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APPLICATION OF SOFT COMPUTING

The reliability analysis based on the generalized intuitionistic fuzzy two-parameter Pareto distribution

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Abstract

In this paper, the two-parameter Pareto lifetime distribution is considered with vague shape and scale parameters, where parameters are set as generalized intuitionistic fuzzy numbers. A new L-R type intuitionistic fuzzy number is introduced, and cuts of the new fuzzy set are provided. The generalized intuitionistic fuzzy reliability characteristics such as reliability, conditional reliability, hazard rate and mean time to failure functions are defined, along with the special case of the two-parameter Pareto generalized intuitionistic fuzzy reliability analysis. Furthermore, the series and parallel system reliability are evaluated by the generalized intuitionistic fuzzy sets. Finally, for certain cases of the fuzzy shape and scale parameters and cut set values, the generalized intuitionistic fuzzy reliability characteristics are provided and compared through several illustrative plots.

Keywords Generalized L-R type intuitionistic fuzzy numbers · (α_1, α_2)-cut set · Generalized intuitionistic fuzzy reliability · Generalized intuitionistic fuzzy probability · Two-parameter Pareto distribution

1 Introduction

The fuzzy sets (FSs) theory as a generalization of the classical theory of sets provides the uncertainty associated with classification or imprecision. In the FSs, elements are defined by their membership function, which represents the possibility of the occurrence of an event to accommodate the uncertainty. In the last decades, several developments of the FSs are recommended, containing the L-fuzzy, interval-valued fuzzy, rough and intuitionistic fuzzy sets (IFSs). The application of IFSs instead of FSs means providing another degree of freedom into a set description. In other words, the IFSs are equipped by the degree of hesitation, which handles the ambiguity and vagueness along with the membership, non-membership and hesitancy functions.

The IFSs conception has been applied in a wide range of branches, such as reliability (Shu et al. 2006; Aikhuele 2020), transportation problem (Mahmoodirad et al. 2019; Mishra and Kumar 2020), data envelopment analysis (Puri and Yadav 2015; Arya and Yadav 2019) and decision making (Yang et al. 2021; Pękala et al. 2021).

Atanassov (2017) provided a comparison study among the type-1 fuzzy sets and IFSs and transformed some concepts from the IFSs theory to the type-1 fuzzy sets theory. The theory included new operations, relations, and operators that extend the operators defined over the type-1 fuzzy sets.

The triangular intuitionistic fuzzy number (TIFN) is introduced by Mahapatra and Roy (2009) for reliability analysis purposes. Afterward, Mahapatra and Mahapatra (2010) reported the intuitionistic fuzzy fault tree using the arithmetic operation of trapezoidal intuitionistic fuzzy number (TrIFN), which are evaluated based on the (α, β)-cuts method.

Varghese and Rosario (2021) introduced the Pendant, Hexant and Octant fuzzy numbers along with the α-cuts are defined and mathematical operations. The reliability analysis based on different fuzzy numbers was compared via the numerical examples, and defuzzification was performed using various approaches, including the signed distance, graded mean integration and centroid methods, with special attention to the reliability of the weaving machine.

Feng et al. (2020) concentrated on the generalizations of the expectation score function called Minkowski score functions of intuitionistic fuzzy values (IFV) and ranking IFV from a geometric perspective in decision-making issues.

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They provided a new algorithm for solving decision-making problems based on the Minkowski weighted score function and the maximizing deviation method under the IFSs.

Due to uncertainty in medical diagnosis, incomplete evidence and imprecise information, Kozae et al. (2020) introduced a new definition of IFS and evaluated its implementation in the Covid-19 pandemic.

Citakoglu et al. (2014) estimated the monthly mean reference evapotranspiration through the adaptive network-based fuzzy inference system and artificial neural network models, and Cobaner et al. (2014) estimated the means of maximum, minimum and average monthly temperatures as a function of geographical coordinates and month number for any location in Turkey by the artificial neural networks, adaptive neuro-fuzzy inference system and multiple linear regression models (see also Citakoglu 2017; Citakoglu 2015).

The classical reliability analysis is based on the crisp information on lifetime data and cannot cover the uncertainty environments regarding the randomness, vagueness, ambiguity, and imprecision with different and specific characteristics. The uncertainties in the reliability fields are concerned with the components, parameters, phenomena and underlying assumptions. The estimation methods for reliability characteristics must be modified based on the fuzzy lifetimes to attain a more realistic analysis and exploit the uncertainty or imprecision in the data. The concept of the FS has also received considerable attention from system reliability analysis researchers such as Mahapatra and Roy (2012), Pan et al. (2015), Pramanik et al. (2019) and El-Damcese et al. (2014).

The fuzzy reliability analysis is illustrated based on various lifetime distributions, for instance, exponential (Baloui Jamkhaneh 2011), Weibull (Baloui Jamkhaneh 2014), Rayleigh (Pak et al. 2014) and three-parameter Weibull, Pareto and Gamma (Shafiq et al. 2017).

Liu et al. (2007) illustrated the fuzzy reliability analysis and mean time to failure of series, parallel, series-parallel, parallel-series and cold standby systems. Kumar et al. (2013) extended the fuzzy set semantics to IFS and analyzed IFS reliability based on the profust reliability theory, where the failure rate is represented by a time-dependent TIFN. Sharma et al. (2012) provided the fuzzy reliability of systems by IFS and implemented the TIFN and its arithmetic operations. Akbari and Hesamian (2020) considered the intuitionistic fuzzy random variable with crisp parameters and reported a procedure for constructing time-dependent reliability systems. The uncertainty of the number of failures is modulated with the aid of a fuzzy framework by Husniah and Supriatna (2021), where Weibull failure distribution is considered with the fuzzy shape parameter.

The new generalized intuitionistic fuzzy sets (GIFS_B) along with some operators over GIFS_B and the new generalized intuitionistic fuzzy number (GIFN_B) based on the GIFS have been, respectively, introduced by Baloui Jamkhaneh and Nadarajah (2015) and Shabani and Baloui Jamkhaneh (2014). Baloui Jamkhaneh (2016) represented the values and indeterminacy of the degree of membership and non-membership functions of GIFS_B and Baloui Jamkhaneh (2017) considered the generalized intuitionistic fuzzy exponential lifetime distribution and the reliability analysis based on GIFS_B. Ebrahimnejad and Baloui Jamkhaneh (2018) and Roohanizadeh et al. (2021), respectively, considered system reliability of Rayleigh and Pareto distributions with GIFN_B.

The heavy-tailed univariate Pareto distribution has been used often to model reliability and continuous lifetime data, which was first proposed as a model for rare events as the survival function slowly decreases in comparison to other life distributions. The Pareto distribution has been applied in modeling various phenomena in the description of hydrology, insurance, scientific, finance, and actuarial science, which can be found in the works of Amin (2008), Fu et al. (2012), Prakash (2017), Lee and Kim (2018) and Ghitany et al. (2018). According to practical purposes and research, several kinds of Pareto distribution are introduced. In this paper, we concentrate on two-parameter Pareto distribution with fuzzy scale and fuzzy shape parameters.

The purpose of the paper has twofold. First, we provide a new generalized L-R type intuitionistic fuzzy number with corresponding (α_1, α_2) cut sets. The second principal aim is extending the reliability characteristics in the GIFS environment, which was introduced by Baloui Jamkhaneh and Nadarajah (2015), with special attention to the two-parameter Pareto distribution. We consider Pareto distribution, which has the uncertainty in its lifetime scale and shape parameters by the GIFN. The vagueness in the reliability characteristics is represented perfectly by parameter fuzzification into the GIFN_B, and the generalized intuitionistic fuzzy reliability (GIFR) modeling is introduced via the generalized intuitionistic fuzzy probabilities (GIFP). Several characteristics such as conditional reliability, hazard and mean time to failure functions are obtained via generalized intuitionistic fuzzy parameters. Also, the fuzzy reliability of the series and parallel system has been represented separately.

The structure of the present paper is organized as follows. In Sect. 2, we report some basic concepts of GIFN_B. The GIFP is introduced in Sect. 3, where parameters are set as the GIFN_B. In Sect. 4, we obtain the GIFR characteristics, which include the reliability, conditional reliability, hazard and mean time to failure functions, and as a special case, we consider the two-parameter Pareto distribution with GIFN_B scale and shape parameters. Section 5 concentrates on GIFR for both series and parallel systems. Finally, in Sect. 6, the graphical illustration and numerical example confirm the theoretical outcomes.

2 Preliminaries

In this section, we concentrate on the GIFS_B with basic GIFN_B elements that are summarized in the next Definitions. Also, a new generalized L-R type intuitionistic fuzzy number is provided, which is used throughout the paper.

Definition 1 (Baloui Jamkhaneh and Nadarajah 2015) The generalized intuitionistic fuzzy set ($\mathrm{GIFS}_B(X)$) A in X, is defined as follows

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\},$$

where X is a non-empty set and $\mu_A : X\to[0,1]$, $\nu_A : X\to[0,1]$ denote the degree of membership and non-membership functions of x in A, respectively. Also, $0\le\mu_A^{\delta}(x)+\nu_A^{\delta}(x)\le 1$, $\forall x\in X$ and $\delta=n$ or $\frac{1}{n}$, $n=1,2,\ldots,N$.

Afterward, Shabani and Baloui Jamkhaneh (2014) introduced the GIFN_B based on the $\mathrm{GIFS}_B(X)$ defined in Definition 1. We review the GIFN_B in the next Definition.

Definition 2 (Shabani and Baloui Jamkhaneh 2014) Consider $\mathrm{GIFS}_B(X)$ from the real number domain, a generalized L-R type intuitionistic fuzzy number A is defined with the following membership $\mu_A(x)$ and non-membership $\nu_A(x)$ functions

$$\mu_A(x)=\begin{cases} f^L(x), & a\le x\le b\\ u, & b\le x\le c\\ f^R(x), & c\le x\le d\\ 0, & o.w\end{cases},$$

$$\nu_A(x)=\begin{cases} g^L(x), & a_1\le x\le b\\ w, & b\le x\le c\\ g^R(x), & c\le x\le d_1\\ 1, & \text{o.w}\end{cases},$$

such that bound values must be satisfied in $a_1\le a\le b\le c\le d\le d_1$ constraint and

$$0\le\mu_A^{\delta}(x)+\nu_A^{\delta}(x)\le 1,\ \forall x\in X.$$

The basis left $\left(f^L(x),g^L(x)\right)$ and right $\left(f^R(x),g^R(x)\right)$ are continuous monotone membership and non-membership functions, where $f^L(x)$, $g^R(x)$ are increasing and $f^R(x)$, $g^L(x)$ are decreasing functions.

A class of generalized L-R type intuitionistic fuzzy number (GIFN_B) A is defined as

$$\mu_A(x)=\begin{cases} \left(\dfrac{(x-a)\mu}{b-a}\right)^{\frac{1}{\delta}}, & a\le x\le b\\ \mu^{\frac{1}{\delta}}, & b\le x\le c\\ \left(\dfrac{(d-x)\mu}{d-c}\right)^{\frac{1}{\delta}}, & c\le x\le d\\ 0, & \text{o.w}\end{cases},$$

$$\nu_A(x)=\begin{cases} \left(1-\dfrac{(1-\nu)(x-a_1)}{b-a_1}\right)^{\frac{1}{\delta}}, & a_1\le x\le b\\ \nu^{\frac{1}{\delta}}, & b\le x\le c\\ \left(1-\dfrac{(1-\nu)(d_1-x)}{d_1-c}\right)^{\frac{1}{\delta}}, & c\le x\le d_1\\ 1, & \text{o.w}\end{cases}.$$

with the condition $\mu+\nu\le 1$.

The GIFN_B A is denoted as $A=(a_1,a,b,c,d,d_1,\mu,\nu,\delta)$, where $\mu_A(x)$ and $1-\nu_A(x)$ are fuzzy numbers. Two parameters $\mu^{\frac{1}{\delta}}$ and $\nu^{\frac{1}{\delta}}$ reflect the confidence level and non-confidence level of the A, respectively.

The α-cut of a fuzzy set is the classical set that includes all the elements of the set with greater than or equal to the specified value of α membership degree. Baloui Jamkhaneh (2016) introduced the (α_1,α_2)-cut of GIFN_B, which is briefly explained in Definition 3.

Hereafter in the paper, we select the fixed numbers $\alpha_1,\alpha_2\in[0,1]$ such that both hold in the constraint $0\le\alpha_1\le\mu^{\frac{1}{\delta}},\nu^{\frac{1}{\delta}}\le\alpha_2\le 1$ and $0\le\alpha_1^{\delta}+\alpha_2^{\delta}\le 1$, to avoid repetition.

