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ERRATUM

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Erratum to: Finite deformation of a pressurized magnetoelastic membrane in a stationary dipole field

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Equations (58)–(60) should read:

$$\int_{\Omega} (G_{\mathbf{f}} \cdot \nabla \dot{\mathbf{r}} \ + G_{\mathbf{e}} \cdot \dot{\mathbf{e}}) dA = \int_{\Omega} p\alpha \mathbf{n} \cdot \dot{\mathbf{r}}\, dA + \mu_0 \int_{\Omega} \dot{\mathbf{r}} \cdot \ \rho_\kappa (grad \mathbf{h}_a) \boldsymbol{\mu} dA, \tag{58}$$

$$G_{\mathbf{e}} = \mathbf{0} \quad \text{and} \quad div(G_{\mathbf{f}}) + p\alpha \mathbf{n} + \mu_0 \rho_\kappa (grad \mathbf{h}_a) \boldsymbol{\mu} = \mathbf{0} \quad \text{in} \quad \Omega, \tag{59}$$

$$div(\epsilon G_{\mathbf{f}}) + P\alpha \mathbf{n} + \mu_0 \hat{\rho}_\kappa (grad \mathbf{h}_a) \boldsymbol{\mu} = \mathbf{0}. \tag{60}$$

In Eq. (84), μ_0 needs to be changed to μ_0^2, resulting in:

$$\mathbf{g} := \mu_0 \hat{\rho}_\kappa (grad \mathbf{h}_a) \boldsymbol{\mu} = \frac{\mu_0^2 \epsilon \rho_\kappa^2}{A} (grad \mathbf{h}_a) \mathbf{h}_a. \tag{84}$$

In Eq. (85), ϱ^6 needs to be changed to ϱ^8, resulting in:

$$\mathbf{g} = \frac{H}{\varrho^8} \left\{ (\varrho \mathbf{a} \cdot \mathbf{k}) \mathbf{k} - [1 + 4(\mathbf{a} \cdot \mathbf{k})^2] (\varrho \mathbf{a}) \right\}, \tag{85}$$

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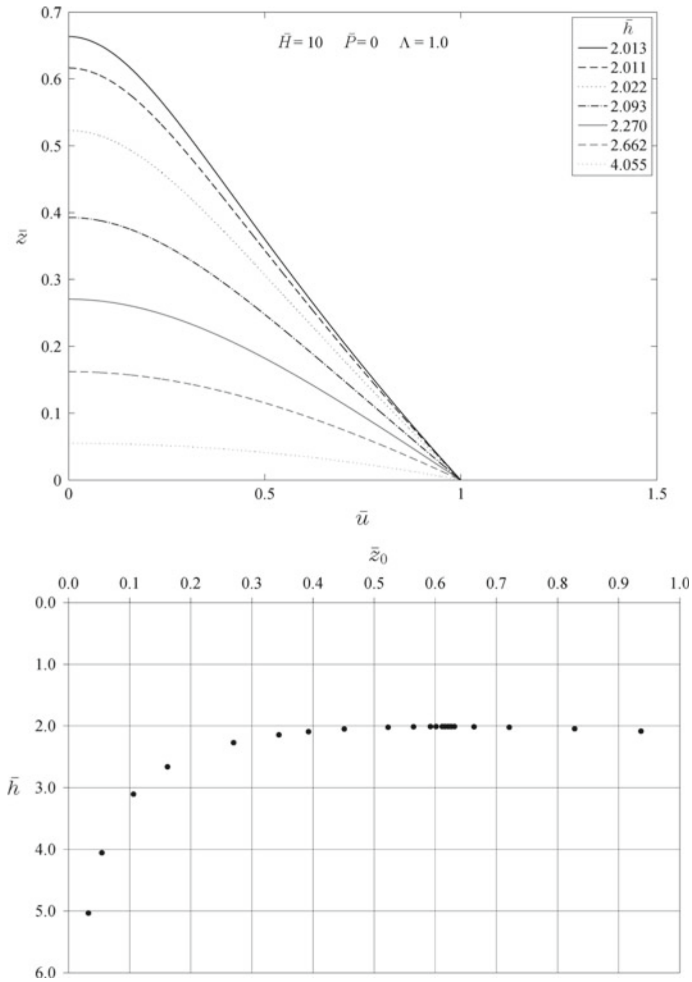

Fig. 2 Deformation induced by varying the height of a dipole source of fixed intensity ($\bar{H} = 10$). There is no pre-stretch

