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Explicit Renormalization of Nuclear Chiral EFT and Nonperturbative Effects

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Abstract Nucleon-nucleon interaction is studied within chiral effective field theory at next-to-leading order in the chiral expansion. The leading order interaction is resummed nonperturbatively, whereas the next-to-leading-order terms are taken into account in a perturbative manner. Explicit renormalizability of such a scheme is analyzed in several important cases. The possibility to absorb the power-counting breaking terms originating from the integration regions with large momenta is studied for both perturbative and nonperturbative regimes. A comparison of the schemes with a finite and an infinite cutoff is performed.

1 Introduction

One of the most powerful tools for studying hadronic systems at low energies is the effective field theory (EFT) approach, which allows one to perform systematically improvable calculations in accordance with a well defined power counting. To take advantage of all symmetries of the Standard Model, one typically employs chiral EFT that contains nucleon and pion fields as degrees of freedom.

The chiral power counting corresponds to an expansion of observables in terms of the ratio of the soft and the hard scale $Q = \frac{q}{\Lambda_b}$. The soft scale q is determined by the pion mass M_π and the external particle 3-momenta $|\mathbf{p}|$. The chiral expansion breakdown scale Λ_b can be associated with the ρ-meson mass.

The main ideas of chiral nucleon-nucleon (NN) EFT were formulated in the works of Weinberg [1,2]. Recent progress in the field is reviewed in Refs. [3–6]. Compared to the purely pionic and in the single-nucleon sector, where one can calculate obserbables in a strictly perturbative manner using the so-called chiral perturbation theory, in the NN sector, a nonperturbative (at least to some extent) approach is necessary.

The nonperturbative treatment of the NN amplitude requires a regularization of an infinite number of divergent Feynman diagrams. The most practical approach to realize it is to introduce a finite (of the order of the hard scale Λ_b) cutoff Λ in momentum space, see Refs. [7–9] for recent applications.

To rigorously justify such a scheme from the fundamental point of view, one has to deal with the issue of renormalization and power counting violation caused by the appearance of positive powers of the cutoff in the amplitude. Such contributions come from the loop momenta of the order of the cutoff Λ. As was shown in Refs. [10,11], such positive powers of Λ in the leading-order (LO) amplitude are compensated by the negative powers of the scale $\Lambda_V \sim \Lambda_b$ coming from the LO potential. Moreover, at next-to-leading (NLO) EFT order, the power counting breaking terms can be absorbed by a renormalization (shift) of lower order contact interactions under certain conditions.

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An alternative approach is represented by the infinite-cutoff or "renormalization group (RG)-invariant" scheme, which assumes a finite limit and cutoff independence of the scattering amplitude at each EFT order separately for $\Lambda \to \infty$, i.e. $\Lambda \gg \Lambda_b$, see Refs. [6,12] for reviews.

In this talk, we report on various issues related with the renormalization and the nonperturbative effects in both finite- and infinite-cutoff schemes.

2 Chiral EFT Expansion for the NN Amplitude

Chiral EFT is based on the effective Lagrangian represented as a series of all possible terms consistent with the symmetries of the Standard Model [13]. The expansion of the Lagrangian is performed in terms of the quark masses and the derivatives of fields:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \dots , \tag{1}$$

where the superscripts denote chiral orders, and the subscripts π, πN and NN denote the purely pionic, single-nucleon and two-nucleon parts of the Lagrangian, respectively.

The chiral expansion of the NN amplitude in terms of the small parameter Q is performed according to the Weinberg's power counting [2] (although certain modifications based on phenomenological arguments are allowed, e.g. promotion of some higher order contributions to lower orders). The chiral order for a potential (two-nucleon-irreducible) contribution is determined by a sum over all vertices i in the diagram:

$$D = 2L + \sum_i \left(d_i + \frac{n_i}{2} - 2 \right) , \tag{2}$$

where L is the number of loops, n_i is the number of nucleon lines in vertex i and d_i is the number of derivatives and the pion-mass insertions at vertex i, whereas the chiral order of a $2N$-reducible diagram is equal to the sum of the orders of its irreducible components.