Definition 3 (Baloui Jamkhaneh (2016)) Consider the set of (α_1,α_2)-cut generated by a GIFN_B A defined by

$$A[\alpha_1,\alpha_2,\delta]=\big\{\langle x,\mu_A(x)\ge\alpha_1,\ \nu_A(x)\le\alpha_2\rangle : x\in X\big\}.$$

$A[\alpha_1,\alpha_2,\delta]$ is defined as the crisp set of elements x which belong to A at least α_1 degree and does not belong to A at most α_2 degree. The α_1-cut set of a GIFN_B A is a crisp subset of real number domain $\mathbb{R}$, which is defined as

$$A[\alpha_1,\delta]=\{\langle x,\mu_A(x)\ge\alpha_1\rangle : x\in X\}=[L_1(\alpha_1),U_1(\alpha_1)],$$

where

$$L_1(\alpha_1)=a+\frac{(b-a)\alpha_1^{\delta}}{\mu},\quad U_1(\alpha_1)=d-\frac{(d-c)\alpha_1^{\delta}}{\mu}.$$

Analogously, the α_2-cut set of a GIFN_B A is a crisp subset of $\mathbb{R}$ as

$$A[\alpha_2,\delta]=\{\langle x,\nu_A(x)\le\alpha_2\rangle : x\in X\}=[L_2(\alpha_2),U_2(\alpha_2)],$$

where

$$L_2(\alpha_2)=a_1+\frac{(b-a_1)(1-\alpha_2^{\delta})}{1-\nu},$$
$$U_2(\alpha_2)=d_1-\frac{(d_1-c)(1-\alpha_2^{\delta})}{1-\nu}.$$

Similarly,

$$A[\alpha_1, \alpha_2, \delta] = \{\langle x, \mu_A(x) \geq \alpha_1, 1 - \nu_A(x) \geq 1 - \alpha_2 \rangle : x \in X\}.$$

If set $\alpha_1 = 1 - \alpha_2 = \alpha$, then

$$A[\alpha_1, \alpha_2, \delta] = \{\langle x, \mu_A(x) \geq \alpha, 1 - \nu_A(x) \geq \alpha \rangle : x \in X\}.$$

The GIFN$_B$ based on the α_1-cut and α_2-cut sets is shown as

$$A(\alpha_1, \alpha_2, \delta) = (A_\mu[\alpha_1, \delta], A_\nu[\alpha_2, \delta]).$$

Definition 4 Consider two α-cut sets $[a, b]$ and $[c, d]$, some relations and operations on α-cut sets are defined as follows:

(i) The relation $[a, b] \preccurlyeq [c, d]$ is hold, if and only if $a \leq c$ and $b \leq d$,

(ii) If $k > 0$, then $k \otimes [a, b] = [ka, kb]$ and if $k < 0$, then $k \otimes [a, b] = [kb, ka]$,

(iii) $k \oplus [a, b] = [k + a, k + b]$ and $k \ominus [a, b] = [k - b, k - a]$,

(iv) $[a, b] \oplus [c, d] = [a + c, b + d]$.

Definition 5 Several relations and operations on GIFN$_B$s are listed as below:

(i) $A(\alpha_1, \alpha_2, \delta) \oplus B(\alpha_1, \alpha_2, \delta) = \big(A_\mu[\alpha_1, \delta] \oplus B_\mu[\alpha_1, \delta], A_\nu[\alpha_2, \delta] \oplus B_\nu[\alpha_2, \delta]\big)$,

(ii) $k \otimes A(\alpha_1, \alpha_2, \delta) \oplus b = \big(k \otimes A_\mu[\alpha_1, \delta] \oplus b,\ k \otimes A_\nu[\alpha_2, \delta] \oplus b\big)$,

(iii) $b \ominus A(\alpha_1, \alpha_2, \delta) = (b \ominus A_\mu[\alpha_1, \delta], b \ominus A_\nu[\alpha_2, \delta])$,

(iv) $A(\alpha_1, \alpha_2, \delta) \preccurlyeq B(\alpha_1, \alpha_2, \delta)$, if and only if $A_\mu[\alpha_1, \delta] \preccurlyeq B_\mu[\alpha_1, \delta]$ and $A_\nu[\alpha_2, \delta] \preccurlyeq B_\nu[\alpha_2, \delta]$,

(v) $A(\alpha_1, \alpha_2, \delta) = B(\alpha_1, \alpha_2, \delta)$, if and only if $A_\mu[\alpha_1, \delta] = B_\mu[\alpha_1, \delta]$ and $A_\nu[\alpha_2, \delta] = B_\nu[\alpha_2, \delta]$.

where $A(\alpha_1, \alpha_2, \delta)$ and $B(\alpha_1, \alpha_2, \delta)$ are two GIFN$_B$s.

3 Generalized intuitionistic fuzzy probability

The uncertainty in lifetime data may be caused by the random variables or parameters of the model. Here, we focus on the imprecise parameters modeled by fuzzy numbers. We introduce the fuzzy probability where parameters of the model are considered as the GIFN$_B$.

Consider the continuous random variable X from a density function $f(x, \tilde{\theta}, \tilde{\beta})$ where $\tilde{\theta}$ and $\tilde{\beta}$ are GIFN$_B$. Then, α_1-cut set of membership and α_2-cut set of non-membership functions of the GIFP of C is defined as

$$\begin{aligned} P_j(C)[\alpha_i, \delta] &= \{P(C) \mid \theta \in \theta_j[\alpha_i, \delta], \beta \in \beta_j[\alpha_i, \delta]\} \\ &= \Big[P_j^L(C)[\alpha_i],\ P_j^U(C)[\alpha_i]\Big], \\ (i, j) &= (1, \mu), (2, \nu), \end{aligned}$$

where $P(C)$ is the crisp probability defined as $P(C) = \int_C f(x, \theta)\, dx$, and

$$P_j^L(C)[\alpha_i] = \inf_{\substack{\theta \in \theta_j[\alpha_i, \delta] \\ \beta \in \beta_j[\alpha_i, \delta]}} P(C),$$

$$P_j^U(C)[\alpha_i] = \sup_{\substack{\theta \in \theta_j[\alpha_i, \delta] \\ \beta \in \beta_j[\alpha_i, \delta]}} P(C), \ (i, j) = (1, \mu), (2, \nu).$$

Consequently,

$$\begin{aligned} \tilde{P}(C) &= P(C)(\alpha_1, \alpha_2, \delta) \\ &= \Big(P_\mu(C)[\alpha_1, \delta],\ P_\nu(C)[\alpha_2, \delta]\Big), \end{aligned}$$

which is the GIFN$_B$ and (α_1, α_2)-cut set of GIFP of C is defined as

$$P(C)[\alpha_1, \alpha_2, \delta] = \Big\{w, w \in P_\mu(C)[\alpha_1, \delta] \cap P_\nu(C)[\alpha_2, \delta]\Big\}.$$

Corollary 1 *Consider the GIFP as $P(C)$, then*

(i) $P(C^c)(\alpha_1, \alpha_2, \delta) = 1 \ominus P(C)(\alpha_1, \alpha_2, \delta)$,
(ii) *If* $C_1 \subset C_2$ *then* $P(C_1)(\alpha_1, \alpha_2, \delta) \preccurlyeq P(C_2)(\alpha_1, \alpha_2, \delta)$.

Proof (i) Regarding to the definition of GIFP, for $(i, j) = (1, \mu), (2, \nu)$ we have

$$\begin{aligned} & P_j\big(C^c\big)[\alpha_i, \delta] \\ & = \Big\{1 - P(C) \mid \theta \in \theta_j[\alpha_i, \delta], \beta \in \beta_j[\alpha_i, \delta]\Big\} \\ & = \Big[P_j^L\big(C^c\big)[\alpha_i], P_j^U\big(C^c\big)[\alpha_i]\Big] \\ & = \left[\inf_{\substack{\theta \in \theta_j[\alpha_i, \delta] \\ \beta \in \beta_j[\alpha_i, \delta]}} (1 - P(C)),\ \sup_{\substack{\theta \in \theta_j[\alpha_i, \delta] \\ \beta \in \beta_j[\alpha_i, \delta]}} (1 - P(C))\right] \end{aligned}$$

$$= \left[1 - \sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} P(C), 1 - \inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} P(C) \right]$$
$$= 1 \ominus \left[P_j^L(C)[\alpha_i], P_j^U(C)[\alpha_i] \right],$$

which is verified by Definition 5-v. (ii) Since $P(C_1) \leq P(C_2)$, so

$$P_j(C_1)[\alpha_i,\delta]$$
$$= \left[\inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} P(C_1), \sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} P(C_1) \right]$$
$$\preccurlyeq \left[\inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} P(C_2), \sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} P(C_2) \right]$$
$$= P_j(C_2)[\alpha_i,\delta],$$

and based on Definition 5-iv, the proof is completed. □

The fuzzification of some statistical concepts, including expectation and variance, can be induced by the specification of GIFP.
A set of α_1-cut of membership and α_2-cut set of non-membership functions of generalized intuitionistic fuzzy expectation (GIFE) $\tilde{E}(g(X))$ is determined as

$$E_j(g(X))[\alpha_i,\delta]$$
$$= \{E(g(X)) \mid \theta \in \theta_j[\alpha_i,\delta], \beta \in \beta_j[\alpha_i,\delta]\}$$
$$= \left[E_j^L(g(X))[\alpha_i], E_j^U(g(X))[\alpha_i] \right],$$

where $(i, j) = (1, \mu), (2, \nu)$ and based on the crisp expectation $E(g(X))$, we have

$$E_j^L(g(X))[\alpha_i] = \inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} E(g(X)),$$
$$E_j^U(g(X))[\alpha_i] = \sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} E(g(X)).$$

Consequently, it is concluded that

$$E(g(X))(\alpha_1,\alpha_2,\delta)$$
$$= \left(E_\mu(g(X))[\alpha_1,\delta], E_\upsilon(g(X))[\alpha_2,\delta] \right),$$

and (α_1, α_2)-cut set of GIFE of $g(X)$ is described as

$$E(g(X))[\alpha_1,\alpha_2,\delta]$$
$$= \left(E_\mu(g(X))[\alpha_1,\delta] \cap E_\upsilon(g(X))[\alpha_2,\delta] \right).$$

Remark 1 The GIFE of $X(\tilde{\mu})$ and generalized intuitionistic fuzzy variance of $X(\tilde{\sigma}^2)$ are obtained by the assumptions $g(X) = X$ and $g(X) = (X - E(X))^2$, respectively.

Corollary 2 *Consider a, b, c as constant numbers, then*

(i) $\tilde{E}(c) = c$,
(ii) $\tilde{E}(ag(X) + b) = a \otimes \tilde{E}(g(X)) \oplus b$,
(iii) $\tilde{\sigma}^2(c) = 0$,
(iv) $\tilde{\sigma}^2(aX + b) = a^2 \otimes \tilde{\sigma}^2(X)$.

Proof (i) and (iii) are obvious and the proofs are omitted. The proof of (ii) is obtained as follows:

$$\inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} E(ag(X) + b) = a \inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} E(g(X)) + b,$$
$$\sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} E(ag(X) + b) = a \sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} E(g(X)) + b,$$
$$(i, j) = (1, \mu), (2, \nu).$$

Also, (iv) is concluded by

$$\inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} \sigma^2(aX + b) = a^2 \inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} \sigma^2(X),$$
$$\sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} \sigma^2(aX + b) = a^2 \sup_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} \sigma^2(X),$$
$$(i, j) = (1, \mu), (2, \nu),$$

and the proof is completed. □

4 Generalized intuitionistic fuzzy reliability analysis

The intuitionistic fuzzy approach for reliability parameter analysis leads to more flexible information that can capture subjective, uncertain, and ambiguous information.

Consider X as a lifetime variable of a component with a density function $f(x, \tilde{\theta})$ where the vector of the parameters $\tilde{\theta}$ is the GIFN$_B$ and the GIFR characteristic (GIFRC) denoted by $\tilde{g}(t)$. A set of α_1-cut of membership and α_2-cut set of non-membership functions of GIFRC are denoted by $g_j(t)[\alpha_i,\delta]$ as

$$g_j(t)[\alpha_i,\delta] = \{g(t) \mid \theta \in \theta_j[\alpha_i,\delta], \beta \in \beta_j[\alpha_i,\delta]\}$$
$$= \left[g_j^L(t)[\alpha_i], \ g_j^U(t)[\alpha_i] \right],$$

where

$$g_j^L(t)[\alpha_i] = \inf_{\substack{\theta \in \theta_j[\alpha_i,\delta] \\ \beta \in \beta_j[\alpha_i,\delta]}} g(t),$$

$$g_j^U(t)[\alpha_i] = \sup_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}} g(t), \quad (i,j) = (1,\mu),(2,\nu).$$

The function $g(t)$ can be considered as the reliability, conditional reliability, hazard rate, cumulative risk and reverse hazard functions. It can be shown that $g(\alpha_1,\alpha_2,\delta) = (g_\mu(t)[\alpha_1,\delta], g_\nu(t)[\alpha_2,\delta])$ and the (α_1,α_2)-cut set of GIFRC is defined as

$$g(t)[\alpha_1,\alpha_2,\delta] = \left\{w, w\in g_\mu(t)[\alpha_1,\delta]\cap g_\nu(t)[\alpha_2,\delta]\right\}.$$

In the next subsection, we provide different reliability characteristics, comprehensively. Also, the fuzzy reliability characteristics of the two-parameter Pareto lifetime distribution with the scale parameter λ and shape parameter γ is provided as a special case.

4.1 Generalized intuitionistic fuzzy reliability function

The fuzzy reliability accounts for the uncertainty of the membership and non-membership grades of the component's reliability. In this section, the GIFR as the GIFP of surviving beyond time t, denoted by $\tilde{S}(t)$, is constructed based on the lifetime GIFN$_B$ parameters.

The α_1-cut set of membership and α_2-cut set of non-membership functions of GIFR of component denoted by $S_j(t)[\alpha_i,\delta]$, are obtained as

$$\begin{aligned} S_j(t)[\alpha_i,\delta] &= \{S(t) \mid \theta\in\theta_j[\alpha_i,\delta], \beta\in\beta_j[\alpha_i,\delta]\} \\ &= \left[S_j^L(t)[\alpha_i],\ S_j^U(t)[\alpha_i]\right], \\ &\qquad (i,j) = (1,\mu),(2,\nu), \end{aligned}$$

where $S(t)$ is the crisp reliability function and

$$S_j^L(t)[\alpha_i] = \inf_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}} S(t),$$
$$S_j^U(t)[\alpha_i] = \sup_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}} S(t), \quad (i,j) = (1,\mu),(2,\nu),$$

and it can be shown as

$$S(\alpha_1,\alpha_2,\delta) = (S_\mu(t)[\alpha_1,\delta], S_\nu(t)[\alpha_2,\delta]).$$

The (α_1,α_2)-cut set of GIFR is defined as follows:

$$S(t)[\alpha_1,\alpha_2,\delta] = \left\{w, w\in S_\mu(t)[\alpha_1,\delta]\cap S_\nu(t)[\alpha_2,\delta]\right\},$$

where $S_j(t)[\alpha_i,\delta]$, $(i,j) = (1,\mu),(2,\nu)$ are two-variate functions in terms of α_i, $i = 1,2$ and t. For t_0, $\tilde{S}(t_0)$ is the GIFN$_B$.

