A. M. Badalian · B. L. G. Bakker

The Regge Trajectories and Leptonic Widths of the Vector $s\bar{s}$ Mesons

Abstract The spectrum of the $s\bar{s}$ mesons is studied performing a phenomenological analysis of the Regge trajectories defined for the excitation energies. For the $\phi(3\,^3S_1)$ state the mass $M(\phi(3S)) = 2100(20)$ MeV and the leptonic width $\Gamma_{ee}(\phi(3S)) = 0.27(2)$ keV are obtained, while the mass of the $2\,^3D_1$ state, $M(\phi(2\,^3D_1)) = 2180(5)$ MeV, appears to be in agreement with the mass of the $\phi(2170)$ resonance, and its leptonic width, $\Gamma_{ee}(2\,^3D_1) = 0.20 \pm 0.10$ keV, has a large theoretical uncertainty, depending on the parameters of the flattened confining potential.

1 Introduction

Recently BES III observed a new resonant structure in the $J/\psi \rightarrow \phi\eta'\eta$ decays in the $\phi\eta'$ invariant mass distribution, denoted as $X(2000)$ [1]. The quantum numbers of $X(2000)$ are not fixed yet and two possibilities are presented. First, assuming the state $J^P = 1^-$, the mass $M(X) = 2002.1 \pm 27.5 \pm 15.0$ MeV and the width $\Gamma = 129 \pm 17 \pm 7$ MeV with the significance 5.3σ were obtained, while assuming the state $J^P = 1^+$, the larger mass $M(X) = 2062.8 \pm 13.1 \pm 4.2$ MeV of the structure with the width $\Gamma = 177 \pm 36 \pm 20$ MeV and the significance 4.9σ were determined. This new structure was already analyzed in several theoretical studies, where in the conventional $s\bar{s}$ picture the resonance $X(2000)$ is considered as a candidate of the vector $\phi(3\,^3S_1)$ state [2] or, in Ref. [3], it was interpreted as the second excitation of the axial-vector $h_1(1380)$ meson. A different conception of $X(2000)$ was suggested in Refs. [4,5], where $X(2000)$ is assumed to be a candidate of the $ss\bar{s}\bar{s}$ tetra-quark with $J^{PC} = 1^{+-}$.

The $X(2000)$ together with $\phi(2170)$ represent a special interest for the theory, being the highest excitations in the $s\bar{s}$ system observed up to now, although the properties and decays of the $s\bar{s}$ excitations were studied for decades [6–15] in different approaches: in the framework of relativistic potential models (RPM) [7,8], the Regge trajectories (RTs) [9–12], and the QCD sum rules [13–15]. Comparison of the predicted masses shows that the masses of high $\phi(nS)$ excitations differ by $\sim(100–150)$ MeV, even if the masses of the low states practically coincide. Such differences can be easily understood taking into account that the properties of high excitations are very sensitive to the chosen values of the constituent quark masses and the parameters of the quark–antiquark interaction. This statement can be illustrated by the masses of the nS and nD states, collected

A. M. Badalian
Institute of Theoretical and Experimental Physics, Moscow, Russia
E-mail: badalian@itep.ru

B. L. G. Bakker (✉)
Faculty of Science, Vrije Universiteit, Amsterdam, The Netherlands
E-mail: b.l.g.bakker@vu.nl

Table 1 The masses of the $\phi(nS)$ and $\phi(n^3D_1)$ states (in MeV)

State	GI [7]	EFG [8]	MGI [2]	Experiment [16]
$\phi(1S)$	1016	1038	1030	1020
$\phi(2S)$	1687	1698	1687	1680(20)
$\phi(3S)$	2200	2119	2149	2002(42) ?
$\phi(4S)$	2622	2472	2498	Abs.
$\phi(1D)$	1876	1845	1869	Abs
$\phi(2D)$	2337	2258	2276	2188(10) ?
$\phi(3D)$	2725	2607	2593	Abs.

in Ref. [2] and given in Table 1, where MGI refers to the modified Godfrey–Isgur model with the screened confining potential [2].

