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APPLICATION OF SOFT COMPUTING

Note on modular equations in the theory of signature 4

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Abstract

In 2009, M. S. M. Naika et al. obtained Schl$\breve{a}$fli type modular equations of degree 4. Motivated by their work, in the present work, we prove a class of modular equations in Ramanujan's alternative theory of elliptic functions using theory of signature 4. In particular, the composite degrees of modular equations $\{1, 2, 4\}$, $\{1, 3, 9\}$, $\{1, 5, 25\}$ and $\{1, 7, 49\}$.

Keywords Modular equation · Theta functions

1 Introduction

Ramanujan (1914, 1957, 1962) stated some results which are now called as identities on the theories of elliptic functions to alternative bases corresponding to the classical theory and later these identities are proved by Berndt (1991). Some of the results in alternative theories are examined by Borwein and Borwien (1991) and Venkatachaliengar (1988). Ramanujan offered many beautiful series for $1/\pi$, and he concluded that 14 of these series belong to alternative theories in which the base q in the classical theory of elliptic functions is replaced by one or the other of the functions. Let

$$z(r; x) := {}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}; 1; x\right)$$

and

$$q_r := q_r(x) := exp\left(-\pi csc(\pi/r)\frac{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}, 1; 1-x\right)}{{}_2F_1\left(\frac{1}{r}, \frac{r-1}{r}, 1, x\right)}\right)$$
$$0 < x < 1,$$

where $r = 2, 3, 4$ and 6, and ${}_2F_1$ is the Gaussian hypergeometric function defined as follows:

$${}_2F_1(\alpha, \beta; \gamma; z) = \sum_{m=0}^{\infty}\frac{(\alpha)_m(\beta)_m}{(\gamma)_m m!}z^m, |z| < 1$$

and

$$(\alpha)_m = \alpha(\alpha+1)\ldots(\alpha+m-1).$$

In fact on six pages, of his second notebook (Ramanujan 1914, pp. 257–262), Ramanujan gave nearly 50 results without proof in theories. In 1995, Berndt et al. (1995) proved the same identities with base q_r calling it as theory of signature r. Among these results, the following hypergeometric transformations play a key role in the development of the alternative theories of signature 4 especially in establishing numerous modular equations have been proved by different methods in Berndt et al. (1995) using Maple, parametrization

Dedicated to Prof. Chandrashekar Adiga on his 65th birthday.

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and evaluation theorems.

$$
\begin{aligned}
&{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\frac{2\sqrt{x}}{1+\sqrt{x}}\right)\\
&=\sqrt{1+\sqrt{x}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};1;x\right), \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&{}_2F_1\left(\frac{1}{4},\frac{3}{4};1;1-\left(\frac{1-\sqrt{x}}{1+3\sqrt{x}}\right)^2\right)\\
&=\sqrt{1+3\sqrt{x}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};1;x\right), \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&\sqrt{27-18\sqrt{x}-x}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};1;\alpha\right)\\
&=3\sqrt{3+6\sqrt{x}-x}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};1;\beta\right) \qquad (3)
\end{aligned}
$$

where

$$\alpha=\frac{64\sqrt{x}}{(3+6\sqrt{x}-x)^2} \quad \text{and} \quad \beta=\frac{64\sqrt[3]{x}}{(27-18\sqrt{x}-x)^2}.$$

Ramanujan, while outlining the theory of signature 4, indicates a device of deducing formulas in the theory of signature 4 from the corresponding formulas in the classical theory. In fact, if we let $x \to \frac{2\sqrt{x}}{1+\sqrt{x}}$, then $z(2;x)$ changes to $\sqrt{1+\sqrt{x}}\,z(4;x)$ and $q_2^2(x)$ changes to $q_4(x)$ and any formula

$$\Omega(x,q_2^2,z(2))=0$$

yields the formula

$$\Omega\left(\frac{2\sqrt{x}}{1+\sqrt{x}},q_4,\sqrt{1+\sqrt{x}}\,z(4)\right)=0$$

gives the theory of signature 4. The above equation (1) is same as Entry 33(i) of Chapter 2 of Ramanujan (1957). In Berndt et al. (1995), (2) has been proved using (1) and an associate formula based on Entry 33(iv) of Chapter 2 of Ramanujan (1957). Further, the proof of (3) given in Berndt et al. (1995) is based on parametrization results in the theory of signature 4.