As a special case, we consider the two-parameter Pareto distribution and provide each corresponding reliability characteristic, respectively. Consider the random variable X from the two-parameter Pareto lifetime distribution

$$f(x,\lambda) = \frac{\lambda\gamma^\lambda}{x^{\lambda+1}}, \qquad x>\gamma,\ \lambda,\gamma>0,$$

which has the uncertainty in both scale and shape parameters and the vagueness are represented by fuzzifying the parameter values into a GIFN$_B$. Set the generalized intuitionistic fuzzy lifetime scale parameter

$$\tilde{\lambda} = (a_{11}, a_1, b_1, c_1, d_1, d_{11}, \mu, \nu, \delta),$$

and shape parameter

$$\tilde{\gamma} = (a_{21}, a_2, b_2, c_2, d_2, d_{21}, \mu, \nu, \delta),$$

then, the cut sets of GIFR function for $(i,j) = (1,\mu),(2,\nu)$ is obtained as follows

$$S_j(t)[\alpha_i,\delta] = \left\{\left(\frac{\gamma}{t}\right)^\lambda \mid \lambda\in\lambda_j[\alpha_i,\delta], \gamma\in\gamma_j[\alpha_i,\delta]\right\}.$$

Since $(\frac{\gamma}{t})^\lambda$ is a monotonically decreasing with respect to λ and increasing with respect to γ, the reliability bands are given by

$$S_\mu(t)[\alpha_1,\delta] = \left[\left(\frac{a_2 + \frac{(b_2-a_2)\alpha_1^\delta}{\mu}}{t}\right)^{d_1 - \frac{(d_1-c_1)\alpha_1^\delta}{\mu}}, \left(\frac{d_2 - \frac{(d_2-c_2)\alpha_1^\delta}{\mu}}{t}\right)^{a_1 + \frac{(b_1-a_1)\alpha_1^\delta}{\mu}}\right],$$

$$S_\nu(t)[\alpha_2,\delta] = \left[\left(\frac{a_{21} + \frac{(b_2-a_{21})(1-\alpha_2^\delta)}{1-\nu}}{t}\right)^{d_{11} - \frac{(d_{11}-c_1)(1-\alpha_2^\delta)}{1-\nu}}, \left(\frac{d_{21} - \frac{(d_{21}-c_2)(1-\alpha_2^\delta)}{1-\nu}}{t}\right)^{a_{11} + \frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}}\right].$$

In this method, for every specially α_{10} and α_{20}, shapes of $S_j(t)[\alpha_{i0},\delta]$, $(i,j) = (1,\mu),(2,\nu)$ are like bands with upper and lower curves. For $(i,j) = (1,\mu),(2,\nu)$, this reliability bands has the following properties

(i) $S_j(0)[\alpha_{i0},\delta]=[1,1]$, i.e., no one starts off dead,
(ii) $S_j(\infty)[\alpha_{i0},\delta]=[0,0]$, i.e., everyone dies eventually,
(iii) $S_j(t_1)[\alpha_{i0},\delta]\succcurlyeq S_j(t_2)[\alpha_{i0},\delta]$ if and only if $t_1\le t_2$, i.e., bands of $S_j(t)[\alpha_{i0},\delta]$ declines monotonically.

4.2 Generalized intuitionistic fuzzy conditional reliability function

In reliability analysis, conditional reliability is the probability of an item surviving for the time t, given that it has already survived until time τ.

Here, we extend the conditional reliability function to the uncertain case by the GIFS concept. The generalized intuitionistic fuzzy conditional reliability (GIFCR) function of the component is denoted by $\tilde{S}(t|\tau)$. The α_1-cut set of membership and α_2-cut set of non-membership functions of $\tilde{S}(t|\tau)$ are represented as

$$S_j(t|\tau)[\alpha_i,\delta]=\{S(t|\tau)|\theta\in\theta_j[\alpha_i,\delta],\beta\in\beta_j[\alpha_i,\delta]\}$$
$$=\left[S_j^L(t\mid\tau)[\alpha_i],\ S_j^U(t\mid\tau)[\alpha_i]\right],\quad (i,j)=(1,\mu),(2,\nu),$$

where $S(t|\tau)$ is the crisp conditional reliability function and

$$S_j^L(t|\tau)[\alpha_i]=\inf_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}}S(t|\tau),$$
$$S_j^U(t|\tau)[\alpha_i]=\sup_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}}S(t|\tau),\quad (i,j)=(1,\mu),(2,\nu).$$

Subsequently, we have

$$S(\alpha_1,\alpha_2,\delta)=(S_\mu(t|\tau)[\alpha_1,\delta],\ S_\nu(t|\tau)[\alpha_2,\delta]).$$

The (α_1,α_2)-cut set of GIFCR function is defined as

$$S(t|\tau)[\alpha_1,\alpha_2,\delta]=\left\{w,w\in S_\mu(t|\tau)[\alpha_1,\delta]\cap S_\nu(t|\tau)[\alpha_2,\delta]\right\},$$

where $S_j(t|\tau)[\alpha_i,\delta]$, $(i,j)=(1,\mu),(2,\nu)$ are two-variate functions in terms of α_i, $i=1,2$ and t.

For t_0, $\tilde{S}(t_0|\tau)$ is the GIFN$_B$. In this method, for every specially α_{10} and α_{20}, shapes of $S_j(t|\tau)[\alpha_{i0},\delta]$, $(i,j)=(1,\mu),(2,\nu)$ are like bands with upper and lower curves.

Consider the two-parameter Pareto distribution, the cut sets of GIFCR function, for $(i,j)=(1,\mu),(2,\nu)$ are represented by

$$S_j(t|\tau)[\alpha_i,\delta]=\left\{\left(\frac{\tau}{t+\tau}\right)^{\lambda}|\lambda\in\lambda_j[\alpha_i,\delta],\gamma\in\gamma_j[\alpha_i,\delta]\right\}.$$

Since $\left(\frac{\tau}{t+\tau}\right)^{\lambda}$ is a monotonically decreasing function with respect to λ, the conditional reliability bands are computed as

$$S_\mu(t\mid\tau)[\alpha_1,\delta]=\left[\left(\frac{\tau}{t+\tau}\right)^{d_1-\frac{(d_1-c_1)\alpha_1^\delta}{\mu}},\ \left(\frac{\tau}{t+\tau}\right)^{a_1+\frac{(b_1-a_1)\alpha_1^\delta}{\mu}}\right],$$

$$S_\nu(t\mid\tau)[\alpha_2,\delta]=\left[\left(\frac{\tau}{t+\tau}\right)^{d_{11}-\frac{(d_{11}-c_1)(1-\alpha_2^\delta)}{1-\nu}},\ \left(\frac{\tau}{t+\tau}\right)^{a_{11}-\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}}\right].$$

For every especial t_0, membership and non-membership functions of $\tilde{S}(t_0\mid\tau)$ are given as

$$\mu_{S(t_0|\tau)}(x)=\begin{cases}\left(\dfrac{\mu\left(d_1-\dfrac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}\right)}{d_1-c_1}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{d_1}\le x\le\left(\frac{\tau}{t_0+\tau}\right)^{c_1}\\ \mu^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{c_1}\le x\le\left(\frac{\tau}{t_0+\tau}\right)^{b_1}\\ \left(\dfrac{\mu\left(\dfrac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}-a_1\right)}{b_1-a_1}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{b_1}\le x\le\left(\frac{\tau}{t_0+\tau}\right)^{a_1}\\ 0, & \text{o.w.}\end{cases},$$

$$\nu_{S(t_0|\tau)}(x)=\begin{cases}\left(\dfrac{d_{11}-c_1+(1-\nu)\left(\dfrac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}-d_{11}\right)}{d_{11}-c_1}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{d_{11}}\le x\le\left(\frac{\tau}{t_0+\tau}\right)^{c_1}\\ \nu^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{c_1}\le x\le\left(\frac{\tau}{t_0+\tau}\right)^{b_1}\\ \left(\dfrac{b_1-a_{11}+(1-\nu)\left(a_{11}-\dfrac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}\right)}{b_1-a_{11}}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{b_1}\le x\le\left(\frac{\tau}{t_0+\tau}\right)^{a_{11}}\\ 1, & \text{o.w.}\end{cases}.$$

4.3 Generalized intuitionistic fuzzy hazard function

Another fuzzy character of the lifetime distribution is the fuzzy hazard function (or fuzzy failure rate). We propose the generalized intuitionistic fuzzy hazard (GIFH) function of component as $\tilde{h}(t)$ and it means the probability of an item failing at the time interval Δt if it operated until t. The α_1-cut set of membership and α_2-cut set of non-membership functions of GIFH of the component are illustrated as

$$h_j(t)[\alpha_i,\delta] = \{h(t)|\theta\in\theta_j[\alpha_i,\delta],\beta\in\beta_j[\alpha_i,\delta]\} = \left[h_j^L(t)[\alpha_i], h_j^U(t)[\alpha_i]\right],$$

where $h(t)$ is the crisp hazard rate function and

$$h_j^L(t)[\alpha_i] = \inf_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}} h(t),$$
$$h_j^U(t)[\alpha_i] = \sup_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}} h(t), \quad (i,j)=(1,\mu),(2,\nu).$$

It can be shown that

$$h(\alpha_1,\alpha_2,\delta) = (h_\mu(t)[\alpha_1,\delta], h_\nu(t)[\alpha_2,\delta]),$$

and the (α_1,α_2)-cut set of GIFH function is defined by

$$h(t)[\alpha_1,\alpha_2,\delta] = \left\{w, w\in h_\mu(t)[\alpha_1,\delta]\cap h_\nu(t)[\alpha_2,\delta]\right\},$$

where $h_j(t)[\alpha_i,\delta], (i,j)=(1,\mu),(2,\nu)$ are two-variate functions in terms of $\alpha_i, i=1,2$ and t.

Remark 2 Same as the GIFR, for every especially α_{10} and α_{20}, the shapes of $S_j(t|\tau)[\alpha_{i0},\delta]$ and $h_j(t)[\alpha_{i0},\delta]$, $(i,j)=(1,\mu),(2,\nu)$ are like bands with upper and lower curves and for every especially t_0, $\tilde{S}(t_0|\tau)$ and $\tilde{h}(t_0)$ are the GIFB$_B$s.

Remark 3 If $\mu=1$ and $\nu=0$, then our method changes to its special case; in addition, if $\delta=1$, then our method is named intuitionistic fuzzy reliability evaluation. If $\alpha_1=1-\alpha_2$, $a=a_1$ and $d=d_1$, then it changes to fuzzy reliability evaluation. Finally, if assumption $a=b=c=d$ is added, it agrees to classical reliability theory.

For $(i,j)=(1,\mu),(2,\nu)$, the cut set of GIFH function for two-parameter Pareto lifetime distribution is demonstrated as

$$h_j(t)[\alpha_i,\delta] = \left\{\frac{\lambda}{t} \mid \lambda\in\lambda_j[\alpha_i,\delta], \gamma\in\gamma_j[\alpha_i,\delta]\right\} = \left[h_j^L(t)[\alpha_i], h_j^U(t)[\alpha_i]\right],$$

where

$$h_j^L(t)[\alpha_i] = \inf\left\{\frac{\lambda}{t} \mid \lambda\in\lambda_j[\alpha_i,\delta], \gamma\in\gamma_j[\alpha_i,\delta]\right\},$$
$$h_j^U(t)[\alpha_i] = \sup\left\{\frac{\lambda}{t} \mid \lambda\in\lambda_j[\alpha_i,\delta]\, \gamma\in\gamma_j[\alpha_i,\delta]\right\},$$
$$(i,j)=(1,\mu),(2,\nu).$$

Therefore,

$$h_\mu(t)[\alpha_1,\delta] = \left[\frac{a_1}{t}+\frac{(b_1-a_1)\alpha_1^\delta}{\mu t}, \frac{d_1}{t}-\frac{(d_1-c_1)\alpha_1^\delta}{\mu t}\right],$$

$$h_\nu(t)[\alpha_2,\delta] = \left[\frac{a_{11}}{t}+\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{(1-\nu)t}, \frac{d_{11}}{t}-\frac{(d_{11}-c_1)(1-\alpha_2^\delta)}{(1-\nu)t}\right].$$

As can be seen, the GIFH function for generalized intuitionistic fuzzy two-parameter Pareto distribution is decreasing with respect to time t. The membership and non-membership functions of $\tilde{h}(t_0)$ are reported as follows:

$$\mu_{h(t_0)}(x) = \begin{cases} \left(\dfrac{\mu(t_0x-a_1)}{b_1-a_1}\right)^{\frac{1}{\delta}}, & \frac{a_1}{t_0}\le x\le\frac{b_1}{t_0} \\ \mu^{\frac{1}{\delta}}, & \frac{b_1}{t_0}\le x\le\frac{c_1}{t_0} \\ \left(\dfrac{\mu(d_1-t_0x)}{d_1-c_1}\right)^{\frac{1}{\delta}}, & \frac{c_1}{t_0}\le x\le\frac{d_1}{t_0} \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_{h(t_0)}(x) = \begin{cases} \left(\dfrac{b_1-a_{11}+(1-\nu)(a_{11}-t_0x)}{b_1-a_{11}}\right)^{\frac{1}{\delta}}, & \frac{a_{11}}{t_0}\le x\le\frac{b_1}{t_0} \\ \nu^{\frac{1}{\delta}}, & \frac{b_1}{t_0}\le x\le\frac{c_1}{t_0} \\ \left(\dfrac{d_{11}-c_1-(1-\nu)(d_{11}-t_0x)}{d_{11}-c_1}\right)^{\frac{1}{\delta}}, & \frac{c_1}{t_0}\le x\le\frac{d_{11}}{t_0} \\ 1, & \text{o.w.} \end{cases}.$$

4.4 Some fuzzy reliability characteristics properties

In this section, we provided some relations and properties of the reliability characteristics, with special attention to fuzzy two-parameter Pareto reliability.

