A Spherical Indentation Study on the Mechanical Response of Selected Rocks in the Range from Very Hard to Soft with Particular Interest to Drilling Application

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Abstract

The focus of this work is toward an investigation of the mechanical response of a variety of rocks to indentation loading as a close condition to drilling application. A rock classification method is introduced based on the mechanical response of the rock when loaded by a spherical indenter. Spherical shape is selected for the indenter as a common geometry, which to some extent represents most of the new or worn inserts in drill bits. Both of the force–penetration and fragmentation responses are studied and the results are categorized accordingly. Indentation loading of a quasi-brittle medium like rock contains a complex three-dimensional stress state, in which the compressive strength, tensile strength, compaction behavior and the brittleness of the rock all together are reflected in its mechanical response. Therefore, depending on the type of the rock and its properties, this response is also very diverse for different rocks. Eight types of rocks are investigated and the results from the force–penetration and fragmentation responses are summarized into three different classes.

Keywords Indentation test · Rock classification · Granite · Limestone · Sandstone · Diabase

1 Introduction

The knowledge of the mechanical response of rocks subjected to indentation loading plays a critical role in how to effectively break them in drilling applications. The fracture system and the load capacity of brittle and semi-brittle materials loaded by an indenter has been widely investigated in the literature (Wagner and Schümann 1971; Lawn and Wilshaw 1975; Hagan 1979; Yoffe 1982; Cook et al. 1984; Wijk 1989; Cook and Pharr 1990; Pang and Goldsmith 1990; Tan et al. 1998; Rouxel et al. 2010). To analyze the fracture system, the focus is obviously on the formation and propagation of different types of cracks during the indentation process, together with the understanding of the development of a compacted zone created immediately beneath the indenter. Depending on the brittleness of the material and based on its microstructure, and the amount of defects

such as pores and microcracks, the boundary between the crushed zone and the fractured zone is changed. The crushed zone is believed to develop due to high compressive stresses very early during the indentation process and then transmits the force to the rest of the material (Lindqvist and Hai-Hui 1983). The formation of different types of cracks including radial, conical and median cracks in the fractured zone, however, is connected to tensile stresses generated due to indentation and is observed during the loading stage (Lawn and Wilshaw 1975; Hagan 1979; Cook et al. 1984; Cook and Pharr 1990). On the other hand, the initiation of lateral cracks, also called side cracks, is suggested to be related to the expansion of the fractured rock material under the indenter as the load is increased (Wagner and Schümann 1971). However, the main reason for the further formation of the lateral cracks during the unloading stage is suggested to be driven by residual stresses at the boundary of the compacted zone (Hagan 1979; Cook and Pharr 1990). Different cracking patterns were observed in sharp indentation of glass both at loading and unloading stages depending on the glass composition, which reflects itself in ductility or brittleness of the material (Rouxel 2015).

The load-bearing capacity of rocks during a flat punch indentation test, also called stamp test, has been historically

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used in the mining industry to characterize the rock properties and its response at drilling and excavation processes (Wagner and Schümann 1971; Wijk 1989). The stamp strength σ_{ST} , which is the mean contact pressure at failure (load capacity), of rocks has been found to be affected by the indenter size. The effect of this size dependency is more pronounced in case of brittle rocks as compared to ductile rocks (Wagner and Schümann 1971). The stamp strength together with the crater volume formed in the rock during the stamp test is used for design of efficient drilling tools and also for the prediction of the drilling rate (Wijk 1989). Substantial efforts were made to predict the load capacity of the rock, during a stamp test, based on elasticity theory together with different failure criteria (Cook et al. 1984). It was observed that cracks and microcracks are formed in the rock prior to the final failure. While the well-known Hertzian crack is formed due to the first principal stress during loading, the crack propagates in a stable manner and the crack front becomes concealed in the region with intense microcracking (Cook et al. 1984). Therefore, the load capacity and final failure were connected to the initiation of a subsurface median crack due to the tensile hoop (circumferential) stress, which is the second principal stress except in a region centrally below the indenter. However, the suggested model is not able to predict the load capacity of the rock in the indentation test with an acceptable accuracy (Cook et al. 1984). The force–penetration response of rocks under sharp indenters has also been widely investigated and the fragmentation mechanism is explained (Michalowski 1985; Pang et al. 1989). The sharp conical indentation test has the advantage of self-similarity in the problem compared to the flat punch indentation of different sizes, but this will also lead to a situation where plastic (irreversible) deformation is introduced immediately at contact.