A relation between α and β given by the following identity:

$$
n\frac{{}_2F_1\left(\frac{1}{r},\frac{r-1}{r};1;1-\alpha\right)}{{}_2F_1\left(\frac{1}{r},\frac{r-1}{r};1;\alpha\right)}
=\frac{{}_2F_1\left(\frac{1}{r},\frac{r-1}{r};1;1-\beta\right)}{{}_2F_1\left(\frac{1}{r},\frac{r-1}{r};1;\beta\right)}. \qquad (4)
$$

is called as modular equation of degree n. Then in the theory of elliptic functions of signature n, a modular equation of degree n is a relation between α and β induced by (4). We often say that β is of degree n over α and

$$m=\frac{z(r,\alpha)}{z(r,\beta)}=\frac{{}_2F_1\left(\frac{1}{r},\frac{r-1}{r};1;\alpha\right)}{{}_2F_1\left(\frac{1}{r},\frac{r-1}{r};1;\beta\right)}$$

is called the multiplier. A modular equation (Berndt 1991) of degree n in the theory of signature 2, is a relation between α and β given by the relation

$$
n\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;1-\alpha\right)}{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\alpha\right)}
=\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;1-\beta\right)}{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\beta\right)},
$$

and we say that β has degree n over α. Then, the multiplier m associates α and β induced by

$$m=\frac{z(2,\alpha)}{z(2,\beta)}=\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\alpha\right)}{{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\beta\right)}.$$

If we let $r=4$ in (4), we obtain a modular equation of degree n in the theory of signature 4. All through this paper, we have assumed $|q|<1$. The standard q-shifted factorial is defined as

$$(a;q)_0:=1, \qquad (a;q)_n:=\prod_{i=1}^{n}(1-aq^{i-1})$$
$$\text{and} \qquad (a;q)_\infty:=\prod_{i=0}^{\infty}(1-aq^{i}).$$

Ramanujan Berndt (1991),Ramanujan (1957) has defined one of theta function as follows:

$$
\begin{aligned}
f(-q):=f(-q,-q^2)&=\sum_{i=0}^{\infty}(-1)^i q^{i(3i-1)/2}\\
&+\sum_{i=1}^{\infty}(-1)^i q^{i(3i+1)/2}=(q;q)_\infty.
\end{aligned}
$$

For convenience, we write $f(-q^n)$ by f_n. (Ramanujan 1957, Vol II) in his notebooks documented some modular

equations. Further these are proved by Berndt (1991), by parameterization, modular forms and the techniques known to Ramanujan. Also Adiga et al. (2003), Adiga et al. (2017), Baruah (2002),Baruah (2003), Berndt et al. (1995), Berndt et al. (2001), S. Bhargava et al. (2003), Garvan (1995), Naika (2006), Naika and Chandan Kumar (2010), Saikia and Chetry (2019), S. P. Singh and Vijay Yadav [18], Srivatsa Kumar and Shruthi (2020) and Vijay Yadav and S. Chandankumar [21] deduced some modular equations of signature 3.
Inspired by the above work, in the present work, we establish some modular equation of signature 4. In Sect. 2 of this paper, we document some P-Q type theta function identities which will be utilized to demonstrate modular equations of signature 4 in Sect. 3.

2 Preliminaries

Lemma 1 *If*

$$P=\frac{f_1}{q^{1/8}f_4} \quad \text{and} \quad Q_n=\frac{f_n}{q^{n/8}f_{4n}},$$

then we have

$$(PQ_2)^4+\frac{256}{(PQ_2)^4} = \left(\frac{Q_2}{P}\right)^{12}-16\left(\frac{P}{Q_2}\right)^4-16\left(\frac{Q_2}{P}\right)^4. \tag{5}$$

$$PQ_3+\frac{4q}{PQ_3}=\left(\frac{Q_3}{P}\right)^2+q\left(\frac{P}{Q_3}\right)^2. \tag{6}$$

$$(PQ_5)^2+\frac{16}{(PQ_5)^2}=\left(\frac{P}{Q_5}\right)^3 +\left(\frac{Q_5}{P}\right)^3-5\left(\frac{P}{Q_5}+\frac{Q_5}{P}\right). \tag{7}$$

$$(PQ_7)^3+\frac{64}{(PQ_7)^3}+7\left((PQ_7)^2+\frac{16}{(PQ_7)^2}\right) +28\left(PQ_7+\frac{4}{PQ_7}\right)+70 =\left(\frac{P}{Q_7}\right)^4+\left(\frac{Q_7}{P}\right)^4. \tag{8}$$

For the proof of (5)–(8), one may refer Theorem 3.2 Naika et al. (2009). Also (7) is documented in Ramanujan (1957), p. 55.

