Impact of Stress-Induced Rock Damage on Elastic Symmetry: From Transverse Isotropy to Orthotropy

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Abstract

In this article we explore the change in elastic symmetry and anisotropy of two geological materials with stress-induced damage. Two independent uniaxial deformation experiments on two layered and natural geomaterials, a shale and a sandstone, support this analysis. Both samples were loaded along their bedding planes at a constant strain rate up to mechanical failure. During deformation, an array of ultrasonic P- and S-wave transducers were employed to monitor the evolution of the ultrasonic velocities up to and beyond sample failure. We report here the impact on elastic symmetry and anisotropy of a uniaxial load applied parallel to the bedding plane of the transversely isotropic shale and sandstone samples. Based on symmetry considerations we analyse whether this load and the resulting stress-induced damage preserve the original transverse isotropy of the rock prior to loading, or lead to orthotropy (orthorhombic symmetry). Our results show that the two transverse isotropic samples retained their transverse isotropy in the initial stages of loading/damage. However for the more anisotropic shale sample the main failure mode was splitting of the bedding planes. In contrast the sandstone sample failed along a shear plane inclined to the bedding plane. In addition, the experimental data are further analysed using continuum damage mechanics to identify the evolution of the general fourth order anisotropic damage tensor during loading. We found that the highest damage variable of the damage tensor was D_{11} for the shale and D_{55} for the sandstone sample. The next highest damage variables for the shale sample were related to the bedding plane (x_2 – x_3 plane) symmetry: D_{23} , D_{32} , D_{44} . However the damage variables obtained for the less anisotropic and more brittle sandstone sample were harder to interpret due to the mixed mode of fracturing present.

Highlights

- Transverse isotropic shale and sandstone samples were loaded parallel to their bedding planes.
- Ultrasonic measurements during loading was used to quantify the resulting anisotropic damage based on continuum damage mechanics. The damage variables obtained were compared with previous studies of transverse isotropic shale samples loaded perpendicular to their bedding plane.
- For the more anisotropic shale sample its main failure mode was splitting of the bedding planes. The less anisotropic sandstone sample failed along a shear plane inclined to the bedding plane.

Keywords Continuum damage mechanics · Anisotropy · Ultrasonics stiffness reduction · Rock mechanics · Orthotropy · Transverse isotropy

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1 Introduction

Anisotropy is an important factor in both composite manufacturing and unconventional oil and gas extraction. In many geotechnical applications the initial anisotropy of the rock can impact on the stability of structures such as cavities, wellbores or hydraulic fractures. In particular shale

anisotropy has been known to be a significant problem for seismic exploration for many years (Dodds et al. 2007).

To date there have been many numerical and experimental studies on the influence of single or multiple flaws, pores and joints localised at the macroscale level on the mechanical behaviour of isotropic rocks loaded under either uniaxial or triaxial conditions (Mondal et al. 2019, 2020a, b; Griffiths et al. 2017; Wasantha et al. 2021; Tang and Kou 1998; Xue et al. 2021; Xu et al. 2013). We extend this analysis to consider the influence of bedding planes of initially transverse isotropic rocks and its orientation with respect to the uniaxial stress applied. In this case the bedding planes have a laminated structure which can occur at a global scale and ranges from the micro to macroscopic level.

Griffiths et al. (2017) demonstrated that sandstones are systematically weaker when they are deformed parallel to their bedding plane rather than perpendicular to it. The ratio of unconfined compressive strengths of sandstones loaded perpendicular vs parallel to their bedding planes ranged between 1.12–1.54. We extend this analysis further to also consider the mode of fracturing and damage evolution for samples loaded parallel to their bedding plane.

Browning et al. (2018) conducted true triaxial experiments on initially isotropic sandstone samples where they explored the effect of crack damage memory by cycling the principal load in three orthogonal directions. Browning et al. (2017) created two end-member crack distributions in sandstone samples; one displaying cylindrical transverse isotropy (under conventional triaxial loading), and the other planar transverse isotropy (under true triaxial loading). We also consider two end-member cases in Fig. 1 for two transverse isotropic rocks loaded in a direction parallel to their bedding plane: hypothesis 1 where the rocks lose their transverse isotropy and become orthotropic and hypothesis 2 where the rocks retain their transverse isotropy. Through these past experiments (Browning et al. 2017, 2018) and the results of this paper we show that crack damage evolution can be anisotropic and must be considered as a three-dimensional problem.

Triaxial and uniaxial loading experiments of shales (Sarout et al. 2007), sandstones (King et al. 1995; Scott et al. 1993) and composites (Castellano et al. 2017; Marguères and Meraghni 2013; Audoin and Baste 1994) have already shown how materials can undergo stress-induced damage to change their symmetry or magnitude of anisotropy. These simpler cases have already been discussed in previous papers (Olsen-Kettle 2018b, 2019). We consider a more complex loading scenario of uniaxial loading up to failure along the bedding plane (i.e. plane of isotropy) of two transverse isotropic geomaterials: a shale and a sandstone sample. We found that such boundary conditions changed the mode of fracturing. When the uniaxial loading is perpendicular to the bedding plane generally tensile fractures parallel to the load

are produced similar to the isotropic case. When the uniaxial loading is parallel to the bedding plane in-plane splitting of the bedding planes can also occur.

Figure 1 shows three schematic cases we consider to identify the impact on elastic symmetry from loading aligned with a material's bedding plane. The first case would be to treat the shale or sandstone sample as initially isotropic and in that case we expect that uniaxial loading cause microcracks to open perpendicular to the loading resulting in transverse isotropic material symmetry. However because we assume that both the considered materials are initially transverse isotropic we consider two alternative hypotheses based on symmetry considerations. For the case referred to as hypothesis 1 the uniaxial loading aligned with the bedding plane causes randomly oriented vertical tensile cracks to open perpendicularly to the loading direction and the material becomes orthotropic. However in the third case referred to as hypothesis 2, the uniaxial loading causes splitting along the bedding planes predominantly and the material retains its transverse isotropy. Another possibility is that the cracking occurs in a mixed mode between the two cases in hypotheses 1 and 2. We also investigate whether the mode of cracking depends on the material under consideration (shale or sandstone) and the magnitude of the initial anisotropy. We design an array of 18 P- and S-wave transducers to measure ultrasonic velocities along ray paths of known orientation with respect to the isotropy axis to identify if the material retains its transverse isotropy under uniaxial loading aligned with its bedding plane or becomes orthotropic.

Ultrasonic techniques provide fast and non-destructive methods for reliable measurement of elastic properties and their change with damage (Marguères and Meraghni 2013). We measure ultrasonic wave velocities in shale and sandstone samples to detect a change in anisotropy of these specimens. We also show how to use the ultrasonic measurements to quantify the time-evolution of the fourth order anisotropic damage tensor characterizing the internal damage using continuum damage mechanics. We employ a general fourth order damage tensor to describe the internal damage (Olsen-Kettle 2018a, b, 2019). We extend this analysis to include the case of an initially transverse isotropic material undergoing stress-induced damage to become orthotropic (or higher symmetry) in Appendix A. Assuming the initial material to be transverse isotropic can simplify the analytical derivation of the damage tensor and some elastic moduli, when compared with the more general case of an initially orthotropic material derived in Olsen-Kettle (2018a). We show how this simplified analysis can reduce the propagation of errors in the calculation of the internal damage variables in Appendix B.