There are many standard tests and methods to classify rocks for drilling application. For instance, unconfined compressive strength (UCS), tensile strength from Brazilian or bending tests, porosity and texture, and the level of joints and faults are commonly used to classify different types of rocks (Howarth and Rowlands 1987; Thuro 1997; Kahraman et al. 2003; Tanaino 2005; Hoseinie et al. 2008, 2009; Dahl et al. 2012; Yarali and Soyer 2013; Taheri et al. 2016). Furthermore, these studies provide multiple drillability and penetrability indexes that can be used to classify different types of rocks and accordingly predict the drilling rate. As mentioned earlier, the main focus in these works is to find some correlation between the above-mentioned indexes to one or more standard properties of rocks. However, in drilling and excavation applications, the tool penetrates into the rock and breaks it and therefore the classification system must properly represent the loading condition. In fact, the indentation process in these applications occurs at a speed of up to 10 m/s, which means that the dynamic response may

be neglected to some extent in such a small affected volume of rock (Weddfelt et al. 2017). Therefore, investigation of rocks' responses at quasi-static indentation is still crucial to better understand the drilling process (Kahraman et al. 2000; Yagiz 2009; Shariati et al. 2019).

Brittleness is known as one of the important properties of rocks that affect the drilling efficiency (Yagiz 2009; Altindag 2010; Tarasov and Potvin 2013; Kaunda and Asbury 2016). But there is neither a general consensus on its definition nor a standardized measurement method to define it. In tension, most of the geomaterials are quite brittle and the opening of cracks is directly related to the fracture energy. When it comes to compression, the criteria for rock brittleness estimation should be obtained under triaxial compression (Tarasov and Potvin 2013). This is based on the concept of energy stored during the loading and energy released during the unloading associated with failure. It is well known that brittleness, under this definition, is a property that varies with the state of confinement. In this context, a rock that is very brittle under low confinement can lose its brittleness and even become ductile if subject to large hydrostatic pressures.

In drilling application, the drill bits produce high local confined pressures in the rock and therefore the compressive aspect of brittleness could be significant. However, high hydrostatic pressure occurs mainly locally under the tool, in the crushed/compacted zone. In the regions where tensile fracture occurs, which is indicative of good excavation conditions, most of the geomaterials are quite brittle. Therefore, the brittleness definition should include both of these regions and the outcome corresponds to the overall fragmentation response of the material at indentation process. As stated earlier, indentation loading of a quasi-brittle medium like rock contains a complex three-dimensional stress state, in which the compressive strength, tensile strength, compaction behavior and accordingly the brittleness of the rock all together are reflected in its mechanical response.

In this work, the focus is toward an investigation of the mechanical response of a variety of rocks to indentation loading as being closely related to drilling applications and possibly interpret this response based on contact mechanics knowledge. Both the force–penetration and fragmentation responses from different types of rocks are studied and compared. A spherical shape is selected for the indenter as a common geometry, as this to some extent represents most of the new or worn inserts in drill bits. Compared with the stamp test using cylindrical flat indenters, as a traditional method to classify rocks for drilling application (Wijk 1989), the contact area increases monotonically with the increase of the indentation depth for spherical indenters as is the case at drilling. Consequently, the location of the tensile stress components on the surface, as well as inside the rock up to a depth of a few times larger than the contact radius, changes constantly during the indentation process. These

dissimilarities may have an effect on the chipping process and the fracture pattern.

2 Theoretical Background

The stress state in rocks subjected to indentation loading plays a crucial role when investigating their fragmentation response. Depending on the mechanical behavior of the rock from hard and brittle up to soft and ductile, the indentation response also changes from nearly elastic up to fully plastic. This affects the whole stress state in the rock including the location and amplitude of the tensile stresses, which are mainly responsible for opening the tensile cracks and forming chips, and accordingly explains the fragmentation response. A brief initial theoretical discussion of the elastic and elastic–plastic contact is presented in this section that will be used for interpretation of the experimental results.

