FUZZY SYSTEMS AND THEIR MATHEMATICS

A novel linguistic decision making approach based on attribute correlation and EDAS method

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Abstract

One of characteristics of large-scale linguistic decision making problems is that decision information with respect to decision making attributes is derived from multi-sources information. In addition, the number of decision makers, alternatives or criteria of decision making problems in the context of big data are increasingly large. Correlation analysis between decision making attributes has become an important issue of large-scale linguistic decision making problems. In the paper, we concentrate on correlation analysis between decision making attributes to deal with large-scale decision making problems with linguistic intuitionistic fuzzy values. Firstly, we proposed a new similarity measure between two linguistic intuitionistic fuzzy sets to formally define correlation between decision making attributes. Then we propose linguistic intuitionistic fuzzy reducible weighted Maclaurin symmetric mean (LIFRWMSM) operator and linguistic intuitionistic fuzzy reducible weighted dual Maclaurin symmetric mean (LIFRWDMSM) operator to aggregate linguistic intuitionistic fuzzy value decision information of correlational decision making attributes, and analyze several important properties of the two operator. Inspired by evaluation based on distance from average solution (EDAS) method, we design a solution scheme and decision steps to deal with large-scale linguistic intuitionistic fuzzy decision making problems. To show the effectiveness and usefulness of the proposed decision method, we employ the choice of buying a house and the selection of travel destination to demonstrate our method and make comparative analysis with others aggregation operators or methods.

Keywords Linguistic intuitionistic fuzzy sets · Correlational Attribute · Reducible weighted MSM operator · EDAS method · Multiple attribute decision making

1 Introduction

The main aim of classical multi-attributes decision making (MADM) is to select the best alternative from a set of alter-

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natives according to decision making methods, in which the scale of attributes is small and the complexity is not high. In the era of big data, MADM problems are becoming more and more complex than classical MADM problems. Meanwhile, the scale of decision makers, alternatives or decision making attributes are becoming greater in number (also called as large-scale decision making problems), such as decision making in building a large hydropower station (Liu et al. 2015), large groups emergency decision making (Xu et al. 2019) and decision making in social network (Wang et al. 2021). Recently, many researchers have be attracted to solve large-scale decision making problems, such as Zhou et al. (2020) proposed a hierarchical selection algorithm to deal with MADM problems with large-scale alternatives. Wu and Liu (2016) introduced clustering analysis and aggregation operator to handle MADM problems with large-scale decision makers.

Generally, decision information with respect to decision making attributes in a large-scale decision making prob-

lem is derived from multi-sources information. In many cases, several decision making attributes may be correlation in the large-scale decision making problem, such as in the green environmental behavior decision making problem (Li et al. 2020), the software trust worthiness evaluation problem (Yang et al. 2009) and the multi-attributes emergency decision making problem (Chen and Wang 2018). In fact, correlation analysis is an important issue in multi-sources information fusion. In large-scale decision making problems, correlation between decision making attributes is objective and pervasive due to multi-sources decision information; hence, correlation analysis between decision making attributes is necessary; otherwise, the decision making results of large-scale decision making problems will be affected by correlation between decision making attributes. Based on the assumption that decision making attributes may be correlation each other, linguistic intuitionistic fuzzy Einstein Heronian mean operator (Liu and You 2018), linguistic intuitionistic fuzzy Muirhead mean operaror (Rong et al. 2020), linguistic intuitionistic fuzzy Maclaurin symmetric mean (LIFMSM) operator and linguistic intuitionistic fuzzy weighted Maclaurin symmetric mean (Liu and Qin 2017) have been proposed to aggregate correlational decision information and deal with multi-attributes decision making problems. However, in real-world large-scale decision making problems, correlation between decision making attributes is different understood by different decision makers. In other words, several decision making attributes are correlation, but others are independence in a large-scale decision making problem. To our knowledge, few researches pay attention to such correlation between decision making attributes and the corresponding large-scale decision making problems. In addition, because linguistic intuitionistic fuzzy sets with linguistic memberships and nonmemberships (Chen et al. 2015; Ou et al. 2018) can be utilized to represent more uncertainty information than classical fuzzy sets (Zadeh 1975), intuitionistic fuzzy sets (Atanassov 1986) and fuzzy linguistic terms (Pedrycz et al. 2011; Pei et al. 2010; Zhang and Zhao 2020; Li et al. 2020; Liu et al. 2020; Yan and Pei 2022), many researchers pay close attention to linguistic intuitionistic fuzzy decision making methods or aggregation operators. Compared with the TOPSIS and VIKOR decision making method (Pei et al. 2019; Li et al. 2017), many researchers are attracted to the EDAS method (Keshavarz Ghorabaee et al. 2015; Fan et al. 2020; Karasan and Kahraman 2018; Liang 2020; Kundakci 2019; Zhan et al. 2020; Wang et al. 2019; Li et al. 2019; Fan et al. 2019; Mishra et al. 2020 due to only the positive distance from average alternative (PDA) and the negative distance from average alternative (NDA) are needed in the EDAS method. Furthermore, Feng et al. (2018) extended the EDAS method to the extended hesitant fuzzy linguistic environment. Li et al. (2019) designed a MAGDM procedure by improving the EDAS method. Yanmaz et al. (2020)

extended the EDAS method to interval-valued Pythagorean fuzzy environment to deal with the car selection problem.

In the paper, we propose a novel linguistic decision making approach to aggregate correlational decision information represented by linguistic intuitionistic fuzzy sets and deal with large-scale multi-attributes decision making problems inspired by the EDAS method, major contributions of the paper are concentrated on:

1. A new similarity measure between two decision making attributes is proposed according to linguistic intuitionistic fuzzy set decision information in a decision matrix. Then the correlation between two decision making attributes is formally defined by the similarity measure. Accordingly, LIFRWMSM and LIFRWDMSM operators are presented to aggregate decision information with correlation;
2. Inspired by EDAS method, a new decision procedure is designed to deal with the large-scale linguistic decision making problems with correlation between decision making attributes. The choice of buying a house and the selection of travel destination are employed to demonstrate the proposed method and compare with LIFWA, LIFWG, LIFHA, WLIFMM, WLIFDMM and WLIFMSM methods which are utilized in the two examples.

The rest of the paper is structured as follows: In Sect. 2, we review some fundamental notions including linguistic intuitionistic fuzzy sets and MSMs operators. In Sect. 3, we define a new linguistic intuitionistic fuzzy similarity measure, which is applied to analyze correlation between two decision making attributes. In Sect. 4, we present LIFRWMSM and LIFRWDMSM operators to aggregate correlational decision information and analyze their properties. In Sect. 5, we design a novel decision procedure by combining correlation analysis of decision making attributes and EDAS method to deal with large-scale linguistic decision making problems with correlation between decision making attributes. In Sect. 6, we employ the choice of buying a house and the selection of travel destination to demonstrate the proposed decision making method and test the superiority of our method by comparing with LIFWA, LIFWG, LIFHA, WLIFMM, WLIFDMM and WLIFMSM methods. We conclude the paper in Sect. 7.

2 Preliminaries

Linguistic terms $\mathcal{S} = \{s_0, s_1, \ldots, s_h\}$ are always utilized to represent qualitative intuitionistic uncertainty of alternatives in linguistic decision making, for example, $\mathcal{S}=\{s_0$(extremely dissatisfied), s_1(very dissatisfied), s_2 (dissatisfied), s_3 (slightly dissatisfied), s_4 (medium), s_5 (slightly satisfied), s_6 (satisfied), s_7(very satisfied), s_8 (extremely sat-

isfied)}. Xu (2004) have extended the discrete linguistic term set $\mathcal{S}$ to a continuous and completely ordered linguistic term set $\tilde{\mathcal{S}} = \{s_\alpha | \alpha \in [0, h]\}$ based on the 2-tuple fuzzy linguistic representation model (Herrera and Martinez 2000). If the index α of s_α is a natural number from 0 to h, then s_α is an original linguistic term of $\mathcal{S}$. Otherwise s_α is deem to a virtual linguistic term. For convenience, we use linguistic term $s_\alpha \in \tilde{\mathcal{S}} = \{s_\alpha | \alpha \in [0, h]\}$ to represent continuous linguistic decision information in the paper. The concept of linguistic intuitionistic fuzzy set (LIFS) defined in $\tilde{\mathcal{S}} = \{s_\alpha | \alpha \in [0, h]\}$ is formalized as follows.

Definition 1 (Zhang 2014) Let U be a discourse domain and $\tilde{\mathcal{S}} = \{s_\alpha | \alpha \in [0, h]\}$ be a set of continuous linguistic terms. A is called as a LIFS on U, i.e.,

$$A = \{(x, s_\alpha(x), s_\beta(x)) | x \in U\} \tag{1}$$

where $s_\alpha(x), s_\beta(x) \in \tilde{\mathcal{S}}$ represents the linguistic membership grade and linguistic non-membership grade of $x \in A$, respectively. For any $x \in U$, $s_\alpha(x)$ and $s_\beta(x)$ always satisfy the condition $0 \leq \alpha + \beta \leq h$, and $s_\pi(x)$ denote linguistic hesitant grade, where $\pi = h - \alpha - \beta$.

The following operations are always utilized to deal with LIFSs information (Chen et al. 2015): let $\vartheta_1 = (s_{\alpha_1}, s_{\beta_1})$, $\vartheta_2 = (s_{\alpha_2}, s_{\beta_2})$, $\vartheta = (s_\alpha, s_\beta)$ be linguistic intuitionistic fuzzy values (LIFVs). For any constant $\delta > 0$, we have

1. $\vartheta_1 \oplus \vartheta_2 = (s_{\alpha_1}, s_{\beta_1}) \oplus (s_{\alpha_2}, s_{\beta_2}) = \left(s_{\alpha_1 + \alpha_2 - \frac{\alpha_1 \alpha_2}{h}}, s_{\frac{\beta_1 \beta_2}{h}}\right)$;
2. $\vartheta_1 \otimes \vartheta_2 = (s_{\alpha_1}, s_{\beta_1}) \otimes (s_{\alpha_2}, s_{\beta_2}) = \left(s_{\frac{\alpha_1 \alpha_2}{h}}, s_{\beta_1 + \beta_2 - \frac{\beta_1 \beta_2}{h}}\right)$;
3. $\delta \vartheta = \delta(s_\alpha, s_\beta) = \left(s_{h - h(1 - \frac{\alpha}{h})^\delta}, s_{h(\frac{\beta}{h})^\delta}\right)$;
4. $\vartheta^\delta = (s_\alpha, s_\beta)^\delta = \left(s_{h(\frac{\alpha}{h})^\delta}, s_{h - h(1 - \frac{\beta}{h})^\delta}\right)$.

To order LIFSs on U, Zhang (2014) provides the linguistic score result $\mathscr{S}(\vartheta)$ and the linguistic accuracy result $\mathscr{H}(\vartheta)$ of ϑ:

$$\mathscr{S}(\vartheta) = \frac{h + \alpha - \beta}{2}, \quad \mathscr{H}(\vartheta) = \alpha + \beta. \tag{2}$$

Then for any two LIFVs $\vartheta_1 = (s_{\alpha_1}, s_{\beta_1})$, $\vartheta_2 = (s_{\alpha_2}, s_{\beta_2})$, we have

1. If $\mathscr{S}(\vartheta_1) < \mathscr{S}(\vartheta_2)$, then $\vartheta_1 < \vartheta_2$;
2. If $\mathscr{S}(\vartheta_1) = \mathscr{S}(\vartheta_2)$, then,

   - If $\mathscr{H}(\vartheta_1) > \mathscr{H}(\vartheta_2)$, then $\vartheta_1 > \vartheta_2$;
   - If $\mathscr{H}(\vartheta_1) = \mathscr{H}(\vartheta_2)$, then $\vartheta_1 = \vartheta_2$.

To fuse n inputs with correlation derived from multi-sources information, Maclaurin symmetric mean (MSM) operator has been proposed as follows.

Definition 2 (Maclaurin 1729) Let $\vartheta_1, \ldots, \vartheta_n$ be nonnegative real numbers, then the MSM operator is

$$\text{MSM}^{(k)}(\vartheta_1, \ldots, \vartheta_n) = \left(\frac{\sum_{1 \leq i_1 < \cdots < i_k \leq n} \left(\prod_{j=1}^{k} \vartheta_{i_j} \right)}{\mathcal{C}_n^k} \right)^{\frac{1}{k}} \tag{3}$$

where $k = 1, \ldots, n$, $i_1, \ldots, i_k$ is any k-tuple permutation of $\{1, \ldots, n\}$ and $\mathcal{C}_n^k = \frac{n!}{k!(n-k)!}$.