Details of the uniaxial loading experiment and the shale and sandstone specimens are in Sect. 2.1. We show the structural evolution undergone by the specimen through X-Ray Computed Tomography images obtained before and after loading in Sect. 2.2. We plot the time-evolution of the elastic

moduli in Sects. 3.1 and 3.2. In Sect. 4 and Appendix B we plot the time-evolution of the tensorial damage variables for the shale and sandstone specimens with their bedding plane parallel to the uniaxial load applied. We compare the damage variables obtained under the assumption of different initial material symmetries: transverse isotropy or orthotropy. Depending on the initial material symmetry present we show that the measurement errors can be reduced significantly if the material is assumed to be initially transverse isotropic using the analytical derivations in Appendix B.

2 Stress-Induced Anisotropy in Shale and Sandstone Samples

2.1 Materials and Methods

In this section we perform uniaxial compression of a shale and sandstone sample. The shale sample is an Opalinus clay originating from the Mont-Terri Underground Research

Laboratory operated by NAGRA (Switzerland). Details on the mineralogy and key petrophysical characteristics of this shale can be found in Sarout et al. (2014). It had a diameter of 38 mm, height of 85.37 mm and a dry mass of 242.61 g. The sandstone sample originates from the Jurassic eolian Nugget sandstone formation originating from a quarry in Utah. Its depositional environment confers to the Nugget reservoir intrinsic anisotropy supported by alternating sand and granulometry along the bedding planes. It had a diameter of 38 mm, height of 84.76 mm and a dry mass of 226.77 g. Both samples were dried at 105°C for 48 h prior to testing and then weighed. The ultrasonic wave velocities were measured during the loading to quantify their change with the stress-induced damage.

The specially-designed ultrasonic transducer locations are shown in Fig. 2. The numbered circles show the location of P-wave transducers and the numbered rectangles show the location and orientation of the S-wave transducers. Table 1 summarizes the ultrasonic velocities measured where we are using the notation: V_{ij} to denote the speed of an ultrasonic wave propagating in the x_i -direction with particle displacements in the x_j -direction (longitudinal if $i = j$ and shear if $i \neq j$). $V_{ij/kl}$ are quasi-longitudinal or quasi-transverse waves propagating in the direction $(e_i + e_j)$ with particle displacements in a plane parallel to the plane formed by x_k and x_l .

When measuring ultrasonic wave speeds we took multiple measurements along different paths for the same expected wave speed according to the material's symmetry axes under damage: the samples initial axis of isotropy (x_1) and the loading axis (x_3). For example, in Table 1, when we measured the P-wave speed in the x_1 -direction, 4 transducers are used (transducers 1,3,5 and 7 shown in Fig. 2),

Table 1 Ultrasonic wave speed measurements and transducer locations in Fig. 2

Transducers	Ultrasonic wave speed	Transducers	Ultrasonic wave speed
1–3, 5–7	$V_{1/1}$	2–4, 6–8	$V_{2/2}$
11–12	$V_{3/3}$	2–8, 4–6	$V_{23/23}$
9–10	$V_{12/12}$	3–5, 1–7	$V_{13/13}$
13–14	$V_{2/3}$	17–18	$V_{1/3}$
15–16	$V_{1/2}$		

Fig. 1 Possible scenarios with uniaxial loading of an isotropic or transverse isotropic material with bedding plane aligned with the loading direction

Fig. 2 The ultrasonic transducer locations on shale sample where the numbered circles show the location of P-wave transducers and the numbered rectangles show the location of the S-wave transducers polarized along the rectangle lengths

and hence we obtained 4 different measurements of the P-wave in the x_1 -direction. This enabled us to calculate the standard error from these different measurements of the same expected wave speed. It is expected that there would be some natural variability in rock samples and using multiple transducers gives more confidence in using only one sample for shale and sandstone in this study. For each of the samples, and at each stress condition, we recorded a large number of ultrasonic wave velocities along multiple ray paths directions/positions within the sample, hence the large amount of data, and the confidence in them for the interpretations provided. The initial CT scans in Figs. 3 and 4 also show the degree of homogeneity of the materials. Ultrasonic investigations of materials undergoing uniaxial loading (Castellano et al. 2017; Marguères and Meraghni 2013) often rely on single or a limited number of samples due to the complexity of the experimental set-up and the post-processing of the data. Mondal et al. (2020b) showed using numerical simulations of damage that the overall peak stress for a model of sandstone undergoing uniaxial loading did not exhibit much variation with different realizations of rock heterogeneity. We expect that during the initial stages of damage before the final failure that there would be less variability in the materials response, and that the rock heterogeneity may have a bigger impact on the final stages of macrocracking.

We apply uniaxial compression at a constant axial strain rate of 10^{-5} s^{-1} to the samples where the applied stress in the x_3 -direction is coaxial with the bedding plane of the sample. We consider two cases: assume that the sample is initially transverse isotropic with an axis of isotropy in the x_1 -direction in Appendix B, or assume that the sample is initially orthotropic with the material axes aligned with the axes shown in Fig. 2 (as well as aligned with the applied

uniaxial compressive stress). This means that in both cases the material axes of the damage tensor and damaged stiffness tensor are the same as the original undamaged shale's material axes, and coaxial with the applied principal stresses. In the case of an initially transverse isotropic shale with an axis of isotropy in the x_1 -direction we assume that the loading in the x_3 -direction could break the symmetry in the x_2 - x_3 -plane. This is assumed because Curie's principle states that the physical effects are at least as symmetric as the causes and we expect uniaxial compression in the x_3 -direction to either cause the sample to retain its transverse isotropy or become orthotropic (Rasolofosaon 1998).