2.1 Elastic Contact

The stress state in an elastic body subjected to indentation loading can be analytically solved for different shapes of indenters. For a spherical indenter as the selected geometry in this work, the normal stress distribution under the indenter is first given by Hertz (Hertz 1881). Furthermore, the complete stress field on the surface and interior of the body is also given (e.g., in Johnson 1987, 2006).

On the surface of the specimen, beneath the indenter, all three principal stresses are compressive. The radial stress is the first principal stress on the surface. Outside the contact circle but still on the surface, the radial stress becomes tensile with a maximum value at the edge of the

contact circle. This stress is responsible for the formation of Hertzian cone cracks. The hoop stress, on the surface, is always a principal stress because of symmetry. It is nearly always the second principal stress inside the contact circle, but outside the contact circle it is equal in magnitude to the radial stress with opposite sign. The normal stress is the third principal stress on the surface of the specimen inside the contact region, and its magnitude becomes zero outside the contact region since it acts normal to a free surface in this region.

Within the interior of the specimens, the stresses are analytically calculated (Huber 1904; Lawn et al. 1974, 2006). Beneath the surface along the axis of symmetry z at ($r=0$), the first two principal stresses are equal. They are compressive up to a depth of about 1.3 times the contact radius a (for a Poisson's ratio $\nu = 0.25$) and then immediately become tensile (see Fig. 1). As presented in Zeng et al. (1992) and 2006, the sign of the compressive stresses is negative in contour plots as shown in Fig. 1. From this depth, the first principal stress extends tensile all the way up to the surface at the contact boundary and onward the contact region (see Fig. 1a). On the surface of the specimen, this principal stress is in the radial direction as stated before. This stress is responsible for the formation of Hertzian cone cracks. The second principal stress, however, extends tensile downward and sideward from this depth in a circular shape (see Fig. 1b). Cracks corresponding to this stress component are also suggested to extend downward and sideward in a circular shape, which are called median cracks in the literature (Lawn et al. 1980; Cook and Pharr 1990). Some efforts have been devoted to connect the load capacity of hard rocks at stamp test using cylindrical indenters to the same stress component (Weddfelt et al. 2017).

Fig. 1 Stress contours of the principal stresses calculated for Poisson's ratio $\nu = 0.25$ and normalized by maximum contact pressure 1.5 pm (p_m is contact force divided by projected contact area). **a** First

principal stress, **b** second principal stress, **c** third principal stress (Zeng et al. 1992, 2006; Louapre and Breder 2015)

2.2 Elastic–Plastic Contact

Analytical treatments of the indentation stress fields for elastic–plastic contact are complex due to the presence of plastic deformation under the indenter. There has been a lot of attention to this topic in the literature leading to multiple theories (Marsh 1964; Johnson 1970; Chiang et al. 1982a, b; Yoffe 1982; Biwa and Storåkers 1995; Hill 1998). However, these difficulties can be avoided by use of the finite element (FE) method which can provide numerical results for complex geometries and material properties.

The indentation response of an elastic–plastic material can generally be divided into three regimes, from fully elastic to constrained plastic and finally fully plastic, depending on the significance of the plastic deformation under the indenter. The first regime, in which the response is elastic, occurs at the initial stage of loading and the response can be predicted by Hertzian theory. Theoretical treatment of regime two is difficult because of the uncertainty in size and shape of the evolving plastic zone. In regime three, the plastic region is significant and reaches the top free surface of the specimen. If ideal plasticity is assumed in this regime, little or no increase in the contact pressure is expected when increasing the indenter load, leading to a linear force–penetration response (Storåkers et al. 1997).

