A surrogate merit function developed for structural weight optimization problems

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Abstract

In this paper, a surrogate merit function (SMF) is proposed to be evaluated instead of the traditional *merit functions* (i.e., penalized weight of the structure). The standard format of the conventional *merit functions* needs several expensive trial-and-error tuning processes to enhance optimization convergence quality, retuning for different structural model configurations, and final manual local search in case optimization converges to infeasible vicinity of global optimum. However, on the other hand, SMF has no tunning factor but shows statistically stable performance for different models, converges directly to outstanding feasible points, and shows other superior advantages such as reduced required iterations to achieve convergence. In other words, this new function is a no-hassle one due to its brilliant user-friendly application and robust numerical results. SMF might be a revolutionary step in commercializing design optimization in the real-world construction market.

Keywords Constraints handling · Structural weight optimization · Penalty techniques · *Fuzzy* membership function · *Merit function* · Fitness function

1 Introduction

Structural optimization belongs to the scope of constrained optimization problems in which the objective function $F(\overrightarrow{X})$ is the weight of the structure $W(\overrightarrow{X})$ ($\overrightarrow{X} = \overrightarrow{A}$ and $\overrightarrow{A}$ is the vector of cross sections of the beams and columns) and design criteria are the constraints of the problem $C(\overrightarrow{X})$ determined using the design codes (see Table 1). However, current optimization algorithms are for unconstrained problems, and a *merit function or fitness function,* which is a co-function of objective function $F(\overrightarrow{X})$ and constraints $C(\overrightarrow{X})$, is required to properly handle the constraints (for consistency in the text's terminology, the *merit function* is used everywhere else in this paper). Penalty methods are the traditional concept in structural engineering. However,

due to the hassles of using them (as discussed in next Sect. 1.3.1), and unavailability of other effective methods, the authors decided to propose a new one.

The main goal is to provide a user-friendly concept as simple as a *merit function* formula, which is general and true on every structural model, straightforward to use and has a higher efficiency compared to penalties. Since no commercial software (e.g., ETABS or SAP2000) offers an optimization option due to penalty hassles, authors expect SMF to be a breakthrough or at least a beginning step in facilitating the presence of an optimization option in commercial software soon.

1.1 Traditional penalty-based format

The standard format of the traditional *merit function* formula for structural optimization includes the weight of the structure $W(\overrightarrow{\text{X}})$ multiplied by some penalty rule (called "Constraint Handling Technique") which means that the *merit function* is like:

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Table 1 General optimization statement and its equivalent statement in structural optimization

General optimization statement	Equivalent statement in structural weight optimization
$Find \overrightarrow{X} = [X_1, X_2, X_3, \ldots, X_n]$	$Find \overrightarrow{A} = [A_1, A_2, \ldots, A_n]$
To minimize $F(\overrightarrow{X})$	To minimize $W(\overrightarrow{X})$ while $\overrightarrow{X} = A$
Subject to $C(\overrightarrow{X})$: constraints of problem	Subject to $C(\overrightarrow{X})$: design criteria vector

$$Mer(\overrightarrow{X}) = f(W(\overrightarrow{X}), P(\overrightarrow{X}, \overrightarrow{C}, t, \alpha, \beta, \varphi, \ldots, a, b, c, \ldots)) \tag{1}$$

where, $\overrightarrow{X}$ is a proposed solution vector obtained by the optimization algorithm (e.g., suggested cross-section list for a steel frame design), $\overrightarrow{C} = [g_1, g_2, g_3, \ldots, g_n]$ is the vector of constraints in which each g_i is a constraint of the problem to be satisfied. The rule of constraint-handling part $P(\overrightarrow{X}, \overrightarrow{C}, \alpha, \beta, \varphi, \ldots, a, b, c, \ldots)$ is the key to composing the *Merit function*, and factors like α, β, φ, a, b, c, and so on are included in the formulas as tunning factors for convergence quality control purposes. For example, the formula by Joines and Houck (1994)) includes α, β, and c as the tuning factor:

$$Mer\left(\overrightarrow{X}\right) = F\left(\overrightarrow{X}\right) + (c \times t)^{\alpha} \times \sum_1^n \max(0, g_i(\overrightarrow{X}))^{\beta} \tag{2}$$

Here, n is the number of constraints of the problem. t is the current iteration number throughout the optimization process.

The $F(\overrightarrow{X})$ for structural engineers is usually the weight of the structure, $W(\overrightarrow{X})$. Also, the engineers consider the design criteria of the codes as the constraints $g_i(\overrightarrow{X})$ and denote them in normalized format when doing structural optimization (for simplicity, we denote $g_i(\overrightarrow{X})$ as C_i in the remaining of the text). For example, the most common criterion for members is a strength capacity check, according to AISC (2010) provisions, and stated as

$$\frac{Demand}{Capacity} =$$
$$\begin{cases} \dfrac{Pu}{2\varphi cPn} + \dfrac{Mux}{\varphi bMnx} + \dfrac{Muy}{\varphi bMny} \leq 1 \, For \, \dfrac{Pu}{\varphi cPn} < 0.2 \\ \dfrac{Pu}{\varphi cPn} + \dfrac{8}{9}\left(\dfrac{Mux}{\varphi bMnx} + \dfrac{Muy}{\varphi bMny}\right) \leq 1 \, For \, \dfrac{Pu}{\varphi cPn} \geq 0.2 \end{cases}$$
$$\tag{3}$$

$$\to \text{Capacity Index} = \frac{Demand}{Capacity} \leq 1 \quad \begin{array}{c} \text{Normalized} \\ \text{Format} \end{array}$$

which is originally written in a normalized format in the design code. However, some other criteria need to be re-written in normalized format. For instance, for story s, inter-story drift is stated as Eq. 4. like:

$$\frac{\Delta_{s+1} - \Delta_s}{h_s} \leq RIRI = 0.0025(AISC, 2010) \tag{4}$$

$$Drift \, Index = \frac{\Delta_{s+1} - \Delta_s}{RI \times h_s} \leq 1 \, \text{Normalized Format}$$

where Δ_s and Δ_{s+1} are the lateral displacements of a presumed spot on the stories s and $s + 1$, respectively. h_s is the height of the story s, and RI is the code limit. But engineers change it to a normalized format to apply to structural optimization. In this paper, the capacity index (CI) and drift index (DI) are selected as the main design constraints and implemented in the optimization process.

1.2 Three factual strategies to develop SFM

In the current paper, SMF is developed in two steps based on three factual strategies.

Step1: Initiating Formulation Based on *Fuzzy* Logic

First Factual Strategy is to Use *Fuzzy* Platform to Develop the Formula: SFM must have a logical format with no clash with the nature of the search space to perform well. Fortunately, the fact is that the structures' combinatorial search space is a *fuzzy* one; thus, it has the potential to attribute a *fuzzy* membership value for each candidate structure in it (as described in Sects. 3.1, 3.2, and 3.3). Since this fuzziness is an intrinsic property of the search space, a *fuzzy* membership format sounds to be the best fit for the formulation as the platform. However, it must be converted to a proper *merit function* afterward.

Step2: Converting *Fuzzy* Membership Function into a Useful *Merit Function*

Second and third factual strategies are to convert the ***fuzzy membership function*** into proper ***merit function*** to be efficient in structural optimization.

Second Factual Strategy is to Set the Minimum of SMF on the Feasibility Boundary: The fact is that an efficient *merit function* must be capable of helping algorithm push the exploration away from overdesign structures toward on-the-edge feasible structures to meet global optimum. The authors' proposed strategy here is to set the minimum of SFM on the feasibility boundary by assigning zero to on-

the-edge feasible elements and a positive value for any of others.