Definition 3 (Qin and Liu 2015) Let $\vartheta_1, \ldots, \vartheta_n$ be nonnegative real numbers, then the DMSM operator is

$$\text{DMSM}^{(k)}(\vartheta_1, \vartheta_2, \ldots, \vartheta_n)$$
$$= \frac{\prod_{1 \leq i_1 < \cdots < i_k \leq n} \left(\sum_{j=1}^{k} \vartheta_{i_j} \right)^{\frac{1}{C_n^k}}}{k} \tag{4}$$

where $k = 1, \ldots, n$, $i_1, \ldots, i_k$ is any k-tuple permutation of $\{1, 2, \ldots, n\}$ and $\mathcal{C}_n^k = \frac{n!}{k!(n-k)!}$.

In decision making analysis, MSM and DMSM operators have been applied in multi-attributes decision making problems to aggregate decision information with correlation (Feng and Geng 2019; Li et al. 2016; Liu and Liu 2018; Liu and Qin 2018). After then, Shi and Xiao (2019) proposed the reducible weighted Maclaurin symmetric mean (RWMSM) and reducible weighted dual Maclaurin symmetric mean (RWDMSM) operators to deal with MADM problems.

Definition 4 (Shi and Xiao 2019) Let $\vartheta_1, \ldots, \vartheta_n$ be nonnegative real numbers, $\mathscr{W} = (\varpi_1, \ldots, \varpi_n)^T$ be a vector of weights, where $\varpi_i > 0$ and $\sum_{i=1}^{n} \varpi_i = 1$. For any $k \in \{1, \ldots, n\}$, the RWMSM operator is

$$\text{RWMSM}^{(k)}(\vartheta_1, \ldots, \vartheta_n)$$
$$= \left(\frac{\sum_{1 \leq i_1 < \cdots < i_k \leq n} \mathcal{P}_{i_k} \left(\prod_{j=1}^{k} \vartheta_{i_j} \right)}{\mathcal{P}_k} \right)^{\frac{1}{k}} \tag{5}$$

where $\mathcal{P}_{i_k} = \prod_{j=1}^{k} \varpi_{i_j}$, $\mathcal{P}_k = \sum_{1 \leq i_1 < \cdots < i_k \leq n} \mathcal{P}_{i_k}$.

Definition 5 (Shi and Xiao 2019) Let $\vartheta_1, \ldots, \vartheta_n$ be nonnegative real numbers, $\mathscr{W} = (\varpi_1, \ldots, \varpi_n)^T$ be a vector of weights, where $\varpi_i > 0$ and $\sum_{i=1}^{n} \varpi_i = 1$. For any $k \in \{1, \ldots, n\}$, the RWDMSM operator is

$$\text{RWDMSM}^{(k)}(\vartheta_1, \vartheta_2, \ldots, \vartheta_n)$$

$$= \frac{\prod_{1 \leq i_1 < \cdots < i_k \leq n} \left(\sum_{j=1}^{k} \vartheta_{i_j} \right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}}{k} \tag{6}$$

where $i_1, \ldots, i_k$ is any k-ary arrangement of $\{1, \ldots, n\}, \mathcal{C}_n^k = \frac{n!}{k!(n-k)!}$, and $\mathcal{M}_{i_k} = \sum_{j=1}^{k} \varpi_{i_j}, \mathcal{M}_k = \sum_{1 \leq i_1 < \cdots < i_k \leq n} \mathcal{M}_{i_k}$.

3 Correlation between decision making attributes in decision making matrix

In real-world large-scale decision making problems, correlation between decision making attributes is different understood by different decision makers, such as the correlation is determined by expert judgment, information sources correlation or semantic of decision making attributes. In the section, we present correlation analysis between decision making attributes based on decision information, which is provided by decision makers according to decision making attributes and represented by LIFSs. Intuitively, suppose $A = (A_1, A_2, \ldots, A_m)$ is the collection of alternatives of a large-scale MADM problem, $B = (B_1, B_2, \ldots, B_n)$ $(n \geq 20)$ is the collection of decision making attributes with weights $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ such that $0 \leq \varpi_i \leq 1$ and $\sum_{i=1}^{n} \varpi_i = 1$, the set of linguistic terms is $\tilde{S} = \{s_\alpha | \alpha \in [0, h]\}$, which are used by decision makers to evaluate alternatives according to decision making attributes. All of LIFS decision information provided by a decision maker can be represented in a decision matrix $\Psi = (\vartheta_{ij})_{m \times n}$, i.e.,

$$\Psi = \begin{matrix} & B_1 & B_2 & \cdots & B_n \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{pmatrix} \vartheta_{11} & \vartheta_{12} & \cdots & \vartheta_{1n} \\ \vartheta_{21} & \vartheta_{22} & \cdots & \vartheta_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \vartheta_{m1} & \vartheta_{m2} & \cdots & \vartheta_{mn} \end{pmatrix}, \tag{7}$$

where every ϑ_{ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is a linguistic intuitionistic fuzzy value $\vartheta_{ij} = (s_{\alpha_{ij}}, s_{\beta_{ij}})$, that is, the decision maker provides $(s_{\alpha_{ij}}, s_{\beta_{ij}})$ to evaluate alternative $A_i \in A$ with respect to the attribute $B_j \in B$. Accordingly, every attribute $B_j \in B$ can be represented by a LIFS on $A = (A_1, A_2, \ldots, A_m)$, i.e.,

$$B_j = \{(A_i, s_{\alpha_{ij}}(A_i), s_{\beta_{ij}}(A_i)) | A_i \in A\}.$$

More generally, we provide the similarity measure between two LIFSs on discrete discourse domain $U = \{x_1, x_2, \ldots, x_n\}$ as follows.

Definition 6 Let continuous linguistic terms be $\tilde{S} = \{s_\alpha | \alpha \in [0, h]\}$. Two LIFSs on $U = \{x_1, \ldots, x_n\}$ are $A_1 = $
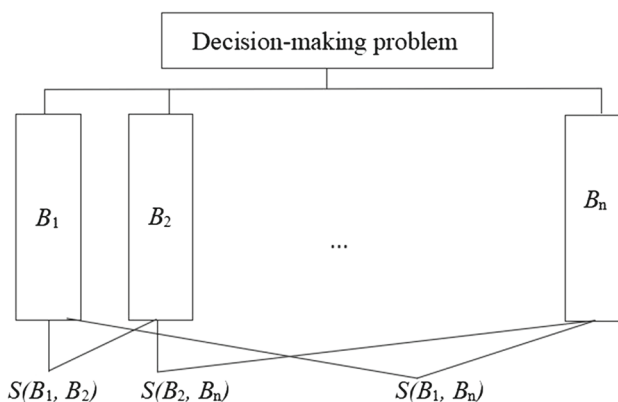

Fig. 1 The similarity measure between two decision making attributes in a decision matrix Ψ

$\{(x_i, s_{\alpha_{i1}}(x_i), s_{\beta_{i1}}(x_i)) | x_i \in U\}$ and $A_2 = \{(x_i, s_{\alpha_{i2}}(x_i), s_{\beta_{i2}}(x_i)) | x_i \in U\}$. Then the similarity measure between A_1 and A_2 is defined by

$$S(A_1, A_2) = h - \left[\frac{1}{2n} \sum_{i=1}^{n} \left(|\alpha_{i1} - \alpha_{i2}|^\gamma + |\beta_{i1} - \beta_{i2}|^\gamma \right. \right.$$
$$\left. \left. + |\pi_{i1} - \pi_{i2}|^\gamma \right) \right]^{\frac{1}{\gamma}} \tag{8}$$

where γ is a constant such that $\gamma > 0$. The indexes $\alpha_{i1}, \beta_{i1}, \alpha_{i2}$ and β_{i2} of linguistic terms $s_{\alpha_{i1}}, s_{\beta_{i1}}, s_{\alpha_{i2}}$ and $s_{\beta_{i2}}$ are such that $0 \leq \alpha_{i1} + \beta_{i1} \leq h, 0 \leq \alpha_{i2} + \beta_{i2} \leq h, \pi_{i1} = h - \alpha_{i1} - \beta_{i1}$ and $\pi_{i2} = h - \alpha_{i2} - \beta_{i2}$.

According to Eq. (8) of Definition 6, the following property can be easily proved.

Proposition 1 *Let continuous linguistic terms be* $\tilde{S} = \{s_\alpha | \alpha \in [0, h]\}$. *For any two LIFSs* $A_1 = \{(x_i, s_{\alpha_{i1}}(x_i), s_{\beta_{i1}}(x_i)) | x_i \in U\}$ *and* $A_2 = \{(x_i, s_{\alpha_{i2}}(x_i), s_{\beta_{i2}}(x_i)) | x_i \in U\}$ *on* $U = \{x_1, \ldots, x_n\}$, *we have*

1. $0 \leq S(A_1, A_2) \leq h$;
2. $S(A_1, A_2) = h$ *if and only if* $A_1 = A_2$;
3. $S(A_1, A_2) = S(A_2, A_1)$.

Based on the similarity measure between two LIFSs, we can present correlation analysis between decision making attributes. Formally, because every attribute $B_j \in B$ is a LIFS on $A = (A_1, A_2, \ldots, A_m)$, we can calculate the similarity measure between two decision making attributes in a decision matrix $\Psi = (\vartheta_{ij})_{m \times n}$, i.e.,

Definition 7 In any decision matrix $\Psi = (\vartheta_{ij})_{m \times n}$, attributes B_i and B_k are correlation if and only if the similarity measure between them is such that

$$S(B_i, B_k) \geq \varrho, \tag{9}$$

where $0 < \varrho \le h$ is a threshold determined by decision makers.

Formally, Property 1 shows that correlation between two attributes decided by Eqs. (9) and (8) is a similar relations on the set of attributes B (Fig. 1), i.e., correlation between two attributes provides a partition $\{C_1, C_2, \ldots, C_t\}$ of B for a fixed threshold $0 < \varrho \le h$, in which each subset $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}(t' = 1, 2, \ldots, t)$ of B is consisted of decision making attributes that are correlation each other.

Example 1 Let continuous linguistic terms be $\tilde{S} = \{s_\alpha | \alpha \in [0, 8]\}$. Assume the following decision matrix of a multi-attributes decision making problem

$$\Psi = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \\ \begin{pmatrix} (s_5, s_1) & (s_1, s_7) & (s_2, s_6) & (s_5, s_3) & (s_4, s_4) \\ (s_4, s_4) & (s_6, s_1) & (s_5, s_2) & (s_4, s_4) & (s_4, s_4) \\ (s_6, s_2) & (s_6, s_2) & (s_6, s_1) & (s_6, s_1) & (s_7, s_1) \end{pmatrix} \end{array},$$

Accordingly, decision making attributes B_1, B_2, B_3, B_4 and B_5 are represented by LIFSs on the set $\{A_1, A_2, A_3\}$ of alternatives, i.e., $B_1 = \{(s_5, s_1), (s_4, s_4), (s_6, s_2)\}$, $B_2 = \{(s_1, s_7), (s_6, s_1), (s_6, s_2)\}$, $B_3 = \{(s_2, s_6), (s_5, s_2), (s_6, s_1)\}$, $B_4 = \{(s_5, s_3), (s_4, s_4), (s_6, s_1)\}$, $B_5 = \{(s_4, s_4), (s_4, s_4), (s_7, s_1)\}$. In the example, we fix $\gamma = 1$, then according to Eq. (8), we have

$$S(B_1, B_2) = 8 - \left[\frac{4 + 6 + 2 + 2 + 3 + 1 + 0 + 0 + 0}{6} \right]$$
$$= 5.0,$$

Similarly, we have $(B_1, B_3) = 5.3$, $S(B_1, B_4) = 7.0$, $S(B_1, B_5) = 6.7$, $S(B_2, B_3) = 7.0$, $S(B_2, B_4) = 5.3$, $S(B_2, B_5) = 5.7$, $S(B_3, B_4) = 6.3$ and $S(B_3, B_5) = 7.3$.

If the threshold is fixed by $\varrho = 6.6$, then the partition of B is $C_1 = \{B_1, B_4, B_5\}$ and $C_2 = \{B_2, B_3\}$, i.e., decision making attributes in $C_1 = \{B_1, B_4, B_5\}$ (or $C_2 = \{B_2, B_3\}$) are correlation each other. If the threshold is fixed by $\varrho = 5$, then the partition of B is itself, i.e., $C_1 = \{B_1, B_2, B_3, B_4, B_5\}$.