The presence of the plastic zone significantly changes the near-field indentation stresses, while the far-field stresses appear to be little changed compared with the elastic case (2006). For instance, including plasticity would lead in some cases to circumferential tension right outside the contact region on the surface (Johnson 1987; Biwa and Storåkers 1995), which is opposite to the elastic case in which the radial stress is tensile in this region. The change from radial tension under elastic conditions to circumferential tension under elasto-plastic is largely responsible for the change of fracture pattern from a ring crack in a very brittle material such as glass to a radial crack in a semi-brittle material such as Perspex (Johnson 1987) and granite (Shariati 2019). It should be emphasized that this discussion is applicable to brittle materials that exhibit shear-driven plasticity at indentation (2006). Porous materials behave differently, as their response to indentation is mainly crushing and compaction due to the indentation pressure.

3 Experimental Work and Discussion of Results

Indentation loading of a quasi-brittle medium like rock induces a complex three-dimensional stress state, in which the compressive strength, tensile strength, compaction behavior and the brittleness of the rock all together are reflected in its mechanical response. This response can be

primarily summarized into two aspects: (a) force–penetration (F – d) and (b) fragmentation (cracking) mechanism.

- (a) The force–penetration response is mainly governed by the load-carrying part of the material under the indenter, which is either compacted due to high hydrostatic pressure, or deformed due to compressive and shear stresses. In addition, the tensile failure and cracking may also be reflected in the form of load drops or degradation of the force–penetration slope.
- (b) The fragmentation mechanism includes different failure zones, from the compacted and crushed material due to compressive and shear stresses up to the chips and fragments that are cracked due to tensile stresses. These failure zones all together form a crater in the material that contributes in an accumulative manner to the drilling rate. Depending on the type of the rock and its properties, each of these failure zones has different significance, which means that the mechanical response to indentation is also very diverse for different types of rock.

In this study, both of the force–penetration and fragmentation responses from different types of rocks at indentation are analyzed and compared and the results are categorized accordingly.

3.1 Experimental Setup

The experimental work contains multiple indentation experiments on different types of rocks using an 11 mm spherical indenter made of tungsten carbide. The elastic modulus of the indenter is one order of magnitude higher than that of the rocks and therefore can be approximated as rigid. The experiments were performed in quasi-static conditions and in displacement control at a rate of 0.01 mm/s using an MTS 500 kN load frame (see Fig. 2). The rock specimens were carefully saw-cut blocks of size $200 \times 200 \times 100 \text{ mm}^3$. Multiple indentations were made on the same block, but the indentation results with far enough spacing from each other were selected for this study to avoid any interaction effect. The tests were stopped at an indentation depth around 1.5–2 mm to avoid the split of the whole block.

3.2 Force–Penetration (F – d) Response

The force–penetration (F – d) response of different rocks when subjected to spherical indentation loading is discussed in this section. The F – d response is divided into two parts: (a) up to the load capacity; (b) post-load capacity. The load capacity is defined as the maximum force level that a rock can take during indentation before the major load drop occurs and the force level drops almost to zero (Weddfelt

Fig. 2 The experimental setup for the indentation tests

et al. 2017). This phenomenon is more distinct in hard rocks such as granite and diabase. During the indentation process, the load increases gradually by increasing the indentation depth. However, before the load capacity is reached, a number of smaller load drops may usually be observed due to chipping of the material. The combination of the F - d slope, severity of the load drops and the load capacity level is a unique characteristic of each rock and an essential feature how to break it.

Figure 3 indicates the F - d curves obtained from the indentation tests on a variety of rocks. The circle symbols in the plots correspond to the maximum force level in each test, while the cross symbol indicates the last result captured before the test was stopped. A summary of the indentation test results on different rocks is reported in Table 1.

