iay) + L_1 B_{10} f_1, \tag{29}$$

$$\psi_2(x,y,t) = \sum_{n=1}^{3} H_{1n} R_n \exp(-k_n x + i\omega t + iay) + L_1 B_{12} f_1, \tag{30}$$

$$\theta(x,y,t) = \sum_{n=1}^{3} H_{2n} R_n \exp(-k_n x + i\omega t + iay) + L_1 B_{11} f_1. \tag{31}$$

Here

$$H_{1n} = \frac{-b_3 k_n}{(k_n^2 - B_4)}, \quad n = 1,2,3, \quad H_{2n} = \frac{(k_n^2 - B_1) - B_2 H_{1n} k_n}{B_3}, \quad n = 1,2,3$$

$$L_1 = -\frac{1}{\gamma^6 - B_7\gamma^4 + B_8\gamma^2 - B_9}, \quad f_1 = \left(1 - \frac{t}{t_0}\right)\exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right)$$

where $R_n (n = 1,2,3)$ are some undefined coefficients.

To obtain the components of the displacement vector, substituting (29) and (30) into (12) gives

$$u(x,y,t) = \sum_{n=1}^{3} M_{1n} R_n \exp(-k_n x + i\omega t + iay) - \left(\gamma I_1 + \frac{2yI_2}{r^2}\right)\exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right), \tag{32}$$

$$v(x,y,t) = \sum_{n=1}^{3} M_{2n} R_n \exp(-k_n x + i\omega t + iay) + \left(-\gamma I_2 + \frac{2yI_1}{r^2}\right)\exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right) \tag{33}$$

where $M_{1n} = -k_n + iaH_{1n}, \quad M_{2n} = ia + k_n H_{1n}, \quad n = 1,2,3.$

To get the components of the stress tensor, substitute (32), (33) and (31) into (17)–(20),

$$\sigma_{xx}(x,y,t) = \sum_{n=1}^{3} H_{3n} R_n \exp(-k_n x + i\omega t + iay) + I_4 \exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right), \tag{34}$$

$$\sigma_{yy}(x,y,t) = \sum_{n=1}^{3} H_{4n} R_n \exp(-k_n x + i\omega t + iay) + I_5 \exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right), \tag{35}$$

$$\sigma_{zz}(x,y,t) = \sum_{n=1}^{3} H_{5n} R_n \exp(-k_n x + i\omega t + iay) + I_6 \exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right), \tag{36}$$

$$\sigma_{xy}(x,y,t) = \sum_{n=1}^{3} H_{6n} R_n \exp(-k_n x + i\omega t + iay) + I_7 \exp\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \gamma x\right). \tag{37}$$

Here

$$H_{3n} = -b_4 k_n M_{1n} + iaM_{2n}(b_1 - 1) - b_2 H_{2n}, \quad H_{4n} = ib_4 a M_{2n} - k_n M_{1n}(b_1 - 1) - b_2 H_{2n},$$

$$H_{5n} = ia(b_1 - 1)M_{2n} - (b_1 - 1)k_n M_{1n} - b_2 H_{2n}, \quad H_{6n} = -k_n M_{2n} + iaM_{1n}, \quad n = 1,2,3,$$

$$I_1 = -B_{10}\left(1 - \frac{t}{t_0}\right)L_1, \quad I_2 = \frac{B_{12}}{B_{10}} I_1, \quad I_3 = \frac{B_{11}}{B_{10}} I_1,$$

$$I_4 = b_4\gamma\left(\gamma I_1 + \frac{2y}{r^2} I_2\right) - \frac{2}{r^2} b_1\left(I_1 + 2y\gamma I_2 - \frac{2y^2}{r^2} I_1\right) - b_2 I_3,$$

$$I_5 = (b_1 - 1)\gamma\left(\gamma I_1 + \frac{2y}{r^2} I_2\right) - \frac{2}{r^2} b_4\left(I_1 + 2y\gamma I_2 - \frac{2y^2}{r^2} I_1\right) - b_2 I_3,$$

