BULLETIN OF THE BRAZILIAN MATHEMATICAL SOCIETY

Bull Braz Math Soc, New Series 46(1), 159-161

ISSN: 1678-7544 (Print) / 1678-7714 (Online)

Corrigendum to "On the solutions of a parametric family of cubic Thue equations" [Bull Braz Math Soc, New Series 39 (4) (2008), 537–554]

Alain Togbé*

Abstract. In our original paper [4], Theorem 2.1 of [2], that we quoted, was not properly proved. This was done by Lee-Louboutin [3]. The gap doesn't affect the final result of the paper.

Keywords: Parametric Thue equations, Baker's method.

Mathematical subject classification: 11D59, 11Y50.

0. Corrigendum

In [4], using Baker's method we studied the family of Thue equations

$$\begin{aligned}\Phi_n(x,y) = x^3 &- n(n^2+n+3)(n^2+2)x^2y \\ &- (n^3+2n^2+3n+3)xy^2 - y^3 = \pm 1,\end{aligned} \tag{0.1}$$

for $n \geq 0$. To do so, we considered the number field $\mathbb{K}_n$ related with $\phi_n(x)$ defined by

$$\phi_n(x) = x^3 - n(n^2+n+3)(n^2+2)x^2 - (n^3+2n^2+3n+3)x - 1. \tag{0.2}$$

See also [2]. One can see that ϕ_n has three real roots $\theta^{(1)}$, $\theta^{(2)}$, $\theta^{(3)}$. For a solution (x, y) of (0.1), we have

$$\Phi_n(x,y) = \prod_{j=1}^{3}\left(x - \theta^{(j)}y\right) = N_{\mathbb{Q}(\theta^{(1)})/\mathbb{Q}}\left(x - \theta^{(j)}y\right) = \pm 1. \tag{0.3}$$

Received 31 October 2014.

*The author was supported in part by Purdue University North Central.

This means theat $x - \theta^{(j)} y$ is a unit in the order $\mathcal{O} := \mathbb{Z}[\theta^{(1)},\ \theta^{(2)}]$. Kishi proved the following theorem (see Theorem 2 of [2]).

Theorem 0.1. *Let θ and θ' be two distinct roots of $f_n(X)$. Then $\{\theta, \theta'\}$ is a system of fundamental units of the order $\mathbb{Z}[\theta, \theta']$. Let E denote the unit group of $\mathcal{O}_{\mathbb{K}_n}$ and put*

$$N := \frac{(n^2+3)(n^4+n^3+4n^2+3)}{4^{\delta_1} \cdot 9^{\delta_2}}$$

where

$$\delta_1 = \begin{cases} 0 & \text{if } n \text{ is even}, \\ 1 & \text{if } n \text{ is odd}; \end{cases} \qquad \delta_2 = \begin{cases} 0 & \text{if } n \equiv 2 (\text{mod } 3), \\ 1 & \text{if } n \not\equiv 2 (\text{mod } 3). \end{cases}$$

Suppose N is squarefree. Then the index $[E : \langle -1, \theta, \theta' \rangle]$ is equal to 1, that is, $\{\theta, \theta'\}$ is a system of fundamental units of $\mathbb{K}_n$, except for $n = \pm 1, -2$. For $n = \pm 1$, we have $[E : \langle -1, \theta, \theta' \rangle] = 7$, and for $n = -2$, $[E : \langle -1, \theta, \theta' \rangle] = 3$.

We deduced that $[E : \langle -1, \theta, \theta' \rangle] \leq 2$ (see Remark 2.2 on page 540 of [4]). There was a gap in the proof of Theorem 0.1 due to the misinterpretation of Cusick's result, see Proposition 2.1 of [1]. Lee and Louboutin filled the gap and confirmed Remark 2.2, for any n by proving Lemma 6.1, page 290 of [3]. As Remark 2.2 on page 540 of [4] is correct, this doesn't affect the main result obtained in our paper.

Remark 0.2. The example given on page 288 of [3] has no link with polynomial (0.2).

Acknowledgments. We thank Professor Stéphane Louboutin for sending us the manuscript of their paper.

References

[1] T. Cusick. *Lower bounds for regulators*, in "Number Theory, Noordwijkerhout, 1983," Lecture Notes in Mathematics, Vol. 1068, pp. 63–73, Springer-Verlag, Berlin/New York, (1984).

[2] Y. Kishi. *A family of cyclic polynomials whose roots are systems of fundamental units*, Journal of Number Theory, **102** (2003), 90–106.

[3] Jun Ho Lee and Stéphane R. Louboutin. *On the fundamental units of some cubic orders generated by units*, Acta Arith., **165** (2014), 283–299.

[4] A. Togbé. *On the solutions of a parametric family of cubic Thue equations.* Bull. Braz. Math. Soc., New Series, **39** (2008), 537–554.

Alain Togbé
Mathematics Department
Purdue University North Central
1401 S, U.S. 421
Westville IN 46391
USA

E-mail: atogbe@pnc.edu