Correction to: Infinitely Many Congruences for k -Regular Partitions with Designated Summands

Robson da Silva¹ · James A. Sellers²

Published online: 11 December 2019

© Sociedade Brasileira de Matemática 2019

Correction to: Bull Braz Math Soc, New Series

<https://doi.org/10.1007/s00574-019-00156-x>

In our original paper da Silva and Sellers (2019), Eq. (42) was not correctly quoted from one of the references, which led to some minor errors that do not affect the final results.

In da Silva and Sellers (2019) we proved infinitely many congruences for the number of k -regular partitions with designated summands, denoted by $PD_k(n)$. In order to do so, we made use of some known 2- and 3-dissections of certain quotients of eta functions. One of the 3-dissections was wrongly quoted from Chan (2010), namely Eq. (42). Below we indicate the small changes that are necessary to correct the minor errors caused by having wrongly quoted Eq. (6) from Chan (2010).

1. Eq. (42) should be replaced by

$$\frac{1}{f_1 f_2} = \frac{f_9^9}{f_3^6 f_6^2 f_{18}^3} + q \frac{f_9^6}{f_3^5 f_6^3} + 3q^2 \frac{f_9^3 f_{18}^3}{f_3^4 f_6^4} - 2q^3 \frac{f_{18}^6}{f_3^3 f_6^5} + 4q^4 \frac{f_{18}^9}{f_3^2 f_6^6 f_9^3}.$$

The original article can be found online at <https://doi.org/10.1007/s00574-019-00156-x>.

✉ Robson da Silva
silva.robson@unifesp.br

James A. Sellers
sellersj@psu.edu

¹ Universidade Federal de São Paulo-UNIFESP, Av. Cesare M. G. Lattes, 1201, São José dos Campos, SP 12247-014, Brazil

² Department of Mathematics, Penn State University, 104 McAllister Building, University Park, PA 16802, USA

2. The two identities after (46) should be replaced by

$$\sum_{n=0}^{\infty} PD_6(n)q^n = \frac{f_6^2 f_{12} f_{18}}{f_3 f_{36}} \left(\frac{f_9^9}{f_3^6 f_6^2 f_{18}^3} + q \frac{f_9^6}{f_3^5 f_6^3} + 3q^2 \frac{f_9^3 f_{18}^3}{f_3^4 f_6^4} - 2q^3 \frac{f_{18}^6}{f_3^3 f_6^5} + 4q^4 \frac{f_{18}^9}{f_3^2 f_6^6 f_9^3} \right),$$

and

$$\sum_{n=0}^{\infty} PD_6(3n+1)q^{3n+1} = q \frac{f_9^6 f_{12} f_{18}}{f_3^6 f_6 f_{36}} + 4q^4 \frac{f_{12} f_{18}^{10}}{f_3^3 f_6^4 f_9^3 f_{36}}.$$

3. The proof of (13) should be replaced by:

In order to prove (13), we replace (42) in (49) to get

$$\sum_{n=0}^{\infty} PD_{12}(3n+1)q^{3n+1} = \frac{f_6 f_{12} f_{24} f_{36}}{f_3 f_{72}} \left(q \frac{f_9^6}{f_3^5 f_6^3} + 4q^4 \frac{f_{18}^9}{f_3^2 f_6^6 f_9^3} \right),$$

from which we obtain

$$\begin{aligned} \sum_{n=0}^{\infty} PD_{12}(3n+1)q^n &= \frac{f_2 f_4 f_8 f_{12}}{f_1 f_{24}} \left(\frac{f_3^6}{f_1^5 f_2^3} + 4q \frac{f_6^9}{f_1^2 f_2^6 f_3^3} \right) \\ &= \frac{f_3^6 f_4 f_8 f_{12}}{f_1^6 f_2^2 f_{24}} + 4q \frac{f_4 f_6^9 f_8 f_{12}}{f_1^3 f_2^5 f_3^3 f_{24}} \\ &\equiv \frac{f_1^2 f_4 f_6^4 f_8 f_{12}}{f_3^2 f_2^6 f_{24}} + 4q f_1 f_3 \frac{f_6^7 f_8 f_{12}}{f_2^5 f_{24}} \pmod{8}, \end{aligned}$$

using the fact that, by (43),

$$\frac{f_4 f_6^9 f_8 f_{12}}{f_1^4 f_2^5 f_3^4 f_{24}} \equiv \frac{f_6^7 f_8 f_{12}}{f_2^5 f_{24}} \pmod{2}.$$

Now, using (32) and (34) it follows that

$$\begin{aligned} \sum_{n=0}^{\infty} PD_{12}(6n+4)q^n &\equiv -2 \frac{f_4^2 f_6^2}{f_1^4} + 4 \frac{f_3^6 f_4^3 f_6^5}{f_1^4 f_2^2 f_{12}^3} \\ &\equiv -2 \frac{f_4^2 f_6^2}{f_1^4} + 4 \frac{f_4^3 f_6^8}{f_1^4 f_2^2 f_{12}^3} \pmod{8}, \end{aligned} \quad (1)$$

since, by (43),

$$\frac{f_3^6 f_4^3 f_6^5}{f_1^4 f_2^2 f_{12}^3} \equiv \frac{f_4^3 f_6^8}{f_1^4 f_2^2 f_{12}^3} \pmod{2}.$$

From (25), using (23) and (24), we have

$$\frac{1}{f_1^4} = \frac{f_4^{14}}{f_2^{14} f_8^4} + 4q \frac{f_4^2 f_8^4}{f_2^{10}}. \quad (2)$$

Hence the odd part of $\frac{1}{f_1^4}$ is divisible by 4. Thus taking the odd part on both sides of (52), we obtain (13) when $k = 1$.

The original article was updated.

References

- Chan, H.C.: Ramanujan's cubic continued fraction and a generalization of his most beautiful identity. *Int. J. Num. Theory* **6**, 673–680 (2010)
- da Silva, R., Sellers, J.A.: Infinitely many congruences for k -regular partitions with designated summands. *Bull. Braz. Math. Soc. New Ser.* (2019). <https://doi.org/10.1007/s00574-019-00156-x>

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.