4 Linguistic intuitionistic fuzzy RWMSM and RWDMSM operators

In this section, we proposed linguistic intuitionistic fuzzy RWMSM (LIFRWMSM) operator and linguistic intuitionistic fuzzy RWDMSM (LIFRWDMSM) operator based on correlation analysis between decision making attributes, which can be utilized to aggregate correlational linguistic intuitionistic fuzzy decision information and deal with large-scale linguistic intuitionistic fuzzy MADM problems with correlation between decision making attributes. Meanwhile,

several properties of LIFRWMSM and LIFRWDMSM operators are also analyzed.

According to the above analysis, suppose that t $(t \le n)$ correlational attributes subsets are obtained, i.e., $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}(t' = 1, 2, \ldots, t)$ and $|C_{t'}|$ is the number of correlational attributes in $C_{t'}$. Before LIFRWMSM and LIFRWDMSM operators, we need to redetermine weights of the aggregated attributes according to initial weights $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ of decision making attributes, here for any correlational attribute subset $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}$, the weight of attribute $B_{t'j} \in C_{t'}$ is redetermined by

$$\varpi_{t'j} = \frac{\varpi_j}{\sum_{j=1}^{|C_{t'}|} \varpi_j}, \tag{10}$$

where ϖ_j is the initial weight of $B_{t'j}$ in B.

Example 2 In Example 1, suppose the weight vector of attributes be $\varpi = \{0.1, 0.2, 0.3, 0.2, 0.2\}$, then in the correlational attribute subsets $C_1 = \{B_1, B_4, B_5\}$, weights of B_1, B_4 and B_5 become

$$\varpi_{11} = \frac{0.1}{0.1 + 0.2 + 0.2} = 0.2,$$
$$\varpi_{12} = \frac{0.2}{0.1 + 0.2 + 0.2} = 0.4,$$
$$\varpi_{13} = \frac{0.2}{0.1 + 0.2 + 0.2} = 0.4.$$

Similarly, in the correlational attribute subsets $C_2 = \{B_2, B_3\}$, weights of B_2 and B_3 become $\varpi_{21} = \frac{0.2}{0.2 + 0.3} = 0.4$ and $\varpi_{22} = \frac{0.3}{0.2 + 0.3} = 0.6$.

Due to each $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}(t' = 1, 2, \ldots, t)$ of B is such that decision making attributes of $C_{t'}$ are correlation each other, the RWMSM operator can be utilized to aggregate linguistic intuitionistic fuzzy evaluation values with respect to subset $C_{t'}$ of B.

Definition 8 For any decision matrix $\Psi = (\vartheta_{ij})_{m \times n}$ with LIFVs as decision information of alternatives, let $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}$ be a correlational attributes subset of B by Eqs. (8) and (9). The LIFV $\vartheta_{i_j} = (s_{\alpha_{ij}}, s_{\beta_{ij}})$ is decision information of the alternative $A_i \in A$ with respect to the attribute $B_{t'j}$ and $\varpi_{t'} = (\varpi_{t'1}, \ldots, \varpi_{t'|C_{t'}|})^T$ be a vector of weights decided by Eq. (10). Then the LIFRWMSM operator is defined by

$$\text{LIFRWMSM}^{(k)}(\vartheta_{i_1}, \vartheta_{i_2}, \ldots, \vartheta_{i_{|C_{t'}|}})$$
$$= \left(\frac{\bigoplus_{1 \le i_1 < \cdots < i_k \le |C_{t'}|} \mathcal{P}_{i_k} \left(\bigotimes_{j=1}^{k} \vartheta_{i_j} \right)}{\mathcal{P}_k} \right)^{\frac{1}{k}}, \tag{11}$$

where $t' = 1, 2, \ldots, t, k \in \{1, \ldots, |C_{t'}|\}$, $\mathcal{P}_{i_k} = \prod_{j=1}^k \varpi_{t'i_j}$ and $\mathcal{P}_k = \sum_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \mathcal{P}_{i_k}$.

Formally, according to operations $\oplus$ and $\otimes$ on LIFVs, the aggregation result of the LIFRWMSM operator is still a LIFV, i.e.,

$$
\left(\frac{\bigoplus\limits_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \mathcal{P}_{i_k} \left(\bigotimes\limits_{j=1}^k \vartheta_{i_j} \right)}{\mathcal{P}_k} \right)^{\frac{1}{k}}
$$

$$
= \left(\begin{array}{l} s_{h\left(1 - \left(\prod\limits_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \left(1 - \frac{\prod_{j=1}^k \alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}} \right)^{\frac{1}{\mathcal{P}_k}} \right)^{\frac{1}{k}}}, \\ s_{h-h\left(1 - \left(\prod\limits_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \left(1 - \prod_{j=1}^k \left(1 - \frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}} \right)^{\frac{1}{\mathcal{P}_k}} \right)^{\frac{1}{k}}} \end{array} \right).
$$
(12)

Example 3 In Example 2, for the correlational attribute subsets $C_1 = \{B_1, B_4, B_5\}$ and weights of ϖ_{11}, ϖ_{12} and ϖ_{13}, decision information of A_1 with respect to C_1 is $\vartheta_{1_1} = (s_5, s_1)$, $\vartheta_{1_2} = (s_5, s_3)$ and $\vartheta_{1_3} = (s_4, s_4)$, then we have

$$
\text{LIFRWMSM}^{(3)}(\vartheta_{1_1}, \vartheta_{1_2}, \vartheta_{1_3})
$$

$$
= \left(\begin{array}{l} s_{8*\left(1 - \left(\left(1 - \frac{5*5*4}{8^3}\right)^{0.2*0.4*0.4}\right)^{\frac{1}{0.2*0.4*0.4}}\right)^{\frac{1}{3}}}, \\ s_{8-8*\left(1 - \left(\left(1 - \left(1-\frac{1}{8}\right)\left(1-\frac{3}{8}\right)\left(1-\frac{4}{8}\right)\right)^{0.2*0.4*0.4}\right)^{\frac{1}{0.2*0.4*0.4}}\right)^{\frac{1}{3}}} \end{array} \right)
$$

$$
= \left(\begin{array}{l} s_{8*\left(\left(\frac{5}{8}\right)*\left(\frac{5}{8}\right)*\left(\frac{4}{8}\right)\right)^{\frac{1}{3}}}, \\ s_{8-8*\left(\left(1-\frac{1}{8}\right)\left(1-\frac{3}{8}\right)\left(1-\frac{4}{8}\right)\right)^{\frac{1}{3}}} \end{array} \right)
$$

$$
= (s_{4.6}, s_{2.8}).
$$

The proof of Eq. (12) is provided in Appendix 1. In addition, the following properties of the LIFRWMSM operator can be proved according to operations $\oplus$ and $\otimes$ on LIFVs.

Theorem 1 (Idempotency) *The LIFRWMSM operator [Eq. (11)] is idempotent, i.e, if $\vartheta_{i_1} = \cdots = \vartheta_{i_{|C_{t'}|}} = (s_\alpha, s_\beta)$, then*

$$
\text{LIFRWMSM}^{(k)}(\vartheta_{i_1}, \vartheta_{i_2}, \ldots, \vartheta_{i_{|C_{t'}|}}) = (s_\alpha, s_\beta)
$$

Theorem 2 (Monotonicity) *The LIFRWMSM operator [Eq. (11)] is monotonic, i.e, for any LIFVs $\vartheta_{i_j} = (s_{\alpha_{ij}}, s_{\beta_{ij}})$ and $\vartheta'_{i_j} = (s_{\alpha'_{ij}}, s_{\beta'_{ij}})$, if $s_{\alpha_{ij}} \geq s_{\alpha'_{ij}}, s_{\beta_{ij}} \leq s_{\beta'_{ij}}$, then*

$$
\text{LIFRWMSM}^{(k)}(\vartheta_{i_1}, \vartheta_{i_2}, \ldots, \vartheta_{i_{|C_{t'}|}})
$$

$$
\geq \text{LIFRWMSM}^{(k)}(\vartheta'_{i_1}, \vartheta'_{i_2}, \ldots, \vartheta'_{i_{|C_{t'}|}})
$$

Theorem 3 (Boundedness) *The LIFRWMSM operator [Eq. (11)] is bounded, i.e, for LIFVs $\vartheta_{i_1} = (s_{\alpha_{i1}}, s_{\beta_{i1}}), \ldots, \vartheta_{i_{|C_{t'}|}} = (s_{\alpha_{i|C_{t'}|}}, s_{\beta_{i|C_{t'}|}})$, let $\vartheta_{ij}^+ = (s_{max\alpha_{ij}}, s_{min\beta_{ij}})$ and $\vartheta_{ij}^- = (s_{min\alpha_{ij}}, s_{max\beta_{ij}})$ $(j = 1, \ldots, |C_{t'}|)$, then*

$$
\vartheta_{ij}^- \leq \text{LIFRWMSM}^{(k)}(\vartheta_{i_1}, \ldots, \vartheta_{i_{|C_{t'}|}}) \leq \vartheta_{ij}^+
$$

Theorem 4 (Commutativity) *The LIFRWMSM operator [Eq. (11)] is commutative, i.e, let $(\vartheta_{i_1}^T, \ldots, \vartheta_{i_{|C_{t'}|}}^T)$ be a permutation of $(\vartheta_{i_1}, \ldots, \vartheta_{i_{|C_{t'}|}})$, then*

$$
\text{LIFRWMSM}^{(k)}(\vartheta_{i_1}^T, \ldots, \vartheta_{i_{|C_{t'}|}}^T)
$$

$$
quad = \text{LIFRWMSM}^{(k)}(\vartheta_{i_1}, \ldots, \vartheta_{i_{|C_{t'}|}})
$$

The proofs of these theorems are provided in Appendixes 2–5, respectively. In addition, based on the partition $\{C_1, C_2, \ldots, C_t\}$ of B of B for a fixed threshold $0 < \varrho \leq h$, the aggregation process of LIFSs decision information in the decision matrix $\Psi = (\vartheta_{ij})_{m \times n}$ by using the LIFRWMSM operator shows different cases. In some degree, the cases can be considered as reducibility of the LIFRWMSM operator in aggregating LIFSs in $\Psi = (\vartheta_{ij})_{m \times n}$, i.e., in Eq. (12),

Case 1 *If $C_{t'} = 1$ (it is equal to $k = 1$), the LIFRWMSM operator is reduced to*

$$
\text{LIFRWMSM}^{(1)}(\vartheta_{i_1}, \ldots, \vartheta_{i_{|C_{t'}|}})
$$

$$
= \text{LIFRWMSM}^{(1)}(\vartheta_{i_1})
$$

$$
= \varpi_j \vartheta_{ij} = \text{LIFWA}(\vartheta_{i_1})
$$

The case also means that decision making attributes $B = (B_1, B_2, \ldots, B_n)$ are independent each other, and LIFWA aggregation operator can be directly used to aggregate LIFSs decision information in $\Psi = (\vartheta_{ij})_{m \times n}$, i.e., for each alternative $A_i (i = 1, 2, \ldots, m)$, we have

$$
\text{LIFWA}(\vartheta_{i1}, \ldots, \vartheta_{in}) = \oplus_{j=1}^n \varpi_j \vartheta_{ij}.
$$

Case 2 *If $C_{t'} = |B|$, i.e., decision making attributes $B = (B_1, B_2, \ldots, B_n)$ are correlation each other, then the proposed LIFRWMSM operator is reduced to linguistic intuitionistic fuzzy geometric (LIFG) operator,*

$$\mathrm{LIFRWMSM}^{(|B|)}(\vartheta_{i_1}, \vartheta_{i_2}, \ldots, \vartheta_{i_{|B|}})$$

$$= \left(s_{h\left(1-\left(\prod\limits_{1\le i_1 <\cdots < i_{|B|}\le |B|}\left(1-\frac{\prod_{j=1}^{|B|}\alpha_{i_j}}{h^{|B|}}\right)^{\mathcal{P}_k}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{|B|}}}, s_{h-h\left(1-\left(\prod\limits_{1\le i_1 <\cdots < i_n\le |B|}\left(1-\prod\limits_{j=1}^{|B|}\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_k}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{|B|}}} \right)$$

$$= \left(s_{h\prod\limits_{j=1}^{|B|}\left(\frac{\alpha_{i_j}}{h}\right)^{\frac{1}{|B|}}}, s_{\left(h-h\prod\limits_{j=1}^{|B|}(1-\frac{\beta_{i_j}}{h})^{\frac{1}{|B|}}\right)} \right) = \mathrm{LIFG}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{in}).$$