Measurements of the ultrasonic velocities beyond the elastic regime of deformation can reveal the progression of damage and if a change in the anisotropy has occurred. Once the ultrasonic velocities are measured the coefficients of the damaged elastic stiffness tensor can be calculated from the well-known Christoffel's equations for elastic wave propagation (Van Buskirk et al. 1981):

$$\begin{aligned}\tilde{E}_{11} &= \rho V_{1/1}^2, \\ \tilde{E}_{22} &= \rho V_{2/2}^2, \\ \tilde{E}_{33} &= \rho V_{3/3}^2, \\ \tilde{E}_{44} &= \rho V_{2/3}^2 = \rho V_{3/2}^2, \\ \tilde{E}_{55} &= \rho V_{1/3}^2 = \rho V_{3/1}^2, \\ \tilde{E}_{66} &= \rho V_{1/2}^2 = \rho V_{2/1}^2,\end{aligned}\quad (1)$$

where ρ is the density of the shale or sandstone. The above equations hold when the ultrasonic velocities are recorded along the sample's material symmetry axes. However the off-axis ultrasonic measurements of the quasi P-wave velocities gives us the observed group velocities ($V_{\phi}^{ij/ij}$) measured

Fig. 3 The CT scan for the shale sample before and after uniaxial loading in the x_3 direction (perpendicular to the axis of initial isotropy shown as X in blue)

at a ray path angle, ϕ , of 45° between the material axes i and j . Since determination of $\tilde{E}_{12}$, $\tilde{E}_{13}$, and $\tilde{E}_{23}$ requires knowledge of the phase velocity, $V_{\theta}^{ij/ij}$, it is necessary to solve for the eigenvalues of the Christoffel equations (Tsvankin 2001). Eigensolutions to the Christoffel equation for orthotropic materials are given by setting the following determinants to zero:

Fig. 4 The CT scan for the sandstone sample before and after uniaxial loading in the x_3 direction (perpendicular to the axis of initial isotropy shown as X in red)

$$\begin{vmatrix} \tilde{E}_{11} \sin^2 \theta + \tilde{E}_{55} \cos^2 \theta - \rho(V_{\theta}^{13/13})^2 & (\tilde{E}_{13} + \tilde{E}_{55}) \sin \theta \cos \theta \\ (\tilde{E}_{13} + \tilde{E}_{55}) \sin \theta \cos \theta & \tilde{E}_{33} \sin^2 \theta + \tilde{E}_{55} \cos^2 \theta - \rho(V_{\theta}^{13/13})^2 \end{vmatrix} = 0, \\ \begin{vmatrix} \tilde{E}_{22} \sin^2 \theta + \tilde{E}_{44} \cos^2 \theta - \rho(V_{\theta}^{23/23})^2 & (\tilde{E}_{23} + \tilde{E}_{44}) \sin \theta \cos \theta \\ (\tilde{E}_{23} + \tilde{E}_{44}) \sin \theta \cos \theta & \tilde{E}_{44} \sin^2 \theta + \tilde{E}_{33} \cos^2 \theta - \rho(V_{\theta}^{23/23})^2 \end{vmatrix} = 0, \quad (2) \\ \begin{vmatrix} \tilde{E}_{11} \sin^2 \theta + \tilde{E}_{66} \cos^2 \theta - \rho(V_{\theta}^{12/12})^2 & (\tilde{E}_{12} + \tilde{E}_{66}) \sin \theta \cos \theta \\ (\tilde{E}_{12} + \tilde{E}_{66}) \sin \theta \cos \theta & \tilde{E}_{66} \sin^2 \theta + \tilde{E}_{22} \cos^2 \theta - \rho(V_{\theta}^{12/12})^2 \end{vmatrix} = 0.$$

The group velocities ($V_{\phi}^{ij/ij}$) are related to the phase velocities ($V_{\theta}^{ij/ij}$) by (Berryman 1979; Thomsen 1986; Tsvankin 2001; Dewhurst and Siggins 2006):

$$V_{\phi}^2 = V_{\theta}^2 + \left(\frac{\partial V_{\theta}}{\partial \theta} \right)^2$$

$$\tan \phi = \frac{V_{\theta} \sin \theta + \frac{\partial V_{\theta}}{\partial \theta} \cos \theta}{V_{\theta} \sin \theta - \frac{\partial V_{\theta}}{\partial \theta} \cos \theta} \quad (3)$$

where the observed group angle is $\phi = 45^\circ$ for our ultrasonic measurements and we are measuring the group velocities in the x_1 – x_2 , x_1 – x_3 and x_2 – x_3 planes, and the sample's initial transverse isotropic axis of symmetry is x_1 and the uniaxial loading direction is x_3 .

Equations (2) and (3) give us three nonlinear equations to solve for $V_{ij/ij}^{\theta}$, $\tilde{E}_{ij}$ and θ for each $ij = 12, 13$ or 23 . These variables $V_{ij/ij}^{\theta}$, $\tilde{E}_{ij}$ and θ were solved using the nonlinear solver, *vpasolve*, in matlab for each measured off-axis group velocity recorded. Once the damaged stiffness tensor, $\tilde{E}$, has been calculated using Eqs. (1)–(3) we can calculate the associated damage tensor and elastic moduli. Details of the derivation for the damaged tensor are given in Appendix A.

2.2 CT Scans of Shale and Sandstone Samples Before and After Loading

Figures 3 and 4 show the X-Ray Computed Tomography images of the two samples obtained before and after loading along the x_3 axis (shown as Z-axis here). The slices are reported along the main planes investigated with our velocity arrays - i.e. X–Y (x_1 – x_2), X–Z (x_1 – x_3) and Y–Z (x_2 – x_3) planes. Please note that the axis of isotropy for both samples (x_1) is defined as X and is in blue for the shale sample Fig. 3, and is in red for the sandstone sample in Fig. 4.

The CT image for the shale sample in Fig. 3 shows that the predominant mode of fracturing is extension fractures between the bedding planes or possibly some sliding of the bedding layers, and that in the microcracking stage it is likely that the shale sample retained its transverse isotropy. Calculation of the Young's and shear moduli in the following Sect. 3.1 in Fig. 6 using ultrasonic measurements confirm this hypothesis as well. However there is more ambiguity in interpreting the change in the Poisson's ratios in Fig. 7 which may be because the initial magnitudes of the Poisson's ratio for the shale sample more closely resemble an orthotropic material symmetry initially in Fig. 5b. However the ultrasonic measurements for the shear and Young's moduli more closely resemble the assumed transverse isotropic material symmetry in Fig. 5a with axis of isotropy in the x_1 direction.

The CT image for the sandstone sample in Fig. 4 shows some initial extension or shear fracturing between the bedding planes before the final failure of shear fracturing at an angle to the loading as observed commonly in isotropic

Fig. 5 Calculated values of initial elastic moduli in **a** and Poisson's ratio in **b** with different assumptions of initial material anisotropy (transverse isotropic or orthotropic) for the shale sample

samples. This mixed mode of fracturing might be more likely for the sandstone because overall it is less anisotropic than the shale sample.

3 Evolution of the Elastic Moduli

Using the measured ultrasonic wave speeds we can invert the well-known Christoffel equations (1) and (2) for the stiffness tensor to calculate the compliance tensor and the associated elastic moduli. In all the figures in this paper we plot the 95% confidence interval using the approximate formula for propagation of error for a multivariable function from

Fig. 6 The change in material parameters for the shale sample subject to uniaxial loading in the x_3 direction in **a** and **b** where the shale's bedding plane (x_2 – x_3 plane) is *parallel* to the uniaxial loading direction. The change in material parameters in **c** and **d** from the analysis in Olsen-Kettle (2018b) using the experimental results of Sarout and

Guéguen (2008) (see Fig. 14) where a dry shale rock specimen is subject to triaxial loading with a confining pressure of 15 MPa with the maximum principal stress applied *perpendicular* to the bedding plane (x_1 – x_2 plane). The x_3 -direction is the direction of the maximum principal stress in both cases

Hughes and Hase (2010) (equation 4.16). This formula has some limitations and assumes that the magnitude of the error is small. In all the figures we have only plotted the 95% confidence interval for every tenth data point for the shale sample and every fifth data point for the sandstone sample so the figures are clearer. It is important to note that the only source of error considered for the calculation of the confidence interval is the measurement errors from the ultrasonic velocity calculations for a single sample. There may be other sources of error due to natural variability and heterogeneity of the individual samples especially when considering small cylinder dimensions.