$$I_6 = (b_1 - 1)\gamma \left(\gamma I_1 + \frac{2y}{r^2} I_2\right) - \frac{2}{r^2}(b_1 - 1)\left(I_1 - 2y\gamma I_2 + \frac{2y^2}{r^2} I_1\right) - b_2 I_3,$$

$$I_7 = -I_2 \left(\frac{2}{r^2} + \gamma^2 + \frac{4y^4}{r^4}\right).$$

4 Boundary conditions

In this section, we determine the constants $R_n (n = 1, 2, 3)$. The boundary conditions under consideration should suppress the positive exponentials to avoid unboundedness at infinity. The coefficients R_1, R_2, R_3 are chosen such that the boundary conditions on the surface at $x = 0$ are

(i) The mechanical boundary conditions

$$\sigma_{xx} = -p_1 \exp(\omega t + iay), \quad \sigma_{xy} = 0. \tag{38}$$

(ii) The thermal boundary condition on the surface of the half space

$$\frac{\partial \theta}{\partial x} = 0 \tag{39}$$

where p_1 is the magnitude of the mechanical force.

Substituting the expressions of the considered variables in the above boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^{3} H_{3n} R_n = -p_1, \tag{40}$$

$$\sum_{n=1}^{3} H_{6n} R_n = 0, \tag{41}$$

$$\sum_{n=1}^{3} -k_n H_{2n} R_n = 0. \tag{42}$$

Invoking the boundary conditions (38) and (39) at the surface $x = 0$ of the plate, we get a system of three equations (40)–(42). Applying the inverse of the matrix method, we then obtain the values of the three coefficients $R_n (n = 1, 2, 3)$.

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} H_{31} & H_{32} & H_{33} \\ H_{61} & H_{62} & H_{63} \\ -k_1 H_{21} & -k_2 H_{22} & -k_3 H_{23} \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ 0 \end{pmatrix}. \tag{43}$$

Hence, we obtain the expressions for the displacements, the temperature distribution, and the other physical quantities of the plate surface.

5 Numerical results and discussion

For numerical computations, following Dhaliwal and Singh [20] the magnesium material was chosen. All the units of the parameters used in the calculation are given in SI units. The constants of the problem are taken as

$$\lambda = 2.17 \times 10^{10}\,\text{N/m}^2, \quad \mu = 3.278 \times 10^{10}\,\text{N/m}^2, \quad K = 1.7 \times 10^2\,\text{W/m K}, \quad \rho = 1.74 \times 10^3\,\text{kg/m}^3,$$
$$\beta = 2.68 \times 10^6\,\text{N/m}^2\,\text{K}, \quad C_e = 1.04 \times 10^3\,\text{J/kg K}, \quad \omega_1^* = 3.58 \times 10^{11}/\text{s}, \quad \alpha_t = 1.78 \times 10^{-5}\,\text{K}^{-1},$$
$$T_0 = 298\,\text{K}.$$

The laser pulse parameters are

$$I_0 = 10^2\,\text{J/m}^2, \quad r = 0.2\,\mu\text{m}, \quad \gamma = 25/\text{m}, \quad t_0 = 10\,\text{ns}.$$

The comparisons were carried out for

$$p_1 = 0.25\,\text{N/m}^2,\quad k^* = 100\,\text{W/m K},\quad a = 0.5\,\text{m},\quad \omega = 2.9\,\text{rad/s},\quad x = 2\,\text{m},\quad t = 0.9\,\text{s},\quad g = 9.8\,\text{m/s}^2, \quad \text{and}\quad 0 \le x \le 2.5\,\text{m}.$$

The comparisons are established for the cases:

1. Different values of the gravity [$g = 9.8, 3\,\text{m/s}^2$ and $t = 0.9\,\text{s}$].
2. G–N theory type II and type III [$g = 9.8\,\text{m/s}^2$ and $t = 0.9\,\text{s}$].

These values are used for the distribution of the real parts of the displacement components, the temperature, and the stresses with the distance x for (G–N) theory of both types II and III in different values of the gravity effect $g = 9.8, 3$, and $t = 0.9$.

Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14 show the changes in the behavior of the physical quantities as functions of the distance x in 2D for different values of the gravity and time.

Figure 1 represents the distribution of the displacement components u in the case of $g = 9.8$ and $g = 3$ in the context of both types II and III of (G–N) theory. It is noticed that the distribution of u decreases with the increase in the gravity for $x > 0$, in both types of (G–N) theory II and III. Figure 2 illustrates that the

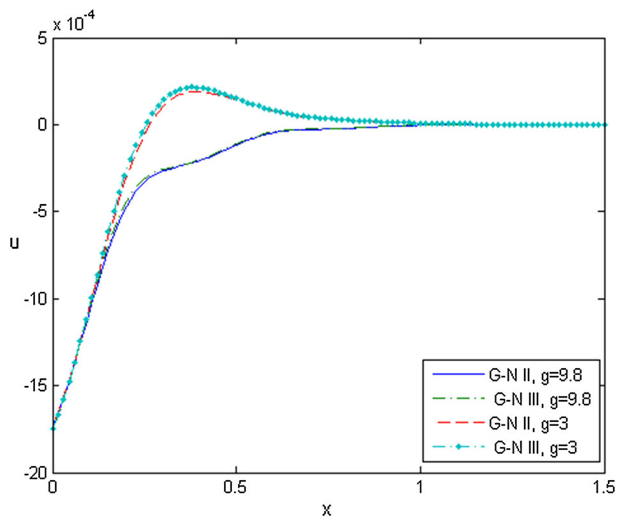

Fig. 1 Distribution of the displacement u against x

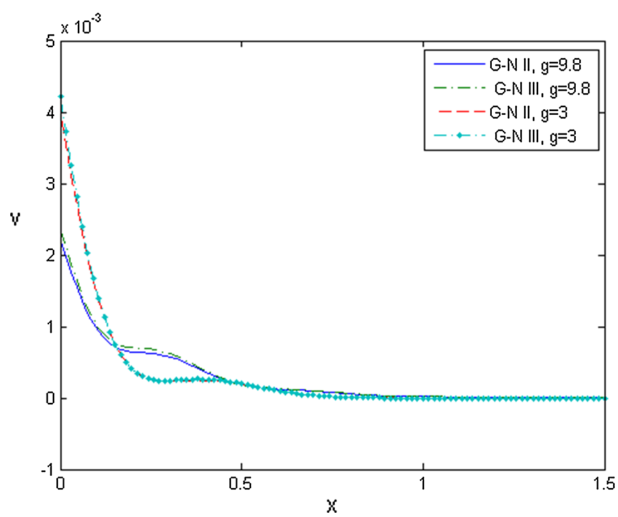

Fig. 2 Distribution of the displacement v against x

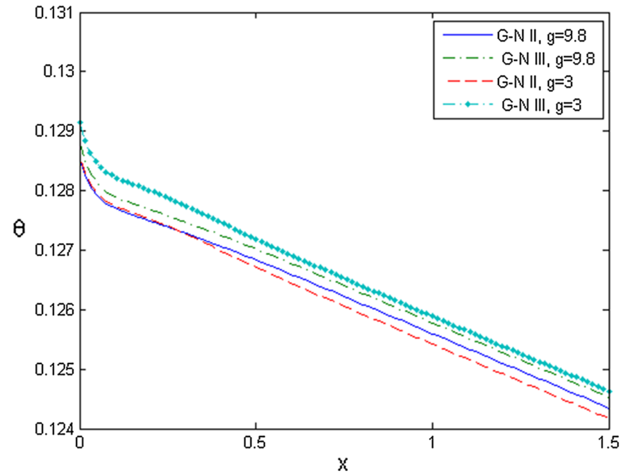

Fig. 3 Distribution of the temperature θ against x

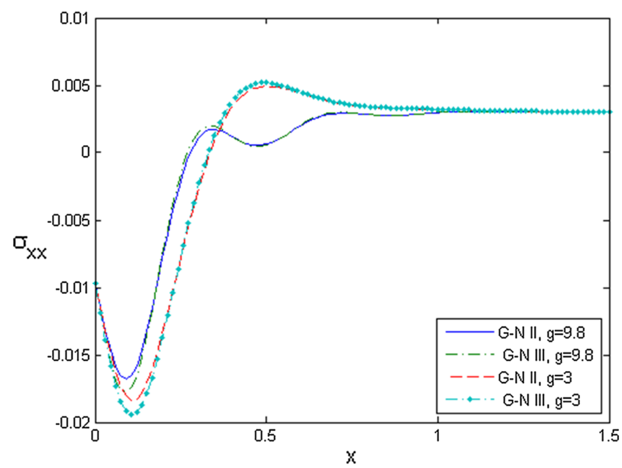

Fig. 4 Distribution of the stress component σ_{xx} against x

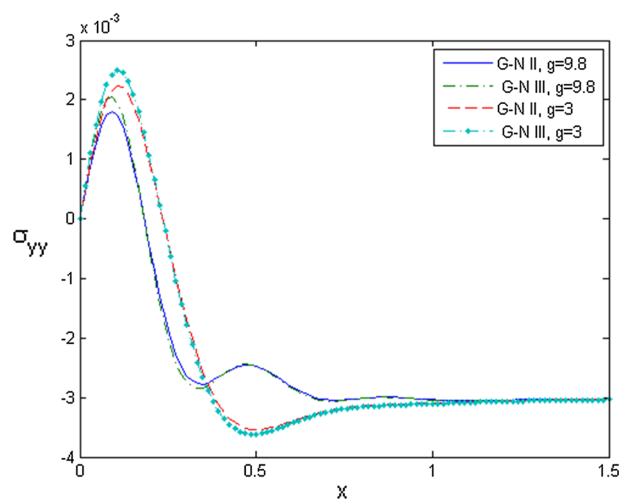

Fig. 5 Distribution of the stress component σ_{yy} against x

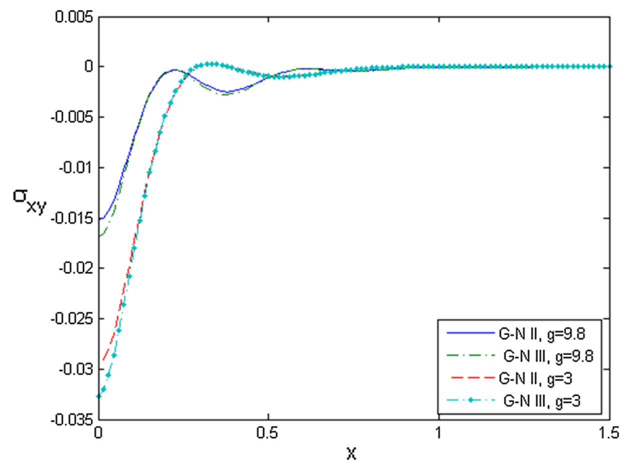

Fig. 6 Distribution of the stress component σ_{xy} against x

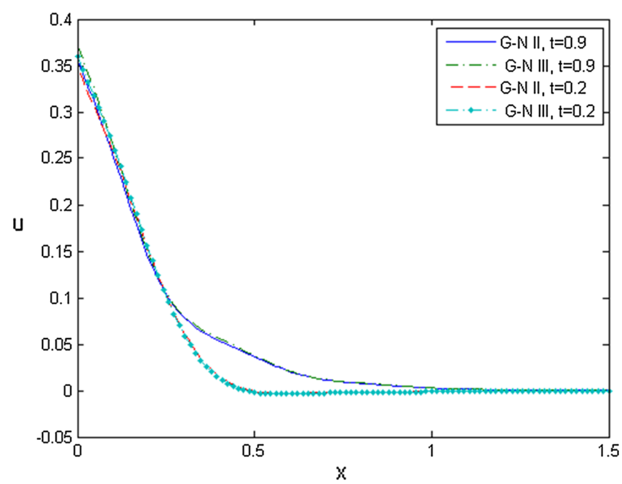