Bare potentials $V^{(i)}$, where i is a chiral order are split into the renormalized parts V_i and the counter terms δV_i:

$$V^{(i)} = V_i + \delta V_i \, , \quad \delta V_i = \delta V_i^{(2)} + \delta V_i^{(3)} + \delta V_i^{(4)} + \dots . \tag{3}$$

The counter terms $\delta V_i^{(j)}$ $(j > i)$ absorb the divergent and power counting violating terms appearing at order $O(Q^j)$.

The LO potential V_0 appears at order $O(Q^0)$ and is represented by the sum of the one-pion-exchange potential and the short-range part:

$$V_0(\vec{p}\,', \vec{p}\,) = V_{1\pi}^{(0)}(\vec{p}\,', \vec{p}) + V_{\text{short}}^{(0)}(\vec{p}\,', \vec{p}), \tag{4}$$

where the short-range part $V_{\text{short}}^{(0)}$ may contain momentum-independent contact terms as well as the contact terms quadratic in momenta.

Since the LO potential V_0 is of order $O(Q^0)$, it has to be iterated an infinite number of times. To make the iterations of V_0 finite, V_0 is regulated using a cutoff Λ.

The next-to-leading-order ($O(Q^2)$) potential $V_2(\vec{p}\,', \vec{p})$ contains the short-range part and the leading two-pion-exchange potential:

$$V_2(\vec{p}\,', \vec{p}) = V_{2\pi}^{(2)}(\vec{p}\,', \vec{p}) + V_{\text{short}}^{(2)}(\vec{p}\,', \vec{p}). \tag{5}$$

In the partial wave lsj basis, the NN potential and the NN scattering amplitude become $n_{\text{PW}} \times n_{\text{PW}}$ matrices, where $n_{\text{PW}} = 1$ ($n_{\text{PW}} = 2$) for the uncoupled (coupled) partial waves. The series for the partial wave LO amplitude T_0 and for the unrenormalized NLO amplitude T_2 are given by

$$T_0 = \sum_{n=0}^{\infty} T_0^{[n]}, \qquad T_0^{[n]} = V_0 K^n = \bar{K}^n V_0, \tag{6}$$

$$T_2 = \sum_{m,n=0}^{\infty} T_2^{[m,n]}, \qquad T_2^{[m,n]} = \bar{K}^m V_2 K^n, \tag{7}$$

where G is the free two-nucleon propagator and

$$K = GV_0, \qquad \bar{K} = V_0 G. \tag{8}$$

In the nonperturbative regime for the LO potential, these equations generalize to

$$T_0 = V_0 R = \bar{R} V_0, \tag{9}$$

$$T_2 = \bar{R} V_2 R, \tag{10}$$

where R ($\bar{R}$) is the resolvent of the Lippmann-Schwinger equation (LSE)

$$R = \frac{1}{\mathbb{1} - K}, \qquad \bar{R} = \frac{1}{\mathbb{1} - \bar{K}}. \tag{11}$$

The renormalized expression for the NLO amplitude $\mathbb{R}(T_2)$ is obtained by adding the relevant counter term:

$$\mathbb{R}(T_2) = \bar{R}\left(V_2 + \delta V_0^{(2)}\right) R. \tag{12}$$

The explicit form of the LSE, $T_0 = V_0 + V_0 G T_0$, reads

$$(T_0)_{l'l}(p', p; p_{\rm on}) = \sum_{l''} \int \frac{p''^2 dp''}{(2\pi)^3} (V_0)_{l'l''}(p', p'') G(p''; p_{\rm on})(T_0)_{l''l}(p'', p; p_{\rm on}),$$

$$G(p''; p_{\rm on}) = \frac{m_N}{p_{\rm on}^2 - p''^2 + i\epsilon}, \tag{13}$$

where the indices l, l', l'' denote the orbital angular momentum of the NN system, $p_{\rm on}$ is the on-shell c.m. nucleon momentum and p (p') are the initial (final) off-shell c.m. momenta.