3 Modular equations in the theory of signature 4

Theorem 1 *If 2 and 4 are the degrees of β and γ, over α and let*

$$u=q^{1/16}\left(\frac{\sqrt{\beta}(1-\sqrt{\alpha})}{\sqrt{\alpha}(1-\sqrt{\beta})}\right)^{1/8}$$

and

$$v=q^{1/8}\left(\frac{\sqrt{\gamma}(1-\sqrt{\beta})}{\sqrt{\beta}(1-\sqrt{\gamma})}\right)^{1/8},$$

then we have

$$\left(u^{16}t-v^8s\right)\left(v^{16}s-u^8t\right)=256u^{16}v^{16}\left(1-u^8v^8\right)^2,$$

where

$$s=16u^{16}-16u^8-1 \quad \text{and} \quad t=16v^{16}+16v^8-1.$$

Proof From (Berndt 1991, p. 124), we have

$$f_1=2^{-1/6}\sqrt{z}\left\{\frac{x(1-x)^4}{q}\right\}^{1/24} \tag{9}$$

and

$$f_4=2^{-2/3}\sqrt{z}\left\{\frac{x^4(1-x)}{q^4}\right\}^{1/24}. \tag{10}$$

From (9) and (10), we deduce

$$\frac{f_1}{q^{1/8}f_4}=\sqrt{2}\left(\frac{1-x}{x}\right)^{1/8}. \tag{11}$$

After letting $x\to 2\sqrt{x}/(1+\sqrt{x})$ in (11), we deduce

$$\frac{f_1}{q^{1/16}f_4}=2^{3/8}\left(\frac{1-\sqrt{x}}{\sqrt{x}}\right)^{1/8}. \tag{12}$$

Let

$$A=\frac{f_1}{q^{1/8}f_4},\qquad B=\frac{f_2}{q^{1/4}f_8} \quad \text{and} \quad C=\frac{f_4}{q^{1/2}f_{16}}. \tag{13}$$

Using (13) in (5), we have

$$(AB)^4+\frac{256}{(AB)^4}=\left(\frac{B}{A}\right)^{12}-16\left(\frac{A}{B}\right)^4-16\left(\frac{B}{A}\right)^4 \tag{14}$$

and

$$(BC)^4 + \frac{256}{(BC)^4} = \left(\frac{C}{B}\right)^{12} - 16\left(\frac{B}{C}\right)^4 - 16\left(\frac{C}{B}\right)^4. \tag{15}$$

Using (12) in (13), we have

$$\frac{A}{B} = q^{1/16}\left(\frac{\sqrt{\beta}(1-\sqrt{\alpha})}{\sqrt{\alpha}(1-\sqrt{\beta})}\right)^{1/8} = u \tag{16}$$

and

$$\frac{B}{C} = q^{1/8}\left(\frac{\sqrt{\gamma}(1-\sqrt{\beta})}{\sqrt{\beta}(1-\sqrt{\gamma})}\right)^{1/8} = v, \tag{17}$$

where the degrees of β and γ are 2 and 4, respectively, over α. On using (16) and (17) in (14) and (15), respectively, we have

$$u^{16}B^{16} + \left(16u^{16} + 16u^8 - 1\right)B^8 + 256u^8 = 0 \tag{18}$$

and

$$v^8B^{16} + \left(16v^{16} + 16v^8 - 1\right)B^8 + 256v^{16} = 0. \tag{19}$$

From (18) and (19), we observe that

$$\begin{aligned}&\frac{B^{16}}{256v^{16}\left(16u^{16}-16u^8-1\right) - 256u^8\left(16v^{16}+16v^8-1\right)}\\ &\quad= \frac{B^8}{256u^8v^8\left(1-u^8v^8\right)}\\ &\quad= \frac{1}{u^{16}\left(16v^{16}+16v^8-1\right) - v^8\left(16u^{16}-16u^8-1\right)}.\end{aligned}$$

On solving, we obtain

$$B^{16} = \frac{256\left(v^{16}\left(16u^{16}-16u^8-1\right) - u^8\left(16v^{16}+16v^8-1\right)\right)}{u^{16}\left(16v^{16}+16v^8-1\right) - v^8\left(16u^{16}-16u^8-1\right)} \tag{20}$$

and

$$B^8 = \frac{256u^8v^8\left(1-u^8v^8\right)}{u^{16}\left(16v^{16}+16v^8-1\right) - v^8\left(16u^{16}-16u^8-1\right)}. \tag{21}$$