Example 4 Based on Examples 1 and 2, when we fix $\varrho = 7.5$, the partition of B is $C_1 = \{B_1\}$, $C_2 = \{B_2\}$, $C_3 = \{B_3\}$, $C_4 = \{B_4\}$ and $C_5 = \{B_5\}$, i.e., $C_{t'} = k = 1$, then for LIFSs decision information of alternative A_1 in the decision matrix of Examples 1,

$$\mathrm{LIFWA}(\vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}, \vartheta_{15})$$

$$= \left(s_{8-8*\left(1-(\frac{5}{8})\right)^{0.1}*\left(1-(\frac{1}{8})\right)^{0.2}*\left(1-(\frac{2}{8})\right)^{0.3}*\left(1-(\frac{5}{8})\right)^{0.2}*\left(1-(\frac{4}{8})\right)^{0.2}}, s_{8*\left(\frac{1}{8}\right)^{0.1}*\left(\frac{7}{8}\right)^{0.2}*\left(\frac{6}{8}\right)^{0.3}*\left(\frac{3}{8}\right)^{0.2}\left(\frac{4}{8}\right)^{0.2}} \right) = (s_{3.4}, s_{4.2}).$$

when we fix $\varrho = 5$, the partition of B is $C_1 = \{B_1, B_2, B_3, B_4, B_5\}$, i.e., $C_{t'} = |B|$, then we have

$$\mathrm{LIFRWMSM}^{(k)}(\vartheta_{i_1}, \vartheta_{i_2}, \ldots, \vartheta_{i_{|C_{t'}|}})$$

$$= \frac{\bigotimes\limits_{1\le i_1 <\cdots < i_k \le |C_{t'}|}\left(\bigoplus\limits_{j=1}^{k}\vartheta_{i_j}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}}{k}, \tag{13}$$

where $t' = 1, 2, \ldots, t$, $k \in \{1, \ldots, |C_{t'}|\}$, $\mathcal{M}_{i_k} = \sum_{j=1}^{k}\varpi_{i_j}$, $\mathcal{M}_k = \sum_{1\le i_1 <\cdots < i_k \le |C_{t'}|}\mathcal{M}_{i_k}$.

$$\mathrm{LIFRWMSM}^{(5)}(\vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}, \vartheta_{15})$$

$$= \left(s_{8*\left(1-\left(\left(1-\frac{5*1*2*5*4}{8^3}\right)^{0.1*0.2*0.3*0.2*0.2}\right)^{\frac{1}{0.1*0.2*0.3*0.2*0.2}}\right)^{\frac{1}{5}}}, s_{8-8*\left(1-\left(\left(1-\left(1-\frac{1}{8}\right)\left(1-\frac{7}{8}\right)\left(1-\frac{6}{8}\right)\left(1-\frac{3}{8}\right)\left(1-\frac{4}{8}\right)\right)^{0.1*0.2*0.3*0.2*0.2}\right)^{\frac{1}{0.1*0.2*0.3*0.2*0.2}}\right)^{\frac{1}{5}}} \right).$$

$$= \left(s_{8*\left(\left(\frac{5}{8}\right)*\left(\frac{1}{8}\right)*\left(\frac{2}{8}\right)*\left(\frac{5}{8}\right)*\left(\frac{4}{8}\right)\right)^{\frac{1}{5}}}, s_{8-8*\left(\left(1-\frac{1}{8}\right)\left(1-\frac{7}{8}\right)\left(1-\frac{6}{8}\right)\left(1-\frac{3}{8}\right)\left(1-\frac{4}{8}\right)\right)^{\frac{1}{5}}} \right).$$

$$= (s_{2.9}, s_{4.9}) = \mathrm{LIFG}(\vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}, \vartheta_{15}).$$

Definition 9 For any decision matrix $\Psi = (\vartheta_{ij})_{m\times n}$ with LIFVs as decision information of alternatives, let $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}$ be a correlational attributes subset of B by Eqs. (8) and (9). The LIFV $\vartheta_{i_j} = (s_{\alpha_{ij}}, s_{\beta_{ij}})$ is decision information of the alternative $A_i \in A$ with respect to the attribute $B_{t'j}$ and $\varpi_{t'} = (\varpi_{t'1}, \ldots, \varpi_{t'|C_{t'}|})^T$ be a vector of weights decided by Eq. (10). The LIFRWDMSM operator is

Formally, according to operations $\oplus$ and $\otimes$ on LIFVs, the aggregation result of the LIFRWDMSM operator is still a LIFV, i.e.,

$$\frac{\underset{1\leq i_1<\cdots<i_k\leq|C_{t'}|}{\bigotimes}\left(\underset{j=1}{\overset{k}{\bigoplus}}\vartheta_{i_j}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}}{k}=\left(\begin{array}{c}s_{h-h\left(1-\left(\underset{1\leq i_1<\cdots<i_k\leq|C_{t'}|}{\prod}\left(1-\prod_{j=1}^{k}\left(1-\frac{\alpha_{i_j}}{h}\right)\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}},\\[2em]s_{h\left(1-\left(\underset{1\leq i_1<\cdots<i_k\leq|C_{t'}|}{\prod}\left(1-\frac{\prod_{j=1}^{k}\beta_{i_j}}{h^k}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}}\end{array}\right)\tag{14}$$

Example 5 In Example 3, for the correlational attribute subsets $C_1=\{B_1,B_4,B_5\}$ and weights of ϖ_{11},ϖ_{12} and ϖ_{13}, decision information of A_1 with respect to C_1 are $\vartheta_{11}=(s_5,s_1),\vartheta_{12}=(s_5,s_3)$ and $\vartheta_{13}=(s_4,s_4)$, then we have

$$\text{LIFRWDMSM}^{(k)}(\vartheta_{11},\vartheta_{12},\vartheta_{13})=\left(\begin{array}{c}s_{8-8*\left(1-\left(\left(1-\left(1-\frac{5}{8}\right)\left(1-\frac{5}{8}\right)\left(1-\frac{4}{8}\right)\right)^{0.2+0.4+0.4}\right)^{\frac{1}{0.2+0.4+0.4}}\right)^{\frac{1}{3}}},\\[1.5em]s_{8*\left(1-\left(\left(1-\frac{1*3*4}{8^3}\right)^{0.2+0.4+0.4}\right)^{\frac{1}{0.2+0.4+0.4}}\right)^{\frac{1}{3}}}\end{array}\right)$$

$$=\left(s_{8-8*\left(\left(1-\left(\frac{5}{8}\right)\right)*\left(1-\left(\frac{5}{8}\right)\right)*\left(1-\left(\frac{4}{8}\right)\right)^{\frac{1}{3}}\right)},s_{8*\left(\left(\frac{1}{8}\right)\left(\frac{3}{8}\right)\left(\frac{4}{8}\right)\right)^{\frac{1}{3}}}\right)=(s_{4.7},s_{2.3}).$$

Equation (14) can be similarly proved in Appendix 6. In addition, the following properties of the LIFRWDMSM operator can be proved according to operations $\oplus$ and $\otimes$ on LIFVs.

Theorem 5 (Idempotency) *The LIFRWDMSM operator [Eq. (13)] is idempotent, i.e, if $\vartheta_{i_1}=\cdots=\vartheta_{i_{|C_{t'}|}}=(s_\alpha,s_\beta)$, then,*

$$\text{LIFRWDMSM}^{(k)}(\vartheta_{i_1},\vartheta_{i_2},\ldots,\vartheta_{i_{|C_{t'}|}})=(s_\alpha,s_\beta)$$

Theorem 6 (Monotonicity) *The LIFRWDMSM operator [Eq. (13)] is monotonic, i.e, for any LIFVs $\vartheta_{i_j}=(s_{\alpha_{ij}},s_{\beta_{ij}})$ and $\vartheta'_{i_j}=(s_{\alpha'_{ij}},s_{\beta'_{ij}})$, if $s_{\alpha_{ij}}\geq s_{\alpha'_{ij}},s_{\beta_{ij}}\leq s_{\beta'_{ij}}$, then*

$$\text{LIFRWDMSM}^{(k)}(\vartheta_{i_1},\vartheta_{i_2},\ldots,\vartheta_{i_{|C_{t'}|}})$$
$$\geq\text{LIFRWDMSM}^{(k)}(\vartheta'_{i_1},\vartheta'_{i_2},\ldots,\vartheta'_{i_{|C_{t'}|}})$$

Theorem 7 (Boundedness) *The LIFRWDMSM operator [Eq. (13)] is bounded, i.e, for LIFVs $\vartheta_{i_1}=(s_{\alpha_{i_1}},s_{\beta_{i_1}}),\ldots,$ $\vartheta_{i_{|C_{t'}|}}=(s_{\alpha_{i_{|C_{t'}|}}},s_{\beta_{i_{|C_{t'}|}}})$, let $\vartheta^+_{ij}=(s_{max\alpha_{ij}},s_{min\beta_{ij}})$ and $\vartheta^-_{ij}=(s_{min\alpha_{ij}},s_{max\beta_{ij}})\,(j=1,\ldots,|C_{t'}|)$, then*

$$\vartheta^-_{ij}\leq\text{LIFRWDMSM}^{(k)}(\vartheta_{i_1},\ldots,\vartheta_{i_{|C_{t'}|}})\leq\vartheta^+_{ij}$$

Theorem 8 (Commutativity) *The LIFRWDMSM operator [Eq. (13)] is commutative, i.e, let $(\vartheta^T_{i_1},\ldots,\vartheta^T_{i_{|C_{t'}|}})$ be a permutation of $(\vartheta_{i_1},\ldots,\vartheta_{i_{|C_{t'}|}})$, then*

$$\text{LIFRWDMSM}^{(k)}(\vartheta^T_{i_1},\ldots,\vartheta^T_{i_{|C_{t'}|}})$$
$$=\text{LIFRWDMSM}^{(k)}(\vartheta_{i_1},\ldots,\vartheta_{i_{|C_{t'}|}})$$

These theorems can be similarly proved in Appendixes 2–5. The proof of Theorem 5 in Appendix 7. In addition, like the cases of the LIFRWMSM operator, we have the following cases of reducibility of the LIFRWDMSM operator, in Eq. (12),

Case 3 *If $k=|C_{t'}|=1$, the proposed LIFRWDMSM operator is reduced to*

$$\text{LIFRWDMSM}^{(1)}(\vartheta_{i_1},\ldots,\vartheta_{i_{|C_{t'}|}})$$
$$=\text{LIFRWDMSM}^{(1)}(\vartheta_{i_1})=(\vartheta_{ij})^{\varpi_j}=\text{LIFWG}(\vartheta_{i_1}).$$

In such case, LIFSs decision information of alternative A_i in the decision matrix $\Psi=(\vartheta_{ij})_{m\times n}$ can be aggregated by LIFWG operator, i.e.,

$$\text{LIFWG}(\vartheta_{i1},\ldots,\vartheta_{in})=\otimes_{j=1}^{n}(\vartheta_{ij})^{\varpi_j}.$$

Case 4 *If $C_{t'}=|B|$, the proposed LIFRWDMSM operator is reduced to the LIFA operator, i.e.,*

$\text{LIFRWDMSM}^{(n)}(\vartheta_{i_1}, \vartheta_{i_2}, \ldots, \vartheta_{i_{|B|}})$

$$= \left(s_{h-h\left(1-\left(\prod\limits_{1 \le i_1 < \cdots < i_{|B|} \le |B|}\left(1-\prod\limits_{j=1}^{|B|}\left(1-\frac{\alpha_{i_j}}{h}\right)\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{|B|}}}, s_{h\left(1-\left(\sum\limits_{1 \le i_1 < \cdots < i_{|B|} \le |B|}\left(1-\frac{\prod_{j=1}^{|B|}\beta_{i_j}}{h^{|B|}}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{|B|}}} \right)$$

$$= \left(s_{h-h\prod\limits_{1 \le i_1 \le |B|}\left(1-\frac{\alpha_{i_j}}{h}\right)^{\frac{1}{|B|}}}, s_{h\prod\limits_{1 \le i_1 \le |B|}\left(\frac{\beta_{i_j}}{h}\right)^{\frac{1}{|B|}}} \right) = \text{LIFA}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{in})$$

Example 6 Similar with Example 4, for $\varrho = 7.5$ and LIFSs decision information of alternative A_1, we have