In Eq. (86), μ_0 needs to be changed to μ_0^2, resulting in:

$$H := 3D^2 \frac{\mu_0^2 \epsilon \rho_\kappa^2}{A}. \tag{86}$$

In Eqs. (89.1) and (89.2) ϱ^6 needs to be changed to ϱ^8, resulting in:

$$\mathbf{v}_2 \cdot \mathbf{g} = \frac{H}{\varrho^8} \left\{ 4(h-z)\frac{(h-z)^2}{\varrho^2} \sin\phi + \left[1 + 4\frac{(h-z)^2}{\varrho^2} \right] u \cos\phi \right\}$$

and

$$\mathbf{v}_2 \cdot \mathbf{g} = \frac{H}{\varrho^8} \left\{ 4(h-z)\frac{(h-z)^2}{\varrho^2} \cos\phi - \left[1 + 4\frac{(h-z)^2}{\varrho^2} \right] u \sin\phi \right\}. \tag{89}$$

In Eq. (91.8) r_0^4 needs to be changed to r_0^6, resulting in:

$$\bar{H} = H/\epsilon G r_0^6. \tag{91}$$

The text beginning with the last paragraph on p. 12 and concluding at the bottom of p. 17 contains numerical errors. The corrected text, including corrected figures, is appended here:

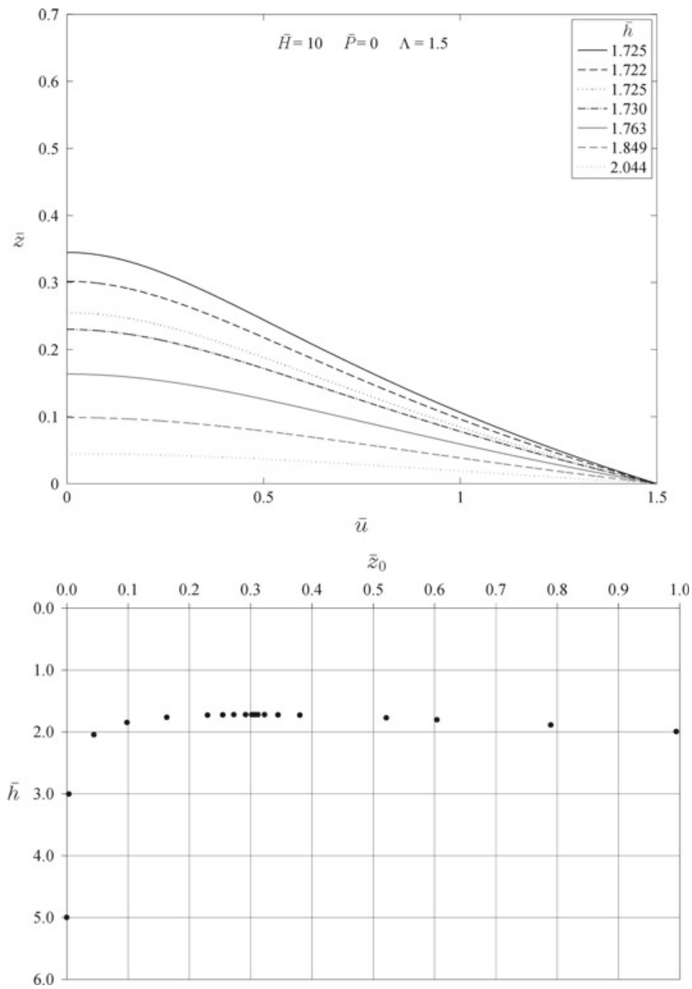

Fig. 3 Deformation induced by varying the height of a dipole source of fixed intensity ($\bar{H} = 10$). The pre-stretch is $\Lambda = 1.5$