From Table 1 one can see that in all RPMs with constituent quark masses, the mass $M(\phi(3S))$ appears to be by (120–200) MeV larger than the mass of $X(2000)$ with $J^P = 1^-$ in experiment. Also in RPMs the first excitation $\phi(2D)$ has a mass larger than the mass of $\phi(2170)$ by ~ 100 MeV, although in the MGI model, where a screened confining potential (CP) is used, the masses of the $\phi(3S)$ and $\phi(2D)$ states are about 50 MeV smaller than those in the GI model [7], where the purely linear CP is used. Note that for $\phi(4S)$ and $\phi(3D)$ the GI model predicts values of the masses, which are already by ~ 120 MeV larger, which means that within the same model the choice of the parameters of the confining potential at large distances is crucially important to describe higher excitations. The situation is different, if an analysis of the spectrum is performed with the help of Regge trajectories, defined for excitation energies, denoted as ERT [17], where the parameters of the ERT can be extracted from experiment and the predicted masses of high excitations appear to be smaller than those in RPMs. In our paper we perform a phenomenological analysis of the $\phi(nS)$ and $\phi(nD)$ resonances, using the ERT, introduced by Afonin and Pusenkov [13–15], as it was done in the analysis of the heavy-quarkonia spectra in Ref. [17]. We will also discuss how the parameters of the ERT depend on the mass of the s-quark.

2 The Radial ERT of $\phi(nS)$ Mesons

In heavy quarkonia, as well as in the $s\bar{s}$ system, the ERT are defined for the excitation energies $E(nJ)$ [13–15],

$$M(nJ) - 2m_s = E(nJ). \tag{1}$$

First, we consider the $\phi(n^3S_1)$ radial trajectory, which needs special consideration since its radial slope is larger than that of the ERT for the states with $l \neq 0$ [17]; this effect is seen in the RT of light mesons [9–12,18–21], as well as in heavy quarkonia [17]. The reason why the radial slope is larger in the S-wave mesons is explained by the stronger gluon–exchange (GE) interaction in the states with $l = 0$ than in those with $l \neq 0$ [22].

The radial ERT of $\phi(nS)$ can be presented as,

$$E^2(nS) = a_S + \beta_S n_r. \tag{2}$$

where n_r is the radial quantum number. The s-quark mass m_s, the intercept a_S, and the slope b_S can be extracted from experiment, if there are enough experimental data on the $\phi(nS)$ excitations, measured with great accuracy. However, the existing experimental data do not allow to extract m_s at low scale and here we take m_s at low scale, using the relation for the running mass in pQCD [23,24] and the conventional value of $m_s(q = 2\,\text{GeV}/c) = 96(5)\,\text{MeV}$ [16], defined at the scale $q = 2\,\text{GeV}/c$. The sizes of high $s\bar{s}$ mesons are large, > 1.0 fm, and therefore their dynamics are determined by small characteristic momenta, $q \lesssim 1\,\text{GeV}$. The following mass relations, $m_s(q = 1\,\text{GeV}/c) = 1.27m_s(q = 2\,\text{GeV}/c) = 122$ MeV and $m_s(q = 0.5\,\text{GeV}/c) = 1.97m_s(q = 2\,\text{GeV}/c) = 189\,\text{MeV}$ were obtained in Ref. [23], i.e., $m_s \sim (120-180)\,\text{MeV}$ at low scale. Just the value $m_s = 180$ MeV was used in the analysis of the decay constants of the strange mesons D_s and B_s [24].

Calculations with the use of the ERT show that in both cases, when $m_s(\sim 0.5\,\text{GeV}) = 180$ MeV or $m_s(\sim 1\,\text{GeV}) = 125$ MeV are taken, the masses of the $\phi(nS)$ mesons differ only within (10–20) MeV, i.e., weakly depend on the chosen value of m_s, although the intercept and the radial slope of the radial ERTs Eq. (2) are different (see their values in Table 2). In Table 1 for comparison we give also the masses $M(\phi(nS))$ from

Table 2 The masses of $\phi(nS)$ (in GeV) from the ERT and the parameters a_S, b_S (in GeV^2) for $m_s = 0.125$ GeV and $m_s = 0.180$ GeV and the masses from Ref. [2] and from experiment [16]

	$m_s = 0.125$ GeV	$m_s = 0.180$ GeV	$m_s = 0.387$ GeV [2]	Experiment
a_s	0.593	0.4356		
b_s	1.465	1.30		
$\phi(1S)$	1.020	1.020	1.030	1.020
$\phi(2S)$	1.680	1.677	1.687	1.680(20)
$\phi(3S)$	2.121	2.102	2.149	2.002(42) ?
$\phi(4S)$	2.477	2.442	2.498	abs.

Ref. [2], where the MGI model with screened confining potential is used, giving the masses of the $3S$ and $4S$ excitations by ~ 50 MeV larger than those in our analysis of the ERT.