3.1 Experimental Results for the Shale Sample

Figure 5 show the calculation of the initial elastic moduli for the shale sample under the assumption of an initially transverse isotropic specimen and under the assumption of an initially orthotropic specimen. It is clear that the elastic

moduli calculated under an assumption of an initially transverse isotropic symmetry for the shale sample matches the experimental data well for the Young's and the shear moduli.

Figures 6a, b and 7a, b show the change in the elastic moduli as the uniaxial loading increases when the bedding plane is *parallel* to the axial load. For clarity we also plot the initial values of the elastic moduli on the left of each figure calculated under the assumption of initial transverse isotropy for the shale sample. For comparison we also plot the change in elastic moduli in Figs. 6c, d and 7c from the analysis in Olsen-Kettle (2018b) using the experimental results of Sarout and Guéguen (2008) (see Fig. 14) where a dry shale rock specimen is subject to triaxial loading with a confining pressure of 15 MPa with the maximum principal stress applied *perpendicular* to the bedding plane (x_1 – x_2 plane).

We observe salient differences in the evolution of the elastic moduli of the shale sample when we compare the evolution with stress-induced damage of the Young's and the shear moduli in Fig. 6a, b where its bedding plane is

Fig. 7 **a** The change in Poisson's ratios for the shale sample subject to uniaxial loading in the x_3 direction in **a** and **b** where the shale's bedding plane (x_2 – x_3 plane) is *parallel* to the uniaxial loading direction. The change in Poisson's ratios in **c** from the analysis in Olsen-Kettle (2018b) using the experimental results of Sarout and Guéguen (2008) (see Fig. 14) where a dry shale rock specimen is subject to triaxial loading with a confining pressure of 15 MPa with the maximum principal stress applied *perpendicular* to the bedding plane (x_1 – x_2 plane). The x_3 -direction is the direction of the maximum principal stress in both cases

parallel to the direction of the principal stress applied, with those obtained from a (dry) shale sample with its bedding plane perpendicular to the direction of the maximum principal stress applied as in Fig. 6c, d. For the case studied in this paper where the bedding planes are parallel to the loading direction (x_3) we see that the shear moduli in the bedding plane (G_{yz}) decreases significantly due to splitting of these bedding planes in Fig. 6b. The Young's moduli E_x , E_y and E_z remain fairly constant and only decrease after failure in Fig. 6a, showing that splitting of the bedding planes is more important for damage accumulation when the bedding plane is parallel to the applied stress. In contrast in Fig. 6c, d where the bedding plane is perpendicular to the applied stress both the Young's moduli $E_x (= E_y)$ and the shear modulus in the bedding plane G_{xy} decrease due to the tensile cracking parallel to the principal stress in the x_3 direction. We observe in both cases that the shear moduli in the bedding plane show significant reductions (G_{yz} when the bedding plane is parallel to the axial load in Fig. 6b and G_{xy} when the bedding plane is perpendicular to the axial load in Fig. 6d).

Similarly we compared the evolution of the Poisson's ratios of the shale sample with stress-induced damage in Fig. 7a, b where its bedding plane is parallel to the direction of the principal stress applied, with those obtained from a (dry) shale sample with its bedding plane perpendicular to the direction of the maximum principal stress applied in Fig. 7c. For the evolution of the Poisson's ratio in Fig. 7 we observed that the Poisson's ratio ν_{xy} and ν_{yx} decrease under loading whereas the other Poisson's ratios are relatively unaffected or increase due to the loading for both cases (bedding plane parallel or perpendicular to the axial load). This could be because the prominent mode of initial failure is splitting of (in the case of the axial load perpendicular to bedding plane) or expansion (in the case of the axial load parallel to the bedding plane) in the bedding planes which may impact on Poisson's ratio in the x_1 – x_2 plane causing the shale to become more compressible and hence decrease in this plane.

When a load is applied parallel to the shale's bedding plane it appears that the splitting of the bedding plane is the predominant mode of failure and microcracking, and that in this case the shale retains its transverse isotropy. This corresponds to hypothesis 2 in Fig. 1.

3.2 Experimental Results for the Sandstone Sample

Figure 8 shows the calculation of the initial elastic moduli under the assumption of an initially transverse isotropic specimen and under the assumption of an initially

Fig. 8 Calculated values of initial elastic moduli in **a** and Poisson's ratio in **b** with different assumptions of initial material anisotropy (transverse isotropic or orthotropic) for the sandstone sample

orthotropic specimen. It is clear that the elastic moduli calculated under an assumption of an initially orthotropic sandstone specimen have smaller confidence intervals for some moduli than those calculated under an assumption of an initially transverse isotropic sandstone specimen in Fig. 8a, b. For this reason we consider two cases: (i) assume the sandstone specimen is initially transverse isotropic, and (ii) consider that the sandstone is initially orthotropic to better match the experimental observations in Fig. 8. We compare these two assumptions in Figs. 13 and 14 in Sect. 4.2.

Figures 9 and 10 show the change in the elastic moduli as the uniaxial loading increases. For clarity we also plot the initial values of the elastic moduli on the left of each figure calculated under the assumption of initial orthotropy for the sandstone sample.

Fig. 9 The change in Young's moduli in **a** and shear moduli in **b** for the sandstone sample subject to uniaxial loading in the x_3 direction

In contrast for the sandstone sample we observed a very different trend in the evolution of the shear moduli with loading where the shear moduli in the bedding plane (G_{yz}) remained fairly constant and only decreased slightly, whereas G_{xz} decreased more. After the sandstone sample had started to fail the Young's moduli E_x and E_z decreased. So in the case of the sandstone sample it would appear that both extensional fracturing and splitting of the bedding planes can occur in the early onset of damage. However as shown in the CT image in Fig. 4 the final failure occurs as a result of shear fracturing at an angle to the x_1 - x_3 -plane which is similar to fracturing of an isotropic sample under uniaxial loading. This could be why, in contrast to the shale sample, the Young's modulus E_z also decreased at the end in a similar way to E_x . This

Fig. 10 The change in Poisson's ratios for the sandstone sample subject to uniaxial loading in the x_3 direction for ν_{zy} , ν_{zx} and ν_{yx} in **a** and ν_{yz} , ν_{xz} and ν_{xy} in **b**

corresponds to a mixture of the modes of fracturing proposed in hypothesis 1 and 2 in Fig. 1. Again for the evolution of the Poisson's ratio in Fig. 10 we observed relatively little change in the Poisson's ratio until the final failure event at the end where ν_{zx} and ν_{xz} decreased the most. We observe that the elastic moduli of the sandstone sample show relatively small changes up until the final and sudden failure event. Note that for the sandstone, the inelastic regime of deformation where damage accumulates is brief and only allows for a couple of velocities measurements to be successfully recorded. In comparison, the evolution of the elastic moduli of the shale sample exhibits less brittleness than the sandstone sample under load.