The effect of varying the height of a dipole source of fixed intensity ($\bar{H} = 10$) above the base plane $z = 0$ is depicted in Fig. 2. The hoop stretch at the outer radius is $\Lambda = 1$, corresponding to the absence of pre-stretch. The upper figure shows the shape of the deformed meridian of the membrane at various values of the dipole height $\bar{h}$. As expected, the membrane is more severely deformed as the height is decreased (for a monotonically decreasing sequence of height values), due to the ever closer proximity of the source to the membrane material. The variation of $\bar{h}$ with elevation of the pole above the base plane, denoted by z_0, is shown in the lower figure. This response exhibits a turning point at around $\bar{h} = \bar{h}_{\min} = 2.01$—the minimum source height possible in equilibrium—followed thereafter by increasing values of $\bar{h}$ (source farther from the base plane) as z_0 continues to increase. In a purely mechanical analogue to this problem, we would interpret the field exerted on the membrane as a surrogate for a distributed mechanical force. Our results would then indicate that a limit-point instability is encountered as the source height approaches its minimum. It is thus appropriate to conclude that a limit-point instability is encountered in the present problem and thus that the decreasing branch of the response curve ($\bar{h}$ increasing), after $\bar{h}_{\min}$ is encountered, correspond to unstable equilibrium. We conjecture that a full dynamic analysis of this problem would predict that, after the equilibrium limit point is reached, the membrane is pulled into the dipole source dynamically. The membrane is substantially strained at $\bar{h} = \bar{h}_{\min}$, with a maximum stretch of 1.22 occurring in the meridional direction.

The effect of pre-stretch ($\Lambda = 1.5$) is shown in Fig. 3. The features of the response to a dipole source of the same strength ($\bar{H} = 10$) are qualitatively similar to those for the problem with no pre-stretch. In the present case, the stable branch of the equilibrium response is stiffer, and the turning point is encountered later (i.e., when the source is closer to the base plane), at $\bar{h} = \bar{h}_{\min} = 1.72$. At this stage, the maximum stretch in the membrane is 1.58 and again occurs in the meridional direction. Figure 4 exhibits the computed response

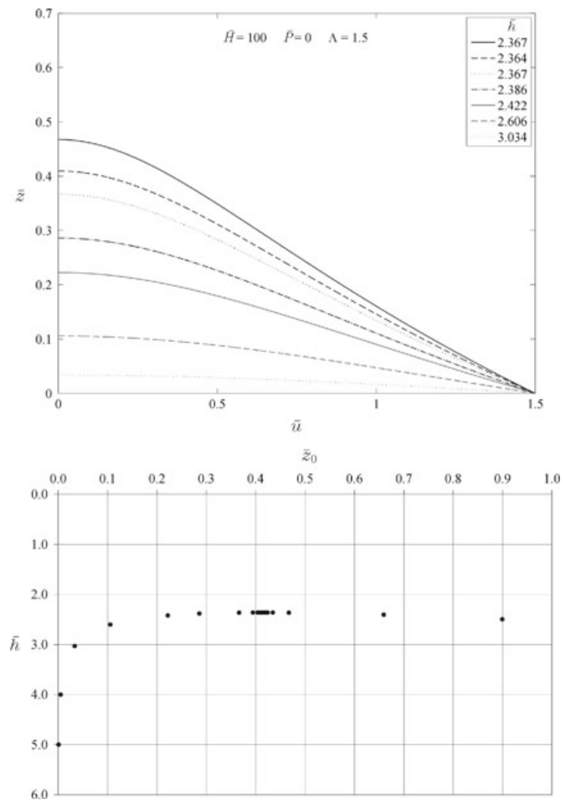

Fig. 4 Deformation induced by varying the height of a dipole source of fixed intensity ($\bar{H} = 100$). The pre-stretch is $\Lambda = 1.5$

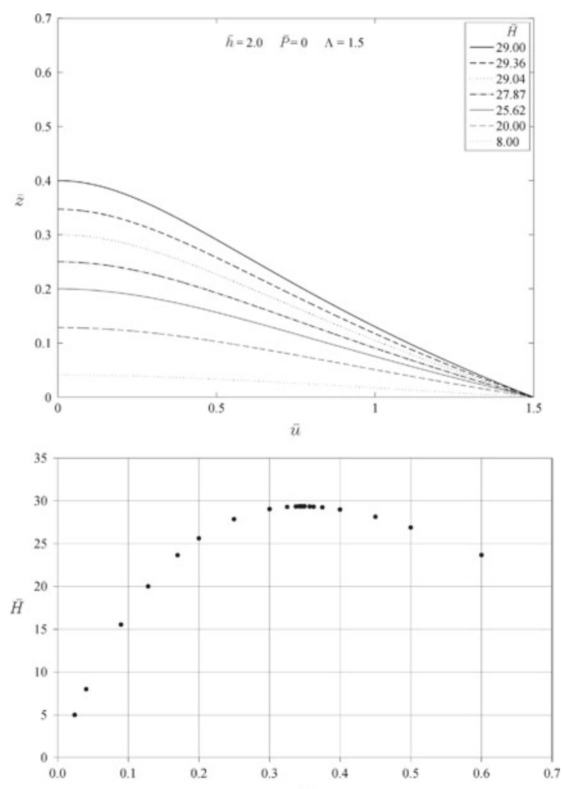

Fig. 5 Deformation generated by a sequence of dipoles of varying strength at fixed height ($\bar{h} = 2.0$). The pre-stretch is $\Lambda = 1.5$