From Table 1 one can see that the masses, defined by the ERT with $m_s = 125$ MeV and $m_s = 180$ MeV, coincide within ~ 10 MeV accuracy for $\phi(1S)$ and $\phi(2S)$ and differ only by ~ 20 MeV for the higher $3S$ and $4S$ excitations, although their intercepts and the radial slopes are different. Notice that for a smaller $m_s = 125$ MeV the values of the intercept and the radial slope are practically equal to those for light mesons [12,20–22], while for $m_s = 180$ MeV they are closer to the values in heavy quarkonia [17].

Notice that in our analysis, where $M(\phi(3S)) = 2102$ MeV, as well as in RPMs (see Table 1), the mass of $\phi(3\,^3S_1)$ is larger than the experimental mass of $X(2000)$ [1]. However, one cannot exclude a large hadronic shift-down of the $3\,^3S_1$ state due to the $P-$wave $\phi\phi$ threshold (with $M(\text{thresh.}) = 2039$ MeV) and then this state could be a candidate to be the $X(2000)$ resonance, as it is assumed in Ref. [2].

3 The Generalized ERT of the $s\bar{s}$ Resonances

The experimental masses of the ground states $\phi(1S)$, $f_2'(1525)$ and $\phi_3(1850)$ ($J = 1, 2, 3$) allow to define the orbital parameter b_J of the leading ERT with $J = l + 1$, $n_r = 0$:

$$E^2(J, n_r = 0) = a + b_J J. \tag{3}$$

First, we take $m_s = 180$ MeV and by definition of the ERT the mass differences can be written as

$$M(\phi(1S)) - 2m_s = \sqrt{a + b_J} = 0.660\,\text{GeV}; \quad \sqrt{a + 2b_J} - \sqrt{a + b_J} = 0.505\,\text{GeV}, \tag{4}$$

giving the following intercept and orbital slope of the leading ERT,

$$a = -0.4644\,\text{GeV}^2; \quad b_J = 0.905\,\text{GeV}^2, \quad (m_s = 0.180\,\text{GeV}, \quad J = l + 1). \tag{5}$$

From here the masses of the $s\bar{s}$ states with $J = l + 1$ are following,

$$M(\phi(1S)) = 1.020\,\text{GeV}, \quad M(f_2'(1P)) = 1.518\,\text{GeV}, \quad M(\phi_3(1D)) = 1.859\,\text{GeV},$$
$$M(f_4(1F)) = 2.135\,\text{GeV}, \quad M(\phi_5(1G)) = 2.374\,\text{GeV}, \tag{6}$$

where the masses of $\phi(1S)$, $f_2'(1P)$, and $\phi_3(1D)$ are in precise agreement with the experimental masses of $\phi(1020)$, $f_2'(1525)$, and $\phi_3(1850)$), respectively. At the same time, the calculated mass $M(f_4(1P)) = 2131$ MeV is by ~ 100 MeV larger than $M(f_4(2050)) = 2018(11)$ MeV ($\Gamma = 237 \pm 18$ MeV) [16]. One can present arguments that the resonance $f_4(2050)$ has a large hadronic shift and therefore does not lie on a linear ERT (or on a conventional RT). First, from the mass difference $\mu^2 = M^2(f_4(2050)) - M^2(\phi_3(1850) = 0.635\,\text{GeV}^2$ one obtains a very small orbital slope for the conventional RT, $\mu^2 = 0.635\,\text{GeV}^2$, and an even smaller slope $b_J = 0.517\,\text{GeV}^2$ for the ERT. Secondly, the mass difference between the $s\bar{s}$ ground state and corresponding ground state of a light meson is typically ~ 200 MeV, e.g., $M(\phi(1020)) - M(\rho(775)) = 245$ MeV, $M(f_2'(1525) - M(a_2(1320)) = 207$ MeV, while the masses of $M(f_4(2050))$ and $M(a_4(2040)$ are almost equal (their mass difference is 23(18) MeV). This may occur if $f_4(2050)$ has a large hadronic shift, possibly, due to the nearby $\phi\phi$ threshold.