Third Factual Strategy is Securing a Feasible Exploration by Feasibility Factor: The fact is that saving the feasibility of exploration flow over the structural search space is necessary since the optimization algorithms are not built for constrained problems and will not be able to stay on the proper feasible path on the search space. To tackle this fact, a feasibility factor is defined and embedded in SFM formula in Sect. 3.5, which tends to be equal to one for 100% feasible structure and takes values higher than 1 in any other case according to the feasibility level of the structure.

Those three factual strategies are discussed in detail in Sect. 3, and SFM is developed thereafter.

1.3 Motivation behind proposing SFM

The motivation to do the current research is originated from two main streams: first, difficulties of using the traditional penalty-based formats; second, reviewed literature.

1.3.1 The hassles of using penalty based merit in structural optimization problems

The produced value by penalty formulas is sensitive to the tuning factors and requires several try and errors to be adjusted, which are expensive due to the analysis time cost of finite element models of the structures with a high number of degrees of freedom. Other than such hassle, the settings for tuning factors must usually be readjusted when applying to another structure. Moreover, when an optimization process shows convergence and stops, especially when using metaheuristic algorithms, the resulted point is in the vicinity of the feasible global optima but sometimes infeasible. Therefore, the user needs to conduct a little bit of local search manually (e.g., for the steel frames, the engineer plays with the offered result by convergence and removes slight infeasibility between the elements and obtains a feasible lightweight structure with some struggle).

All those drawbacks prevent the structural optimization from being commercialized, and none of the current software has included it. According to the authors' knowledge, penalties have been the common concept utilized by structural engineers, and the research on this topic seems to be virgin. From this viewpoint, the structural engineering field lacks a proper research record.

1.3.2 Inspiration from the literature review

A comprehensive literature review is provided in Sect. 2 on constraints-handling techniques. There, the constraint-handling importance in structural optimization has been reviewed and main review articles to read about constraint-handling techniques are mentioned (Sect. 2.1), available ideas are briefly mentioned (Sect. 2.2), other miscellaneous ideas are mentioned (Sect. 2.3), studies with a focus on structures are cited (Sect. 2.4), and finally concluding an inspiration is discussed (Sect. 2.5). Upon that review, the authors understood that those other potential ideas even though effective in other ways, not suitable enough to remove the hassles of penalty tunning and the best choice is the penalties to be replaced by a new surrogate *merit function*.

1.4 Conceptual comparison with related works

A conceptual list of the differences and advances of SMF which are not available in the similar works is mentioned in Table 2.

2 Comprehensive literature review

2.1 Constraint-handling background and available review articles

As mentioned earlier in the Introduction section, structural optimization is a constrained problem in which the constraints are design criteria mentioned in the related codes. In contrast, most optimization methods are developed to explore the search space to find extremums of objective functions with no constraints. Genetic algorithm (GA) (Holland 1975), differential evolution (DE)) Storn and Price 1997), particle swarm optimization (PSO) (Eberhart and Kennedy 1995), bat algorithm (BA) proposed by Yang (2016), ant colony optimization algorithm (ACO) (Dorigo and Stützle 2004), cuckoo search (CS) (Yang and Deb 2009), tabu search (Glover 1989 and 1990), imperialist competitive algorithm (ICA) (Gargari & Lucas 2007), big bang-big crunch (BB-BC) (Erol and Eksin 2006), artificial bee-colony algorithm (ABC) (Karaboğa 2005), harmony search (HS) (Geem et al. 2001), charged system search algorithm (CSS) (Kaveh and Talatahari 2010a, b), and chaos game optimization (CGO) (Talatahari and Azizi 2020) all perform in this way. Therefore, handling design criteria has been of utmost interest among structural engineers. Structural engineers and other academicians face difficulties solving a related constraint optimization problem. Therefore, so many miscellaneous techniques applicable to all or limited types of problems have been

Table 2 Main features of surrogate merit function (SMF)

Formulated based on factual properties which means:

  1. Utilizes a combinatorial *fuzzy* search space, letting both feasible and infeasible part of structure enroll in the formula (penalties simply penalize by infeasible constraints)

  2. Set the minimum of Merit intentionally on the feasible boundary (no minimum is set by penalties)

  3. Defining a feasibility saving factor (the only common part with penalties. Penalties are in fact feasibility savers)

Specified for structural optimization problem only:

  1. Applicable to all the structures

  2. Not Applicable to problems other than structures

Is User Friendly by:

  1. Not having the hassles of previous methods, such as penalties

  2. Not a complicated computer programming required

  3. Reducing required number of iterations

Has a focus on:

  1. Expediting convergence by a compatible membership with optimization

but has **No** focus on:

  1. Reducing computational cost by improving finite element concept or efficient programming

  2. Handling constraints by constrained version of algorithms

developed until now. Coello (2001) and Montes and Coello (2011) are comprehensive review surveys on different constraint handling techniques. Typically, three main groups of constraint handling techniques are available in the literature: (a) penalty-based techniques, (b) modified evolutionary algorithms (EAs) or hybrid algorithms such as co-evolutionary algorithms (CoEAs), and (c) miscellaneous techniques.

2.2 Available approaches

The most well-known methods are penalty techniques that have different forms such as static penalties, dynamic penalties, death penalty, adaptive penalty. Some other researchers have focused on the manipulated versions of the algorithms to directly embed a constraints-handling technique into the algorithm's logic directly. Yousefi et al. (2012) have improved ICA with a feasibility-based ranking embedded algorithm applied for handling design constraints of heat-transferring plates. Nema et al. (2011) have presented a hybrid co-evolutionary version of PSO combined with the gradient search and used an augmented Lagrangian method as the constraints handling technique. Mun and Cho (2012) explained a modified HS algorithm with an embedded fitness priority-based ranking method (FPBRM) to handle design constraints. Zade et al. (2017) introduced a hybridized cuckoo search with box-complex method to handle design constraints and increase the convergence rate and computation speed. Mendez and Coello (2009) utilized a selection mechanism and incorporated it into a DE algorithm to handle constraints. Liu et al. (2018)

presented a modified PSO using a subset constrained boundary narrower (SCBN) method cooperating with a sequential quadratic programming for finding near-boundary feasible answers for solving engineering problems. Lee and Kang (2015) worked on handling constraints in water resources optimization problems with modifying an EA called shuffled complex evolution (SCE) with an adaptive penalty. Stripinis et al. (2019) presented a constraint handling technique incorporating a two-step selection procedure and a penalty function called *the direct type* constraint handling technique. Although efficient and valuable, these algorithms are usually sophisticated in logic or need exhausting programming and might not be simply available for users.

2.3 Other miscellaneous ideas

Other miscellaneous ideas are available in the literature. Chehouri et al. (2016) presented a method, so-called violation constraint handling (VCH) technique, for GAs by utilizing a violation factor without tunable parameters displaying a consistent performance. Guan et al. (2008) have developed a repairing procedure added on GAs for handling constraints applying to water resources optimization problems. Mallipeddi and Suganthan (2010) utilized four different constraint handling techniques, superiority of feasible solutions (SF), self-adaptive penalty (SP), ε-constraint (EC), and stochastic ranking (SR), simultaneously cooperating according to some outlined rules and called it ensemble of constraint handling techniques (ECHT) which outperforms all the four techniques

when utilized individually. Leguizamón and Coello (2009) developed a boundary-search constraint handling technique utilizing two local and global level exploration, based on an ant colony metaphor.