4 Analysis of Experimental Results Using Continuum Damage Mechanics

In general an eighth order damage tensor is needed to relate the damaged stiffness tensor and the undamaged stiffness tensor. However Cauvin and Testa (1999) showed using the principle of strain equivalence that in general only a fourth order tensor is needed to describe a material undergoing damage. They also showed that the actual number of independent damage parameters in such a tensor is related to the material and damage symmetry. Using symmetry considerations we use the material axes of the original initial transverse isotropy (axis of isotropy in x_1 direction) and the direction of the applied uniaxial load along the x_3 direction as the co-ordinate axes for our model. We show in Appendix B that the damage tensor reduces to a fourth order tensor with 9 independent damage variables: D_{11} , D_{12} , D_{13} , D_{22} , D_{23} , D_{33} , D_{44} , D_{55} , and D_{66} , and 3 dependent damage variables D_{21} , D_{31} , and D_{32} .

The appendix contains detailed derivations of the general fourth order damage tensor for an initially transverse isotropic material undergoing damage to become orthotropic (or higher symmetry) in Appendix A. These derivations extend our previous papers (Olsen-Kettle 2018a, b, 2019) to consider an initially transverse isotropic material becoming orthotropic with stress-induced damage. We derive the quantitative relationship between the macroscopic, empirically observed damaged elastic moduli and the internal damage variables in Sect. 4. Often only a subset of the diagonal elements of the damage tensor are retained (see for example (Gaede et al. 2013; Chow and Wang 1987; Lemaitre et al. 2000) and often the principal directions of a second order or fourth order anisotropic damage tensor are assumed to coincide with the applied principal stress directions (Chow and Wang 1987). However we will show in this section that the off-diagonal elements of the fourth order damage tensor are nonzero using the principal stress directions as our model axes.

4.1 Evolution of Damage Tensor for the Shale Sample

Figure 11 plots the evolution of the internal damage variables with loading for the shale sample. Again similarly to our previous papers (Olsen-Kettle 2018a, b, 2019) we show that the off-diagonal elements of the fourth order damage tensor are significant when we use the principal stress direction and material axes as the model axes. In particular D_{11} , D_{23} , D_{32} and $D_{44}(=D_{2323})$ are significant for the shale sample when the bedding plane is parallel to the axial load. We can observe that the off-diagonal damage variables related to the bedding plane x_2 – x_3 plane are the highest. However the damage variable D_{11} is much higher than D_{22} and this is likely to be due to the splitting of the bedding planes in the x_1 -direction. We

Fig. 11 The internal tensorial damage variables for the shale sample (using the simplified analysis assuming the shale is initially transverse isotropic in Appendix A) subject to uniaxial loading in the x_3 direction

also compared the damage variables when the bedding plane is parallel to the axial load with the damage variables obtained when the bedding plane is perpendicular to the axial load in Fig. 12 using the results in Olsen-Kettle (2018b) for a dry shale sample. When the bedding plane is perpendicular to the axial load $D_{11}(=D_{22})$ has the most significant increase, and $D_{12}(=D_{21}=D_{66}(=D_{1212}))$ are the next highest in magnitude. We observe in this case that the damage variables related to the bedding plane x_1 – x_2 plane are the highest.

As noted earlier in Fig. 6b the shear modulus in the bedding plane (G_{yz}) decreases significantly due to splitting of these bedding planes and this leads to the increase in the damage variable D_{44} . Because the damage variables D_{44} , D_{55} and D_{66} only depend on $\tilde{G}_{yz}$, $\tilde{G}_{xz}$ and $\tilde{G}_{xy}$ it is much easier to analyse the trend in these variables with the shear moduli in

comparison to the other damage variables defined in Eq. (9), which depend on multiple elastic moduli.

4.2 Evolution of Damage Tensor for the Sandstone Sample

Figures 11, 12, 13, 14 plot the evolution of the internal damage variables with loading for the sandstone sample. For the sandstone sample some of the initial elastic moduli indicate that the initial sandstone sample may be better represented using an orthotropic symmetry. For this reason we calculated the damage tensor variables again under the assumption of initial orthotropic material symmetry to compare with those calculated under the assumption of initial transverse isotropy in Figs. 13 and 14. While the overall trend in the damage tensor is similar, as expected from Eq. (5), we

Fig. 12 Comparison of the internal tensorial damage variables for the shale sample subject to uniaxial loading *parallel* to the bedding plane and the damage variables calculated in Olsen-Kettle (2018b) using the experimental results of Sarout and Guéguen (2008) (see Fig. 14) where a dry shale rock specimen is subject to triaxial loading with a confining pressure of 15 MPa with the maximum principal stress applied *perpendicular* to the bedding plane. The x_3 -direction is the direction of the maximum principal stress in both cases

can see that some of the damage tensor variables are either over-estimated or under-estimated for the initially transverse isotropic case when compared to the initially orthotropic case. For the sandstone case D_{11} significantly increased with loading similarly to the shale sample. However D_{12} and D_{21} remained fairly constant. D_{55} and D_{66} increased with loading and D_{33} decreased with loading which is more similar to the isotropic case in the results of Scott et al. (1993) where they conducted triaxial loading of an initially isotropic sandstone specimen. An analysis of their ultrasonic measurements in Olsen-Kettle (2019) revealed a similar decrease in D_{33} due to strain-hardening in the load direction and increase in $D_{66}(=D_{55})$ with loading.

5 Conclusions

These experiments show that when the load is applied parallel to the bedding plane of the shale sample splitting of the bedding plane is the predominant mode of fracture, resulting in the shale retaining its transverse isotropy. In contrast for the less anisotropic sandstone sample the initial mode of fracturing was splitting of the bedding planes followed by rupture along a shear plane inclined to the bedding planes. Rupturing along an inclined angle to the uniaxial load is also commonly observed in isotropic sandstone samples.

We have developed models of anisotropic damage for initially transverse isotropic materials undergoing damage-induced anisotropy to remain transverse isotropic or change to a lower symmetry and become orthotropic. We considered

experiments where the uniaxial stress is perpendicular to the initial transverse isotropy symmetry axis, i.e. parallel to the bedding plane. This symmetry axis remains unchanged in the damaged and undamaged state. We explored the change in anisotropy associated with stress-induced damage for two initially transversely isotropic rock samples: a shale and a sandstone. In the case of the shale sample the predominant failure mode was splitting of the bedding planes, thus retaining its transverse isotropy for longer. However the more isotropic sandstone sample ruptured in a more brittle manner along a shear plane inclined at an angle to the loading axis, where the change in material symmetry during the initial stages of loading/damage was not obvious in the collected data.