Corollary 3 *If $\mu_1\le\mu_2$ and $\nu_1\le\nu_2$, then we have*

(i) $S_{\mu_1}(t)[\alpha_1,\delta]\subset S_{\mu_2}(t)[\alpha_1,\delta]$ *and* $S_{\nu_2}(t)[\alpha_2,\delta]\subset S_{\nu_1}(t)$ $[\alpha_2,\delta]$,
(ii) $S_{\mu_1}(t\mid\tau)[\alpha_1,\delta]\subset S_{\mu_2}(t\mid\tau)[\alpha_1,\delta]$ *and* $S_{\nu_2}(t\mid\tau)[\alpha_2,\delta]\subset S_{\nu_1}(t\mid\tau)[\alpha_2,\delta]$,

(iii) $h_{\mu_1}(t)\,[\alpha_1,\delta] \subset h_{\mu_2}(t)\,[\alpha_1,\delta]$ *and* $h_{\nu_2}(t)\,[\alpha_2,\delta] \subset h_{\nu_1}(t)$
$[\alpha_2,\delta]$.

Corollary 4 *If* $\delta_1 \le \delta_2$, *then we have*

(i) $S_\mu(t)\,[\alpha_1,\delta_1] \subset S_\mu(t)\,[\alpha_1,\delta_1]$ *and* $S_\nu(t)\,[\alpha_2,\delta_2] \subset S_\nu(t)\,[\alpha_2,\delta_1]$,
(ii) $S_\mu(t \mid \tau)\,[\alpha_1,\delta_1] \subset S_\mu(t \mid \tau)\,[\alpha_1,\delta_2]$ *and* $S_\nu(t \mid \tau)\,[\alpha_2,\delta_2] \subset S_\nu(t \mid \tau)\,[\alpha_2,\delta_1]$,
(iii) $h_\mu(t)\,[\alpha_1,\delta_1] \subset h_\mu(t)\,[\alpha_1,\delta_2]$ *and* $h_\nu(t)\,[\alpha_2,\delta_2] \subset h_\nu(t)\,[\alpha_2,\delta_1]$.

Corollary 5 *For every* δ,

$$S\,(t)\left[\mu^{\frac{1}{\delta}},\nu^{\frac{1}{\delta}}\right]=\left[\left(\frac{b_2}{t}\right)^{c_1},\left(\frac{c_2}{t}\right)^{b_1}\right],$$
$$h\,(t)\left[\mu^{\frac{1}{\delta}},\nu^{\frac{1}{\delta}}\right]=\left[\frac{b_1}{t},\frac{c_1}{t}\right],$$
$$S\,(t)\,[0,1,\delta]=\left[\left(\frac{a_2}{t}\right)^{d_1},\left(\frac{d_2}{t}\right)^{a_1}\right],$$
$$h\,(t)\,[0,1]=\left[\frac{a_1}{t},\frac{d_1}{t}\right].$$

Corollary 6 *Consider* $g\,(t_0)\,[\alpha_1,\alpha_2]$ *as* (α_1,α_2)*-cut set of reliability characteristics (GIFR or GIFCR or GIFH) and set*

$$\eta=\frac{1-\frac{1-\alpha_2^\delta}{1-\nu}}{1-\frac{\alpha_1^\delta}{\mu}},\quad z_1=\frac{b-a}{b-a_1},\quad z_2=\frac{d-c}{d_1-c},$$

then

(i) $g(t_0)[\alpha_1,\alpha_2]=\begin{cases}\left[g_\nu^L\,[\alpha_2]\,,g_\nu^U\,[\alpha_2]\right], & \eta<\min(z_1,z_2)\\ \left[g_\mu^L\,[\alpha_1]\,,g_\mu^U\,[\alpha_1]\right], & \eta\ge\max(z_1,z_2)\end{cases}$,
(ii) *if* $\eta=1$ *(i.e.,* $1-\frac{1-\alpha_2^\delta}{1-\nu}=1-\frac{\alpha_1^\delta}{\mu}$*), then* $g\,(t_0)\,[\alpha_1,\alpha_2]=\left[g_\mu^L\,[\alpha_1]\,,g_\mu^U\,[\alpha_1]\right]$,
(iii) *if* $z_1=z_2=\eta$ *then* $g_\mu\,(t_0)\,[\alpha_1]=g_\nu\,(t_0)\,[\alpha_2]=g\,(t_0)\,[\alpha_1,\alpha_2]$,

Corollary 7 *Consider the two-parameter Pareto lifetime distribution, if* $\mu_{g(t_0)}\,(x)=\nu_{g(t_0)}(x)$ *and* $z_1=z_2=z$, *then we have*

(i) $S\,(t_0)\,[\alpha_1,\alpha_2]=S_\mu\,(t_0)\,[\alpha_1]=S_\nu\,(t_0)\,[\alpha_2]=[(\frac{\gamma}{t_0})^\zeta,(\frac{\gamma}{t_0})^\xi]$,
(ii) $h\,(t_0)\,[\alpha_1,\alpha_2]=h_\mu\,(t_0)\,[\alpha_1]=h_\nu\,(t_0)\,[\alpha_2]=\left[\frac{\zeta}{t_0},\frac{\xi}{t_0}\right]$,
(iii) $S\,(t_0\mid\tau)\,[\alpha_1,\alpha_2]=S_\mu\,(t_0\mid\tau)\,[\alpha_1]=S_\nu\,(t_0\mid\tau)\,[\alpha_2]=\left[\left(\frac{\tau}{t_0+\tau}\right)^\zeta,\left(\frac{\tau}{t_0+\tau}\right)^\xi\right]$,
(iv) $\alpha_1=\alpha_2=\left(\frac{(1-\nu)z+\nu}{1+z(\frac{1-\nu}{\mu})}\right)^{\frac{1}{\delta}}$,

where $\zeta=\frac{d_1(\mu-\nu)+c_1(\nu+(1-\nu)z)}{\mu+(1-\nu)z}$ *and* $\xi=\frac{a_1(\mu-\nu)+b_1(\nu+(1-\nu)z)}{\mu+(1-\nu)z}$.

Theorem 1 *Consider the lifetime variables* T_1 *and* T_2 *with the generalized intuitionistic fuzzy density function* $\tilde{f}_1(x,\tilde{\theta},\tilde{\beta})$ *and* $\tilde{f}_2(x,\tilde{\theta},\tilde{\beta})$, *respectively. For every* $t>0$, *if the condition* $\tilde{h}_1(t)\succcurlyeq\tilde{h}_2(t)$ *and* $\tilde{S}_1(\tau)=\tilde{S}_2(\tau)$ *hold, it can be concluded that* $\tilde{S}_1(t|\tau)\preccurlyeq\tilde{S}_2(t|\tau)$.

Proof By using $\tilde{h}_1\,(t)\,(\alpha_1,\alpha_2,\delta)\succcurlyeq\tilde{h}_2\,(t)\,(\alpha_1,\alpha_2,\delta)$ it is induced that

$$\left(h_{1\mu}\,(t)\,[\alpha_1,\delta]\,,h_{1\nu}\,(t)\,[\alpha_2,\delta]\right)\succcurlyeq\left(h_{2\mu}(t)\,[\alpha_1,\delta]\,,h_{2\nu}\,(t)\,[\alpha_2,\delta]\right),$$

which leads to

$$h_{1\mu}\,(t)\,[\alpha_1,\delta]\succcurlyeq h_{2\mu}\,(t)\,[\alpha_1,\delta]\,,$$
$$h_{1\nu}\,(t)\,[\alpha_2,\delta]\succcurlyeq h_{2\nu}\,(t)\,[\alpha_2,\delta]\,.$$

Therefore, for every $\gamma=L,U$, we have

$$h_{1\mu}^\gamma(t)[\alpha_1,\delta]\ge h_{2\mu}^\gamma(t)[\alpha_1,\delta],$$
$$h_{1\nu}^\gamma(t)[\alpha_2,\delta]\ge h_{2\nu}^\gamma(t)[\alpha_2,\delta],$$

consequently,

$$\int_0^{t+\tau}h_{1\mu}^\gamma(x)[\alpha_1,\delta]\mathrm{d}x\ge\int_0^{t+\tau}h_{2\mu}^\gamma(x)[\alpha_1,\delta]\mathrm{d}x,$$
$$\int_0^{t+\tau}h_{1\nu}^\gamma(x)[\alpha_2,\delta]\mathrm{d}x\ge\int_0^{t+\tau}h_{2\nu}^\gamma(x)[\alpha_2,\delta]\mathrm{d}x.$$

Hence, regarding the definition of hazard rate function, we have

$$\int_0^{t+\tau}\frac{f_{1\mu}^\gamma(x)[\alpha_1,\delta]}{1-F_{1\mu}^\gamma(x)[\alpha_1,\delta]}\mathrm{d}x\ \ge\ \int_0^{t+\tau}\frac{f_{2\mu}^\gamma(x)[\alpha_1,\delta]}{1-F_{2\mu}^\gamma(x)[\alpha_1,\delta]}\mathrm{d}x,$$
$$\int_0^{t+\tau}\frac{f_{1\nu}^\gamma(x)[\alpha_2,\delta]}{1-F_{1\nu}^\gamma(x)[\alpha_2,\delta]}\mathrm{d}x\ \ge\ \int_0^{t+\tau}\frac{f_{2\nu}^\gamma(x)[\alpha_2,\delta]}{1-F_{2\nu}^\gamma(x)[\alpha_2,\delta]}\mathrm{d}x,$$

and

$$-\ln\left(1-F_{1\mu}^\gamma\,(t+\tau)\,[\alpha_1,\delta]\right)\ge-\ln\left(1-F_{2\mu}^\gamma\,(t+\tau)\,[\alpha_1,\delta]\right),$$
$$-\ln\left(1-F_{1\nu}^\gamma\,(t+\tau)\,[\alpha_2,\delta]\right)\ge$$

$$-\ln\left(1-F_{2\nu}^{\gamma}\left(t+\tau\right)\left[\alpha_2,\delta\right]\right),$$

subsequently,

$$\begin{aligned}&\left(S_{1\mu}\left(t+\tau\right)\left[\alpha_1,\delta\right],S_{1\nu}\left(t+\tau\right)\left[\alpha_2,\delta\right]\right)\preccurlyeq\\&\quad\left(S_{2}\mu\left(t+\tau\right)\left[\alpha_1,\delta\right],S_{2\nu}\left(t+\tau\right)\left[\alpha_2,\delta\right]\right),\end{aligned}$$

also, $S_1\left(t+\tau\right)\left(\alpha_1,\alpha_2,\delta\right) \preccurlyeq S_2\left(t+\tau\right)\left(\alpha_1,\alpha_2,\delta\right)$ and $\tilde{S}_1(t|\tau)\preccurlyeq\tilde{S}_2(t|\tau)$, which completes the proof. □

Theorem 2 *The increasing condition on the $\tilde{S}(x|t)$ function is a necessary and sufficient condition for $f(x,\tilde{\theta},\tilde{\beta})$ to belong to a class of distribution with a decreasing failure rate (IFR).*

Proof For every $t_1<t_2$ we have $\tilde{S}(x|t_1)\preccurlyeq\tilde{S}(x|t_2)$ and

$$\tilde{S}(x|t_1)(\alpha_1,\alpha_2,\delta)\preccurlyeq\tilde{S}(x|t_2)(\alpha_1,\alpha_2,\delta),$$

we conclude that

$$\begin{aligned}&\left(S_{1\mu}\left(x\mid t_1\right)\left[\alpha_1,\delta\right],S_{1\nu}\left(x\mid t_1\right)\left[\alpha_2,\delta\right]\right)\\&\quad\preccurlyeq\left(S_{2\mu}\left(x\mid t_1\right)\left[\alpha_1,\delta\right],S_{2\nu}\left(x\mid t_2\right)\left[\alpha_2,\delta\right]\right),\end{aligned}$$

then

$$S_{1\mu}\left(x\mid t_1\right)\left[\alpha_1,\delta\right]\preccurlyeq S_{2\mu}\left(x\mid t_2\right)\left[\alpha_1,\delta\right],$$

and

$$S_{1\nu}\left(x\mid t_1\right)\left[\alpha_2,\delta\right]\preccurlyeq S_{2\nu}\left(x\mid t_2\right)\left[\alpha_2,\delta\right].$$

For every $\gamma=L,U$, it can be concluded that

$$S_{\mu}^{\gamma}\left(x\mid t_1\right)\left[\alpha_1,\delta\right]\preccurlyeq S_{\mu}^{\gamma}\left(x\mid t_2\right)\left[\alpha_1,\delta\right],$$

and

$$S_{\nu}^{\gamma}\left(x\mid t_1\right)\left[\alpha_2,\delta\right]\preccurlyeq S_{\nu}^{\gamma}\left(x\mid t_2\right)\left[\alpha_2,\delta\right].$$

Therefore, S_{μ}^{γ} and S_{ν}^{γ} are increasing functions and by using definition of GIFCR function, for $(i,j)=(1,\mu),(2,\nu)$, we have

$$\begin{aligned}S_j^{\gamma}\left(x|t\right)\left[\alpha_i,\delta\right]&=\frac{S_j^{\gamma}(x+t)[\alpha_i,\delta]}{S_j^{\gamma}(t)[\alpha_i,\delta]},\\\frac{\partial S_j^{\gamma}\left(x|t\right)\left[\alpha_i,\delta\right]}{\partial t}&=\frac{-f_j^{\gamma}(x+t)[\alpha_i,\delta]S_j^{\gamma}(t)[\alpha_i,\delta]}{S_j^{\gamma}(t)[\alpha_i,\delta]^2}\\&\quad+\frac{f_j^{\gamma}(t)[\alpha_i,\delta]S_j^{\gamma}(x+t)[\alpha_i,\delta]}{S_j^{\gamma}(t)[\alpha_i,\delta]^2}.\end{aligned}$$