2.4 Studies with focus on structures

Some studies have focused on optimizing skeletal structures. For example, Kaveh and Zolghadr (2012) have applied a penalty function to handle natural frequency constraints on some small/mean scale structures. Their algorithm has beaten all other former literature. The other exterior penalty function utilized by Gholizadeh and Barzegar (2012) is a technique that sequentially provides an unconstrained situation for handling natural frequency constraints. Also, the method presented in a work by Kim et al. (2010) is an improved version of PSO that makes the problem unconstrained. Pholdee and Bureerat (2014) have compared the results of a fuzzy-set penalty function by Cheng and Li (1997) and the three techniques. In the other work by the same authors (i.e., Bureerat and Pholdee 2016), they have introduced a more powerful penalty that outperforms another work by Kaveh and Zolghadr (2012). Their new penalty function shows the best standard deviation among all former techniques while the mean, max, and min of the statistical study are in the scope of others.

2.5 Concluding an inspiration

All the concepts mentioned above are undoubtedly excellent works, but they are not simply available and mostly not specific to structural design problems; the scholars of these studies have not utilized a structural design point of view, and mostly they have had a heuristic or artificial intelligence-based inspiration for their developed techniques. In other words, such techniques manipulate exploration/exploitation in the optimization process and do not use the merit function by inspiration from routines and logic of specific properties of the studied problem (the structural design optimization problem in this paper). Moreover, all the techniques have no statistical evidence of reliable stable performance on large-scale structural design problems. As the vision is general, trial and error for tuning purposes remains the main drawback and prevents the user conveniency, especially in the real market. Hasançebi and Erbatur (2000) have tried to avoid tunable factors of an old penalty technique and eliminated its shortcomings for providing improved performance with a reformulation; but again, designing view is not considered due to basic vision in their research.

The lack of a design vision-based technique specified for the structural optimization motivated the authors to provide a new reliable constraint handling technique. At the first look, one may believe that providing a deal with the computational cost of the optimization process might be a better way than a fitness-based technique to facilitate try-and-error attempts. However, available works on improving computation efficiency from spotlight scholars show that computation expedition methods are not at a level to highly facilitate such attempts in a way to make the process convenient and user-friendly. Here are some examples: Kaveh and Talatahari (2010a, b) presented an improved ACO utilizing the sub-optimization mechanism (SOM) to reduce the size of the pheromone vector, decision vector, search space, and number of fitness evaluations to expedite the optimization process. Hasançebi (2008) attempted to improve the computational performance of adaptive evolution strategies and increase the algorithm's efficiency with an adaptive penalty function. Azad et al. (2013), Azad and Hasançebi (2013), Azad et al. (2014) and Hasançebi and Azad (2015) all struggle to improve the efficiency of various algorithms for solving large scale structures by applying an outstanding strategy called Upper Bound Strategy (UBS). Also, Azad and Hasançebi (2015) attempted to use a guided stochastic search (GSS) technique based on the principle of virtual work to enhance computational efficiency. Kambampati et al. (2018) presented a sparse hierarchical data structure called volumetric dynamic grid (VDG) in combination with the fast-sweeping method and multi-threaded algorithm for faster convergence in the topology optimization of structures by level set method (LSM). The work by Dunning et al. (2016) attempted to overcome computationally difficult and expensive eigenmodes of buckling constraints by reusing available eigenvectors, optimal shift estimates and some other ideas to effectively overcome with many buckling modes in topology optimization. Duarte et al. (2015) and Sanders et al. (2018) introduce two versions of efficient software for computationally fast analysis of polygonal finite element meshes with many degrees of freedom. With an investigation of these references, such expeditions do not eliminate boring tuning situations. In Venkataraman and Haftka (2004), it is illustrated that optimization complexity grows as the computations' possibilities improve and eliminating such costs would be more and more challenging and out of reach.

All said and done, the only way would be to eliminate the tuning necessity with a strategy that basically does not require tuning. The techniques mentioned above may expedite the process, but apply other hassles like hard programming or algorithm manipulation or analysis improvement concepts which don't seem to be user friendly.

3 Factual development of SMF

3.1 A discussion on combinatorial search space

Based on the three facts mentioned in (Sect. 1.4), the new formula gets developed in this section. First, let's elaborate on the combinatorial search space of the structure itself. As shown in Fig. 1, a schematic example of a 3D steel frame may have a different set of sections randomly selected from the list of standard W-shaped sections available in the construction market. Ten sets are shown in Fig. 1., but if the 267 sections are available in the market or can be accessed in a famous software like ETABS or SAP2000 section-properties list, and the frame has n number of elements, then the size of the combinatorial search space is 267^n and the last member of this search space is Sn. For example, the 3D steel Frame in Fig. 1. has 8 beams and columns, so its search space size is equal to 267^8. This number is a big one and gives a sense to the reader about the size of the search space of the larger-scale structures like the 3D numerical examples studied in Sect. 5, which have 267^{1026}, 267^{3860}, and 267^{8272} (i.e., almost 10^{2489}, 10^{9366} and 10^{20000}) size of search space.

3.2 A discussion on *fuzzy* logic

Zadeh (1965) presented the idea of a *fuzzy* point of view on truth-check values. This logic says that truth value variables may be any real number between 0 and 1 (logical response $\forall \mathbb{R} \in [0, 1]$). Figures 2. and 3. show the difference between binary and *fuzzy* logic.

In Fig. 2, the circle (a) is entirely red, the circle (b) is entirely green, the circle (c) is entirely orange, and somebody may say how each circle is colored. It is binary logic and the response to a quote like "the circle is green" receives a clear credit equal to 0 or 1.

However, in Fig. 3, each circle has different colors, and a distinct statement about their color is impossible, but we can say that each of them is somehow red, green, and orange. Figure 3a is 28% red, 47% green, and 22% orange. Figure 3b and c also has different degrees of each color. It is a *fuzzy* point of view. The main benefit of this logic is that a *fuzzy* membership can be attributed value to each circle and distinguish them from each other by using those percentages.

A set in which its members follow this logic in mathematics and logic is called *fuzzy* sets. Zadeh developed the

Fig. 1 The combinatorial search space and sample candidate set of sections Si for 8 member 3D frame

Fig. 2 All the logical checks receive exactly 0% or 100% and no degree in between

100 % Red	100 % Green	100 % Orange
0 % Green	0 % Orange	0 % Red
0 % Orange	0 % Red	0 % Green

Fig. 3 Logical checks cannot be responded to by 0% and 100% only. A mixture of degrees applies

(a)	(b)	(c)
28% red, 47% green, 22% Orange	34% red, 10% green, 56% Orange	17% red, 68% green,

proposal of the "*fuzzy*-set" theory immediately after defining "*fuzzy* logic." The segments on each *fuzzy* member who belongs to a *fuzzy* set are called *fuzzy* segments as they are degree takers of the membership in part. For example, areas with different colors in Fig. 3 are the *fuzzy* areas of each circle.

In the next section, the fuzziness of the combinatorial search space of the structures is discussed, and the initial format of SFM is founded based on that.

3.3 Step1: initiating formulation platform by first factual strategy

When external loading is applied to the structure (e.g., dead, live, earthquake, and wind load cases) and structure is analyzed, in the civil engineering culture, the response of the structure is always stated in the format of feasible (green) or infeasible (red) elements by colors, similar to Fig. 4.

The corresponding feasibility vector $\vec{V}$ is stated with a color in the way that elements with violated constraints ($C_i > 1$), colored in red, is not acceptable and marked as infeasible, which takes value zero, $V_i = 0$, and the elements

with satisfying constraints ($C_i \leq 1$), colored as green, takes value one, $V_i = 1$.