The F - d responses of the whole tests are categorized into the following three different classes:

Class I: The first class contains Bohus granite, Gylsoda diabase and Diamantberget quartzite. A distinguished load

capacity is observed in all three cases, followed by a very dramatic load drop and failure. The failure in this class occurs with a loud sound and flying fragments. The indentation depth and the crater size are limited in this class. The initial slope of the F - d curve is very high and there is also rather small variation in this slope. Multiple small load drops are captured before the load capacity is reached. From a theoretical standpoint, it can be concluded that the contact response in this class is of regime two with constraint plasticity. If it was pure elastic, the force level should have been proportional to $F \sim d^{3/2}$ which is not the case here. But at the same time, the large load drop together with the flying fragments suggests that the plastic deformation is limited. The same conclusion can be drawn from the FE analysis of Bohus granite in this class at indentation, which has been previously performed and reported, in which the inelastic (plastic) response was investigated for this material (Shariati et al. 2019). It should be mentioned that the F - d slope is close to linear even though rigid plasticity is not the case

Fig. 3 The F - d response of different rocks at spherical indentation, **a** class I, **b** class II, **c** class III. Spacing is large enough (around 80 mm would be enough in all cases) to avoid any interaction between the indentations

Fig. 3 (continued)

Table 1 Summary of the indentation test results on a variety of rocks

	Rock type	Initial $F-d$ slope	Load drop level	Crater size
Indentation response				
Class I	Bohus granite	High	Large	Small
	Gylsboda diabase	High	Large	Small
	Diamantberget quartzite	High	Large	Small
Class II	Offerdal quartzite–(schist)	Medium	Medium	Small
	Offerdal (quartzite)–schist	Medium	Medium	Medium
	Ekeberg marble	Medium	Medium	Medium
Class III	Lingulid sandstone	Medium	Small	Medium
	Grå Jämtland limestone	Low	Small	Large

here. This at least suggests that the hardening of the material is minor at a stress range corresponding to indentation problems.

Class II: The second class contains Offerdal quartzite–schist as a layered rock and Ekeberg marble. The failure is much less dramatic in this case and there is also less significant load capacity. In some cases, multiple load drops occurred up to a rather large indentation depth. Quartzite–schist is a layered rock and had different responses depending on which layer was subjected to indentation. Even the initial slope of the F – d was close for both layers, the indentation on schist layer ((quartzite)–schist in Fig. 3) indicated higher maximum loads followed by relatively large but rather smooth load drops, while lower maximum forces and multiple smaller and sharper load drops was captured in the quartzite layer (quartzite–(schist) in Fig. 3). This is interpreted in the way that quartzite as a brittle and strong material had a lot of small chipping, but the force did not get high as the quartzite layer was relatively thin. However the schist layers indicated more crushing and compaction as well as larger chipping. In marble, the initial slope was the same as the rest in this class, but in the remaining of the indentation process, multiple rather smooth load drops were captured, which suggests a relatively large compaction zone but also some chipping as well. The absence of flying fragments suggests that the level of elastic deformation is less than in case of class I materials. At the same time, rigid plasticity does not apply as there are multiple load drops in this class.

Class III: The third class contains Lingulid sandstone and Grå Jämtland limestone. There is no distinguishable load capacity followed by load drops at all, even if the indentation was performed up to relatively large depths. The sandstone has much higher slope in the F – d curve and experiences almost no load drop up to the maximum indentation depth. The high level of porosity together with small particles being compacted together may be one of the reasons for this response. The limestone indicates very low F – d slope and almost gradual failure together with many smooth load drops in the whole range of indentation depth. This suggests a major compaction process together with some chipping of the material. It is concluded that there is major plastic deformation, which means closer to rigid plastic conditions, in this class as there is almost no load drops and flying fragments. Furthermore, the F – d slope is linear indicating low strain hardening. However, the plastic deformation in this case is mainly due to pore collapse and compaction of the material rather than shear driven plasticity as in the case of harder rocks.

3.3 Fragmentation Response

The fragmentation response of the three classes introduced in the F – d section is summarized in this section (see Fig. 4).

Class I: The fragmentation is limited, close to the surface. The compacted zone is noticeable after indentation, in the form of crushed particles compacted to each other. Some small fragments are formed during the loading and the major load drop and the crater size are relatively small. Large radial cracks are not seen on the surface due to the limited amount of plastic deformation.

Class II: Both the crushed and cracked zones are larger than those of class I. Large fragments are formed during loading and the load drops and the size of the crater is relatively large. The indentation depth prior to multiple load drops corresponds approximately to the pre-major load drop indentation depth in class I (~0.5 mm). After this depth, the plastic deformation and accordingly radial cracks are more noticeable in this class.