Due to increasing shape of S_j^{γ} function, so it is induced that $\dfrac{\partial S_j^{\gamma}\left(x\mid t\right)\left[\alpha_i\right]}{\partial t}\geq 0$ and hence

$$f_j^{\gamma}\left(t\right)\left[\alpha_i\right]S_j^{\gamma}\left(x+t\right)\left[\alpha_i\right]\geq f_j^{\gamma}\left(x+t\right)\left[\alpha_i\right]S_j^{\gamma}\left(t\right)\left[\alpha_i\right],$$

so,

$$h_j^{\gamma}\left(t\right)\left[\alpha_i,\delta\right]\geq h_j^{\gamma}\left(x+t\right)\left[\alpha_i,\delta\right],$$

and it is concluded that

$$\begin{aligned}&h_{\mu}^{\gamma}(t)\left[\alpha_i,\delta\right]\geq h_{\mu}^{\gamma}\left(x+t\right)\left[\alpha_i,\delta\right],\\&h_{\nu}^{\gamma}(t)\left[\alpha_i,\delta\right]\geq h_{\nu}^{\gamma}(x+t)\left[\alpha_i,\delta\right],\\&h_{\mu}(t)[\alpha_1,\delta]\succcurlyeq h_{\mu}(x+t)[\alpha_1,\delta],\\&h_{\nu}(t)[\alpha_2,\delta]\succcurlyeq h_{\nu}(x+t)[\alpha_2,\delta].\end{aligned}$$

Finally, we have $h(t)(\alpha_1,\alpha_2,\delta)\succcurlyeq h(x+t)(\alpha_1,\alpha_2,\delta)$ and $\tilde{h}(t)\succcurlyeq\tilde{h}(x+t)$, which completes the proof. □

4.5 Generalized intuitionistic fuzzy mean time to failure for Pareto distribution

The mean time to failure (MTTF) is a reliability character that indicates the expected time span when an unrepairable system is active. The MTTF can be used to evaluate reliability and to improve maintenance and system management strategies. The generalized intuitionistic fuzzy mean time to failure (GIFMTTF) of components is the expected time to failure of the fuzzy system and is denoted by $\widetilde{\mathrm{MTTF}}$. In this section, the GIFMTTF function of any component is provided under the two-parameter Pareto lifetime distribution, which is defined as follows

$$\begin{aligned}&\mathrm{GIFMTTF}_j\left[\alpha_i\right]\\&\quad=\left\{\int_{\gamma}^{\infty}xf(x)\mathrm{d}x\mid\lambda\in\lambda_j[\alpha_i,\delta],\gamma\in\gamma_j[\alpha_i,\delta]\right\}\\&\quad=\left\{\frac{\gamma\lambda}{\lambda-1}\mid\lambda\in\lambda_j\left[\alpha_i,\delta\right],\gamma\in\gamma_j[\alpha_i,\delta]\right\},\quad\lambda>1\\&\quad\quad,(i,j)=(1,\mu),(2,\nu),\end{aligned}$$

then

$$\begin{aligned}\mathrm{GIFMTTF}_j\left[\alpha_1\right]=&\left[\frac{\left(a_2+\frac{(b_2-a_2)\alpha_1^{\delta}}{\mu}\right)\left(d_1-\frac{(d_1-c_1)\alpha_1^{\delta}}{\mu}\right)}{\left(d_1-\frac{(d_1-c_1)\alpha_1^{\delta}}{\mu}\right)-1},\right.\\&\left.\frac{\left(d_2-\frac{(d_2-c_2)\alpha_1^{\delta}}{\mu}\right)\left(a_1+\frac{(b_1-a_1)\alpha_1^{\delta}}{\mu}\right)}{\left(a_1+\frac{(b_1-a_1)\alpha_1^{\delta}}{\mu}\right)-1}\right],\end{aligned}$$

$$\mathrm{GIFMTTF}_j\left[\alpha_2\right]=$$

$$\left[\frac{\left(a_{21}+\frac{(b_2-a_{21})(1-\alpha_2^\delta)}{1-\nu}\right)\left(a_{11}+\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}\right)}{\left(a_{11}+\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}\right)-1},\right.$$
$$\left.\frac{\left(d_{21}-\frac{(d_{21}-c_2)(1-\alpha_2^\delta)}{1-\nu}\right)\left(a_{11}+\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}\right)}{\left(a_{11}+\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}\right)-1}\right],$$

5 GIFR function of series and parallel system

The reliability of a system depends on the manner of relation of each component such as the series or parallel structure. In a series structure, the reliability of the system is the minimum of the reliability of components and the system fails even if an individual component failed. On contrary, for parallel structure, the system works even only one component works and reliability is equal to the maximum of the reliability of components. In this section, we focus on the GIFR of series and parallel systems, such that the failure of any component does not depend on any other component.

5.1 Series system

If n-components are connected in a series manner, then the α_i-cut ($i = 1, 2$) of GIFR with generalized intuitionistic fuzzy distribution is given by

$$\begin{aligned} S_j(t)[\alpha_i,\delta] &= \{P(Y_1>t)\,|\,\theta\in\theta_j[\alpha_i,\delta],\beta\in\beta_j[\alpha_i,\delta]\} \\ &= \{S(t)^n\mid\theta\in\theta_j[\alpha_i,\delta],\beta\in\beta_j[\alpha_i,\delta]\} \\ &= \left[S_j^L(t)[\alpha_i],S_j^U(t)[\alpha_i]\right], \end{aligned}$$

where

$$S_j^L(t)[\alpha_i]=\inf_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}}\left(S(t)\right)^n,$$
$$S_j^U(t)[\alpha_i]=\sup_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}}\left(S(t)\right)^n,\quad (i,j)=(1,\mu),(2,\nu).$$

The α_i-cut ($i = 1, 2$) of GIFR with generalized intuitionistic fuzzy two-parameter Pareto distribution is given by

$$S_j(t)[\alpha_i,\delta]=\left\{\left(\frac{\gamma}{t}\right)^{n\lambda}\mid\lambda\in\lambda_j[\alpha_i,\delta],\gamma\in\gamma_j[\alpha_i,\delta]\right\},$$

and

$$S_\mu(t)[\alpha_1,\delta]=\left[\left(\frac{a_2+\frac{(b_2-a_2)\alpha_1^\delta}{\mu}}{t}\right)^{n\left(d_1-\frac{(d_1-c_1)\alpha_1^\delta}{\mu}\right)},\right.$$
$$\left.\left(\frac{d_2-\frac{(d_2-c_2)\alpha_1^\delta}{\mu}}{t}\right)^{n\left(a_1+\frac{(b_1-a_1)\alpha_1^\delta}{\mu}\right)}\right],$$

$$S_\nu(t)[\alpha_2,\delta]=$$
$$\left[\left(\frac{a_{21}+\frac{(b_2-a_{21})(1-\alpha_2^\delta)}{1-\nu}}{t}\right)^{n\left(d_{11}-\frac{(d_{11}-c_1)(1-\alpha_2^\delta)}{1-\nu}\right)},\right.$$
$$\left.\left(\frac{d_{21}-\frac{(d_{21}-c_2)(1-\alpha_1^\delta)}{1-\nu}}{t}\right)^{n\left(a_{11}+\frac{(b_1-a_{11})(1-\alpha_2^\delta)}{1-\nu}\right)}\right].$$

5.2 Parallel system

If n-components are related in a parallel manner, the α_i-cut ($i = 1, 2$) of GIFR with generalized intuitionistic fuzzy distribution is provided as

$$\begin{aligned} S_j(t)[\alpha_i,\delta] &= \{P(Y_n>t)\,|\,\theta\in\theta_j[\alpha_i,\delta],\beta\in\beta_j[\alpha_i,\delta]\} \\ &= \{1-(1-S(t))^n\,|\,\theta\in\theta_j[\alpha_i,\delta],\beta\in\beta_j[\alpha_i,\delta]\} \\ &= \left[S_j^L(t)[\alpha_i],S_j^U(t)[\alpha_i]\right], \\ &\qquad (i,j)=(1,\mu),(2,\nu), \end{aligned}$$

where

$$S_j^L(t)[\alpha_i]=\inf_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}}(1-(1-S(t)))^n,$$
$$S_j^U(t)[\alpha_i]=\sup_{\substack{\theta\in\theta_j[\alpha_i,\delta]\\ \beta\in\beta_j[\alpha_i,\delta]}}(1-(1-S(t)))^n,\quad (i,j)=(1,\mu),(2,\nu).$$

The α_i-cut ($i = 1, 2$) of GIFR with generalized intuitionistic fuzzy two-parameter Pareto distribution is represented as

$$\begin{aligned} &S_j(t)[\alpha_i,\delta] \\ &=\left\{1-\left(1-\left(\tfrac{\gamma}{t}\right)^\lambda\right)^n\mid\lambda\in\lambda_j[\alpha_i,\delta],\gamma\in\gamma_j[\alpha_i,\delta]\right\}, \end{aligned}$$

and

$$S_\mu(t)[\alpha_1,\delta]=$$
$$\left[1-\left(1-\left(\frac{a_2+\frac{(b_2-a_2)\alpha_1^\delta}{\mu}}{t}\right)^{d_1-\frac{(d_1-c_1)\alpha_1^\delta}{\mu}}\right)^n,\right.$$
$$\left.1-\left(1-\left(\frac{d_2+\frac{(d_2-c_2)\alpha_1^\delta}{\mu}}{t}\right)^{a_1+\frac{(b_1-a_1)\alpha_1^\delta}{\mu}}\right)^n\right],$$

$$S_\nu(t)[\alpha_2,\delta]=$$

$$\Big[1-\Big(1-\big(\frac{a_{21}+\frac{(b_2-a_{21})(1-\alpha_2^{\delta})}{1-\nu}}{t}\big)^{d_{11}-\frac{(d_{11}-c_1)(1-\alpha_2^{\delta})}{1-\nu}}\Big)^n,$$
$$1-\Big(1-\big(\frac{d_{21}-\frac{(d_{21}-c_2)(1-\alpha_2^{\delta})}{1-\nu}}{t}\big)^{a_{11}+\frac{(b_1-a_{11})(1-\alpha_2^{\delta})}{1-\nu}}\Big)^n\Big].$$

6 Numerical example

Let the lifetime of electronic component is modeled by the two-parameter Pareto distribution with generalized intuitionistic fuzzy scale and shape parameters

$$\tilde{\lambda} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.75, 0.25, 2),$$
$$\tilde{\gamma} = (1, 1, 1.25, 1.5, 1.75, 1.75, 0.75, 0.25, 2).$$

Then cut sets of GIFP of $X \le 2$ is obtained, for $(i, j) = (1, \mu), (2, \nu)$, as follows:

$$P_j\,(X \le 2)\,[\alpha_i, 2] = \{1-\left(\frac{\gamma}{2}\right)^{\lambda} | \lambda \in \lambda_j\,[\alpha_i, 2]\,, \gamma \in \gamma_j\,[\alpha_i, 2]\},$$

and

$$P_\mu\,(X \le 2)\,[\alpha_1, 2] = \left[1-\left(\frac{1.75}{2}-\frac{\alpha_1^2}{6}\right)^{0.2+\frac{0.1\alpha_1^2}{0.75}},\right.$$
$$\left.1-\left(\frac{1}{2}+\frac{\alpha_1^2}{6}\right)^{0.5-\frac{0.1\alpha_1^2}{0.75}}\right],$$
$$P_\nu\,(X \le 2)\,[\alpha_2, 2] = \left[1-\left(\frac{1.75}{2}-\frac{(1-\alpha_2^2)}{6}\right)^{0.1+\frac{0.2(1-\alpha_2^2)}{0.75}},\right.$$
$$\left.1-\left(\frac{1}{2}+\frac{1-\alpha_2^2}{6}\right)^{0.6-\frac{0.2(1-\alpha_2^2)}{0.75}}\right].$$

The membership and non-membership functions of the GIFP are represented in Fig. 1. Also, for several values of α_1 and α_2, the α_1-cut set of membership and α_2-cut set of non-membership bands of GIFP and (α_1, α_2)-cut bands of GIFP are reported in Table 1, respectively.

Regarding Table 1, by increasing α_1 and decreasing α_2, the ambiguity in membership and non-membership bands of GIFP is decreased as well as the bands of GIFP.

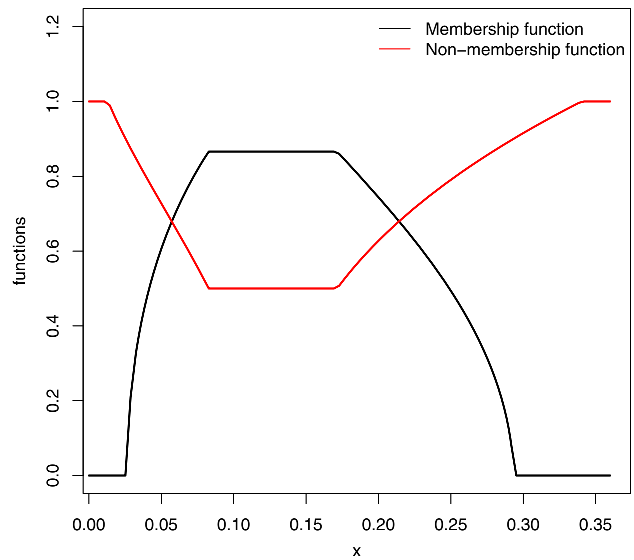

Fig. 1 The membership and non-membership functions of GIFP

Table 1 The α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut band of GIFP, for different values of α_1, α_2

(α_1, α_2)	$P_\mu[\alpha_1]$	$P_\nu[\alpha_2]$	$P[\alpha_1, \alpha_2]$
(0,1)	[0.0263,0.2928]	[0.0132,0.3402]	[0.0263,0.2928]
(0.2,0.9)	[0.0285,0.2856]	[0.0253,0.2932]	[0.0285,0.2856]
(0.3,0.8)	[0.0314,0.2766]	[0.0393,0.2534]	[0.0393,0.2534]
(0.4,0.7)	[0.0357,0.2643]	[0.0541,0.2201]	[0.0541,0.2201]
(0.7,0.6)	[0.0595,0.2098]	[0.0688,0.1931]	[0.0688,0.1931]
$(\sqrt{0.75}, \sqrt{0.25})$	[0.0826,0.1713]	[0.0826,0.1713]	[0.0826,0.1713]

The cut sets of GIFR are given by

$$S_\mu(t)\,[\alpha_1, 2] = \left[\left(\frac{1}{t}+\frac{\alpha_1^2}{3t}\right)^{0.5-\frac{0.1\alpha_1^2}{0.75}},\right.$$
$$\left.\left(\frac{1.75}{t}-\frac{\alpha_1^2}{3t}\right)^{0.2+\frac{0.1\alpha_1^2}{0.75}}\right],$$
$$S_\nu(t)\,[\alpha_2, 2] = \left[\left(\frac{1}{t}+\frac{1-\alpha_2^2}{3t}\right)^{0.6-\frac{0.2(1-\alpha_2^2)}{0.75}},\right.$$
$$\left.\left(\frac{1.75}{t}-\frac{1-\alpha_2^2}{3t}\right)^{0.1+\frac{0.2(1-\alpha_2^2)}{0.75}}\right].$$

Figure 2 represents the surfaces of GIFR for different angles.