3 Finite-Cutoff Scheme

3.1 Renormalization of the Nucleon-Nucleon Amplitude at NLO in the Perturbative Regime

The first step towards explicit renormalization of the NN amplitude within chiral EFT consists in considering the case of a perturbative LO interaction [10]. This means that the series in Eqs. (6) and (7) are convergent, although one may need to take into account an arbitrarily large number of terms in this expansion. For the physical NN system, this situation takes place for most of the P-waves and higher partial waves. The nonperturbative channels correspond to the $^1S_0$, $^3S_1 - {}^3D_1$ and $^3P_0$ partial waves. Nevertheless, the nonperturbative analysis for these channels can be constructed as an extension of the perturbative treatment.

For the LO amplitude, it can be rigorously shown that each term in the expansion in Eq. (6) satisfies the dimensional power counting and is of chiral order $O(Q^0)$ [10]. The proof is based on certain bounds on the potentials and their various subtraction reminders. Similar bounds are used in the analysis of the NLO amlitude and the nonperturbative regime.

The NLO amplitude in P-waves and higher partial waves satisfy the dimensional power counting at each order in V_0 and is of chiral order $O(Q^2)$ up to logarithmic in Λ corrections.

The unrenormalized NLO amplitudes in the S-waves violate the dimensional power counting due to the regions of loop momenta of order Λ and turn out to be of order $O(Q^0)$. Renormalization procedure in these channels constitutes a nontrivial combinatorial problem [10]. In particular, one has to perform subtractions at a fixed, e.g., zero, external momentum in the spirit of the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) renormalization prescription [14–16] to absorb the power-counting breaking terms.

To be specific, one has to make the overall subtractions in all diagrams as well as the subtractions in all nested subdiagrams employing the so-called forest formula for the renormalized NLO amplitude $\mathbb{R}(T_2^{[m,n]})$:

$$\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}, \tag{14}$$

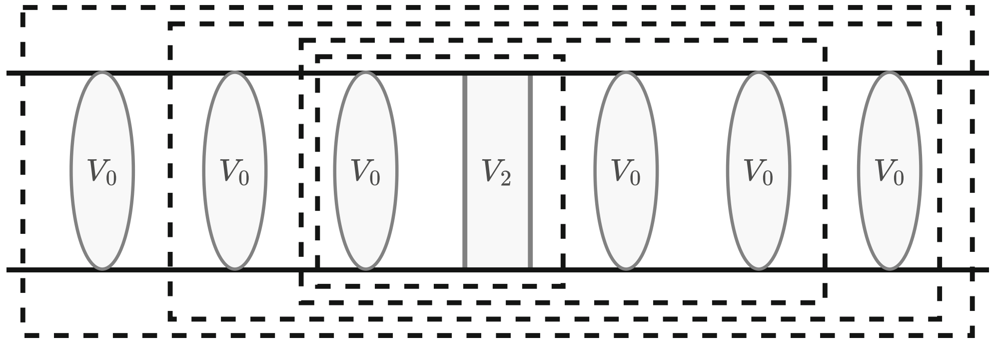

Fig. 1 An example of a forest for the amplitude $T_2^{[3,3]}$

where $\mathcal{F}^{m,n}$ represents the set of all forests, i.e, the set of all possible distinct sequences of nested subdiagrams. The subtraction operation $\mathbb{T}$ replaces an operator X with matrix elements (consider for simplicity an uncoupled channel, the generalization to coupled channels is straightforward) $X(p', p; p_{\rm on})$ with its value at $p = p' = p_{\rm on} = 0$:

$$\mathbb{T}(X) = X(0, 0, 0)V_{\rm ct}, \tag{15}$$

where $V_{\rm ct}$ is the leading contact operator.

An example of a forest that contributes to Eq. (14) for $m = n = 3$ is shown in Fig. 1.