$\text{LIFWG}(\vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}, \vartheta_{15})$

$$= \left(s_{8-8*(1-\left(\left(1-\left(1-\frac{5}{8}\right)\right)^{0.1}\left(1-\left(1-\frac{1}{8}\right)\right)^{0.2}\left(1-\left(1-\frac{2}{8}\right)\right)^{0.3}\left(1-\left(1-\frac{5}{8}\right)\right)^{0.2}\left(1-\left(1-\frac{4}{8}\right)\right)^{0.2}\right)^{\frac{1}{0.1+0.2+0.3+0.2+0.2}}}, \atop s_{8*(1-\left(\left(1-\frac{1}{8}\right)^{0.1}\left(1-\frac{7}{8}\right)^{0.2}\left(1-\frac{6}{8}\right)^{0.3}\left(1-\frac{3}{8}\right)^{0.2}\left(1-\frac{4}{8}\right)^{0.2}\right)^{\frac{1}{0.1+0.2+0.3+0.2+0.2}}}} \right)$$

$$= \left(s_{8*\left(\frac{5}{8}\right)^{0.1}*\left(\frac{1}{8}\right)^{0.2}*\left(\frac{2}{8}\right)^{0.3}*\left(\frac{5}{8}\right)^{0.2}*\left(\frac{4}{8}\right)^{0.2}}, s_{8-8*\left(\left(\frac{7}{8}\right)^{0.1}\left(\frac{1}{8}\right)^{0.2}\left(\frac{2}{8}\right)^{0.3}\left(\frac{5}{8}\right)^{0.2}\left(\frac{4}{8}\right)^{0.2}\right)} \right) = (s_{2.6}, s_{5.2}).$$

For $\varrho = 5$, we have

$\text{LIFRWDMSM}^{(k)}(\vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}, \vartheta_{15})$

$$= \left(s_{8-8*\left(1-\left(\left(1-\left(1-\frac{5}{8}\right)\left(1-\frac{1}{8}\right)\left(1-\frac{2}{8}\right)\left(1-\frac{5}{8}\right)\left(1-\frac{4}{8}\right)\right)^{0.1+0.2+0.3+0.2+0.2}\right)^{\frac{1}{0.1+0.2+0.3+0.2+0.2}}\right)^{\frac{1}{5}}}, \atop s_{8*\left(1-\left(\left(1-\frac{1*7*6*3*4}{8^3}\right)^{0.1+0.2+0.3+0.2+0.2}\right)^{\frac{1}{0.1+0.2+0.3+0.2+0.2}}\right)^{\frac{1}{5}}}} \right)$$

$$= \left(s_{8-8*\left(\left(1-\left(\frac{5}{8}\right)\right)*\left(1-\left(\frac{1}{8}\right)\right)*\left(1-\left(\frac{2}{8}\right)\right)*\left(1-\left(\frac{5}{8}\right)\right)*\left(1-\left(\frac{4}{8}\right)\right)\right)^{\frac{1}{5}}}, s_{8*\left(\left(\frac{1}{8}\right)\left(\frac{7}{8}\right)\left(\frac{6}{8}\right)\left(\frac{3}{8}\right)\left(\frac{4}{8}\right)\right)^{\frac{1}{5}}} \right) = (s_{3.7}, s_{3.5}).$$

$$\theta_{t'} = \frac{\prod_{j=1}^{|C_{t'}|} \varpi_{t'j}}{\sum_{t'=1}^{t} \prod_{j=1}^{|C_{t'}|} \varpi_{t'j}} \tag{15}$$

where $\theta_{t'}$ represents the weight of core attribute $C_{t'}$.

Formally, based on correlation analysis between decision making attributes, the set B of decision making attributes is partitioned into $C = \{C_1, C_2, \ldots, C_t\}(t \le n)$, which can be considered as t independent new attributes, i.e., the set of t independent core attributes of B is C in the paper. By using the proposed LIFRWMSM or LIFRWDMSM operators, LIFV decision information of each $C_{t'}(t' \in \{1, 2, \ldots, t\})$ can be obtained, and weights of independent core attributes $\{C_1, \ldots, C_t\}(t \le n)$ can be renewed as follows

Example 7 In Example 2, for independent core attributes $C_{1'} = \{B_1, B_4, B_5\}$ and $C_{2'} = \{B_2, B_3\}$, due to weights $\varpi_{1'1} = 0.2$, $\varpi_{1'2} = 0.4$, $\varpi_{1'3} = 0.4$ and $\varpi_{2'1} = 0.4$, $\varpi_{2'2} = 0.6$, we have

$$\theta_{1'} = \frac{0.2 \times 0.4 \times 0.4}{0.2 \times 0.4 \times 0.4 + 0.4 \times 0.6} \doteq 0.12,$$

$$\theta_{2'} = \frac{0.4 \times 0.6}{0.2 \times 0.4 \times 0.4 + 0.4 \times 0.6} \doteq 0.88.$$

5 A solution scheme of large-scale linguistic intuitionistic fuzzy decision making problems

Inspired by EDAS decision making method and correlation analysis between decision making attributes, in the section, we present a new solution scheme of large-scale linguistic intuitionistic fuzzy decision making analysis, in which decision information of alternatives are represented by LIFVs and correlation between decision making attributes are existed in decision making process due to multi-sources decision information. Formally, the new solution scheme is consisted by the following steps.

Step 1. Obtaining the decision matrix $\Psi = (\vartheta_{ij})_{m \times n}$ [Eq. (7)] based on multi-sources decision information, in which each ϑ_{ij} is a LIFV.

Step 2. Analyzing correlation between decision making attributes of Ψ based on Eqs. (8) and (9), then obtaining the partition $C = \{C_1, C_2, \ldots, C_t\}(t \leq n)$ of B by the fixed threshold ϱ, in which each core attribute $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}(t' = 1, 2, \ldots, t)$ is such that decision making attributes in $C_{t'}$ are correlation each other.

Step 3. Constructing the new decision matrix $\Psi_{new} = (\vartheta'_{ij})_{m \times t}$, i.e.,

$$\Psi_{new} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} C_1 & C_2 & \cdots & C_t \\ \vartheta'_{11} & \vartheta'_{12} & \cdots & \vartheta'_{1t} \\ \vartheta'_{21} & \vartheta'_{22} & \cdots & \vartheta'_{2t} \\ \vdots & \vdots & \cdots & \vdots \\ \vartheta'_{m1} & \vartheta'_{m2} & \cdots & \vartheta'_{mt} \end{pmatrix}, \tag{16}$$

where each $\vartheta'_{ij}(i = 1, \ldots, m, j = 1, \ldots, t)$ is a LIFV, i.e., $\vartheta'_{ij} = (s_{\alpha_{ij}}, s_{\beta_{ij}})$ is calculated by the LIFRWMSM or LIFRWDMSM operators according to LIFVs ϑ_{ij} in Ψ with respect to $C_{t'} = \{B_{t'1}, \ldots, B_{t'|C_{t'}|}\}$, weights of all $B_{t'|C_{t'}|}$ are determined by Eq. (10).

Step 4. Calculating the average solution $\bar{A}S_j = \bar{\vartheta}_j = (s_{\alpha_j}, s_{\beta_j})$ of each core attribute $C_j(j = 1, \ldots, t)$ in Ψ_{new}, i.e.,

$$\bar{A}S_j = \bar{\vartheta}_j = \frac{1}{m} \oplus_{i=1}^{m} \vartheta'_{ij}.$$

Step 5. Calculating the positive distance from average PDA_{ij} and the negative distance from average NDA_{ij} for each alternative $A_i(i = 1, \ldots, m)$, i.e.,

$$PDA_{ij} = \begin{cases} \mathscr{S}(\vartheta'_{ij}) - \mathscr{S}(\bar{A}S_j), & \text{if } \vartheta'_{ij} \geq \bar{A}S_j; \\ 0, & \text{if } \vartheta_{ij} < AS_j. \end{cases} \tag{17}$$

$$NDA_{ij} = \begin{cases} \mathscr{S}(\bar{A}S_j) - \mathscr{S}(\vartheta'_{ij}), & \text{if } \vartheta'_{ij} < AS_j; \\ 0, & \text{if } \vartheta_{ij} \geq \bar{A}S_j. \end{cases} \tag{18}$$

where $\mathscr{S}(*)$ is defined by Eq. (2).

Step 6. Determining the value of weighted summation of the positive and negative distances from average solution for each alternative $A_i(i = 1, \ldots, m)$, i.e.,

$$SP_i^\theta = \sum_{j=1}^{t} PDA_{ij}\theta_j, \quad SN_i^\theta = \sum_{j=1}^{t} NDA_{ij}\theta_j, \tag{19}$$

where weight θ_j of core attribute $C_j(j = 1, \ldots, t)$ is determined by Eq. (15).

Step 8. Normalizing the values of the weighted PDA and weighted NDA, respectively.

$$SP_i^N = \frac{SP_i^\theta}{\max\{SP_i^\theta | i = 1, 2, \ldots, m\}}, \tag{20}$$

$$SN_i^N = 1 - \frac{SN_i^\theta}{\max\{SN_i^\theta | i = 1, 2, \ldots, m\}}. \tag{21}$$

Step 9. Computing the score $\mathcal{AS}_i$ of each alternative $A_i(i = 1, \ldots, m)$, i.e.,

$$\mathcal{AS}_i = \frac{1}{2}(SP_i^N + SN_i^N)$$

Step 10. Ranking the alternatives according to the decreasing values of the score $\mathcal{AS}_i$, the best alternative $A_{i'}$ is with the highest score $\mathcal{AS}_{i'}$, i.e., $i' = ArgMax_{i \in \{1,2,\ldots,m\}}\{\mathcal{AS}_i\}$.

6 Case study

In this section, we consider two popular practical examples to demonstrate the proposed decision making method.

6.1 Case analysis

With the development of the advancement in people's living standards, more and more people choose to buy quality housing. Generally, the investment to buy a house is an important financial expenditure for a family, the choice of buying a house is a major decision making for the family.

Example 8 Suppose that an agent introduced four houses A_1, A_2, A_3 and A_4 to a family. After careful consideration, the family selected the following attributes to evaluate the four houses, i.e., housing price (B_1), Regional Transportation (B_2), school (B_3), Commercial supermarket (B_4), Hospital (B_5), Park (B_6), Catering (B_7), Community scale (B_8), Community age (B_9), Gated community (B_{10}), Security system configuration (B_{11}), Developer brand (B_{12}), Property brand (B_{13}), service level (B_{14}), Community greening (B_{15}), Parking conditions (B_{16}), architectural style (B_{17}), Number of rooms (B_{18}), Living room area (B_{19}), Unit orientation (B_{20}).

Table 1 Linguistic terms for evaluating alternatives with respect to decision attributes

Linguistic terms	Semantic Description
s_0	Extremely disappointed
s_1	Very disappointed
s_2	Quite disappointed
s_3	Disappointed
s_4	General
s_5	Satisfied
s_6	Quite satisfied
s_7	Very satisfied
s_8	Extremely satisfied

The family comprehensively consider the relevant data provided by the agent to give linguistic assessments according to linguistic terms listed in Table 1, and weights of decision attributes are shown in Table 2. The LIFVs evaluation information (decision matrix) provided by the family is displayed in Table 3. In the example, we fix $k = 2$ and $\gamma = 1$.

Based on the solution scheme of large-scale linguistic intuitionistic fuzzy decision making analysis in Sect. 5, the decision making of Example 8 is as follows.

Step 1. Analyzing correlation between 20 decision attributes based on Eqs. (8) and (9), then obtaining the partition $C = \{C_1, C_2, \ldots, C_t\}(t \le n)$ of B by the fixed $\gamma = 1$ and threshold $\varrho = 6$. In the example, we can obtain core attributes $C_1 = \{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$, $C_1 = \{B_8, B_9,$

$B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}\}$ and $C_3 = \{B_{17}, B_{18}, B_{19}, B_{20}\}$.