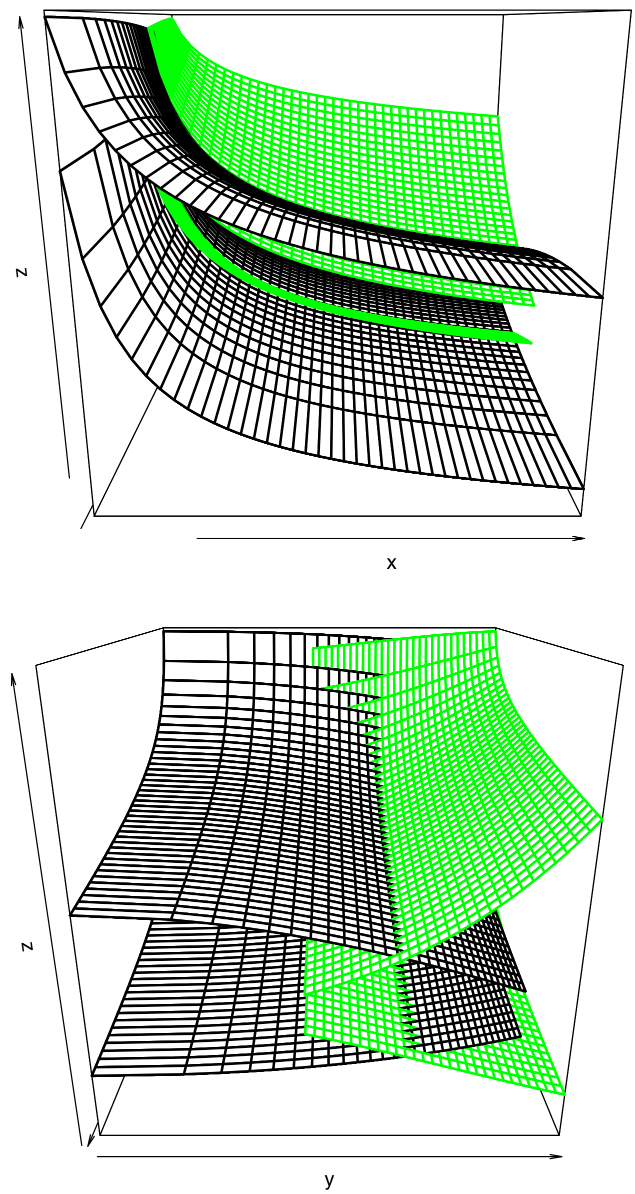

Fig. 2 The surfaces of GIFR function

The bands for $\alpha_1 = \sqrt{0.15}$ and $\alpha_2 = \sqrt{0.25}$ are given by

$$S_\mu(t)\left[\sqrt{0.15}, 2\right] = \left[\left(\frac{1.05}{t}\right)^{0.5-\frac{0.1}{3}}, \left(\frac{1.7}{t}\right)^{0.2+\frac{0.1}{3}}\right],$$

$$S_\nu(t)\left[\sqrt{0.25}, 2\right] = \left[\left(\frac{1.25}{t}\right)^{0.4}, \left(\frac{1.5}{t}\right)^{0.3}\right].$$

The GIFR bands for $\alpha_1 = \sqrt{0.15}$ and $\alpha_2 = \sqrt{0.25}$ are plotted in Fig. 3. As can be seen, the ambiguity of GIFR is increased by increasing time t, due to the increase in both bandwidths of membership and non-membership functions.

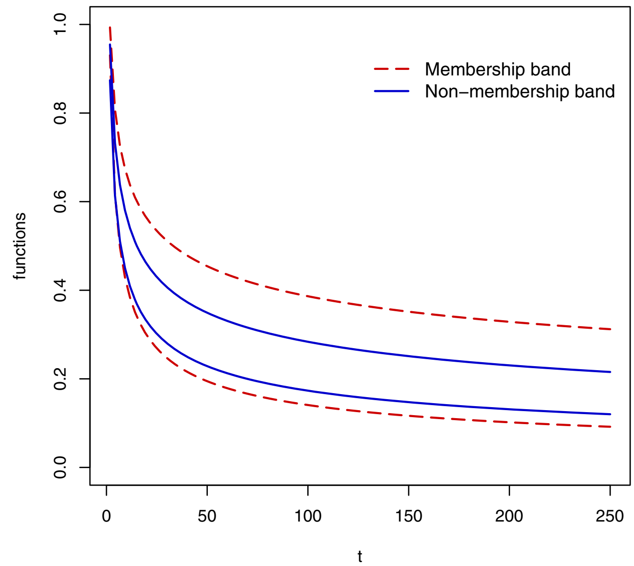

Fig. 3 The GIFR bands for $\alpha_1 = \sqrt{0.15}$ and $\alpha_2 = \sqrt{0.25}$

If set $t = 2$, then cut sets of GIFR are computed as

$$S_\mu(2)\left[\alpha_1, 2\right] = \left[\left(\frac{1}{2} + \frac{\alpha_1^2}{6}\right)^{0.5-\frac{0.1\alpha_1^2}{0.75}}, \left(\frac{1.75}{2} - \frac{\alpha_1^2}{6}\right)^{0.2+\frac{0.1\alpha_1^2}{0.75}}\right],$$

$$S_\nu(2)\left[\alpha_2, 2\right] = \left[\left(\frac{1}{2} + \frac{1-\alpha_2^2}{6}\right)^{0.6-\frac{0.2(1-\alpha_2^2)}{0.75}}, \left(\frac{1.75}{2} - \frac{1-\alpha_2^2}{6}\right)^{0.1+\frac{0.2(1-\alpha_2^2)}{0.75}}\right].$$

The membership and non-membership functions of GIFR are depicted in Fig. 4.

In Table 2, the α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut bands of GIFR are prepared, for different combinations of cuts α_1 and α_2.

Based on Table 2, by increasing α_1 and decreasing α_2, the vagueness in membership and non-membership bands of GIFR and bands of GIFR is decreased.

The cut sets of GIFR are computed as follows:

$$S_\mu(t)\left[\alpha_1, \delta\right] =$$

Table 2 The α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut band of GIFR, for different values of α_1, α_2

(α_1, α_2)	$S_\mu(t)[\alpha_1, 2]$	$S_\nu(t)[\alpha_2, 2]$	$S(t)[\alpha_1, \alpha_2, 2]$
[0,1]	$\left[(\frac{1}{t})^{0.5}, (\frac{1.75}{t})^{0.2}\right]$	$\left[(\frac{1}{t})^{0.6}, (\frac{1.75}{t})^{0.1}\right]$	$\left[(\frac{1}{t})^{0.5}, (\frac{1.75}{t})^{0.2}\right]$
(0.3,0.8)	$\left[(\frac{1.03}{t})^{0.488}, (\frac{1.72}{t})^{0.212}\right]$	$\left[(\frac{1.12}{t})^{0.504}, (\frac{1.63}{t})^{0.196}\right]$	$\left[(\frac{1.12}{t})^{0.504}, (\frac{1.63}{t})^{0.196}\right]$
(0.4,0.7)	$\left[(\frac{1.05}{t})^{0.4786}, (\frac{1.69}{t})^{0.2213}\right]$	$\left[(\frac{1.17}{t})^{0.4639}, (\frac{1.58}{t})^{0.236}\right]$	$\left[(\frac{1.17}{t})^{0.4639}, (\frac{1.58}{t})^{0.236}\right]$
(0.7,0.6)	$\left[(\frac{1.16}{t})^{0.4346}, (\frac{1.58}{t})^{0.2653}\right]$	$\left[(\frac{1.21}{t})^{0.4293}, (\frac{1.53}{t})^{0.2706}\right]$	$\left[(\frac{1.21}{t})^{0.4293}, (\frac{1.53}{t})^{0.2706}\right]$
$(\sqrt{0.75}, \sqrt{0.25})$	$\left[(\frac{1.25}{t})^{0.4}, (\frac{1.5}{t})^{0.3}\right]$	$\left[(\frac{1.25}{t})^{0.4}, (\frac{1.5}{t})^{0.3}\right]$	$\left[(\frac{1.25}{t})^{0.4}, (\frac{1.5}{t})^{0.3}\right]$

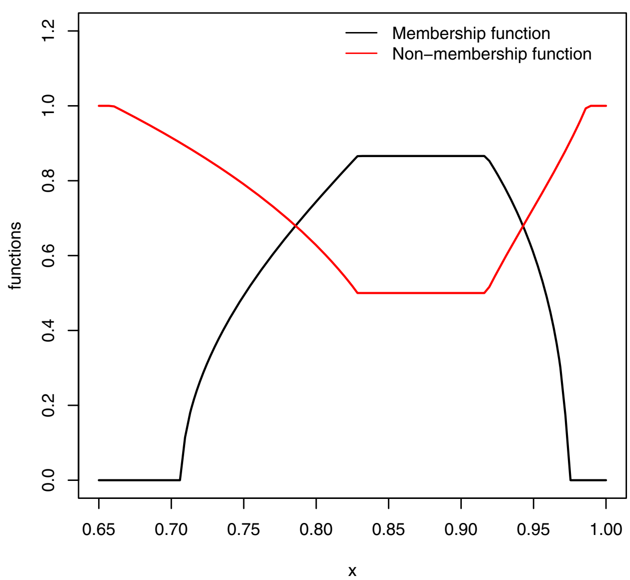

Fig. 4 The membership and non-membership functions of GIFR

$$\left[\left(\frac{1}{t}+\frac{\alpha_1^\delta}{3t}\right)^{0.5-\frac{0.1\alpha_1^\delta}{0.75}}, \left(\frac{1.75}{t}-\frac{\alpha_1^\delta}{3t}\right)^{0.2+\frac{0.1\alpha_1^\delta}{0.75}}\right],$$

$$S_\nu(t)[\alpha_2, \delta] = \left[\left(\frac{1}{t}+\frac{1-\alpha_2^\delta}{3t}\right)^{0.6-\frac{0.2(1-\alpha_2^\delta)}{0.75}}, \left(\frac{1.75}{t}-\frac{1-\alpha_2^\delta}{3t}\right)^{0.1+\frac{0.2(1-\alpha_2^\delta)}{0.75}}\right].$$

The reliability bands for the different values of δ and cut sets (α_1, α_2) are represented in Fig. 5; the large values of the parameter δ lead to less reliability bandwidth and more accurate reliability. Also, by increasing α_1 and decreasing α_2, the uncertainty in reliability bands is reduced. Also, by increasing time t, the uncertainty in GIFR function is increased.

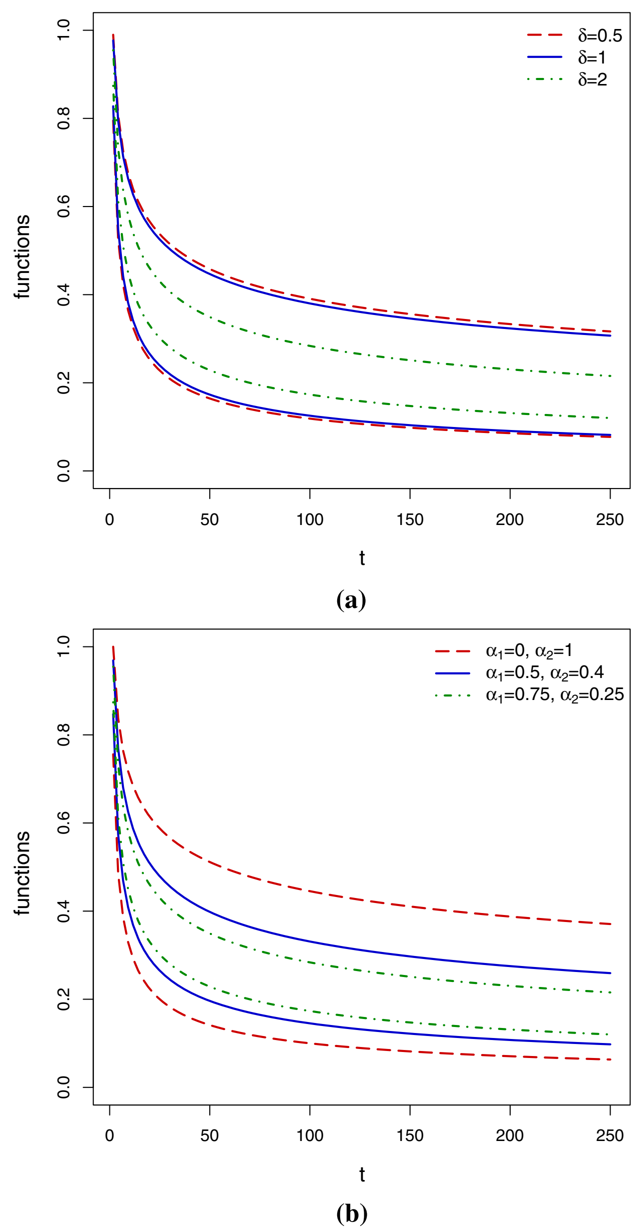

Fig. 5 **a** The reliability bands of $S(t)[0.05, 0.5, \delta]$, **b** The reliability bands of $S(t)[\alpha_1, \alpha_2, 1]$