The structure is represented by degrees of feasibility and infeasibility. Therefore, we can call combinatorial search space a *fuzzy* set because all its members (S_i, S_j, S_k, etc.) have a potential opportunity to be distinguished by a *fuzzy* membership value. S_i, S_j, S_k, etc. are called *fuzzy* members, and structural elements (beams, columns, and braces) of each *fuzzy* member are *fuzzy* segments (because they are value takers upon feasibility or infeasibility). In other words, this *fuzzy* property of the combinatorial search space provides a format platform for a potential SFM.

As the vector V is so simple, and based on the number of feasible and infeasible elements, it might represent several candidates S_i, S_j, S_k, etc. on the combinatorial search space. In terms of mechanics of structure, stiffness matrix K in static equation $F = Kd$ is its unique identity which has a distribution (may be represented by weight distribution vector $\frac{w_i}{W}$) and a scale (may be represented by absolute weight W). Also, a factor related to constraints is required and C vector is available.

As shown in Fig. 5, the C vector and weight distribution vectors compared to the V vector presented in Fig. 4. are
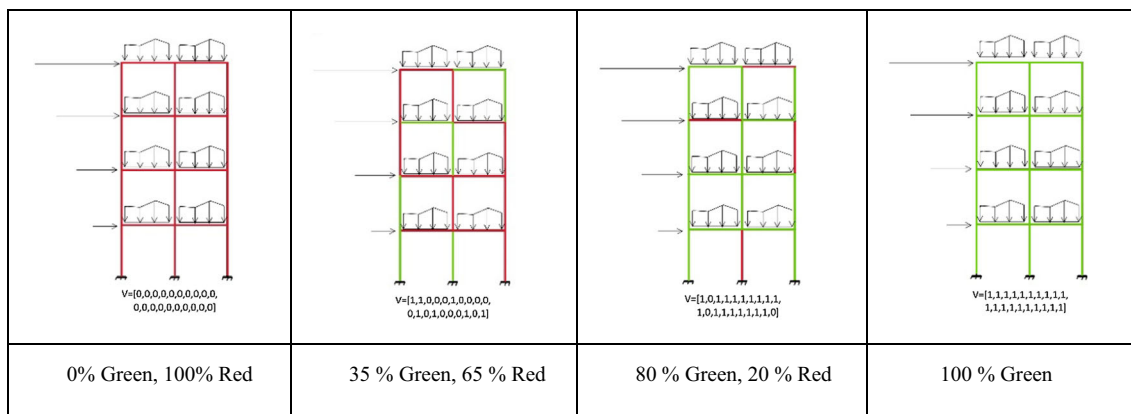
Fig. 4 *Fuzzy* membership vector *V*, attributed to the structure, binary point of view to structural elements. (0 and 1 values for each member) is enough to have different membership values; however, it is not yet sufficient and efficient enough for searching optimum design
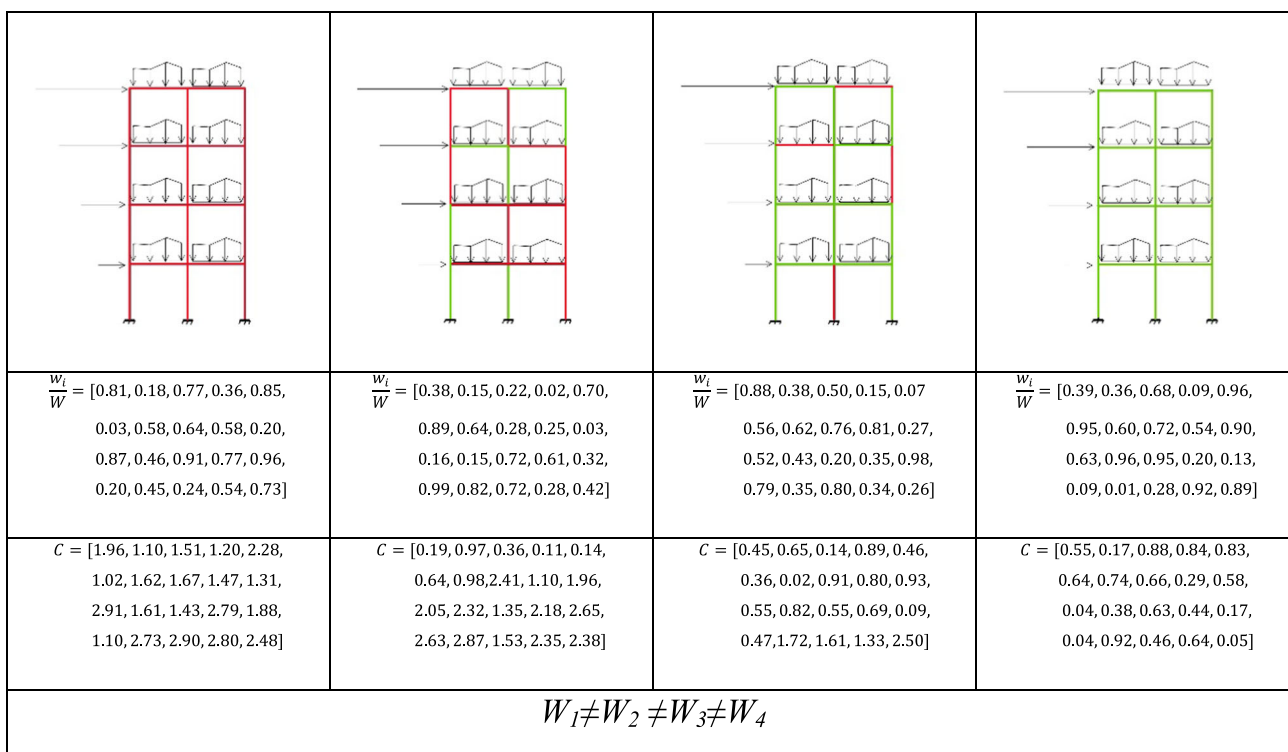

Fig. 5 More distinguished *fuzzy* membership may be attributed to the structure by normalized constraints vector C and weight distribution rate ($\frac{w_i}{W}$)

more unique factors to provide distinguishment among S_i in the combinatorial search space. Therefore, we have:

$$FuzzyMembership(X) \propto \frac{w_i}{W}, W, C \qquad (5)$$

None of the three factors ($\frac{w_i}{W}$), absolute weight *W*, and *C* are unique for each structure but the maximum one of them can be the same and the other ones are certainly different. For example, structures with the same weight *W* have different weight distribution $\frac{w_i}{W}$ and accordingly different C vector, or structures with the same weight distribution have

different weights and are just physical scale of each other and accordingly different C vectors. As a result, we must use at least two of them in the membership function to make sure their product is unique. In this paper, the authors selected weight distribution ($\frac{w_i}{W}$) and C vector and dropped absolute weight W since this factor is used later in feasibility factor in Sect. 3.4, part two, and double embedding in final SMF formula has no sense; and the initial version of the formula is as Eq. 6:
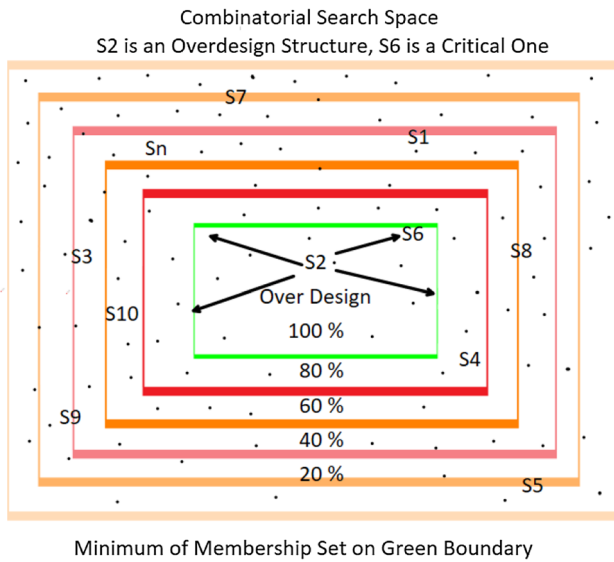
Combinatorial Search Space
S2 is an Overdesign Structure, S6 is a Critical One

Minimum of Membership Set on Green Boundary

Fig. 6 A proper movement from an overdesign sample candidate S_2 to a critically feasible sample structure S6 (green boundary)

$$FuzzyMembership(X) \propto \frac{wi}{W}, C \to FuzzyMembership(X)$$
$$= \sum_{1}^{n} \frac{w_i}{W} \times C_i$$

(6)

This formula is still a *fuzzy* membership function and has no property to support the role of a *merit function* yet. In the following sections, this membership is converted to a *merit function* with two steps: first, by setting function's minimum to the on-the-edge feasible structures (i.e., feasibility boundary) and, second, control the feasibility of the exploration by feasibility factor.