For the shale sample the highest diagonal element of the damage tensor was D_{11} when the bedding plane was parallel to the axial load. In the case studied by Olsen-Kettle (2018b) when the bedding plane is perpendicular to the axial load the highest diagonal elements of the damage tensor were $D_{11} = D_{22}$. The only off-diagonal positive damage variables for both cases of loading of the shale sample were related to the bedding plane symmetry: $x_2 - x_3$ plane when loaded parallel to the bedding plane: D_{23}, D_{32}, D_{44} , and $x_1 - x_2$ plane when loaded perpendicular to the bedding plane: D_{12}, D_{21}, D_{66} . This analysis is useful in identifying which damage variables are important to model when developing theoretical models of damage using continuum damage mechanics.

For the shale sample the shear moduli within the bedding plane, namely G_{yz} (axial load parallel to bedding) and G_{xy} (axial load perpendicular to bedding) were significantly reduced due to either splitting of the bedding planes when the axial load was applied parallel to it or expansion of the bedding planes when the axial load was applied perpendicular to it. For both cases Poisson's ratio in the plane perpendicular to the loading (ν_{xy}, ν_{yx}) decreased due to the splitting or expansion in this $x_1 - x_2$ plane.

Many researchers have defined and analysed phenomenological damage models for anisotropic composite materials (e.g., Audoin and Baste (1994); Hufenbach et al. (2006); Castellano et al. (2017)). Therefore in Appendix A we quantify the relationship between the internal damage variables for a fourth rank orthotropic damage tensor defined using continuum damage mechanics and the empirical damage variables associated with the experimentally-measured reduction in the magnitude of the stiffness tensor components. Our approach defines the damage tensor differently to these purely phenomenological models by explicitly specifying the key characteristics of damage (geometry, distribution). We use a general definition of the damage tensor using the conventional definition for a tensorial damage tensor given by continuum damage mechanics (Cauvin and Testa 1999; Jarić et al. 2012). Compared to published literature, our approach does not limit the damage tensor to be supersymmetric (e.g., Audoin and

Fig. 13 The internal tensorial damage variables for the sandstone sample with different assumptions of initial material anisotropy (transverse isotropic or orthotropic) subject to uniaxial loading in the x_3 direction

Baste 1994; Hufenbach et al. 2006; Castellano et al. 2017). Instead, we define a relationship between the measured stiffness reduction and the internal tensorial damage variables given by continuum damage mechanics.

In Appendix B we show how our simplified derivation of the damage tensor in Appendix A can reduce the

propagation of errors for initially transverse isotropic materials when compared with the more general derivation of the damage tensor given for an initially orthotropic material given by Olsen-Kettle (2018a).

This work helps validate experimentally existing phenomenological continuum damage models. It also paves

Fig. 14 The internal tensorial damage variables for the sandstone sample with different assumptions of initial material anisotropy (transverse isotropic or orthotropic) subject to uniaxial loading in the x_3 direction

the way to build new and more effective phenomenological continuum damage models of anisotropic damage evolution. Future work will extend upon these models of anisotropic damage resulting from well-defined and constrained

loading experiments, which result in either transverse isotropic or orthotropic damage at both the meso and macroscale, to modelling localized damage at the mesoscale level.

Appendix A: Modelling Damage-Induced Orthotropy for Initially Transverse Isotropic Materials

A.1 Fourth Order Damage Tensor

At any given state of damage the elastic part of the material response will be characterized by a fourth order tensor $\tilde{E}$ of the damaged elastic moduli just as the fourth order tensor E describes the elastic response of the virgin (undamaged) material. If the relation between the damaged stiffness tensor ($\tilde{E}$) and the undamaged stiffness tensor (E) is assumed to be linear then an eighth order damage tensor (D_8) can be used to relate the two:

$$\tilde{E} = (I_8 D_8) :: E$$

where I_8 is the eighth order identity tensor.

In the pioneering work of Cauvin and Testa (1999) they showed using the principle of strain equivalence that in general only a fourth order tensor is needed to describe a material undergoing damage. They also showed that the actual number of independent damage parameters in such a tensor is related to the material and damage symmetry. Cauvin and Testa (1999) and Jarić et al. (2012) showed that the general supersymmetry requirements for the damaged elastic stiffness tensor $\tilde{E}$ requires that $\tilde{E}_{ijkl} = \tilde{E}_{jikl} = \tilde{E}_{ijlk} = \tilde{E}_{klij}$, and this requirement places the following constraint on the damage tensor:

$$D_{ijrs}E_{rskl} - D_{klrs}E_{rsij} = 0, \quad (4)$$

where $\tilde{E}_{ijkl} = (I_{ijrs} - D_{ijrs})E_{rskl}$,

$$\text{and } I_{ijrs} = \frac{1}{2}(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr}). \quad (5)$$

where D is the damage tensor, $\tilde{E}$ is the damaged stiffness tensor, E is the original undamaged stiffness tensor and δ_{ij} is the Kronecker delta function. Here we note that the damage tensor is not supersymmetric like $\tilde{E}$ but it does have the same number of independent variables as $\tilde{E}$ (Jarić et al. 2012). Because E is supersymmetric equation (4) implies that D possesses minor symmetries: $D_{ijkl} = D_{jikl} = D_{ijlk}$. The above equations hold for any material symmetry of the initially undamaged and damaged material.

Here we note that we use the conventional definition of the damage tensor using continuum damage mechanics defined as: $\tilde{E} = (I - D) : E$ in Eq. (5) using the double inner product instead of a new definition of the damage tensor used by Audoin and Baste (1994), Hufenbach et al. (2006) and Castellano et al. (2017). Audoin and Baste (1994) originally defined the damage tensor used in these references using an additive form where: $\tilde{E} = E - E_d$, where E_d is their damage

tensor before normalization and imposing supersymmetry requirements.

A.2 Damage Tensor for an Initially Transverse Isotropic Material Becoming Orthotropic with Damage

The compliance tensor in Voigt notation for an initially transverse isotropic material with the axis of isotropy in the x_1 direction is:

$$S = \begin{pmatrix} \frac{1}{E'} & -\nu' & -\nu' & 0 & 0 & 0 \\ -\nu' & \frac{1}{E'} & -\nu' & 0 & 0 & 0 \\ \frac{E'}{E} & \frac{E}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G'} \end{pmatrix}$$

where $E = E_y = E_z$ is the Young's modulus in directions $x_2(y)$ and $x_3(z)$, $E' = E_x$ is the Young's modulus in the x_1 direction; $G' = G_{xy} = G_{xz}$ is the shear modulus for coordinate planes x_1 - $x_2(x-y)$ and x_1 - $x_3(x-z)$; and $\nu' = \nu_{xz} = \nu_{xy}$ and $\nu = \nu_{zy} (= \nu_{yz})$ are the Poisson's ratios characterizing the compressive strain in the direction of the second subscript produced by a tensile stress in the direction of the first subscript. Here we note that Poisson's ratios are not symmetric (i.e. $\nu_{ij} \neq \nu_{ji}$) however they do satisfy $\nu_{ij}/E_i = \nu_{ji}/E_j$ (no summation implied).