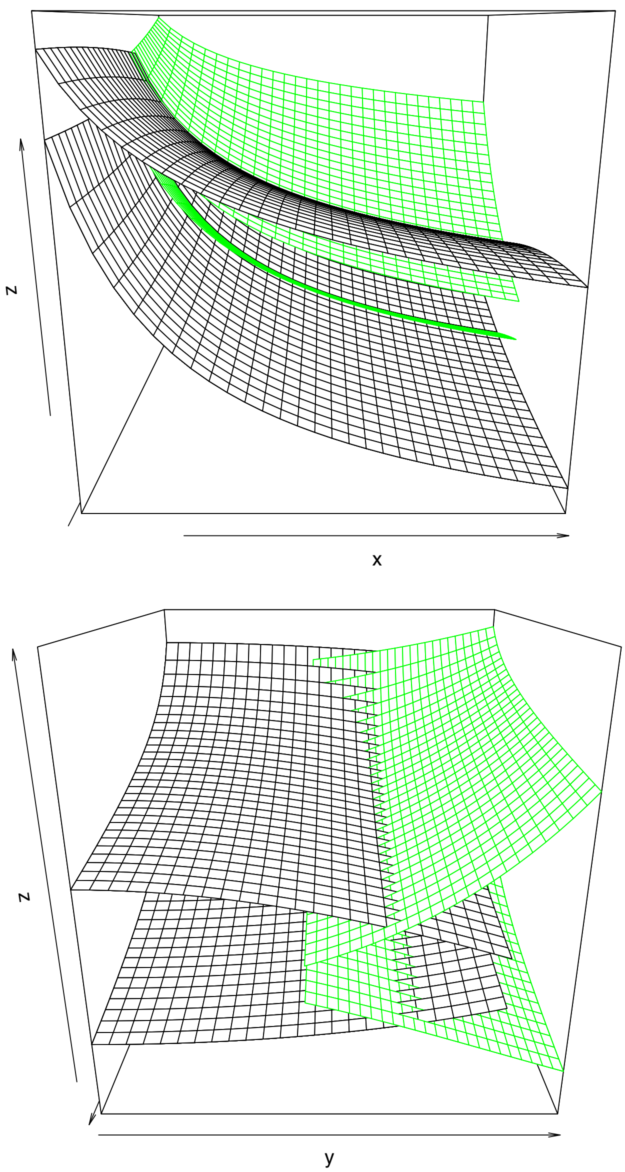

Fig. 6 The surfaces of GIFCR function

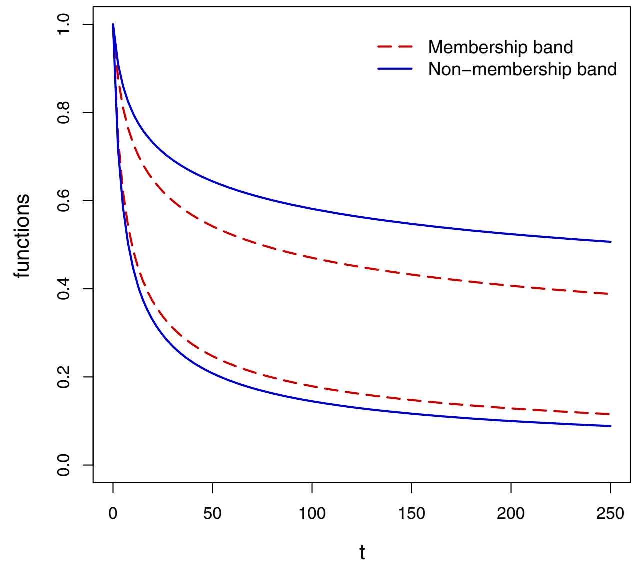

Fig. 7 The GIFCR bands for $\alpha_1 = \sqrt{0.1}$ and $\alpha_2 = \sqrt{0.8}$

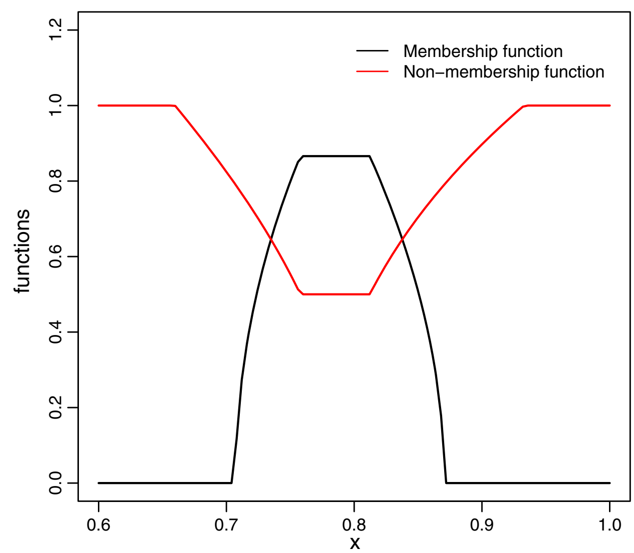

Fig. 8 The membership and non-membership functions of GIFCR

The α_i-cuts of GIFCR for $i = 1, 2$ are given by

$$S_\mu\left(t|\tau\right)[\alpha_1, 2] = \left[\left(\frac{\tau}{t+\tau}\right)^{0.5-\frac{0.1\alpha_1^2}{0.75}}, \left(\frac{\tau}{t+\tau}\right)^{0.2+\frac{0.1\alpha_1^2}{0.75}}\right],$$

$$S_\nu\left(t|\tau\right)[\alpha_2, 2] = \left[\left(\frac{\tau}{t+\tau}\right)^{0.6-\frac{0.2(1-\alpha_2^2)}{0.75}}, \left(\frac{\tau}{t+\tau}\right)^{0.1+\frac{0.2(1-\alpha_2^2)}{0.75}}\right].$$

Figure 6 shows surfaces of the GIFCR function from different angles.

The GIFCR bands with $\tau = 3$ for $\alpha_1 = \sqrt{0.1}$ and $\alpha_2 = \sqrt{0.8}$ are expressed as

$$S_\mu\left(t \mid \tau\right)[0, 2] = \left[\left(\frac{3}{t+3}\right)^{\frac{73}{150}}, \left(\frac{3}{t+3}\right)^{\frac{32}{150}}\right],$$

$$S_\nu\left(t \mid \tau\right)[1, 2] = \left[\left(\frac{3}{t+3}\right)^{\frac{82}{150}}, \left(\frac{3}{t+3}\right)^{\frac{23}{150}}\right].$$

The GIFCR bands for $\alpha_1 = \sqrt{0.1}$ and $\alpha_2 = \sqrt{0.8}$ are depicted in Fig. 7, which indicates that increasing time t leads to increasing the bandwidth which is equivalent to increasing in uncertainty. Let $t_0 = 3$, $\tau = 3$, the membership and non-membership functions of $\tilde{S}(t_0 \mid \tau)$ are obtained as follows:

$$\mu_{S(t_0|\tau)}(x) = \begin{cases} \left(7.5\frac{\ln x}{\ln 2} + 3.75\right)^{\frac{1}{2}}, & \left(\frac{1}{2}\right)^{0.5} \le x \le \left(\frac{1}{2}\right)^{0.4} \\ \sqrt{0.75}, & \left(\frac{1}{2}\right)^{0.4} \le x \le \left(\frac{1}{2}\right)^{0.3} \\ \left(-7.5\frac{\ln x}{\ln 2} - 1.5\right)^{\frac{1}{2}}, & \left(\frac{1}{2}\right)^{0.3} \le x \le \left(\frac{1}{2}\right)^{0.2} \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_{S(t_0|\tau)}(x) = \begin{cases} \left(-3.75\frac{\ln x}{\ln 2} - 1.25\right)^{\frac{1}{2}}, & \left(\frac{1}{2}\right)^{0.6} \le x \le \left(\frac{1}{2}\right)^{0.4} \\ \sqrt{0.25}, & \left(\frac{1}{2}\right)^{0.4} \le x \le \left(\frac{1}{2}\right)^{0.3} \\ \left(3.75\frac{\ln x}{\ln 2} + 1.375\right)^{\frac{1}{2}}, & \left(\frac{1}{2}\right)^{0.3} \le x \le \left(\frac{1}{2}\right)^{0.1} \\ 1, & \text{o.w.} \end{cases}.$$

The $(\sqrt{0.75}, \sqrt{0.25})$-cut set of $\tilde{S}(t|\tau)$ is given by

$$S_\mu(t|\tau)[1, 2] = \left[\left(\frac{\tau}{\tau + t}\right)^{0.4}, \left(\frac{\tau}{\tau + t}\right)^{0.3}\right],$$

$$S_\nu(t|\tau)[0, 2] = \left[\left(\frac{\tau}{\tau + t}\right)^{0.4}, \left(\frac{\tau}{\tau + t}\right)^{0.3}\right],$$

$$S(t|\tau)[\sqrt{0.75}, \sqrt{0.25}] = S_\mu(t|\tau)[1, 2] \cap S_\nu(t|\tau)[0, 2] = \left[\left(\frac{\tau}{\tau + t}\right)^{0.4}, \left(\frac{\tau}{\tau + t}\right)^{0.3}\right].$$

The membership and non-membership functions of GIFCR are represented in Fig. 8.

The α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut bands of GIFCR, for different combinations of cut sets (α_1, α_2) are assembled in Table 3. Based on Table 3, the more accurate bands of membership and non-membership of GIFCR and bands of GIFCR are attained by the maximum value of α_1 and minimum of α_2.

The α_i-cuts of GIFH function for $i = 1, 2$ are given by

$$h_\mu(t)[\alpha_1, 2] = \left[\frac{1.5 + \alpha_1^2}{7.5t}, \frac{3.75 - \alpha_1^2}{7.5t}\right],$$

$$h_\nu(t)[\alpha_2, 2] = \left[\frac{2.75 - 2\alpha_2^2}{7.5t}, \frac{2.5 + 2\alpha_2^2}{7.5t}\right].$$

Figure 9 shows the surfaces of GIFH function from different angles.

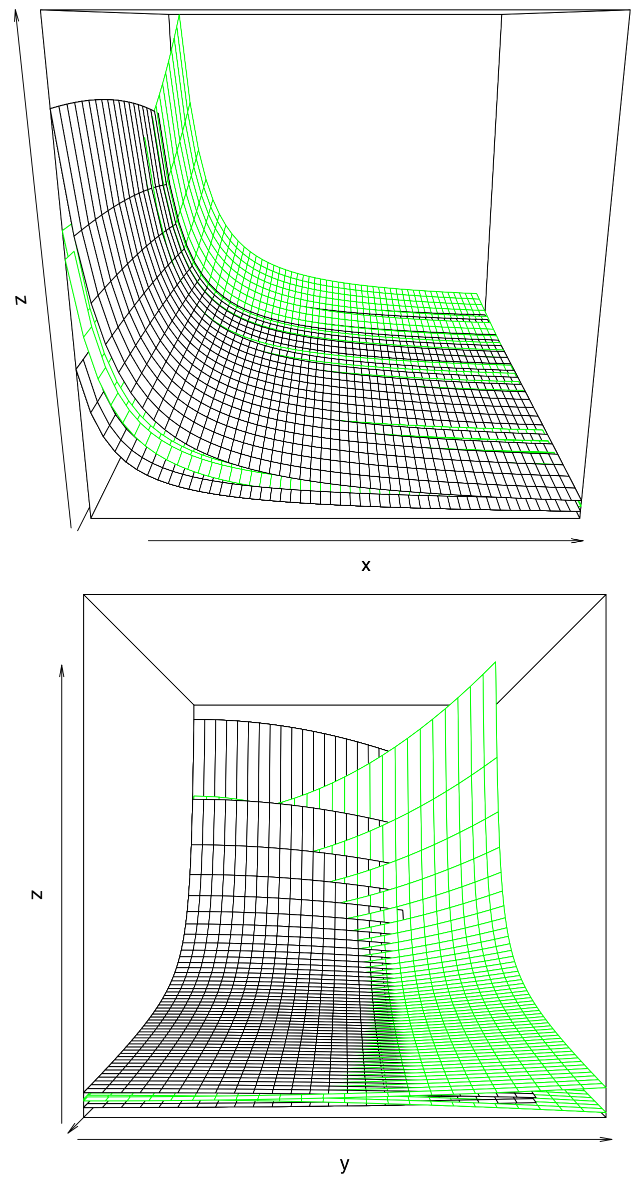

Fig. 9 The surfaces of the GIFH function

The GIFH bands for $\alpha_1 = \sqrt{0.1}$ and $\alpha_2 = \sqrt{0.8}$ are computed as

$$h_\mu(t)\left[\sqrt{0.1}, 2\right] = \left[\frac{1.6}{7.5t}, \frac{3.65}{7.5t}\right],$$

$$h_\nu(t)\left[\sqrt{0.8}, 2\right] = \left[\frac{1.15}{7.5t}, \frac{4.1}{7.5t}\right].$$

The GIFH bands of membership and non-membership functions for $\alpha_1 = \sqrt{0.1}$ and $\alpha_2 = \sqrt{0.8}$ are exhibited in Fig. 10. Analogously, increasing time t causes the more accuracy in GIFH function.