Fig. 7 Distribution of the displacement u against x

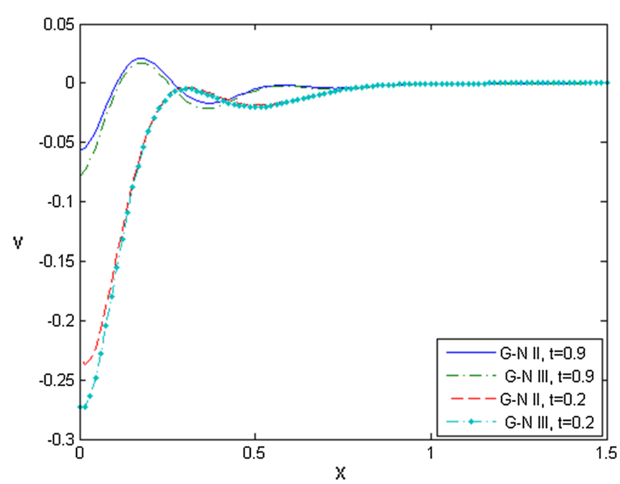

Fig. 8 Distribution of the displacement v against x

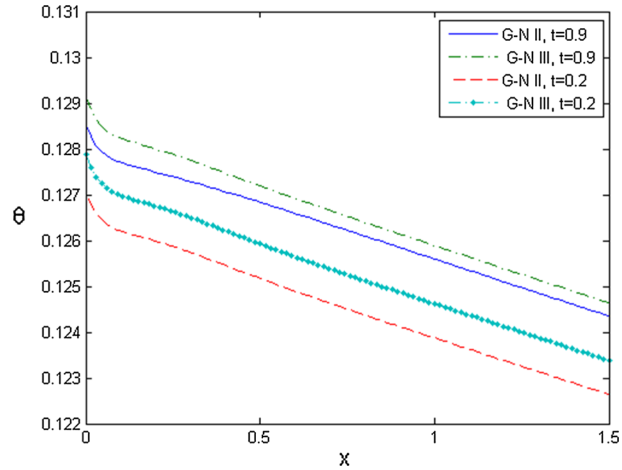

Fig. 9 Distribution of the temperature θ against x

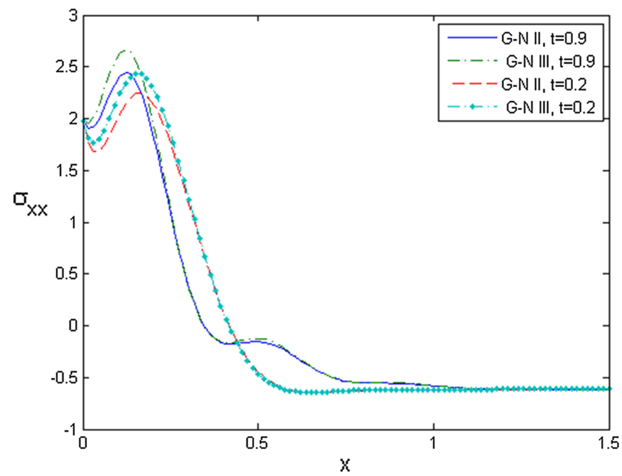

Fig. 10 Distribution of the stress component σ_{xx} against x

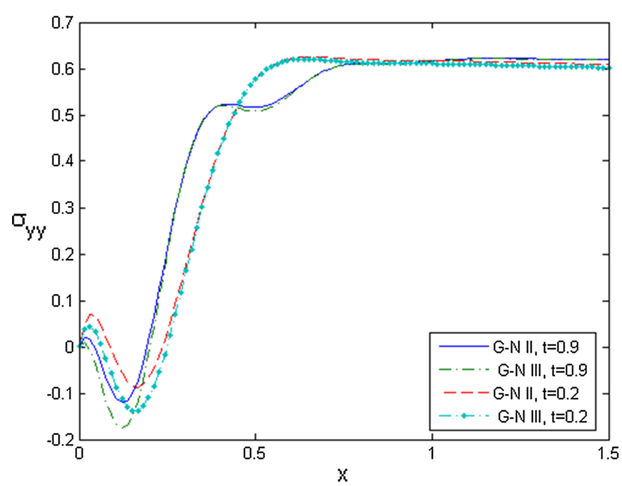

Fig. 11 Distribution of the stress component σ_{yy} against x

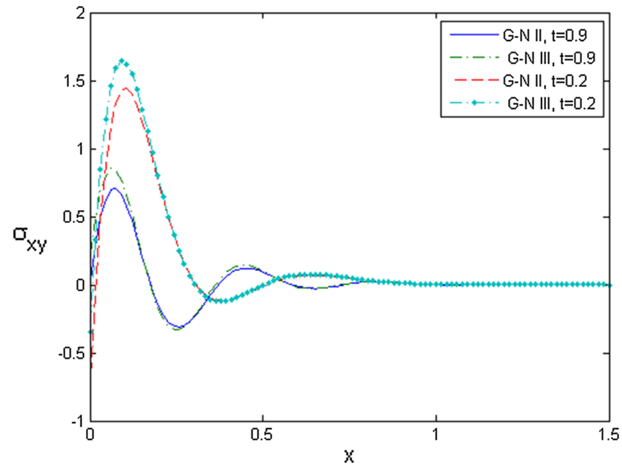

Fig. 12 Distribution of the stress component σ_{xy} against x

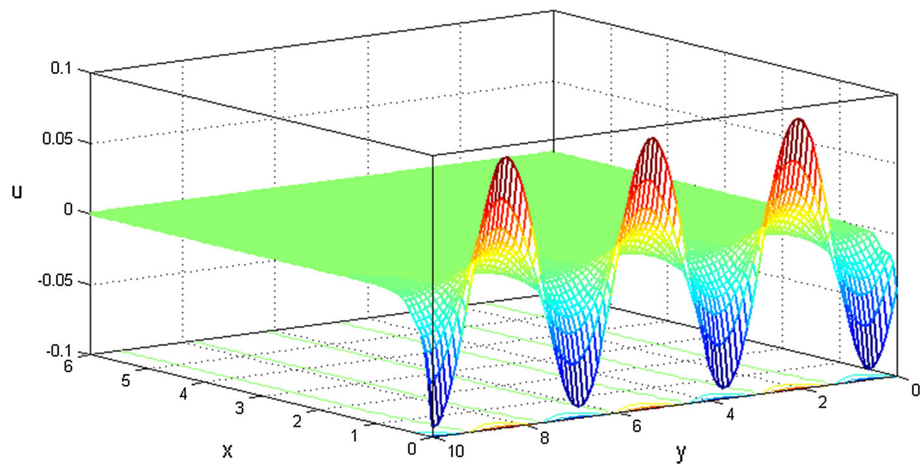

Fig. 13 3D curve of the displacement u versus the components of distance

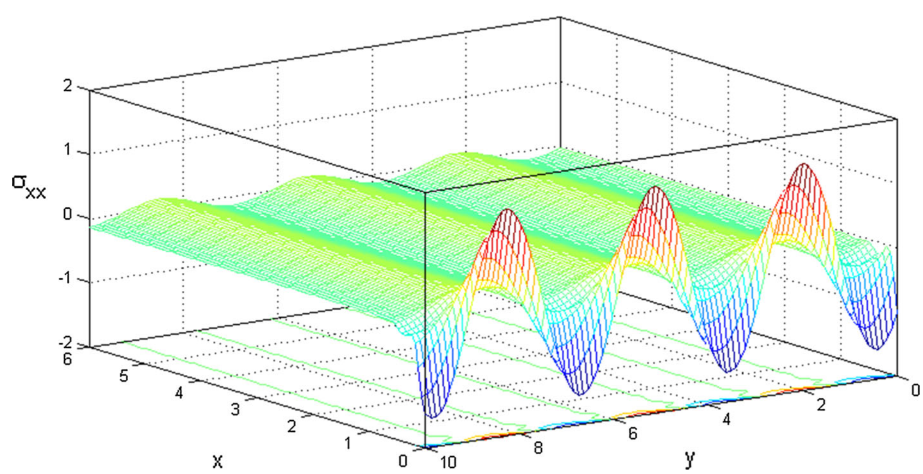

Fig. 14 3D curve of the displacement σ_{xx} versus the components of distances

distribution of v decreases with the increase in the gravity for type II of (G–N) theory, but it increases in the case of type III of (G–N) theory for $x > 0$. Figure 3 explains the distribution of the temperature θ in the case of $g = 9.8$ and $g = 3$ in the context of both types II and III of (G–N) theory. It is noticed that the distribution of