The stiffness tensor for the initially transverse isotropic material E is simply the inverse of the compliance tensor:

$$E = S^{-1} = \begin{pmatrix} \frac{(1-\nu)E'^2}{[(1-\nu)E' - 2Ev^2]} & \frac{EE'\nu'}{[(1-\nu)E' - 2Ev^2]} & \frac{EE'\nu'}{[(1-\nu)E' - 2Ev^2]} & 0 & 0 & 0 \\ \frac{EE'\nu'}{[(1-\nu)E' - 2Ev^2]} & \frac{E(E' - Ev^2)}{[(1-\nu)E' - 2Ev^2]} & \frac{E(vE' + Ev^2)}{(1+\nu)[(1-\nu)E' - 2Ev^2]} & 0 & 0 & 0 \\ \frac{EE'\nu'}{[(1-\nu)E' - 2Ev^2]} & \frac{E(vE' + Ev^2)}{(1+\nu)[(1-\nu)E' - 2Ev^2]} & \frac{E(E' - Ev^2)}{(1+\nu)[(1-\nu)E' - 2Ev^2]} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & G' & 0 \\ 0 & 0 & 0 & 0 & 0 & G' \end{pmatrix} = \begin{pmatrix} E_{11} & E_{13} & E_{13} & 0 & 0 & 0 \\ E_{13} & E_{33} & E_{23} & 0 & 0 & 0 \\ E_{13} & E_{23} & E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{pmatrix}.$$

Here we are using Voigt notation where each pair of indices (ij and kl) is replaced by a single index: $11 \rightarrow 1$, $22 \rightarrow 2$, $33 \rightarrow 3$, $23, 32 \rightarrow 4$, $13, 31 \rightarrow 5$, and $12, 21 \rightarrow 6$.

We assume that the material symmetry axes of the transverse isotropic solid do not change and that the axis of isotropy

becomes a symmetry plane in the damage-induced orthotropic solid. We apply our analysis to initially transverse isotropic shale and sandstone samples undergoing uniaxial loading applied in a direction parallel to the bedding plane. We use symmetry considerations and assume that the lowest symmetry an initially transverse isotropic shale specimen undergoing uniaxial loading applied in a direction parallel to the bedding plane, would attain is orthotropic with its 3 orthogonal symmetry planes aligned with the direction of loading (x_3) and axis of isotropy (x_1).

When deriving the stiffness tensor for the damaged material $\tilde{E}$ and the damage tensor D we use the material axes of the original initial transverse isotropy and the applied principal stresses of the experiment as the model axes. The stiffness tensor for the damaged material becoming orthotropic with the same material symmetry axes (again using the material axes of the initially transverse isotropic material) in Voigt notation is:

$$\tilde{E} = \begin{pmatrix} \tilde{E}_{11} & \tilde{E}_{12} & \tilde{E}_{13} & 0 & 0 & 0 \\ \tilde{E}_{12} & \tilde{E}_{22} & \tilde{E}_{23} & 0 & 0 & 0 \\ \tilde{E}_{13} & \tilde{E}_{23} & \tilde{E}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{E}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{E}_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{E}_{66} \end{pmatrix}.$$

The corresponding general anisotropic damage tensor for a material undergoing orthotropic damage can be written in Voigt notation as:

$$D = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{pmatrix}.$$

We can still write D in Voigt notation as D possesses the minor symmetries: $D_{ijkl} = D_{jikl} = D_{ijlk}$. Here we note that D is not supersymmetric like $\tilde{E}$. However D still has the same number of independent components (nine) as $\tilde{E}$, and D_{21} , D_{31} and D_{32} are given by the supersymmetric constraint for $\tilde{E}$ (Eq. 4):

$$\begin{aligned} D_{21} &= \frac{1}{E_{11}} ((D_{11} - D_{22} - D_{23})E_{13} + D_{13}E_{23} + D_{12}E_{33}), \\ D_{31} &= \frac{1}{E_{11}} (D_{13}E_{33} + (D_{11} - D_{33} - D_{32})E_{13} + D_{12}E_{23}), \\ D_{32} &= \frac{1}{E_{22}} (D_{23}E_{33} + (D_{21} - D_{31})E_{13} + (D_{22} - D_{33})E_{23}). \end{aligned} \quad (6)$$

Using Eq. (5) the damaged stiffness tensor variables for $\tilde{E}$ become:

$$\begin{aligned} \tilde{E}_{11} &= E_{11}(1 - D_{11}) - E_{13}(D_{12} + D_{13}) \\ \tilde{E}_{12} &= -E_{33}D_{12} + E_{13}(1 - D_{11}) - E_{23}D_{13} \\ \tilde{E}_{13} &= -E_{33}D_{13} + E_{13}(1 - D_{11}) - E_{23}D_{12} \\ \tilde{E}_{22} &= E_{33}(1 - D_{22}) - E_{13}D_{21} - E_{23}D_{23} \\ \tilde{E}_{23} &= -E_{33}D_{23} + E_{23}(1 - D_{22}) - D_{21}E_{13} \\ \tilde{E}_{33} &= E_{33}(1 - D_{33}) - E_{23}D_{32} - E_{13}D_{31} \\ \tilde{E}_{44} &= \frac{E(1 - 2D_{44})}{2(1 + \nu)}. \\ \tilde{E}_{55} &= G(1 - 2D_{55}) \\ \tilde{E}_{66} &= G(1 - 2D_{66}). \end{aligned} \quad (7)$$

A.3 Relating Empirical Damage Parameters to Ultrasonic Measurements

We can define empirical damage variables ($D_{E_{ij}}$) which satisfy $\tilde{E}_{ij} = (1 - D_{E_{ij}})E_{ij}$ (no summation over i, j) to compare the measured decrease in ultrasonic elastic wave velocities (and decrease in corresponding stiffness tensor elements $\tilde{E}_{ij}$) with the internal damage variables (D_{ij}):

$$\tilde{E}_{11} = (1 - D_{E_{11}})E_{11},$$

$$\text{using Eq. 7 : } \tilde{E}_{11} = E_{11}(1 - D_{11}) - E_{13}(D_{12} + D_{13}).$$

$$\text{Rearranging for } D_{E_{11}} : D_{E_{11}} = \frac{E_{13}}{E_{11}}(D_{12} + D_{13}) + D_{11},$$

$$\text{and } D_{E_{11}} = 1 - \frac{\tilde{E}_{11}}{E_{11}}.$$

Here we note that the empirical damage parameters are not exactly the same as those used by Baste and Aristiégui (1998); Castellano et al. (2017) etc and we do not need to symmetrize our empirical damage tensor to recover a supersymmetric elastic tensor (Olsen-Kettle 2018a, b, 2019).