On combining (20) and (21) and then simplifying, we get the required result. □

Theorem 2 *If 3 and 9 are the degrees of β and γ over α and let*

$$u = q^{1/8}\left(\frac{\sqrt{\beta}(1-\sqrt{\alpha})}{\sqrt{\alpha}(1-\sqrt{\beta})}\right)^{1/8} \quad \textit{and}$$

$$v = q^{3/8}\left(\frac{\sqrt{\gamma}(1-\sqrt{\beta})}{\sqrt{\beta}(1-\sqrt{\gamma})}\right)^{1/8} \left(u\left(1+v^4\right) - v^3\left(1+u^4\right)\right)$$

$$\left(v\left(1+u^4\right) - u^3\left(1+v^4\right)\right) = u^2v^2\left(1-u^2v^2\right)^2.$$

Proof Let

$$A = \frac{f_1}{q^{1/8}f_4}, \qquad B = \frac{f_3}{q^{3/8}f_{12}} \qquad \textit{and} \qquad C = \frac{f_9}{q^{9/8}f_{36}}. \tag{22}$$

On using (22) in (6), it is observed that

$$AB + \frac{4q}{AB} = \left(\frac{B}{A}\right)^2 + q\left(\frac{A}{B}\right)^2 \tag{23}$$

and

$$BC + \frac{4q}{BC} = \left(\frac{C}{B}\right)^2 + q\left(\frac{B}{C}\right)^2. \tag{24}$$

Using (12) in (22), we observe that

$$\frac{A}{B} = q^{1/8}\left(\frac{\sqrt{\beta}(1-\sqrt{\alpha})}{\sqrt{\alpha}(1-\sqrt{\beta})}\right)^{1/8} = u \tag{25}$$

and

$$\frac{B}{C} = q^{3/8}\left(\frac{\sqrt{\gamma}(1-\sqrt{\beta})}{\sqrt{\beta}(1-\sqrt{\gamma})}\right)^{1/8} = v, \tag{26}$$

where the degrees of β and γ are 3 and 9, respectively, over α. On using (25) and (26) in (23) and (24), we have

$$u^3B^4 - \left(1+u^4\right)B^2 + 4u = 0 \tag{27}$$

and

$$vB^4 - \left(1+v^4\right)B^2 + 4v^3 = 0. \tag{28}$$

From (27) and (28), we deduce

$$\begin{aligned}&\frac{B^4}{4u\left(1+v^4\right) - 4v^3\left(1+u^4\right)}\\ &\quad= \frac{B^2}{4uv\left(1-u^2v^2\right)} = \frac{1}{v\left(1+u^4\right) - u^3\left(1+v^4\right)}.\end{aligned}$$

On solving, we obtain

$$B^4=\frac{4\left(u\left(1+v^4\right)-v^3\left(1+u^4\right)\right)}{v\left(1+u^4\right)-u^3\left(1+v^4\right)} \tag{29}$$

and

$$B^2=\frac{4uv\left(1-u^2v^2\right)}{v\left(1+u^4\right)-u^3\left(1+v^4\right)}. \tag{30}$$

On combining (29) and (30) and simplifying, we obtain the required result. □

Theorem 3 *If 5 and 25 are the degrees of β and γ, over α and let*

$$u=q^{1/4}\left(\frac{\sqrt{\beta}(1-\sqrt{\alpha})}{\sqrt{\alpha}(1-\sqrt{\beta})}\right)^{1/8}$$

and

$$v=q^{5/4}\left(\frac{\sqrt{\gamma}(1-\sqrt{\beta})}{\sqrt{\beta}(1-\sqrt{\gamma})}\right)^{1/8},$$

then we have

$$(ut-v^5s)(u^5t-vs)=16u^2v^2(1-u^4v^4)^2,$$

where

$$s=u^6-5u^4-5u^2+1 \quad \text{and} \quad t=v^6-5v^4-5v^2+1.$$

Proof Let

$$A=\frac{f_1}{q^{1/8}f_4}, \qquad B=\frac{f_5}{q^{5/8}f_{20}} \quad \text{and} \quad C=\frac{f_{25}}{q^{25/8}f_{100}}. \tag{31}$$

On employing (31) in (7), we obtain

$$(AB)^2+\frac{16}{(AB)^2}=\left(\frac{A}{B}\right)^3+\left(\frac{B}{A}\right)^3-5\left(\frac{A}{B}+\frac{B}{A}\right) \tag{32}$$

and

$$(BC)^2+\frac{16}{(BC)^2}=\left(\frac{B}{C}\right)^3+\left(\frac{C}{B}\right)^3-5\left(\frac{B}{C}+\frac{C}{B}\right). \tag{33}$$