3.4 Step2: converting fuzzy membership function into an SMF

3.4.1 Second factual strategy: set the minimum of SMF on the feasibility boundary

To set the minimum of SMF, first, we define overdesign and on-the-edge feasible structures.

Overdesign Structure is a structure on the combinatorial search space Si with excessive material or stiffness and may lose an amount of its weight and remain feasible.

On-the-Edge Feasible Structure is a structure on the combinatorial search space in which any slight loss of material of any element leads to loss of mechanical resistance in a way that at least one infeasible element emerges. These structures are the boundary of the feasible subset of the combinatorial search space.

A schematic of an entirely feasible area (structures with 100% feasibility), edge structures (feasibility boundary), and structures with other degrees of feasibility are shown in Fig. 6.

Accordingly, the exploration may meet the global optimum if we set the minimum of the *fuzzy* membership on the edges and attract algorithms' exploration toward the critical designs and meet the minimums. Therefore, the most critical case scenario $C = 1$ for all the elements is considered to represent the boundary in Eqs. 6 to 1 to set the optima of the *fuzzy* membership function and make it one step forward to be a proper *merit function*. Therefore, the modified formula may be stated as:

$$FuzzyMembership(X) = Merit(X) = \sum_{1}^{n} \frac{w_i}{W} \times |C_i - 1|$$

(7)

Combinatorial Search Space
A Constraints-Handled Exploration Moves

Most of the Moves are Toward the Higher Feasibility or from Inside to Feasibility Edge

Combinatorial Search Space
No Constraints Handled Moves

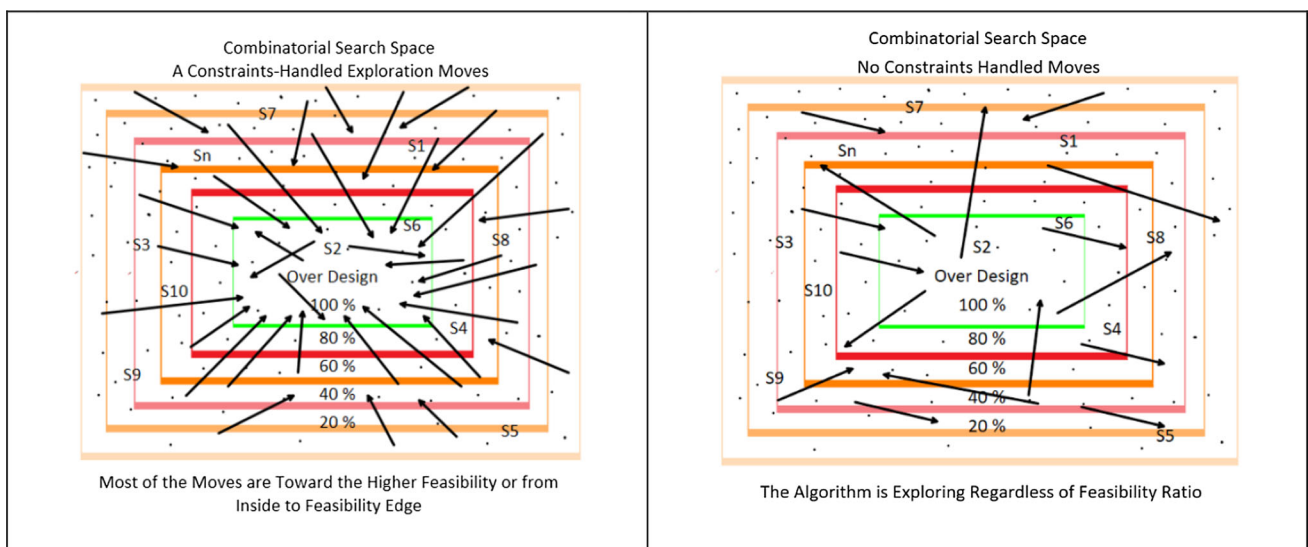The Algorithm is Exploring Regardless of Feasibility Ratio

Fig. 7 Exploration Flow. Various types of movements throughout optimization iterations while an optimization algorithm executes exploration

The set of constraints C might be divided into two feasible and infeasible subsets: C_f and C_{inf}. To boost the exploration, its possible to add a power (e.g., 2) to C_f to push the algorithm to stay more and more near the feasibility boundary and remain far from over-design Si on the combinatorial search space. Therefore, if the structure has a bunch of elements C_f (satisfied constraints) and the remaining with C_{inf} (violated constraints) while $nfeasible + nInfeasible = n$ (n is the total number of structural elements):

$$FuzzyMembership(X) = Merit(X)$$
$$= \sum_{1}^{nfeasible} \frac{w_i}{W} \times (C_f - 1)^2$$
$$+ \sum_{1}^{nInfeasible} \frac{w_j}{W} \times (C_{inf} - 1) \qquad (8)$$

As another booster, we may remove the tense -1 for C_{inf} to let infeasible ones have even higher values and prohibit the algorithm from exploring the infeasible side of the feasibility edge even better. Then we have:

$$FuzzyMembership(X) = Merit(X)$$
$$= \sum_{1}^{nfeasible} \frac{w_i}{W} \times (C_f - 1)^2$$
$$+ \sum_{1}^{nInfeasible} \frac{w_j}{W} \times C_{inf} \qquad (9)$$

It is worth mentioning that the minimum of the function is still set on 1 (feasibility boundary) and the changes in Eqs. 8 and 9 do not affect this property of the formula. The minimum of the function is set, but the feasibility of

Table 3 Parameter summary of the alternative metaheuristic algorithms

Metaheuristic	Parameter	Description	Value
ACO	N_{ant}	Colony size	50
	α	Intensity of pheromone control parameter	1.0
	β	Visibility parameter	0.4
	ρ	Pheromone evaporation	0.2
	ξ	pheromone trail	0.1
ICA	N_{cont}	Countries size	50
	N_{emp}	Number of empires/imperialists	10
	α	Selection pressure	1
	β	Assimilation coefficient	1.5
	p_r	Revolution probability	0.05
	μ	Revolution rate	0.1
	ζ	Colonies mean cost coefficient	0.2
CSS	N_{pop}	Charged particle size	50
	a	Radius of charged sphere	0.1
	$HMCR$	Harmony memory consideration rate	0.85
	PAR	Pitch adjustment rate	0.15
	kt	Attract-repel coefficient	0.9
	N_{cm}	Charged memory size	12
	k_a	Acceleration coefficient	0.5
	k_v	Velocity coefficient	0.5
PSO	N_{pop}	Particle size	50
	w	Intertia coefficient	1
	w_{damp}	Damping ratio of inertia coefficient	0.99
	C_1	Personal acceleration coefficient	2
	C_2	Social acceleration coefficient	2
FA	N_{pop}	Fireflies size	50
	β_0	Attractiveness parameter	1.0
SOS	N_{pop}	Agent size	50
WOA	N_{pop}	Agent size	50
ISA	N_{pop}	Agent size	50