Similarly $\tilde{E}_{12} = (1 - D_{E_{13}})E_{13}$ etc and we can rearrange Eq. (7) to define the empirical damage parameters below:

Fig. 15 The internal diagonal tensorial damage variables for a transverse isotropic shale sample (axis of isotropy initially in x_1 direction) subject to uniaxial loading in the x_3 direction. Here we compare the 95% confidence intervals for the simplified analysis in Sect. 1 to the more general derivation in section 4.2 in Olsen-Kettle (2018a)

$$\begin{aligned}
 D_{E_{11}} &= \frac{E_{13}}{E_{11}}(D_{12} + D_{13}) + D_{11} = 1 - \frac{\tilde{E}_{11}}{E_{11}}, \\
 D_{E_{22}} &= \frac{1}{E_{33}}(E_{13}D_{21} + E_{23}D_{23}) + D_{22} = 1 - \frac{\tilde{E}_{22}}{E_{11}}, \\
 D_{E_{33}} &= \frac{1}{E_{33}}(E_{13}D_{31} + E_{23}D_{32}) + D_{33} = 1 - \frac{\tilde{E}_{33}}{E_{33}}, \\
 D_{E_{12}} &= \frac{1}{E_{13}}(E_{33}D_{12} + E_{23}D_{13}) + D_{11} = 1 - \frac{\tilde{E}_{12}}{E_{13}}, \\
 D_{E_{13}} &= \frac{1}{E_{13}}(E_{23}D_{12} + E_{33}D_{13}) + D_{11} = 1 - \frac{\tilde{E}_{13}}{E_{23}}, \\
 D_{E_{23}} &= \frac{1}{E_{23}}(E_{33}D_{23} + E_{13}D_{21}) + D_{22} = 1 - \frac{\tilde{E}_{23}}{E_{23}}, \\
 D_{E_{44}} &= 2D_{44} = 1 - \frac{\tilde{E}_{44}}{E_{44}}, \\
 D_{E_{55}} &= 2D_{55} = 1 - \frac{\tilde{E}_{55}}{E_{66}}, \\
 D_{E_{66}} &= 2D_{66} = 1 - \frac{\tilde{E}_{66}}{E_{66}}.
 \end{aligned} \tag{8}$$

A.4 Quantifying Internal Damage Variables for an Initially Transverse Isotropic Solid Becoming Orthotropic in Terms of the Empirical Damage Parameters Given by Ultrasonic Measurements

To relate the internal fourth rank damage variables (D_{ij}) to the empirical damage variables ($D_{E_{ij}}$) we can invert Eq. (8) and use the constraints for D_{21} , D_{31} and D_{32} in Eq. (6). The internal damage variables are:

Fig. 16 The internal off-diagonal tensorial damage variables for a transverse isotropic shale sample (axis of isotropy initially in x_1 direction) subject to uniaxial loading in the x_3 direction. Here we compare the 95% confidence intervals bars for the simplified analysis in Sect. 1 to the more general derivation in section 4.2 in Olsen-Kettle (2018a)

$$\begin{aligned}
D_{11} &= \alpha [E'(\nu - 1)D_{E_{11}} + E\nu^2(D_{E_{12}} + D_{E_{13}})] \\
D_{22} &= \frac{\alpha}{1 + \nu} [(E\nu^2 - E')D_{E_{22}} + (E'\nu + E\nu^2)\nu D_{E_{23}} \\
&\quad + E\nu^2(1 + \nu)D_{E_{12}}] \\
D_{33} &= \frac{\alpha}{1 + \nu} [(E'\nu + E\nu^2)\nu D_{E_{23}} + (E\nu^2 - E')D_{E_{33}} \\
&\quad + E\nu^2(1 + \nu)D_{E_{13}}] \\
D_{12} &= E'\nu'\alpha [D_{E_{11}} - D_{E_{12}} + \nu(D_{E_{13}} - D_{E_{11}})] \\
D_{13} &= E'\nu'\alpha [D_{E_{11}} - D_{E_{13}} + \nu(D_{E_{12}} - D_{E_{11}})] \\
D_{23} &= \frac{\alpha}{1 + \nu} [(E' - E\nu^2)\nu D_{E_{22}} - (E'\nu + E\nu^2)D_{E_{23}} \\
&\quad + E\nu^2(1 + \nu)D_{E_{12}}] \\
D_{21} &= \frac{E}{E'^2(1 - \nu^2)} [(E' - E\nu^2)D_{12} + E'\nu'(1 + \nu) \\
&\quad \times (D_{11} - D_{22} - D_{23}) + (E'\nu + E\nu^2)D_{13}] \\
D_{31} &= \frac{E}{E'^2(1 - \nu^2)} [(E' + E\nu^2\nu)D_{13} + E'\nu'(1 + \nu) \\
&\quad \times (D_{11} - D_{23} - \nu D_{22}) + E'\nu'(\nu^2 - 1)D_{33} + (E' - E\nu^2)\nu D_{12}] \\
D_{32} &= D_{23} + \nu(D_{22} - D_{33}) + \frac{E\nu'}{E'}(D_{12} - D_{13}) \\
D_{44} &= \frac{D_{E_{44}}}{2} \\
D_{55} &= \frac{D_{E_{55}}}{2} \\
D_{66} &= \frac{D_{E_{66}}}{2}
\end{aligned} \tag{9}$$

where:

$$\alpha = \frac{1}{(\nu - 1)E' + 2E\nu'^2}$$

and $D_{E_{11}}, D_{E_{22}}, D_{E_{12}}, D_{E_{13}}, D_{E_{23}}, D_{E_{33}}, D_{E_{44}}, D_{E_{55}},$ and $D_{E_{66}}$ are measured experimentally from the ultrasonic elastic wave measurements.

The fourth order damage tensor for an initially transverse isotropic solid undergoing damage to become orthotropic (where the material symmetry axes do not change from undamaged to damaged state) in Voigt notation is:

$$D = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{pmatrix}$$

Appendix B: Comparing Two Analytical Derivations for the Damage Tensor

In this section we assume that the shale and sandstone are initially transverse isotropic (as is often assumed for shale). We assume that the axis of isotropy is in the x_1 -direction. This means that for this section initially there are only 5 independent elastic moduli calculated from the measured ultrasonic velocities (however 9 ultrasonic velocities were actually measured along the 9 independent symmetry directions for an orthotropic solid shown in Fig. 2). The axis of isotropy was assumed to be in the x_1 -direction and this is based on inspection of the bedding planes visible in the two samples (see Figs. 3 and 4).

We calculated the internal damage variables using two analytical derivations for these tensor quantities. The analytical derivation for the damage tensor variables given in this paper in Eq. (9) is termed the simplified case where the shale is assumed to be initially transverse isotropic. As a comparison we also show the results for a different derivation of the damage tensor variables derived for the more general case of an initially orthotropic material given in section 4.2 in Olsen-Kettle (2018a). Figures 15 and 16 show that as expected both methods produce the same values for the damage tensor variable, however the confidence intervals for the simplified derivation in this paper is much smaller than for the more general case. Because the measurement errors of the ultrasonic velocities are propagated through the calculation of the damage tensor variable we can clearly see the advantage of using the simplified derivation shown here for an initially transverse isotropic material.

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Conflict of interest The authors declare that they have no conflict of interest.

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