Using (12) in (31), we have

$$\frac{A}{B}=q^{1/4}\left(\frac{\sqrt{\beta}(1-\sqrt{\alpha})}{\sqrt{\alpha}(1-\sqrt{\beta})}\right)^{1/8}=u \tag{34}$$

and

$$\frac{B}{C}=q^{5/4}\left(\frac{\sqrt{\gamma}(1-\sqrt{\beta})}{\sqrt{\beta}(1-\sqrt{\gamma})}\right)^{1/8}=v, \tag{35}$$

where the degrees of β and γ are 5 and 25, respectively, over α. Employing (34) and (35) in (32) and (33), respectively, we obtain

$$u^5B^8-sB^4+16u=0 \tag{36}$$

and

$$vB^8-tB^4+16v^5=0, \tag{37}$$

where s and t are defined as in Theorem 3.3. From (36) and (37), we deduce

$$\frac{B^8}{16ut-16v^5s}=\frac{B^4}{16uv(1-u^4v^4)}=\frac{1}{u^5t-vs}$$

which implies

$$B^8=\frac{16(ut-v^5s)}{u^5t-vs} \tag{38}$$

and

$$B^4=\frac{16uv(1-u^4v^4)}{u^5t-vs}. \tag{39}$$

On combining (38) and (39) and streamlining the terms, we obtain the required result. □

Theorem 4 *If 7 and 49 are the degrees of β and γ over α and let*

$$u=\frac{2^{3/4}}{q}\left(\frac{(1-\sqrt{\alpha})(1-\sqrt{\beta})}{\sqrt{\alpha\beta}}\right)^{1/8}$$

and

$$v=\frac{2^{3/4}}{q^7}\left(\frac{(1-\sqrt{\beta})(1-\sqrt{\gamma})}{\sqrt{\beta\gamma}}\right)^{1/8},$$

then we have

$$u^4v^4(mu^4-nv^4)(nu^4-mv^4)=(u^8-v^8)^2,$$

where

$$m=s^3+7s^2+16s+14, \quad n=t^3+7t^2+16t+14,$$
$$s=u+\tfrac{4}{u} \quad \text{and} \quad t=v+\tfrac{4}{v}.$$

Proof Let

$$A = \frac{f_1}{q^{1/8} f_4}, \qquad B = \frac{f_7}{q^{7/8} f_{49}} \qquad and \qquad C = \frac{f_{49}}{q^{49/8} f_{196}}. \tag{40}$$

On employing (40) in (8), we have

$$(AB)^3 + \frac{64}{(AB)^3} + 7\left((AB)^2 + \frac{16}{(AB)^2}\right) + 28\left(AB + \frac{4}{AB}\right) + 70 = \left(\frac{A}{B}\right)^4 + \left(\frac{B}{A}\right)^4 \tag{41}$$

and

$$(BC)^3 + \frac{64}{(BC)^3} + 7\left((BC)^2 + \frac{16}{(BC)^2}\right) + 28\left(BC + \frac{4}{BC}\right) + 70 = \left(\frac{B}{C}\right)^4 + \left(\frac{C}{B}\right)^4. \tag{42}$$

Using (12) in (40), we have

$$AB = \frac{2^{3/4}}{q}\left(\frac{(1-\sqrt{\alpha})(1-\sqrt{\beta})}{\sqrt{\alpha\beta}}\right)^{1/8} = u \tag{43}$$

and

$$BC = \frac{2^{3/4}}{q^7}\left(\frac{(1-\sqrt{\beta})(1-\sqrt{\gamma})}{\sqrt{\beta\gamma}}\right)^{1/8} = v, \tag{44}$$

where the degrees of β and γ are 7 and 49 respectively, over α. Employing (43) and (44) in (41) and (42), respectively, we have

$$B^{16} - mu^4 B^8 + u^8 = 0 \tag{45}$$

and

$$B^{16} - nv^4 B^8 + v^8 = 0, \tag{46}$$

where m and n are defined as in Theorem 3.4. From (45) and (46), we deduce

$$\frac{B^{16}}{nu^8 v^4 - mu^4 v^8} = \frac{B^8}{v^8 - u^8} = \frac{1}{mu^4 - nv^4}$$

which implies

$$B^{16} = \frac{u^4 v^4 (nu^4 - mv^4)}{mu^4 - nv^4} \tag{47}$$

and

$$B^8 = \frac{v^8 - u^8}{mu^4 - nv^4}. \tag{48}$$

On combining (47) and (48) and simplifying, we obtain the required result. □

4 Conclusion

In our present investigation, we have systematically studied several modular equations and the associated theta functions. After briefly giving the historical account of modular equations and the theta functions, we have introduced and investigated a new proof of class of modular equations in Ramanujan's alternative theory of elliptic functions using theory of signature 4. It is believed that many of the recent works, which we have chosen to cite in this paper, are potentially useful for indicating directions for further researches on other new signatures.

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Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

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