Table 4 Different utilized penalty methods utilized in the third example; all the formulas are extracted from Coello (2001)

Presenters	Formula	Settings description
Morales and Quezada (1998)	$\text{Fitness}(x) = \begin{cases} F(x) & \text{feasible} \\ K - \sum_1^s \frac{K}{M} & \text{infeasible} \end{cases}$	$K = 1000$ (The author mentions that K is great enough according to dimension of problem, s is the number of satisfied constraints)
Michalewicz and Attia (1994)	$\text{Fitness}(x) = F(x) + F(x) \times \frac{1}{2\tau} \sum_1^n \max(0, g_i(x))^2$	$\tau = 0.99 \times \sqrt{\text{it}}$ (arbitrary chosen)
Hoffmeister and Sprave (1996)	$\text{Fitness}(x) = F(x) + \sqrt{\sum_1^i \max(0, g_i(x))^2}$	
Skalak and Shonkwiler (1998)	$\text{Fitness}(x) = A \times F(x)$	$A = \exp(\frac{-M}{T}) T = \frac{1}{\sqrt{it}} M = \sum_1^i \max(0, g_i(x))^2$ (arbitrary chosen)
Joines and Houck (1994)	$\text{Fitness}(x) = F(x) + (c \times t)^\alpha \times \sum_1^n \max(0, g_i(x))^\beta$	$c = 0.5$ $\alpha = 1$ $\beta = 2$ (These parameters are penalty factors and selected values are inspired from authors of the formula)
Smith and Tate (1993)	$\text{Fitness}(x) = F(x) + (B_{\text{feasible}} - B_{\text{all}})^\alpha \times \sum_1^i \frac{g_i(x)}{b_i}$	$b_i = 0.05$ (b_i is a threshold value for constraints' violation and B is the brevity for 'Best')
Bean and Hadj-Alouane (1992)	$\text{Fitness}(x) = F(x) + \lambda(t) \times \sum_1^n \max(0, g_i(x))^2$ $\lambda(t+1) = \begin{cases} \frac{\lambda(t)}{\beta_1} & \text{best is feasible in last } K \text{ generations} \\ \lambda(t)\beta_2 & \text{best is infeasible in last } K \text{ generation} \\ \lambda(t) & \text{none of above} \end{cases}$	(arbitrary chosen) $\beta_1 = 1.01$ $\beta_2 = 1.05$
Deb (2000)	$\text{Fitness}(x) = \begin{cases} F(x) & \text{feasible} \\ f_{\text{worst}} - r \times \sum_1^n \max(0, g_i(x)) & \text{infeasible} \end{cases}$	$r = f_{\text{worst}}$ (In original form there is no r, but for dimensional objectives in structures, r is included)

[a] G_i

[b] It means iteration in optimization process

exploration is not yet assured which is provided by the feasibility factor described in Part 2.

3.5 Third factual strategy: securing a feasible exploration by feasibility factor

The second important property of the SMF is that it must handle the feasibility of the exploration by algorithms. As already discussed, the optimization algorithms cannot detect the feasibility and must be supported by handling ideas embedded in the *merit function*. The authors here propose to have a simple and near-to-mind feasibility factor, as follows:

$$FeasibilityFactor = \frac{n \times W}{n_f \times w_f} \tag{10}$$

where w_f is cumulative weight of all feasible elements and n_f is the number of feasible elements available in the structure. Other formats for this factor like $\frac{W}{w_f}$ or $\frac{n}{n_f}$ are also available but choosing among them and the one in Eq. 10

is a deliberate one. This factor's value reaches its minimum equal to one when the structure is 100% feasible (i.e., all the members are feasible ones). However, the more infeasible parts in the structure, the bigger value this factor gets and pushes algorithms' exploration toward a higher level of feasibility. Therefore, we can state that:

$$FeasibilityFactor = \frac{n \times W}{n_f \times w_f} \geq 1 \tag{11}$$

and:

$$\lim_{\text{feasibility} \to 100\%} \frac{n \times W}{n_f \times w_f} = 1 \tag{12}$$

Figure 7 shows an unconstrained exploration and a handled exploration by feasibility factor.

Therefore, the final version of the *merit function* may be the product of the feasibility factor and the *fuzzy* membership function, as stated in Eq. 13:
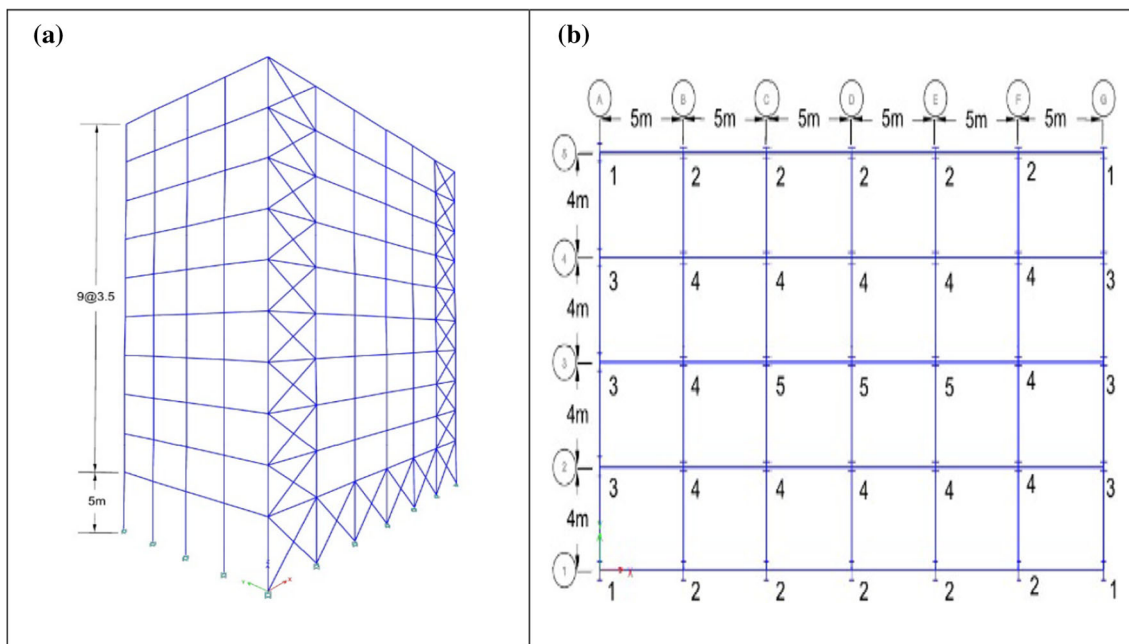
Fig. 8 10-story 1026-element frame: **a** 3D view; **b** plan view and column groupings

$$SMF(X) = FeasibilityFactor \times \left(\sum_{1}^{\text{nfeasible}} \frac{w_f}{W} \times (C_f - 1)^2 \right.$$

$$+ \sum_{1}^{\text{nInfeasible}} \frac{w_{\text{inf}}}{W} \times C_{\text{inf}} \right)$$

$$= \frac{nW}{n_f w_f} \times \left(\sum_{1}^{\text{nfeasible}} \frac{w_f}{W} \times (C_f - 1)^2 \right.$$

$$+ \sum_{1}^{\text{nInfeasible}} \frac{w_{inf}}{W} \times C_{inf} \right)$$

(13)

The feasibility factor assures the feasibility of exploration and prevents algorithm from wasting iterations

Table 5 Design load coefficients according to ASCE (2000)

Number	Combination
1	1.4D[*]
2	1.2D + 1.6L[*]
3	1.2D + 0.5L + 1(E_x[*] or W_x[*])
4	1.2D + 0.5L + 1(E_{ex}[*] or W_{ex}[*])
5	1.2D + 0.5L + 1(E_y or W_y)
6	1.2D + 0.5L + 1(E_{ey} or W_{ey})
7	0.9D + 1(E_x or W_x)
8	0.9D + 1(E_{ex} or W_{ex})
9	0.9D + 1(E_y or W_y)
10	0.9D + 1(E_{ey} or W_{ey})

[*] D, L, E and W denote dead, live, earthquake and wind loads, respectively. x and y are loading directions. ex and ey denote loading direction with eccentricity

exploring infeasible areas on the combinatorial search space. Now, the membership function is fully converted to a *merit function*. SFM performance test results are reported in the following sections by applying to the three large-scale examples.