To describe the radial $s\bar{s}$ excitations with $l \neq 0$, we assume that $f_2(1950)$ is an $s\bar{s}$ state, but not assuming a priori that $\phi(2170)$ is the first excitation of ϕ_3, since its quark structure is still discussed [25,26]. Then the

Table 3 The masses (in MeV) of the $s\bar{s}$ states with $l \neq 0$ from ERT Eq. (7)

State	$n_r = 0$	$n_r = 1$	$n_r = 2$
$n\,^3P_2$	1518	1938	2.268
Experiment	1525(5)	1944(12)	Abs.
$n\,^3D_3$	1858	2203	2492
Experiment	1854(7)	Abs.	Abs.
$n\,^3D_1$	1835	2185	2475
Experiment	Abs.	2188(10)?	Abs.

radial slope of the ERT (for the states with $l \neq 0$) can be extracted from experiment (the parameters a, b_J are given in Eq. (5)), using the mass difference,

$$\sqrt{a + 2b_J + b_n} - \sqrt{a + 2b_J} = 1.944(12) - 1.525(5) = 0.419 \pm 0.017\,\text{GeV}. \tag{7}$$

From here the radial slope $b_n = (1.15 \pm 0.05)\,\text{GeV}^2$ is extracted and the generalized ERT,

$$E^2(J, n_r)(\text{in GeV}^2) = -0.4644 + 0.905\,J + 1.15(5)\,n_r, \quad (J = l + 1) \tag{8}$$

gives the masses, presented in Table 3, where besides the masses of the $n\,^3D_3$, the masses of the $n\,^3D_1$ states are given, which are by $\sim 15(5)\,\text{MeV}$ smaller due to the fine-structure splitting.

In our calculations the mass $M(2\,^3D_1)$ coincides with that of $\phi(2070)$ and this fact indicates that $\phi(2070)$ can have large $s\bar{s}$ component, as it was assumed in Ref. [26].

4 The Leptonic Widths

To define the leptonic widths of the $\phi(1020)$, $\phi(1680)$, $\phi(1\,^3D_1)$, $\phi(2\,^3D_1)$ their wave functions (w.f.s) are calculated, using the relativistic string Hamiltonian (RSH) [18,19,22,27,28] without fitting parameters, where the centroid mass $M_{\text{cog}}(nl)$ is defined by the eigenvalues $M_0(nl)$ of the equation,

$$(2\sqrt{\mathbf{p}^2 + m_s^2} + V_0(r))\varphi_{nl}(r) = M_0(nl)\varphi_{nl}(r). \tag{9}$$

and by two negative corrections, the self-energy and the string corrections [18,19,22,29,30]. Here $m_s = 180\,\text{MeV}$ and the potential $V_0(r)$ is taken as the sum of the confining potential and the gluon–exchange (GE) term, $V_{\text{GE}} = -\frac{4\alpha_V(r)}{3r}$, with the parameters from Ref. [22]. Since the ground states have relatively small sizes, their dynamics is defined by the linear CP, $V_c(r) = \sigma r = 0.18\,r\,\text{GeV}$, while for high excitations, which have large sizes, a flattened CP $V_f(r)$,

$$V_f(r) = \sigma_f(r)r \tag{10}$$

has to be taken [22]. We assume that the self-energy and the string correction do not affect the w.f.s and the radial w.f.s at the origin $R_{nS}(0)$ and $R_{nD}(0)$, which are defined by the solutions $\varphi_{nl}(r = 0)$ of Eq. (8), where for the $n\,^3D_1$ states the w.f. $R_{nD}(0)$ is expressed via the derivative $R''_{nD}(0)$ [31,32],

$$R_{nD}(0) = \frac{5}{2\sqrt{2}} \frac{R''_{nD}(0)}{\omega^2(nD)}, \tag{11}$$

Here the flattened CP $V_f(r)$ is chosen as in Ref. [22] with the string tension, $\sigma_f(r) = \sigma(1 - \gamma f(r))$ and the function $f(r)$,

$$f(r) = \frac{\exp(\sqrt{\sigma}(r - R_0))}{B + \exp(\sqrt{\sigma}(r - R_0))}, \tag{12}$$

defined by the following parameters,

$$\gamma = 0.40, \quad B = 20, \quad R_0 = 6.0\,\text{GeV}^{-1}, \quad \sigma = 0.182\,\text{GeV}^2. \tag{13}$$

Table 4 The r.m.s. (in fm), the quark kinetic energy (in GeV), the w.f. at the origin $R_{nl}(0)$ (in GeV$^{3/2}$), and the leptonic widths $\Gamma_{ee}(nl)$ (in keV) of the $s\bar{s}$ vector mesons ($m_s = 0.180$ GeV)

state	r.m.s (LP)	$\omega(nl)$	$R_{nl}(0)$	Γ_{ee}
$1S$	0.65	0.477	0. 432	1.24
$2S$	1.15	0.596	0.410(10)	0.42(2)
$3S$	1.74	0.613	0.400	0.27(2)
$4S$	2.59	0.63	0.410	0.19 (2)
$1D$	1.18	0.603	0.25(8)	0.13(9)
$2D$	1.81	0.610	0.35(7)	0.20 (10)
$3D$	2.70	0.630	0.44(10)	0.22(12)