Class III: The crushed zone is large for both rocks. For sandstone, the failure mainly contains the crushed material and the crater size is not significant. In the case of limestone, however, the crushed parts extends deep into the material and significant fragments are formed without that much load drops. One limestone sample was broken into pieces during loading and the extension depth of the crushed zone is visible in this case (see Fig. 5). There is extensive plastic deformation for both of the materials in this class, one with pore collapse and particle compaction, and minor cracking (sandstone), and the other with large noticeable radial cracks on the surface (limestone).

4 Discussion and Conclusions

In this work, the focus is toward an investigation of the mechanical response of a variety of rocks to indentation loading as a close condition to drilling application. Percussive drilling is basically a high velocity (up to 10 m/s) indentation of multiple inserts into rocks. In this range of indentation rate, it is suggested that quasi-static indentation can appropriately represent the real application (Weddfelt et al. 2017). In percussive drilling, both rate and size effects on the tensile strength of rocks play an important role in the fragmentation response. However, considering the small values of the effective volume during indentation loading (Weddfelt et al. 2017), the rate dependence of rocks (e.g., Bohus granite based on previous studies performed by the authors (Saadati et al. 2016)) and also the range of strain rates occurring in percussive drilling (Saadati et al. 2014, 2016), the rate effect should be less critical than the size effect (Weddfelt et al. 2017). Furthermore, a single button indentation gives the rock fragmentation response and excludes any effect from the neighboring inserts. This could be advantageous when it comes to the position of the neighboring inserts and to avoid over-crushing of rocks when they are placed too close to each other. Both the force–penetration

Bohus granite

Gylsboda diabase

Diamantberget quartzite

a

Fig. 4 The fragmentation response of different rocks to a spherical indenter before (left) and after (right) cleaning of the indentation imprint, **a** class I, **b** class II, **c** class III

Offerdal quartzite (schist): quartzite layer

Offerdal (quartzite) schist: schist layer

Ekeberg marble

b

Fig. 4 (continued)

Lingulid sandstone

Grå Jämtland limestone

c

Fig. 4 (continued)

and fragmentation responses from different types of rocks are studied and compared. A spherical shape is selected for the indenter as a common geometry, which to some extent represents most of the new or worn inserts in drill bits.

The indentation responses of the whole tests are categorized into three different classes:

Class I: The first class contains Bohus granite, Gylsboda diabase and Diamantberget quartzite. A distinguished load capacity is observed in all three cases, followed by a very dramatic load drop and failure. The failure in this class occurs with a loud sound and flying fragments. The fragmentation is rather limited close to the surface and the crater size is relatively small.

Class II: The second class contains Offerdal quartzite-schist as a layered rock and Ekeberg marble. The failure is much less dramatic in this case and there is also less

significant load capacity. Both the crushed and cracked zones and accordingly the crater are larger than those of class I.

Class III: The third class contains Lingulid sandstone and Grå Jämtland limestone. There is no distinguishable load capacity followed by load drops at all, and even the indentation was performed up to relatively large depths leading to fairly large craters.

These different responses can have significant effect on how to efficiently drill different rocks in practical applications. For instance, class I rocks require a certain threshold force to be passed and a major fragmentation would be expected after that stage. This means in practice that impacting this type of rocks with less force than required does not lead to a proper fragmentation process and the drilling would be partly grinding the rock instead, which is not

Grå Jämtland limestone

Fig. 5 The extension of the crushed zone into the limestone

energy efficient. On the other hand, class III shows gradual failure without a critical threshold force level to be passed to perform efficient drilling. In this class of rocks, more aggressive inserts with sharper geometry may also be advantageous to obtain extra indentation depth and crater volume.

It should also be mentioned that the selection of rocks in this study are in the range from very hard and brittle to relatively soft and ductile. The number of tested rock types is however still somewhat limited and the given classification hence is also somewhat restricted. This means that the same methodology may be used to include more types of rocks in the future and possibly complement different classes.

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Compliance with Ethical Standards

Conflict of interests The authors declare that there is no conflict of interest regarding the publication of this paper.

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