4 Numerical evaluation material

4.1 Optimization methods

Several algorithms are selected to be utilized in this paper to verify the SMF's performance, such as GA (Holland 1975), ACO (Dorigo and Stützle 2004), PSO (Eberhart and Kennedy 1995), CSS (Talatahari and Azizi 2020), interior search algorithm (ICA) (Gargari and Lucas 2007), firefly algorithm (FA) (Yang 2008, 2009), symbiotic organisms search (SOS) (Cheng and Prayogo 2014), upgraded whale optimization algorithm (WOA) (Azizi et al. 2019), and interior search algorithm (ISA) (Gandomi 2014). The constant parameters of these algorithms are set the same as the standard versions provided by the main references, as presented in Table 3. Each algorithm performed thirty optimization runs to have statistical outputs.

4.2 Utilized penalty functions to be compared with SMF

The list of all penalty methods applied to the examples is summarized in Table 4.
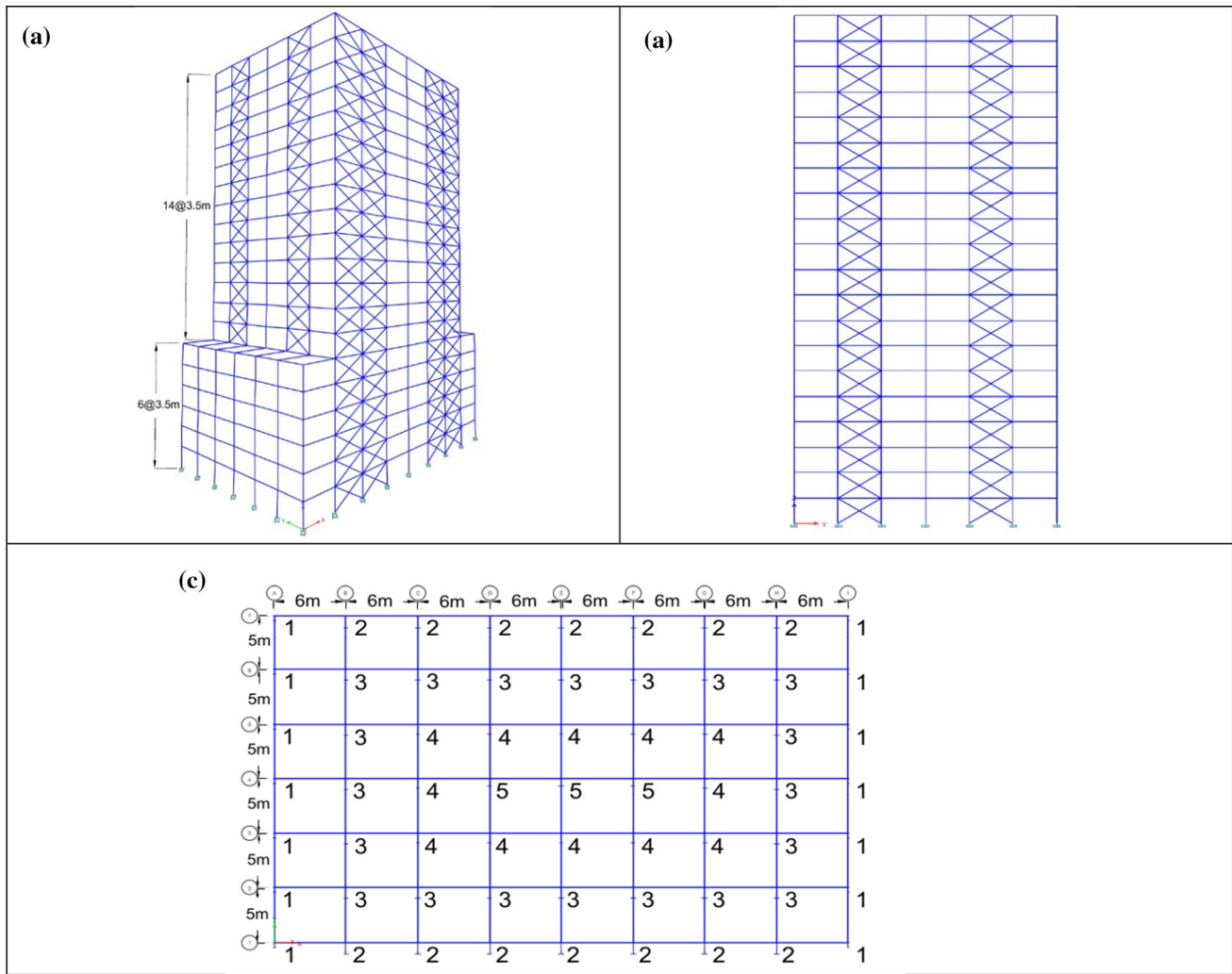
Fig. 9 20-story steel frame: **a** 3D view, **b** elevation views of frames B to H **c** the plan view and columns groupings

4.3 Output clarification

Initially, the method of Hoffmeister and Sprave (1996) is implemented as the penalty to optimize the first and second examples by nine evolutionary algorithms to compare with SMF to prove that SMF is practically hirable and then statistical results are compared. Afterward, by implementing one algorithm (CSS), the results obtained by SMF are statistically compared with the results obtained by the other penalty methods listed in Table 4.

5 Numerical examples and results

5.1 Initial information

This section presents the details of frame structures as well as the numerical results of optimization obtained by different methods. AISC (2010) and ASCE (2000) are the codes for designing and loading of the examples, respectively. In addition, normalized design constraints, the *CI* and *DI*, are selected as criteria.