To analyze all nested subtractions, an appropriate partition of the integration domain into sectors in the multidimensonal momentum space needs to be performed [10].

Finally, the renormalized NLO amplitude obeys the dimensional power counting at each order in V_0 and is of chiral order $O(Q^2)$ up to logarithmic in Λ corrections.

3.2 Renormalization of the Nucleon-Nucleon Amplitude at NLO in the Nonperturbative Regime

An extension of the analysis of the NN EFT to the case when the LO interaction is nonperturbative can be performed by applying the the Fredholm method for solving integral equations [11]. The LO and NLO amplitudes are then represented as the ratios

$$T_0 = \frac{N_0}{D}, \qquad T_2 = \frac{N_2}{D^2}, \tag{16}$$

where the Fredholm determinant is a function of the on-shell momentum only: $D = D(p_{\rm on})$. The quantities N_0, N_2 and D can be expanded into converging series in powers of V_0, so that the power counting can be analyzed by a matching to the perturbative case.

The LO amplitude can be shown to satisfy the dimensional power counting and is of chiral order $O(Q^0)$ provided the Fredholm determinant is natural $D \sim 1$. If the Fredholm determinant is unnaturally small, the LO amplitude is enhanced, which takes place for the ${}^1S_0$ and ${}^3S_1 - {}^3D_1$ channels, where a bound or quasibound state is located close to threshold.

Similarly to the perturbative case, the NLO amplitude for the P-waves and higher satisfy the dimensional power counting and do not have to be further renormalized.

As in the case of perturbative LO interaction, the S-wave NLO amplitude requires renormalization of the LO contact interactions to absorb the power-counting breaking terms coming from integration momenta of the order of Λ.

It turns out that the sum of the series for the renormalized amlitude in Eq. (14) can be obtained explicitly in a closed form [11]:

$$\mathbb{R}(T_2)(p_{\rm on}) = T_2(p_{\rm on}) + \delta C \psi(p_{\rm on})^2, \tag{17}$$

where the vertex function ψ is defined as (for simplicity an uncoupled partial wave is considered)

$$\psi(p_{\rm on}) = 1 + \int \frac{p^2 dp}{(2\pi)^3} G(p; p_{\rm on}) T_0(p, p_{\rm on}; p_{\rm on}) =: \frac{\nu(p_{\rm on})}{D(p_{\rm on})}. \tag{18}$$

The counter term constant is given by

$$\delta C = -\frac{T_2(0)}{\psi(0)^2} = -\frac{N_2(0)}{\nu(0)^2}. \tag{19}$$

Equation (17) can also be obtained directly without referring to the perturbative result from the renormalization condition:

$$\mathbb{R}(T_2)(0) = 0. \tag{20}$$

Therefore, the perturbative and the nonperturbative results match in the regime where both are applicable.

Although Eq. (17) is composed solely in terms of the quantities given by convergent series in V_0, the appearance of the factor $\nu(0)^2$ in the denominator of δC (see Eq. (19)) can potentially destroy renormalizability of the amplitude. This imposes additional constraints on the LO interaction (its short-range part), which were discussed in Ref. [11]. The simplest constraint is the requirement that the quantity $\nu(0)$ be natural, i.e., be bounded not only from above but also from below by some constant of order one:

$$\nu(0) \geq \mathcal{M}_{\nu,\min} \sim 1. \tag{21}$$

In practical calculations, it is easier to verify an equivalent condition that the counter term constant δC has a natural value.

Under the above assumption, the dimensional power counting for the renormalized NLO amplitude is restored also for the channels where the LO amplitude is enhanced. It turns out that for the realistic NN interaction and the cutoff values of the order of the hard scale $\Lambda \sim \Lambda_b$, the renormalizability constraints are fulfilled.