If the flattened CP+GE term is used, then the sizes of the nS and nD excitations increase, see Table 4, where also the w.f.s $R_{nS}(0)$, $R_{nD}(0)$ and the kinetic energies $\omega(nL)$ of the $s-$quark, entering $R_{nD}(0)$, are given. We have observed an interesting effect: if a flattened CP is used, then the kinetic energies $\omega(nl)$ and the w.f. $R_{nS}(0)$ ($n_r \geq 1$) decrease, while the second derivatives $R''_{nD}(0)$ increases. Consequently, the leptonic width of the $\phi(nD)$ also increases. The leptonic width of a vector $s\bar{s}$ meson with the mass $M_V(nl)$ ($l = 0, 2$) (the charge squared $e_s^2 = 1/9$, $\alpha = (137)^{-1}$) is given by the expression [24],

$$\Gamma_{ee}(\phi(nL)) = \frac{4 \, e_s^2 \alpha^2 |R_{nl}(0)|^2 \, \beta_{\text{rel}} \, \beta_{\text{QCD}}}{M_V^2}, \tag{14}$$

where the factor $\beta_{\text{rel}} \cong (0.72-0.74)$ takes into account the relativistic effects [24], and $\beta_{\text{QCD}} = 1 - \frac{16\alpha_s(\mu)}{3\pi} = 0.40$ is the QCD correction, where the strong coupling $\alpha_s(\mu) = 0.353$ at the scale $\mu \sim 1.4$ GeV is taken.

In the GE potential we use the vector coupling constant, which does not contain fitting parameters and takes into account the asymptotic freedom behavior [33], so that the effective coupling of the ground state $\alpha_V(\text{eff.}) = 0.39$ is relatively small, while $\alpha_V = 0.54$ is larger for excited states with $n_r \geq 2$. Details can be found in Ref. [22], where the vector coupling $\alpha_V(n_f = 3)$ is shown to be defined via the QCD vector constant $\Lambda_V(n_f = 3) = 0.455$ GeV, which corresponds to the QCD constant $\Lambda_{\overline{MS}}(n_f = 3) = 330$ MeV from Ref. [34].

5 Conclusions

The spectrum of the $s\bar{s}$ mesons was studied with the use of the ERT trajectories, defined for the excitation energies, $E(nJ) = M(nJ) - 2m_s$ [13–15]. It is shown that the parameters of the ERT depend on the value of the s-quark mass at a low scale. Two values, $m_s = 125$ MeV and $m_s = 180$ MeV, are considered. In both cases the calculated masses coincide within (10–20) MeV accuracy, although for $m_s = 125$ MeV the slope $b(nS) = 1.465$ GeV^2 and the intercept $a(nS) = 0.593$ GeV^2 are larger than those for $m_s = 180$ MeV, and equal to the parameters of the $\rho(nS)$ RT. If $m_s = 180$ MeV is taken, the values $b_n(nS) = 1.30$ GeV^2 and $a(nS) = 0.4356$ GeV^2 are smaller and close to those in heavy quarkonia [17]. For $\phi(3S)$ the leptonic width $\Gamma_{ee} = 0.42(2)$ keV is obtained.

With the use of the ERT the predicted masses of the high excitations appear to be smaller than those calculated in potential models with a constituent s-quark mass. For $\phi(3S)$ the calculated mass $M(\phi(3S)) = 2100(20)$ MeV is larger than that of the $X(2000)$ resonance, recently observed by BES III, but a large hadronic shift down of this resonance is not excluded. For the states with $l \neq 0$ the generalized ERT, which includes the orbital and radial excitations, has the orbital slope $b_J = 0.905$ GeV^2 and the radial slope $b_n = 1.15(5)$ GeV^2. This ERT gives the mass $M(f_2(2P)) = 1938$ MeV in agreement with the mass of $f_2(1950)$ and $M(f_2(3P)) = 2268$ MeV, while the mass $M(2^3D_1) = 2.180(5)$ GeV agrees with the mass of the $\phi(2170)$ resonance, and therefore $\phi(2170)$ could be either the 2^3D_1 state or contain a large $s\bar{s}$ component. The leptonic width of $\phi(2D)$, $\Gamma_{ee} = 0.20(10)$ keV, has a large theoretical uncertainty, which occurs because of the strong sensitivity of the radial w.f. at the origin $R_{2D}(0)$ to the parameters of the flattened (screened) potential.

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