The membership and non-membership functions of $\tilde{h}(2)$ are represented as below:

Table 3 The α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut band of GIFCR, for different values of α_1, α_2

(α_1, α_2)	$S_\mu(t \mid \tau)\,[\alpha_1, 2]$	$S_\nu(t \mid \tau)\,[\alpha_2, 2]$	$S(t \mid \tau)[\alpha_1, \alpha_2]$
(0,1)	$\left[(\frac{\tau}{t+\tau})^{0.5}, (\frac{\tau}{t+\tau})^{0.2}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.6}, (\frac{\tau}{t+\tau})^{0.1}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.5}, (\frac{\tau}{t+\tau})^{0.2}\right]$
(0.3,0.8)	$\left[(\frac{\tau}{t+\tau})^{0.488}, (\frac{\tau}{t+\tau})^{0.212}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.504}, (\frac{\tau}{t+\tau})^{0.196}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.488}, (\frac{\tau}{t+\tau})^{0.212}\right]$
(0.4,0.7)	$\left[(\frac{\tau}{t+\tau})^{0.4786}, (\frac{\tau}{t+\tau})^{0.2213}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.4639}, (\frac{\tau}{t+\tau})^{0.236}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.4639}, (\frac{\tau}{t+\tau})^{0.236}\right]$
(0.7,.6)	$\left[(\frac{\tau}{t+\tau})^{0.4346}, (\frac{\tau}{t+\tau})^{0.2653}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.4293}, (\frac{\tau}{t+\tau})^{0.2706}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.4293}, (\frac{\tau}{t+\tau})^{0.2706}\right]$
$(\sqrt{0.75}, \sqrt{0.25})$	$\left[(\frac{\tau}{t+\tau})^{0.4}, (\frac{\tau}{t+\tau})^{0.3}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.4}, (\frac{\tau}{t+\tau})^{0.3}\right]$	$\left[(\frac{\tau}{t+\tau})^{0.4}, (\frac{\tau}{t+\tau})^{0.3}\right]$

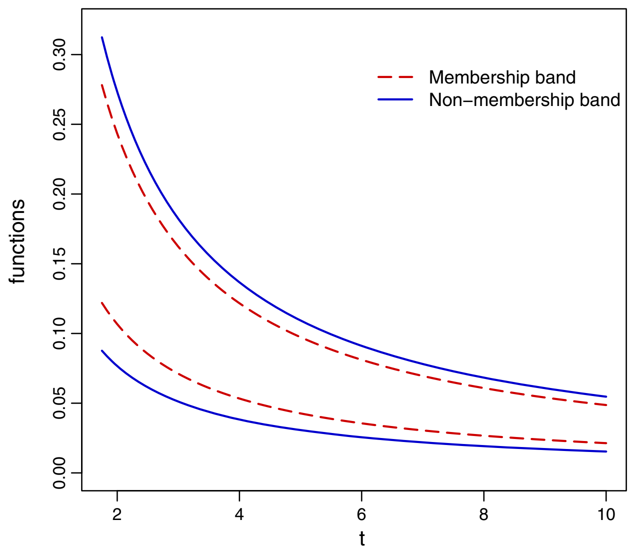

Fig. 10 The GIFH bands for $\alpha_1 = \sqrt{0.1}$ and $\alpha_2 = \sqrt{0.8}$

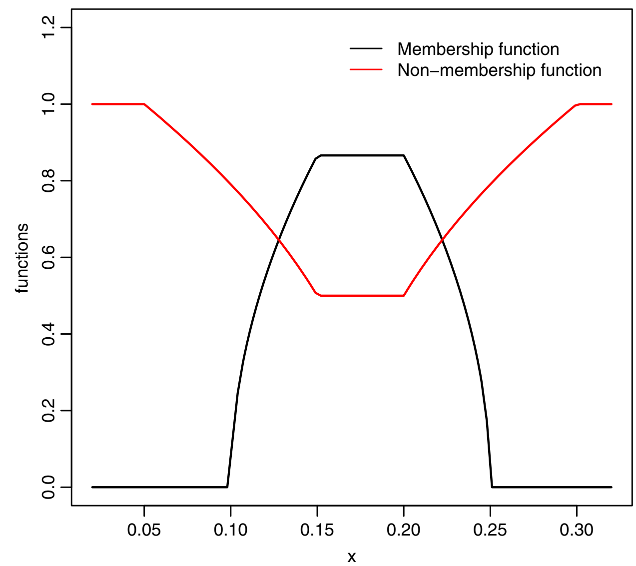

Fig. 11 The membership and non-membership functions of GIFH

$$\mu_{h(t_0)}(x) = \begin{cases} (15x - 1.5)^{0.5}, & 0.1 \le x \le 0.15 \\ \sqrt{0.75}, & 0.15 \le x \le 0.2 \\ (0.75 - 15x)^{0.5}, & 0.2 \le x \le 0.25 \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_{h(t_0)}(x) = \begin{cases} \left(\dfrac{2.75 - 15x}{2}\right)^{0.5}, & 0.05 \le x \le 0.15 \\ \sqrt{0.25}, & 0.15 \le x \le 0.2 \\ \left(\dfrac{15x - 2.5}{2}\right)^{0.5}, & 0.2 \le x \le 0.3 \\ 1, & \text{o.w.} \end{cases}.$$

The membership and non-membership functions of GIFH are displayed in Fig. 11.

Figure 4 reports the α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut bands of GIFH, for different combinations of α_1, α_2, which has the same results as other counterpart tables.

Generally based on Tables 1, 2, 3 and 4, it is inferred that increasing α_1 and decreasing α_2 lead to ambiguity decreasing of the fuzzy reliability characteristics, including GIFR, GIFCR and GIFH bands. Moreover, regarding Figs. 3, 7 and 10, the GIFR, GIFCR and GIFH are decreasing functions with respect to t.

Conclusion

In the present paper, we extend the GIFN$_B$ to analyze the system reliability with the special two-parameter Pareto distribution discussion. Both scale and shape parameters of the two-parameter Pareto distribution are considered as GIFN$_B$, and various generalized intuitionistic fuzzy reliability characteristics are obtained. The reliability characteristics are represented through bands, which attained their most precise bands for large value of the cut set of membership and small value of the cut set of non-membership functions. The the-

Table 4 The α_1-cut of membership, α_2-cut of non-membership bands and (α_1, α_2)-cut band of GIFH, for different values of α_1, α_2

(α_1, α_2)	$h_\mu(t)\,[\alpha_1, 2]$	$h_\nu(t)\,[\alpha_2, 2]$	$h(t)\,[\alpha_1, \alpha_2]$
(0,1)	$[\frac{0.2}{t}, \frac{0.5}{t}]$	$[\frac{0.1}{t}, \frac{0.6}{t}]$	$[\frac{0.2}{t}, \frac{0.5}{t}]$
(0.3,0.8)	$[\frac{0.212}{t}, \frac{0.488}{t}]$	$[\frac{0.196}{t}, \frac{0.504}{t}]$	$[\frac{0.212}{t}, \frac{0.488}{t}]$
(0.4,0.7)	$[\frac{0.2213}{t}, \frac{0.4786}{t}]$	$[\frac{0.236}{t}, \frac{0.464}{t}]$	$[\frac{0.236}{t}, \frac{0.464}{t}]$
(0.5,0.5)	$[\frac{0.2853}{t}, \frac{0.4146}{t}]$	$[\frac{0.2706}{t}, \frac{0.4293}{t}]$	$[\frac{0.2853}{t}, \frac{0.4146}{t}]$
(0.8,0.6)	$[\frac{0.2853}{t}, \frac{0.4146}{t}]$	$[\frac{0.2706}{t}, \frac{0.4293}{t}]$	$[\frac{0.2853}{t}, \frac{0.4146}{t}]$
$(\sqrt{0.75}, \sqrt{0.25})$	$[\frac{0.3}{t}, \frac{0.4}{t}]$	$[\frac{0.3}{t}, \frac{0.4}{t}]$	$[\frac{0.3}{t}, \frac{0.4}{t}]$

oretical results are evaluated by a comprehensive numerical approach. In this context, our study covers several research kinds of literature in fuzzy subjects.

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Data availability Enquiries about data availability should be directed to the authors.

Declarations

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of this paper.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

Aikhuele DO (2020) Intuitionistic fuzzy model for reliability management in wind turbine system. Appl Comput Inform 16(1/2):181–194

Akbari MG, Hesamian G (2020) Time-dependent intuitionistic fuzzy system reliability analysis. Soft Comput 24:14441–14448

Amin ZH (2008) Bayesian inference for the Pareto lifetime model under progressive censoring with binomial removals. J Appl Stat 35(11):1203–1217

Arya A, Yadav SP (2019) Development of intuitionistic fuzzy data envelopment analysis models and intuitionistic fuzzy input-output targets. Soft Comput 23:8975–899

Atanassov KT (2017) Type-1 fuzzy sets and intuitionistic fuzzy sets. Algorithms 10(3):106

Baloui Jamkhaneh E (2011) An evaluation of the systems reliability using fuzzy lifetime distribution. J Appl Math 7(8):73–80

Baloui Jamkhaneh E (2014) Analyzing system reliability using fuzzy Weibull lifetime distribution. Int J Appl Oper Res 4:93–102

Baloui Jamkhaneh E (2016) A value and ambiguity-based ranking method of generalized intuitionistic fuzzy number. Res Commun Math Math Sci 6:89–103

Baloui Jamkhaneh E (2017) System reliability using generalized intuitionistic fuzzy exponential lifetime distribution. Int J Soft Comput Eng 7(4):1–7

Baloui Jamkhane E, Nadarajah S (2015) A new generalized intuitionistic fuzzy sets. Hacettepe J Math Stat 44(6):1537–1551

Citakoglu H (2015) Comparison of artificial intelligence techniques via empirical equations for prediction of solar radiation. Comput Electron Agric 118:28–37

Citakoglu H (2017) Comparison of artificial intelligence techniques for prediction of soil temperatures in Turkey. Theor Appl Climatol 130(1):545–556

Citakoglu H, Cobaner M, Haktanir T, Kisi O (2014) Estimation of monthly mean reference evapotranspiration in Turkey. Water Resour Manag 28(1):99–113

Cobaner M, Citakoglu H, Kisi O, Haktanir T (2014) Estimation of mean monthly air temperatures in Turkey. Comput Electron Agric 109:71–79

Ebrahimnejad A, Baloui Jamkhaneh E (2018) System reliability using generalized intuitionistic fuzzy Rayleigh lifetime distribution. Appl Appl Math Int J 13(1):97–113

El-Damcese M, Faheem F, El-Ghamry E (2014) Reliability analysis of three elements in series and parallel systems under time-varying fuzzy failure rate. Int J Eng Trans A Basics 27(4):553–560

Feng F, Zheng Y, Alcantud JCR, Wang Q (2020) Minkowski weighted score functions of intuitionistic fuzzy values. Mathematics 8(7):1143

Fu J, Xu A, Tang Y (2012) Objective Bayesian analysis of Pareto distribution under progressive type-II censoring. Stat Prob Lett 82(10):1829–1836

Ghitany ME, Gömez-Déniz E, Nadarajah S (2018) A new generalization of Pareto distribution and its application to insurance data. J Risk Financ Manag 11(10):1–14

Husniah H, Supriatna AK (2021) Computing the number of failures for fuzzy Weibull hazard function. Mathematics 9(22):2858

Kozae AM, Shokry M, Omran M (2020) Intuitionistic fuzzy set and its application in Corona Covid-19. Appl Comput Math 9(5):146–154

Kumar M, Yadav SP, Kumar S (2013) Fuzzy system reliability evaluation using time-dependent intuitionistic fuzzy set. Int J Syst Sci 44(1):50–66

Lee S, Kim JHT (2018) Exponentiated generalized Pareto distribution: properties and applications towards extreme value theory. Commun Stat-Theory Methods 48(8):2014–2038

Liu Y, Tang W, Zhao R (2007) Reliability and mean time to failure of unrepairable systems with fuzzy random lifetimes. IEEE Trans Fuzzy Syst 15(5):1009–1026

Mahapatra GS, Mahapatra BS (2010) Intuitionistic fuzzy fault tree analysis using intuitionistic fuzzy numbers. Int Math Forum 5(21):1015–1024

Mahapatra GS, Roy TK (2009) Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations. World Acad Sci Eng Technol 50:574–580

Mahapatra GS, Roy TK (2012) Reliability evaluation of complex system with fuzzy reliability of components using interval nonlinear programming. Electron J Appl Stat Anal 5(2):151–163

Mahmoodirad A, Allahviranloo T, Niroomand S (2019) A new effective solution method for fully intuitionistic fuzzy transportation problem. Soft Comput 23(12):4521–4530

Mishra A, Kumar A (2020) JMD method for transforming an unbalanced fully intuitionistic fuzzy transportation problem into a balanced fully intuitionistic fuzzy transportation problem. Soft Comput 24(20):15639–15654

Pak A, Parham G, Saraj M (2014) Reliability estimation in Rayleigh distribution based on fuzzy lifetime data. Int J Syst Assur Eng Manag 5(4):487–494

Pan CY, Wei WL, Zhan CY, Song LK, Lu C, Liu LJ (2015) Reliability analysis of turbine blades based on fuzzy response surface method. J Intell Fuzzy Syst 29:2467–2474

Pękala B, Grochowalski P, Szmidt E (2021) New transitivity of Atanassov's intuitionistic fuzzy sets in a decision making model. Int J Appl Math Comput Sci 31(4):563–576

Prakash G (2017) Some Bayes estimators for Pareto Type-II progressive censored data. Electron J Appl Stat Anal 10:257–270

Pramanik R, Baidya DK, Dhang N (2019) Implementation of fuzzy reliability analysis for elastic settlement of strip footing on sand considering spatial variability. Int J Geomech 19(12):04019126

Puri J, Yadav SP (2015) Intuitionistic fuzzy data envelopment analysis: an application to the banking sector in India. Expert Syst Appl 42(11):4982–4998

Roohanizadeh Z, Baloui Jamkhaneh E, Deiri E (2021) A novel approach for analyzing system reliability using generalized intuitionistic fuzzy Pareto lifetime distribution. J Math Ext 16(10):1–41

Shabani A, Baloui Jamkhaneh E (2014) A new generalized intuitionistic fuzzy number. J Fuzzy Set Valued Anal 2014:1–10

Shafiq M, Atif M, Viertl R (2017) Parameter and reliability estimation of three-parameter lifetime distributions for fuzzy life times. Adv Mech Eng 9(8):1687814017716887

Sharma MK, Sharma V, Dangwal R (2012) Reliability analysis of a system using intuitionistic fuzzy sets. Int J Soft Comput Eng 2(3):431–440

Shu MH, Cheng CH, Chang JR (2006) Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. Microelectron Reliab 46(12):2139-2148

Varghese PJ, Rosario GM (2021) A study on reliability using pendant, hexant, octant fuzzy numbers. J Reliab Stat Stud. https://doi.org/10.13052/jrss0974-8024.1426

Yang W, Jhang ST, Fu ZW, Ze Shui XuZS, Ma ZM (2021) A novel method to derive the intuitionistic fuzzy priority vectors from intuitionistic fuzzy preference relations. Soft Comput. https://doi.org/10.1007/s00500-020-05472-9