Aggregating $C_1 = \{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$, $C_2 = \{B_8, B_9, B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}\}$ and $C_3 = \{B_{17}, B_{18}, B_{19}, B_{20}\}$ by using the LIFRWDMSM operator [Eq. (14)], in which we fix $k = 2$ to obtain the aggregation results of core attributes C_1, C_2, and C_3, respectively. Then the new decision matrix is as follows.

$$\Psi_{new} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{pmatrix} C_1 & C_2 & C_3 \\ (s_{1.3513}, s_{6.1176}) & (s_{5.5659}, s_{0.7413}) & (s_{7.0000}, s_{1.0000}) \\ (s_{3.0854}, s_{4.5969}) & (s_{5.2739}, s_{1.1189}) & (s_{4.0000}, s_{4.0000}) \\ (s_{5.2024}, s_{2.6861}) & (s_{5.2558}, s_{0.8549}) & (s_{1.0000}, s_{7.0000}) \\ (s_{3.1689}, s_{4.5815}) & (s_{5.6813}, s_{1.2041}) & (s_{1.0000}, s_{5.0000}) \end{pmatrix},$$

According to Eq. (15) and Table 2, weights of core attributes C_1, C_2 and C_3 are $\theta_1 = 0.0071$, $\theta_2 = 0.0011$ and $\theta_3 = 0.9918$, respectively.

Step 2. Calculating the average solution $\bar{A}S_j = \bar{\vartheta}_j (j = 1, 2, 3)$ of core attributes C_1, C_2 and C_3, respectively,

$$\bar{A}S_1 = (s_{3.4158}, s_{4.3132}), \quad \bar{A}S_2 = (s_{4.5678}, s_{0.9613}),$$
$$\bar{A}S_3 = (s_{4.2583}, s_{3.4398})$$

where $\bar{\vartheta}_j = \left(s_{8-8\prod_{i=1}^{4}\left(1-\frac{\alpha_{ij}}{8}\right)^{\frac{1}{4}}}, s_{8\prod_{i=1}^{4}\left(\frac{\beta_{ij}}{8}\right)^{\frac{1}{4}}} \right).$

Step 3. Calculating the positive distance from average PDA_{ij} and the negative distance from average NDA_{ij} of

Table 2 Weights of decision attributes in Example 8

Attribute	Weight	Attribute	Weight	Attribute	Weight	Attribute	Weight
B_1	0.100	B_6	0.030	B_{11}	0.100	B_{16}	0.090
B_2	0.050	B_7	0.020	B_{12}	0.060	B_{17}	0.010
B_3	0.040	B_8	0.030	B_{13}	0.070	B_{18}	0.010
B_4	0.040	B_9	0.030	B_{14}	0.080	B_{19}	0.010
B_5	0.030	B_{10}	0.100	B_{15}	0.080	B_{20}	0.020

Table 3 The LIFVs evaluation information (decision matrix) of Example 8

Alternatives	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
A_1	(s_1, s_7)	(s_1, s_7)	(s_2, s_6)	(s_1, s_5)	(s_2, s_5)	(s_2, s_5)	(s_1, s_6)	(s_6, s_1)	(s_6, s_1)	(s_5, s_1)
A_2	(s_4, s_4)	(s_3, s_5)	(s_4, s_4)	(s_3, s_5)	(s_1, s_5)	(s_2, s_5)	(s_3, s_5)	(s_5, s_2)	(s_5, s_1)	(s_5, s_2)
A_3	(s_6, s_2)	(s_5, s_3)	(s_5, s_3)	(s_4, s_4)	(s_5, s_3)	(s_6, s_1)	(s_4, s_4)	(s_4, s_3)	(s_5, s_1)	(s_5, s_1)
A_4	(s_2, s_5)	(s_4, s_4)	(s_4, s_4)	(s_4, s_4)	(s_4, s_4)	(s_1, s_7)	(s_4, s_4)	(s_6, s_1)	(s_6, s_1)	(s_6, s_1)

	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}
A_1	(s_6, s_1)	(s_6, s_1)	(s_5, s_2)	(s_6, s_1)	(s_5, s_1)	(s_6, s_1)	(s_7, s_1)	(s_7, s_1)	(s_7, s_1)	(s_7, s_1)
A_2	(s_5, s_2)	(s_5, s_2)	(s_5, s_2)	(s_6, s_1)	(s_5, s_1)	(s_6, s_1)	(s_4, s_4)	(s_4, s_4)	(s_4, s_4)	(s_4, s_4)
A_3	(s_5, s_1)	(s_4, s_3)	(s_5, s_1)	(s_6, s_1)	(s_6, s_1)	(s_6, s_1)	(s_1, s_7)	(s_1, s_7)	(s_1, s_7)	(s_1, s_7)
A_4	(s_2, s_6)	(s_6, s_1)	(s_6, s_1)	(s_6, s_1)	(s_5, s_2)	(s_6, s_1)	(s_1, s_5)	(s_1, s_5)	(s_1, s_5)	(s_1, s_5)

Table 4 Weights of decision attributes in Example 9

Attribute	Weight	Attribute	Weight	Attribute	Weight	Attribute	Weight
B_1	0.0400	B_6	0.0300	B_{11}	0.1000	B_{16}	0.0500
B_2	0.0800	B_7	0.1000	B_{12}	0.0400	B_{17}	0.0200
B_3	0.0400	B_8	0.0300	B_{13}	0.0500	B_{18}	0.0200
B_4	0.0200	B_9	0.0400	B_{14}	0.0600	B_{19}	0.0100
B_5	0.1000	B_{10}	0.0400	B_{15}	0.0600	B_{20}	0.0200

four houses A_1, A_2, A_3 and A_4 according to Eqs. (17) and (18), i.e.,

$$PDA = \begin{pmatrix} 0.0000 & 0.6091 & 2.5907 \\ 0.0000 & 0.2742 & 0.0000 \\ 1.7068 & 0.3972 & 0.0000 \\ 0.0000 & 0.4353 & 0.0000 \end{pmatrix},$$

$$NDA = \begin{pmatrix} 1.9345 & 0.0000 & 0.0000 \\ 0.3071 & 0.0000 & 0.4093 \\ 0.0000 & 0.0000 & 3.4093 \\ 0.2576 & 0.0000 & 2.4093 \end{pmatrix}$$

Step 4. Determining the values of weighted sum of the positive and negative distances from average solution according to Eq. (19) and weights θ_j ($j = 1, 2, 3$), i.e.,

$SP_1^\varpi = 2.5701$, $SP_2^\varpi = 0.0003$, $SP_3^\varpi = 0.0126$,
$SP_4^\varpi = 0.0005$;
$SN_1^\varpi = 0.0138$, $SN_2^\varpi = 0.4081$, $SN_3^\varpi = 3.3813$,
$SN_4^\varpi = 2.3913$

Step 5. Normalizing the values of the weighted PDA and weighted NDA according to Eqs. (20) and (21), respectively,

$SP_1^N = 1.0000$, $SP_2^N = 0.0001$, $SP_3^N = 0.0049$,
$SP_4^N = 0.0002$;
$SN_1^N = 0.9959$, $SN_2^N = 0.8793$, $SN_3^N = 0.0000$,
$SN_4^N = 0.2928$

Step 6. Computing the score $\mathcal{AS}_i$ of four houses A_1, A_2, A_3 and A_4,

$\mathcal{AS}_1 = 0.9980$, $\mathcal{AS}_2 = 0.4397$,
$\mathcal{AS}_3 = 0.0025$, $\mathcal{AS}_4 = 0.1465$

Step 7. Ranking four houses A_1, A_2, A_3 and A_4 according to the score $\mathcal{AS}_i$, i.e., $A_1 > A_2 > A_4 > A_3$, house A_1 is the best alternative to improve housing.

With economic development and social progress, people's living standards continue to improve, more and more people choose to travel freely, and the selection of travel destination is an inevitable problem.

Example 9 Suppose four travel destinations A_1, A_2, A_3 and A_4. Tourists select travel destination depending on the following 20 attributes: Household income (B_1), Free time (B_2), Travel motivation (B_3), Physical condition before travel (B_4), Destination visibility and image (B_5), Media promotion status (B_6), Opinions of relatives and friends (B_7), Recommended by travel agencies (B_8), Recommended literary works (B_9), Description or recommendation of travel forums, blogs, personal homepages, etc. (B_{10}), Peer demand (B_{11}), Play time (B_{12}), Play budget (B_{13}), Hotel price (B_{14}), Safety status of travel destinations (B_{15}), Tourist season (B_{16}), Destination tourism resources (B_{17}), Diversity of transportation modes (B_{18}), Diversity of transportation modes (B_{19}), Degree of direct traffic (B_{20}). Suppose that the travel team comprehensively consider the relevant the information collected from multi-sources decision information and use linguistic terms shown in Table 1 to evaluate travel destinations, and weights of decision attributes are shown in Table 4. The LIFVs evaluation information (decision matrix) provided by the travel team is displayed in Table 5. In the example, we fix $k = 2$ and $\gamma = 1$.

Based on the solution scheme of large-scale linguistic intuitionistic fuzzy decision making analysis, the decision making of Example 9 is as follows.

Step 1. Analyzing correlation between 20 decision attributes and obtaining the partition $C = \{C_1, C_2, \ldots, C_t\}$ ($t \leq n$) of B by the fixed $\gamma = 1$ and threshold $\varrho = 6$. In the example, we can obtain core attributes $C_1 = \{B_1, B_2, B_3, B_4\}$, $C_2 = \{B_5, B_6, B_7, B_8, B_9, B_{10}, B_{11}\}$, $C_3 = \{B_{12}, B_{13}, B_{14}, B_{15}, B_{16}\}$ and $C_4 = \{B_{17}, B_{18}, B_{19}, B_{20}\}$.

Step 2. Based on the LIFRWMSM operator [Eq. (11)] and $k = 2$, The LIFVs of C_1, C_2, C_3 and C_4 can be obtained as follows:

Table 5 The LIFVs evaluation information (decision matrix) of Example 9

Alternatives	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
A_1	(s_7, s_1)	(s_7, s_1)	(s_7, s_1)	(s_7, s_1)	(s_6, s_1)	(s_5, s_3)	(s_6, s_2)	(s_5, s_3)	(s_5, s_1)	(s_5, s_2)
A_2	(s_6, s_1)	(s_7, s_1)	(s_7, s_1)	(s_7, s_1)	(s_6, s_2)	(s_4, s_4)	(s_6, s_2)	(s_5, s_1)	(s_5, s_2)	(s_5, s_2)
A_3	(s_1, s_7)	(s_1, s_7)	(s_1, s_7)	(s_1, s_7)	(s_6, s_1)	(s_5, s_3)	(s_5, s_3)	(s_4, s_4)	(s_5, s_3)	(s_6, s_1)
A_4	(s_1, s_5)	(s_1, s_5)	(s_1, s_5)	(s_1, s_5)	(s_5, s_2)	(s_4, s_4)	(s_4, s_4)	(s_4, s_4)	(s_4, s_4)	(s_7, s_1)

	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}
A_1	(s_5, s_3)	(s_1, s_6)	(s_2, s_5)	(s_1, s_6)	(s_1, s_5)	(s_1, s_6)	(s_1, s_6)	(s_1, s_7)	(s_1, s_6)	(s_1, s_6)
A_2	(s_6, s_1)	(s_2, s_5)	(s_2, s_5)	(s_1, s_6)	(s_1, s_5)	(s_1, s_6)	(s_7, s_1)	(s_7, s_1)	(s_6, s_2)	(s_6, s_2)
A_3	(s_4, s_4)	(s_3, s_4)	(s_1, s_5)	(s_1, s_6)	(s_1, s_6)	(s_1, s_6)	(s_6, s_1)	(s_6, s_2)	(s_5, s_1)	(s_5, s_1)
A_4	(s_4, s_4)	(s_3, s_5)	(s_1, s_6)	(s_1, s_6)	(s_2, s_5)	(s_1, s_6)	(s_2, s_6)	(s_1, s_7)	(s_1, s_7)	(s_1, s_7)

$$\Psi_{new} = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{pmatrix} (s_{7.0000}, s_{1.0000}) & (s_{5.5160}, s_{2.0058}) & (s_{1.1799}, s_{5.5912}) & (s_{1.0000}, s_{6.2979}) \\ (s_{6.7701}, s_{1.0000}) & (s_{5.6811}, s_{1.8145}) & (s_{1.3404}, s_{5.4261}) & (s_{6.5762}, s_{1.4238}) \\ (s_{1.0000}, s_{7.0000}) & (s_{5.1121}, s_{2.5961}) & (s_{1.2889}, s_{5.4934}) & (s_{5.5620}, s_{1.2588}) \\ (s_{1.0000}, s_{7.0000}) & (s_{4.6123}, s_{3.2290}) & (s_{1.5148}, s_{5.6225}) & (s_{1.2480}, s_{6.4701}) \end{pmatrix}.$$

According to Eq. (15), weights of core attributes C_1, C_2, C_3 and C_4 are $\theta_1 = 0.8529$, $\theta_2 = 0.0005$, $\theta_3 = 0.1199$ and $\theta_4 = 0.0267$, respectively.