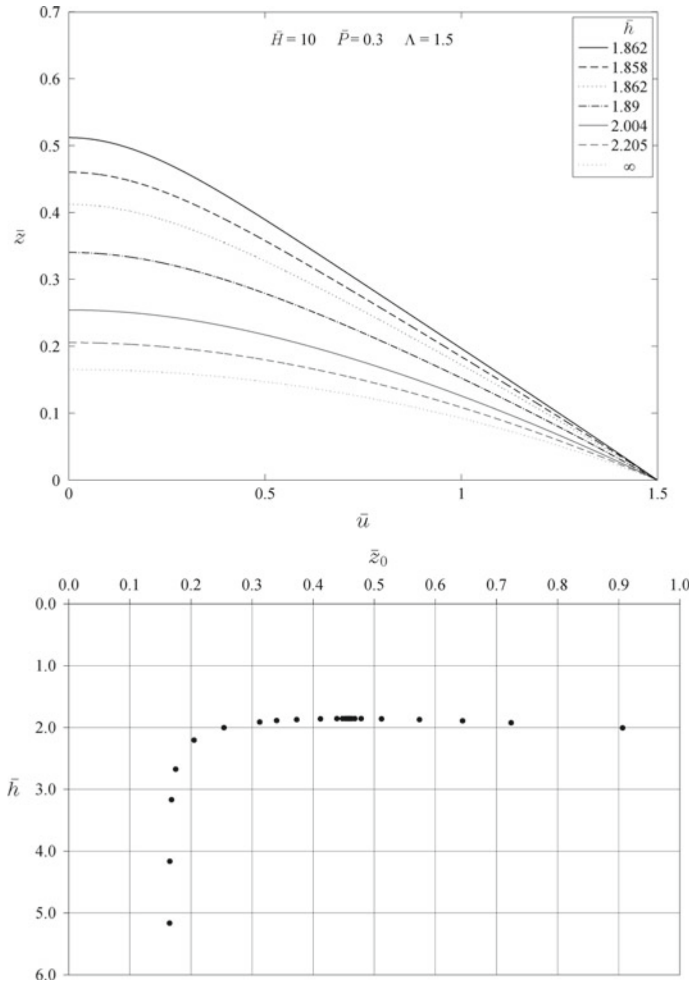

Fig. 6 Deformation of a pressurized membrane ($\bar{P} = 0.3$) induced by varying the height of a dipole of fixed strength ($\bar{H} = 10$). The pre-stretch is $\Lambda = 1.5$

of the pre-stretched membrane to a strong dipole source ($\bar{H} = 100$). The deformation is more severe, as expected, with a maximum (meridional) stretch of 1.64 occurring at the turning point ($\bar{h}_{\min} = 2.37$), which is encountered at a larger distance from the base plane.

Figure 5 illustrates the response of the pre-stretched membrane to dipoles of increasing strength held at a fixed distance ($\bar{h} = 2.0$) above the base plane. Limit-point behavior is again predicted, corresponding to a maximum dipole strength ($\bar{H}_{\max} \approx 29.4$) for which equilibrium is possible, followed by an increase in pole displacement attending a decrease in dipole strength. The response is qualitatively similar to that produced by varying the height of a source of given intensity, producing a peak (meridional) stretch at $\bar{H}_{\max}$ of 1.6.

Figure 6 illustrates the response of the pre-stretched membrane to a uniform pressure of fixed intensity ($\bar{P} = 0.3$) and a dipole source of fixed strength ($\bar{H} = 10$). The height of the source above the plane is varied, as in Fig. 3, and limit-point behavior is again predicted. The effect of pressure is to promote instability, with $\bar{h}_{\min}(= 1.86)$ exceeding that predicted in the case of no pressure. The deformation at the turning point is also more severe, producing a peak meridional stretch of 1.61.

Since the paper was published, subsequent work that confirms the accuracy of the model, and the existence of a limit-point instability, has been conducted.

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