θ decreases with the increase in the gravity for $x > 0$, in type III of (G–N) theory, while an opposite situation takes place in type II of (G–N) theory. Figure 4 depicts the distribution of the stress component σ_{xx} in the context of both types II and III of (G–N) theory for $g = 9.8$ and $g = 3$. It is observed that the distribution of σ_{xx} decreases in the case of (G–N) theory of both types II and III in the range $0 \leq x \leq 0.15$, followed by an increase. Figure 5 explains the distribution of the stress σ_{yy} for $g = 9.8$ and $g = 3$ in the case of (G–N) theory of types II and III; it is seen that σ_{yy} increases then decreases in a small range. The gravity has a decreasing effect on the stress component in the interval $0 \leq x \leq 0.4$. Figure 6 shows the distribution of the stress component σ_{xy} for $g = 9.8$ and $g = 3$ in the case of (G–N) theory of types II and III. It is seen that the distribution of σ_{xy} increases with the increase in the gravity, and then converges to zero. Figure 7 represents the distribution of the displacement component u in the case of $t = 0.9$ and $t = 0.2$ in the context of both types II and III of (G–N) theory; it is noticed that the distribution of u decreases with the increase in time for $x > 0$, in both types of (G–N) theory II and III. Figure 8 shows that the distribution of v increases with the increase in time for both types II and III of (G–N) theory in the range $0 \leq x \leq 0.4$ and decreases in the range $0.4 \leq x \leq 0.7$ for $t = 0.9$ and $t = 0.2$. Figure 9 explains the distribution of the temperature θ for $t = 0.9$ and $t = 0.2$ in the context of both types II and III of (G–N) theory. It is noticed that it increases with the increase in time for both types II and III of (G–N) theory. Figure 10 depicts the distribution of the stress component σ_{xx} in the context of both types II and III of (G–N) theory for $t = 0.9$ and $t = 0.2$; it is observed that the distribution of σ_{xx} increases in the case of (G–N) theory of both types II and III in the range $0 \leq x \leq 0.2$ followed by decreasing behavior. Figure 11 explains the distribution of the stress σ_{yy} for $t = 0.9$ and $t = 0.2$ in the case of (G–N) theory of types II and III; it is seen that σ_{yy} decreases in the range $0 \leq x \leq 0.1$ followed by increasing values in the range $0.1 \leq x \leq 0.4$ for both types II and III of (G–N) theory. Figure 12 determines the distribution of the stress component σ_{xy} for $t = 0.9$ and $t = 0.2$ in the case of (G–N) theory of types II and III; it is noticed that the distribution of σ_{xy} decreases in the range $0 \leq x \leq 0.38$, then increases in the range $0.38 \leq x \leq 1$.

The 3D curve is representing the complete relations between u and the stress components σ_{xx} against both components of the distance x, y as shown in Figs. 13 and 14 in the presence of the gravity under (G–N) theory of type III. This figure is very important to show that the functions are moving in wave propagation.

6 Conclusions

The results of the present work can be summarized as:

(i) The values of all physical quantities converge to zero with increasing the distance x, and all functions are continuous.
(ii) The gravity field as a physical operator has a significant role in the considered physical quantities.
(iii) The laser pulse and the time effect have significant influences on the distribution of the considered physical quantities.

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