Step 3. Calculating the average solution $\bar{A}S_j = \bar{\vartheta}_j (j = 1, 2, 3, 4)$ of each core attribute, i.e.,

$$\bar{A}S_1 = (s_{5.2138}, s_{2.4323}), \quad \bar{A}S_2 = (s_{5.2601}, s_{2.3502}),$$
$$\bar{A}S_3 = (s_{1.3321}, s_{5.5328}), \quad \bar{A}S_4 = (s_{4.4211}, s_{1.8977})$$

where $\bar{\vartheta}_j = \left(s_{8-8\prod_{i=1}^{4}\left(1-\frac{\alpha_{ij}}{8}\right)^{\frac{1}{4}}}, s_{8\prod_{i=1}^{4}\left(\frac{\beta_{ij}}{8}\right)^{\frac{1}{4}}} \right).$

Step 4. Calculating the positive distance from average PDA_{ij} and the negative distance from average NDA_{ij} of each alternative, i.e.,

$$PDA = \begin{pmatrix} 1.6093 & 0.3001 & 0.0000 & 0.0000 \\ 1.4943 & 0.4783 & 0.0575 & 1.3145 \\ 0.0000 & 0.0000 & 0.0000 & 0.8899 \\ 0.0000 & 0.0000 & 0.0465 & 0.0000 \end{pmatrix}$$

$$NDA = \begin{pmatrix} 0.0000 & 0.0000 & 0.1053 & 3.9106 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 4.3907 & 0.1969 & 3.409 & 0.0000 \\ 3.3907 & 0.7633 & 0.0000 & 3.8727 \end{pmatrix}$$

Step 5. Determining the values of weighted sum of the positive and negative distances from average solution.

$SP_1^{\varpi} = 1.3727$, $SP_2^{\varpi} = 1.3167$, $SP_3^{\varpi} = 0.0237$, $SP_4^{\varpi} = 0.0056$;

$SN_1^{\varpi} = 0.1169$, $SN_2^{\varpi} = 0.0000$, $SN_3^{\varpi} = 3.7453$, $SN_4^{\varpi} = 2.9956$

Step 8. Normalizing the values of the weighted PDA and weighted NDA, respectively.

$SP_1^{N} = 1.0000$, $SP_2^{N} = 0.9592$, $SP_3^{N} = 0.0173$, $SP_4^{N} = 0.0041$;

$SN_1^{N} = 0.9688$, $SN_2^{N} = 1.0000$, $SN_3^{N} = 0.0000$, $SN_4^{N} = 0.2002$

Step 9. Computing the score $\mathcal{AS}_i$ of each alternative, i.e.,

$\mathcal{AS}_1 = 0.9844$, $\mathcal{AS}_2 = 0.9796$, $\mathcal{AS}_3 = 0.0086$, $\mathcal{AS}_4 = 0.1021$

Step 10. Ranking four travel destinations A_1, A_2, A_3 and A_4 based on their score $\mathcal{AS}_i$, i.e., $A_1 > A_2 > A_4 > A_3$, travel destination A_1 is the best one.

6.2 Comparison analysis

In the subsection, we compare the proposed LIFRWMSM and LIFRWDMSM aggregation operators with WLIFMM, WLIFMSM, WLIFDMM, LIFWA, LIFWG and LIFHA

Table 6 The comparison with existing MSM operator based on diverse uncertain environment

Aggregating objects	Aggregation operators	Idempotency	Reducibility
LIFSs	WLIFMSM operator (Liu and Qin 2017)	No	No
LIFSs	WLIFDMSM operator (Liu and Qin 2017)	No	No
IFSs	WIFMSM operator (Qin and Liu 2014)	No	No
PFSs	PFSMSM operator (Wei and Lu 2018)	No	No
PFSs	PFSGMSM operator (Wei and Lu 2018)	No	No
HFSs	WHFMSM operator (Qin et al. 2015)	No	No
LIFSs	LIFRWMSM operator	Yes	Yes
LIFSs	LIFRWDMSM operator	Yes	Yes

aggregation operators or decision making methods in Examples 8 and 9. Formally, these aggregation operators are concentrated on aggregating correlational decision information of alternatives with respect to decision making attributes. However, there are differences about the aggregating objects or the properties of operator in these aggregation operators. In addition, the above-mentioned aggregation operators solve different correlation between decision attributes in a real-world decision making problem. These differences in the aggregation operators are compared as follows.

Intuitively, we provide Table 6 to show differences about the aggregating objects or the properties of operator in the aggregation operators, which are used to aggregate correlational decision information of alternatives with respect to decision making attributes. From the aggregating objects point of view, WLIFMSM and WLIFDMM aggregation operators in Liu and Qin (2017) are utilized to aggregate correlational decision information represented by linguistic intuitionistic fuzzy sets (LIFSs), but WIFMSM aggregation operator in Qin and Liu (2014) to aggregate correlational decision information represented by intuitionistic fuzzy sets (IFSs), PFSMSM and PFSGMSM aggregation operators in Wei and Lu (2018) to aggregate correlational decision information represented by Pythagorean fuzzy sets (PFSs) and WHFMSM operators in Qin et al. (2015) to aggregate correlational decision information represented by hesitant fuzzy sets (HFSs). In the paper, LIFRWMSM and LIFRWDMSM aggregation operators are also to aggregate correlational decision information represented by LIFSs. From the properties of aggregation operator point of view, our LIFRWMSM and LIFRWDMSM aggregation operators satisfy idempotency and reducibility. However, the important properties of aggregation operator are not true in the others aggregation operators.

In real-world decision making process, there are differences when the aggregation operators are utilized to aggregate correlational decision information. In fact, LIFRWMSM and LIFRWDMSM aggregation operators are used to aggregate decision information of alternatives with respect to decision making attributes after correlation between

attributes is analyzed, in other words, decision information aggregated by LIFRWMSM and LIFRWDMSM operators are undoubted correlation each other. However, the others aggregation operators are to aggregate decision information of alternatives based on the assumption that decision making attributes may be correlation each other, in other words, independent attributes are with respect to different parameters in the aggregation operators. In our method, independent attributes are represented by the partition $C = \{C_1, C_2, \ldots, C_t\} (t \leq n)$ of decision making attributes B. Different decision results in Examples 8 and 9 are obtained due to different decision steps or decision making methods by using WLIFMM, WLIFMSM, WLIFDMM, LIFWA, LIFWG, LIFHA, LIFRWMSM and LIFRWDMSM aggregation operators, which are shown in Table 7. In reality, the score function is the basis of the decision making methods based on WLIFMM, WLIFMSM, WLIFDMM, LIFWA, LIFWG and LIFHA aggregation operators. Differently, EDAS method is adopted in the proposed decision making method.

7 Conclusion

Due to the fact that decision information with respect to decision making attributes in a large-scale decision making problem is come from multi-sources information, the research is concentrated on dealing with MADM problems with correlation between decision making attributes. Firstly, a new linguistic intuitionistic fuzzy similarity measure is proposed and applied to correlational attribute analysis. Then the LIFRWMSM and LIFRWDMSM operators are proposed to aggregate decision information with respect to correlational decision making attributes. Intuitively, the proposed LIFRWMSM and LIFRWDMSM operators can reduce the dimension of large-scale attributes and avoid the overlap of decision information. Finally, a new solution scheme of large-scale linguistic decision making analysis is designed, in which attribute correlation analysis and the EDAS decision making approach are contained, and the applicability and

Table 7 Quantitative comparisons with other aggregation operators

Exmple	Methods	Parameter	The generalized order	The best option
Example 8	WLIFMM (Rong et al. 2020)	$P = \{\underbrace{1, 0, \ldots, 0}_{20}\}$	$A_3 > A_1 > A_4 > A_2$	A_3
	WLIFDMM (Rong et al. 2020)	$P = \{\underbrace{1, 0, \ldots, 0}_{20}\}$	$A_3 > A_2 > A_4 > A_1$	A_3
	WLIFMSM (Liu and Qin 2017)	$x = 2$	$A_3 > A_1 > A_4 > A_2$	A_3
	LIFWA (Chen et al. 2015)	No	$A_3 > A_1 > A_4 > A_2$	A_3
	LIFWG (Chen et al. 2015)	No	$A_3 > A_2 > A_4 > A_1$	A_3
	LIFHA (Chen et al. 2015)	No	$A_3 > A_1 > A_2 > A_4$	A_3
	The proposed method	$x = 2$	$A_1 > A_2 > A_4 > A_3$	A_1
Example 9	WLIFMM (Rong et al. 2020)	$P = \{\underbrace{1, 0, \ldots, 0}_{20}\}$	$A_2 > A_1 > A_3 > A_4$	A_2
	WLIFDMM (Rong et al. 2020)	$P = \{\underbrace{1, 0, \ldots, 0}_{20}\}$	$A_2 > A_1 > A_3 > A_4$	A_2
	WLIFMSM (Liu and Qin 2017)	$x = 2$	$A_2 > A_1 > A_3 > A_4$	A_2
	LIFWA (Chen et al. 2015)	No	$A_2 > A_1 > A_3 > A_4$	A_2
	LIFWG (Chen et al. 2015)	No	$A_2 > A_1 > A_3 > A_4$	A_2
	LIFHA (Chen et al. 2015)	No	$A_2 > A_1 > A_3 > A_4$	A_2
	The proposed method	$x = 2$	$A_1 > A_2 > A_4 > A_3$	A_1

superiority of our method are demonstrated by two examples and comparative analysis with WLIFMM, WLIFMSM, WLIFDMM, LIFWA, LIFWG and LIFHA aggregation operators.

In the future, our works can be extended. In fact, it is worthy to research correlational attribute analysis with different decision information, such as q-rung orthopair fuzzy set, probabilistic linguistic term sets and hesitant fuzzy numbers. In addition, our method can be utilized to solve hot issues, such as big data analysis and cloud computing.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Appendixes

Appendix 1

The proof of Eq. (12)

$$\bigotimes_{j=1}^{k} \vartheta_{i_j} = \left(s_{\frac{\prod_{j=1}^{k} \alpha_{i_j}}{h^{k-1}}}, s_{h-h \prod_{j=1}^{k} \left(1 - \frac{\beta_{i_j}}{h}\right)} \right)$$

and

$$\mathcal{P}_{i_k}\left(\bigotimes_{j=1}^{k}\vartheta_{i_j}\right)=\left(s_{h-h\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}},s_{h\left(1-\prod_{j=1}^{k}\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}}}\right)$$

$$\bigoplus_{1\le i_1<\cdots<i_k\le|C_{t'}|}\mathcal{P}_{i_k}\left(\bigotimes_{j=1}^{k}\vartheta_{i_j}\right)=\left(s_{h-h\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}},s_{h\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\prod_{j=1}^{k}\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}}}\right)$$

Then

$$\frac{\bigoplus_{1\le i_1<\cdots<i_k\le|C_{t'}|}\mathcal{P}_{i_k}\left(\bigotimes_{j=1}^{k}\vartheta_{i_j}\right)}{\mathcal{P}_k}=\left(s_{h-h\left(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}},s_{h\left(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\prod_{j=1}^{k}\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}}\right)$$

Furthermore

$$\left(\frac{\bigoplus_{1\le i_1<\cdots<i_k\le|C_{t'}|}\mathcal{P}_{i_k}\left(\bigotimes_{j=1}^{k}\vartheta_{i_j}\right)}{\mathcal{P}_k}\right)^{\frac{1}{k}}$$

$$=\left(s_{h\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}},s_{h-h\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\prod_{j=1}^{k}\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}}\right)$$

Thus, Eq. (12) is valid, then we will test the aggregate result is a LIFN, which just need to satisfy the following three conditions.

(i) $0 \le h(1-(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k})^{\mathcal{P}_{i_k}})^{\frac{1}{\mathcal{P}_k}})^{\frac{1}{k}} \le h$;

(ii) $0 \le h - h(1 - (\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}(1 - \prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{h}))^{\mathcal{P}_{i_k}})^{\frac{1}{\mathcal{P}_k}})^{\frac{1}{k}} \le h$;

(iii) $0 \le h(1-(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k})^{\mathcal{P}_{i_k}})^{\frac{1}{\mathcal{P}_k}})^{\frac{1}{k}} + h - h(1-(\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}(1-\prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{h}))^{\mathcal{P}_{i_k}})^{\frac{1}{\mathcal{P}_k}})^{\frac{1}{k}} \le h$;

Proof (i) According to Definition 6, we have $\alpha_{i_j} \in [0, h]$, $\beta_{i_j} \in [0, h]$ and $\alpha_{i_j} + \beta_{i_j} \in [0, h]$, then we can obtain,

$$\frac{\alpha_{i_j}}{h} \in [0, 1] \Rightarrow 1 - \frac{\alpha_{i_j}}{h^k} \in [0, 1]$$

and

$$\prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}} \in [0, 1]$$

then,

$$1 - \prod_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}} \in [0, 1]$$

$$h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^k\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}\right)$$

$$\in[0,h] \qquad\qquad \Box$$

(ii) The proof procedure of (ii) is same as (i), we can also get (ii) is correct.