4 Infinite-Cutoff Scheme

For the NN scattering the "RG-invariant" scheme was shown to possess an infinite-cutoff limit for the LO amlitude [17]. It is also well established that a consistent inclusion of higher-order interactions within the "RG-invariant" scheme is possible only perturbatively, i.e., using the distorted-wave Born approximation, analogously to the treatment within the finite-cutoff scheme considered above. This is because of a singular behaviour of the NLO (and higher order) potential at short distances, which becomes particularly problematic when it is repulsive, see Refs. [12,18–20]. A perturbative inclusion of the NLO terms in spin-triplet channels within the infinite-cutoff scheme was considered in Refs. [21,22] using a regularization scheme in momentum space, see Refs. [23,24] for an analogous approach in coordinate space. In these studies, an indication of a cutoff independence of the NN amplitude at NLO in the limit $\Lambda \to \infty$ was observed.

However, additional constraints on the LO interaction originating from the requirement of the renormalizability of chiral EFT at NLO in the nonperturbative regime discussed above lead to limitations of the use of the infinite-cutoff scheme beyond leading order. It turns out that one cannot fulfill those conditions for all values of the cutoff Λ when passing to the limit $\Lambda \to \infty$. There always exist an infinite number of "exceptional" cutoffs in the vicinity of which the renormalization of the amplitude breaks down [25].

In particular, in the scheme of Long and Yang [21], one introduces for the renormalization of the NLO nucleon-nucleon amplitude in the $^3P_0$ channel, apart from one LO contact term, two NLO contact terms.

To fix the corresponding unknown constants at NLO, $C_0^{\rm NLO}$ and $C_2^{\rm NLO}$, one requires that the empirical phase shifts be exactly reproduced at two energy points E_0 and E_1, where E_0 is the same energy that was used to fix the LO counter term $C_0^{\rm LO}$. Unfortunately, for the exceptional values of the cutoff, the system of such renormalization conditions becomes inconsistent. This is caused by strong oscillations of the LO wave function at short distances, which is typical for the attractive singular potentials such as the unregulated one-pion-exchange potential in the spin-triplet channels, which behaves at short distances as $\sim 1/r^3$.

To be specific, we can write down the NLO amplitude as a sum of the two-pion-exchange amplitude and two contact contributions:

$$T^{\rm NLO}(E) = T_{2\pi}(E) + C_0^{\rm NLO} T_{{\rm ct},0}(E) + C_2^{\rm NLO} T_{{\rm ct},2}(E). \tag{22}$$

For exceptional values of the cutoff, the following condition is satisfied

$$\begin{vmatrix} T_{{\rm ct},0}(E_0) & T_{{\rm ct},2}(E_0) \\ T_{{\rm ct},0}(E_1) & T_{{\rm ct},2}(E_1) \end{vmatrix} = 0, \tag{23}$$

leading to $C_0^{\mathrm{NLO}}, C_2^{\mathrm{NLO}} \to \infty$. As a result, any residual cutoff dependence of the contact parts of the amplitude, $T_{\mathrm{ct},0}(E)$ and $T_{\mathrm{ct},2}(E)$, proportional to $1/\Lambda^\alpha$ with some $\alpha > 0$ is enhanced when multiplied by an arbitrarily large number.

It is important to note that the appearance of an infinite number of exceptional cutoffs does not depend on the choice of renormalization conditions (e.g., the choice of the energies E_0 and E_1), which only determine particular locations of such cutoffs.

5 Summary

Renormalization of nucleon-nucleon Chiral EFT at NLO in the chiral expansion has been analyzed.

In schemes with a finite (of the order of the hard scale Λ_b) cutoff Λ and for a perturbative LO interaction, renormalization works to all orders in the LO potential, i.e., all power counting violating contributions can be absorbed by a redefinition of the LO contact interactions.

In the nonperturbative regime, the requirement of renormalizability imposes certain constraints on the LO potential.

These constraints cannot be fulfilled for some exceptional cutoffs. Therefore, the limit $\Lambda \to \infty$ does not exist and renormalization does not work in the infinite-cutoff scheme beyond leading order.

It seems straightforward to extend the present analysis to other systems (few- and many nucleon systems, electroweak currents) as well as to higher orders in the EFT expansion.

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