(iii) For the condition(iii), according to $0\leq\alpha_{i_j}+\beta_{i_j}\leq h$, we have $0\leq\alpha_{i_j}\leq h-\beta_{i_j}$, then the simplified form can be obtained as:

$$h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^k\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}+h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^k\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}$$

$$\leq h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^k(h-\beta_{i_j})}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}+h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^k\left(1-\frac{\beta_{i_j}}{h}\right)\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}\leq h$$

Evidently, the fused result by the LIFRWMSM operator is still a LIFV. Next, we shall research several properties of LIFRWMSM operator.

Appendix 2

The proof of Theorem 1

$\text{LIFRWMSM}^{(k)}(\vartheta_{i1},\vartheta_{i2},\ldots,\vartheta_{i|C_{t'}|})$

$$=\left(s_{h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^k\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}},s_{h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^k(1-\frac{\beta_{i_j}}{k})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}}\right)$$

$$=\left(s_{h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^k\alpha}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}},s_{h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^k(1-\frac{\beta}{k})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}}\right)$$

$$=\left(s_{h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\alpha^k}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}},s_{h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-(1-\frac{\beta}{h})^k\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}}\right)$$

$$=\left(s_{h\left(1-\left(\left(1-\frac{\alpha^k}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_{i_k}}}\right)^{\frac{1}{k}}},s_{h-h\left(1-\left(\left(1-(1-\frac{\beta}{h})^k\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_{i_k}}}\right)^{\frac{1}{k}}}\right)=\left(s_{h\left(1-\left(1-\frac{\alpha^k}{h^k}\right)\right)^{\frac{1}{k}}},s_{h-h\left(1-\left(\left(1-(1-\frac{\beta}{h})^k\right)\right)\right)^{\frac{1}{k}}}\right)=(s_\alpha,s_\beta).$$

Appendix 3

The proof of Theorem 2 In order to test the monotonicity of
the LIFRWMSM, according to Definition 4, we will prove it via computing their linguistic score result $\mathscr{S}(\vartheta)$ and $\mathscr{S}(T)$, and we can obtain $\vartheta\geq T$. So the process of proof is divided into two steps as below:

Step 1: Since $k\geq 1$ and $s_{\alpha_i}\geq s_{\alpha_i'}\geq 0$, $s_{\beta_i'}\geq s_{\beta_i}\geq 0$, we have $\alpha_i\geq\alpha_i'$, $\beta_i'\geq\beta_i$, then,

$$\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k} \geq \frac{\prod_{j=1}^{k}\alpha'_{i_j}}{h^k} \Rightarrow 1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k} \leq 1-\frac{\prod_{j=1}^{k}\alpha'_{i_j}}{h^k}$$

$$\Rightarrow \left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}$$

$$\leq \left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha'_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}$$

$$\Rightarrow 1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}$$

$$\geq 1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha'_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}$$

$$\Rightarrow h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}$$

$$\geq h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha'_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}$$

Similarly, we have

$$\frac{\beta_i}{h} \leq \frac{\beta'_i}{h} \Rightarrow 1-\frac{\beta_i}{h} \geq 1-\frac{\beta'_i}{h} \Rightarrow \prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{h}) \geq \prod_{j=1}^{k}(1-\frac{\beta'_{i_j}}{h})$$

$$\Rightarrow \left(1-\prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{k})\right)^{\mathcal{P}_{i_k}} \leq \left(1-\prod_{j=1}^{k}(1-\frac{\beta'_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}$$

$$\Rightarrow \left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}$$

$$\leq \left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^{k}(1-\frac{\beta'_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}$$

$$\Rightarrow h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}$$

$$\leq h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^{k}(1-\frac{\beta'_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}$$

Step 2: Suppose

$$\vartheta = \left(\left(h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}\right),\right.$$

$$\left. h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^{k}(1-\frac{\beta_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}\right)$$

$$= (\alpha, \beta)$$

$$\vartheta' = \left(\left(h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\alpha'_{i_j}}{h^k}\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}\right),\right.$$

$$\left. h-h\left(1-\left(\prod_{1\leq i_1<\cdots<i_k\leq|C_{t'}|}\left(1-\prod_{j=1}^{k}(1-\frac{\beta'_{i_j}}{h})\right)^{\mathcal{P}_{i_k}}\right)^{\frac{1}{\mathcal{P}_k}}\right)^{\frac{1}{k}}\right)$$

$$= \left(\alpha', \beta'\right)$$

then we have $\alpha \geq \alpha'$ and $\beta \leq \beta'$, with the aid of Definition 4, we can make a comparison between the above obtained results. So we easily obtain $\mathscr{S}(\vartheta) = \alpha - \beta \geq \mathscr{S}(T) = \alpha' - \beta'$. Then,

(i) If $\mathscr{S}(\vartheta) > \mathscr{S}(\vartheta')$, then $\vartheta > \vartheta'$ via the Definition 4, we have,

$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$$
$$> \text{LIFRWMSM}^{(k)}(\vartheta'_{i1}, \vartheta'_{i2}, \ldots, \vartheta'_{i|C_{t'}|})$$

(ii) If $\mathscr{S}(\vartheta) = \mathscr{S}(\vartheta')$, we have $\vartheta = \vartheta'$, consequently, we get $\mathscr{H}(\vartheta) = \mathscr{H}(\vartheta')$, and thus,

$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$$
$$= \text{LIFRWMSM}^{(k)}(\vartheta'_{i1}, \vartheta'_{i2}, \ldots, \vartheta'_{i|C_{t'}|})$$

Hence, we can imply that,

$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$$
$$\geq \text{LIFRWMSM}^{(k)}(\vartheta'_{i1}, \vartheta'_{i2}, \ldots, \vartheta'_{i|C_{t'}|})$$

Appendix 4

The proof of Theorem 3 With the help of the monotonicity of the LIFRWMSM operator, we get

$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$$
$$\leq \text{LIFRWMSM}^{(k)}(\vartheta^+_{i1}, \vartheta^+_{i2}, \ldots, \vartheta^+_{i|C_{t'}|})$$
$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$$
$$\geq \text{LIFRWMSM}^{(k)}(\vartheta^-_{i1}, \vartheta^-_{i2}, \ldots, \vartheta^-_{i|C_{t'}|})$$

Moreover, in light of the idempotency of the LIFRWMSM operator, we get,

$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}^+, \vartheta_{i2}^+, \ldots, \vartheta_{i|C_{t'}|}^+) = \vartheta_{ij}^+$$
$$\text{LIFRWMSM}^{(k)}(\vartheta_{i1}^-, \vartheta_{i2}^-, \ldots, \vartheta_{i|C_{t'}|}^-) = \vartheta_{ij}^-$$

Obviously, we can imply $\vartheta_i^- \leq \text{LIFRWMSM}^{(k)}(\vartheta_{i1}^+, \vartheta_{i2}^+, \ldots, \vartheta_{i|C_{t'}|}^+) \leq \vartheta_{ij}^+$

Appendix 5

The proof of Theorem 4 According to the above known conditions, we have

$$\left(\frac{\displaystyle\bigoplus_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \mathcal{P}_{i_k}\left(\bigotimes_{j=1}^k \vartheta_{ij}\right)}{\mathcal{P}_k} \right)^{\frac{1}{k}}$$

$$= \left(\frac{\displaystyle\bigoplus_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \mathcal{P}_{i_k}\left(\bigotimes_{j=1}^k \vartheta_{ij}^T\right)}{\mathcal{P}_k} \right)^{\frac{1}{k}}$$

Thus, we have $\text{LIFRWMSM}^{(k)}(\vartheta_{i1}^T, \vartheta_{i2}^T, \ldots, \vartheta_{i|C_{t'}|}^T) = \text{LIFRWMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$.

Appendix 6

The proof of Eq. (14)

$$\bigoplus_{j=1}^k \vartheta_{ij} = \left(s_{h-h\prod_{j=1}^k\left(1-\frac{\alpha_{ij}}{h}\right)}, s_{\frac{\beta_{ij}}{h^{k-1}}} \right)$$

and

$$\left(\bigoplus_{j=1}^k \vartheta_{ij}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}$$

$$= \left(s_{h\left(1-\prod_{j=1}^k\left(1-\frac{\alpha_{ij}}{h}\right)\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}}, s_{h-h\left(1-\frac{\prod_{j=1}^k \beta_{ij}}{h^k}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}} \right)$$

then

$$\bigotimes_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \left(\bigoplus_{j=1}^k \vartheta_{ij}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}$$

$$= \left(s_{h\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1-\prod_{j=1}^k\left(1-\frac{\alpha_{ij}}{h}\right)\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}}, s_{h-h\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1-\frac{\prod_{j=1}^k \beta_{ij}}{h^k}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}} \right)$$

further

$$\frac{\displaystyle\bigotimes_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|} \left(\bigoplus_{j=1}^k \vartheta_{ij}\right)^{\frac{\mathcal{M}_{i_k}}{\mathcal{M}_k}}}{k}$$

$$= \left(s_{h-h\left(1-\left(\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1-\prod_{j=1}^k(1-\frac{\alpha_{ij}}{h})\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}}, s_{h\left(1-\left(\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1-\frac{\prod_{j=1}^k \beta_{ij}}{h^k}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}} \right)$$

Furthermore,

$$0 \leq h - h\left(1 - \left(\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1 - \prod_{j=1}^k(1 - \frac{\alpha_{ij}}{h})\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}$$
$$\leq 1$$

and

$$0 \leq h\left(1 - \left(\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1 - \frac{\prod_{j=1}^k \beta_{ij}}{h^k}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}$$
$$\leq 1$$

Thus,

$$0 \leq h - h\left(1 - \left(\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1 - \prod_{j=1}^k(1 - \frac{\alpha_{ij}}{h})\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}$$
$$+ h\left(1 - \left(\prod_{1 \leq i_1 < \cdots < i_k \leq |C_{t'}|}\left(1 - \frac{\prod_{j=1}^k \beta_{ij}}{h^k}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}$$
$$\leq 1$$

Appendix 7

The proof of Theorem 5

$\text{LIFRWDMSM}^{(k)}(\vartheta_{i1}, \vartheta_{i2}, \ldots, \vartheta_{i|C_{t'}|})$

$$= \left(s_{h-h\left(1-\left(\prod\limits_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\prod\limits_{j=1}^{k}(1-\frac{\alpha_{ij}}{h})\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}}, \quad s_{h\left(1-\left(\prod\limits_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\beta_{ij}}{h^k}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}} \right)$$

$$= \left(s_{h-h\left(1-\left(\prod\limits_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\prod\limits_{j=1}^{k}(1-\frac{\alpha}{h})\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}}, \quad s_{h\left(1-\left(\prod\limits_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\prod_{j=1}^{k}\beta}{h^k}\right)^{\mathcal{M}_{i_k}}\right)^{\frac{1}{\mathcal{M}_k}}\right)^{\frac{1}{k}}} \right)$$

$$= \left(s_{h-h\left(1-\left(\prod\limits_{1\le i_1<\cdots<i_k\le|C_{t'}|}(1-(1-\frac{\alpha}{h})^k)\right)\right)^{\frac{1}{k}}}, \quad s_{h\left(1-\left(\prod\limits_{1\le i_1<\cdots<i_k\le|C_{t'}|}\left(1-\frac{\beta^k}{h^k}\right)\right)\right)^{\frac{1}{k}}} \right)$$

$$= \left(s_{h-h\left((1-\frac{\alpha}{h})^k\right)^{\frac{1}{k}}}, \quad s_{h\left(1-\left(1-\frac{\beta^k}{h^k}\right)\right)^{\frac{1}{k}}} \right)$$

$$= (s_\alpha, s_\beta).$$

The process of proof of monotonicity, boundedness and commutativity of the proposed LIFRWDMSM operator are the same with the process of proof of LIFRWMSM.

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