The proposed technique is tested by three frames optimizations: 10-story frame (Azad et al. 2013), X-braced 20-story tall frame (Azad et al. 2014), and 60-story mega-braced tubed frame (Talatahari et al. 2022). The first and second frames are both benchmark examples in literature. The possible sections for structural members of examples were taken from 267 W-shaped standard profiles. Properties of utilized steel material were $\rho = 7850 \ Kg/m^3$, $E = 200 \ GPa$ and $F_y = 248.2 \ MPa$, as the mass density, modulus of elasticity, and yielding stress, respectively. 10-story and 20-story frames are loaded by dead, live, and seismic loads while the tubed frame is loaded by dead, live, and wind. In the following, each example is described, and numerical results are presented afterward.
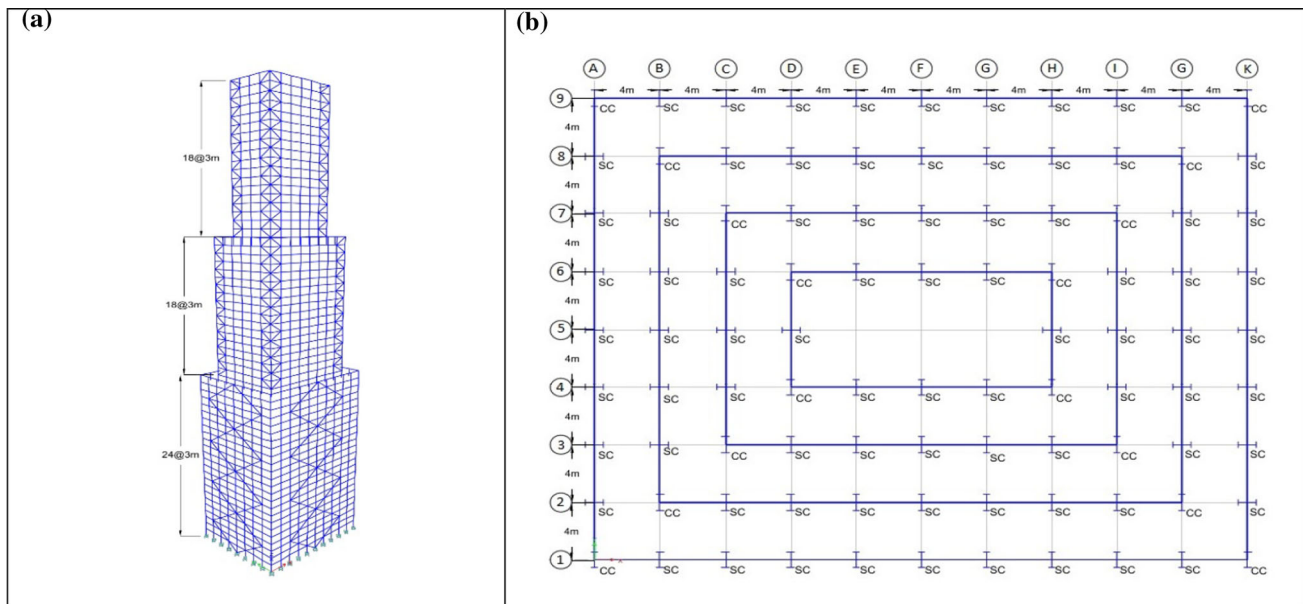
Fig. 10 60-story tubed frame: **a** 3D view, **b** the plan view for 24 initial story, **c** the plan view for stories 25 toward 42 and **d** the plan view for upper stories

5.2 10-story frame

This model, as the first example, is shown in Fig. 8 and was firstly introduced by Azad et al. (2013). This frame contains 1026 elements, 580 beams, 350 columns, and 26 X-type braces. The braces are applied just along x-axes for all bays in the first story and the side-bays of the other stories. Joints are moment-resisting connections for beams and columns and pin connections for braces. Thirty-two different groups of members are available, and they are repeated every three stories from 2nd story toward upwards; 5 column groups, 2 groups for outer and inner beams, and 1 for bracings are the considered groups. It is noteworthy that all the story nodes modeled as rigid diaphragms and 10-story diaphragms modeled for this example. In addition, the effective length factor for buckling stability for columns, braces, and the major bending of beams are all taken equal to one. However, for the bending over the weak bending plane of beams, namely the plane orthogonal to the floor layer, this factor is taken equal to 0.01 since the beams are fully braced by the floors. The loadings applied on the floors are equal to 20KN/m dead and 12KN/m live, respectively. Simultaneously, 15KN/m and 7KN/m are the dead and live loads applied on the roof story. In this paper, the self-weight of the story is considered the effective story weight for calculating the related seismic lateral load and gets updated from time to time while exploration is moved on by algorithms. According to ASCE (2000), the lateral seismic load results as

$$F_x = \frac{w_x h_x^{\ k} V}{\sum_{i=0}^{n} w_i h_i^{\ k}}, V = 0.1W \tag{24}$$

where the w_x and h_s are the effective weight and height of the story s, respectively; k is a function of fundamental period of the structure; and F_s is the lateral seismic load assigned to the mass center of the story s; for this example, the seismic base shear V was taken equal to 10 percentage of the weight of the structure W (i.e., $V = 0.1\ W$). According to ASCE (2000), the fundamental period of the structure could be computed by

$$T = C_T h_n^{\ 3/4} \tag{25}$$

where C_T is taken equal to 0.0853 and H is the height of the structure equal to 36.5 m in this example. According to this formula, the fundamental period of this structure T is equal to 1.267 s. According to ASCE (2000) guideline, the k factor for this value of fundamental period is equal to 1.38. Additionally, different load coefficients applied to 10 load combinations are considered for all the examples as presented in Table 5.

5.3 X-braced 20-story frame

This tall building consists of 8 bays in x-direction and 6 bays in y-direction, but the number of bays in x-direction reduces to 6 from 7th story. Bays are 6 m and 5 m long, in x- and y-directions, respectively. The structure contains 3860 members; 1836 beams; 1064 columns; and 960 bracing elements. This moment frame is braced with X-shaped braces in both x- and y-directions. In this frame,

Table 6 10-story frame results; section properties, the total weight of converged designs, maximum DI and CI, weights in practice resulted utilizing SMF method along with 9 meta-heuristics

Stories	Groups	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
1	CG1*	'W44X262'	'W14X257'	'W24X192'	'W33X221'	'W14X342'	'W40X277'	'W33X354'	'W36X210'	'W24X192'
	CG2	'W21X101'	'W27X178'	'W30X132'	'W21X132'	'W40X235'	'W36X160'	'W27X161'	'W33X130'	'W36X160'
	CG3	'W30X191'	'W18X283'	'W24X104'	'W24X192'	'W14X120'	'W24X117'	'W24X250'	'W30X108'	'W14X99'
	CG4	'W40X249'	'W27X146'	'W12X170'	'W40X392'	'W36X170'	'W27X178'	'W40X211'	'W24X176'	'W27X161'
	CG5	'W30X148'	'W40X321'	'W30X292'	'W30X391'	'W40X297'	'W36X170'	'W14X211'	'W12X305'	'W12X336'
	IB*	'W16X26'	'W8X28'	'W14X74'	'W27X84'	'W8X18'	'W8X28'	'W24X68'	'W24X68'	'W21X73'
	OB*	'W30X90'	'W24X68'	'W27X94'	'W8X40'	'W24X76'	'W30X90'	'W14X48'	'W24X76'	'W16X89'
	BR*	'W18X76'	'W8X24'	'W16X67'	'W8X31'	'W24X62'	'W8X28'	'W21X50'	'W10X68'	'W16X67'
2–4	CG1	'W21X166'	'W30X235'	'W40X297'	'W36X230'	'W30X211'	'W40X372'	'W36X300'	'W40X277'	'W36X300'
	CG2	'W30X116'	'W14X132'	'W12X230'	'W24X192'	'W40X149'	'W40X167'	'W14X176'	'W30X261'	'W14X233'
	CG3	'W27X94'	'W27X217'	'W40X149'	'W21X93'	'W27X102'	'W40X174'	'W40X167'	'W33X118'	'W30X132'
	CG4	'W24X176'	'W27X114'	'W21X122'	'W18X106'	'W33X118'	'W36X160'	'W18X158'	'W24X117'	'W27X114'
	CG5	'W14X99'	'W30X173'	'W36X256'	'W12X210'	'W21X132'	'W21X166'	'W27X114'	'W33X241'	'W18X258'
	IB	'W27X84'	'W18X40'	'W14X26'	'W27X94'	'W24X84'	'W24X55'	'W21X50'	'W16X31'	'W14X22'
	OB	'W16X40'	'W33X130'	'W40X149'	'W18X35'	'W18X46'	'W21X44'	'W24X76'	'W33X118'	'W36X135'
	BR	'W24X76'	'W12X72'	'W8X58'	'W14X74'	'W12X53'	'W14X53'	'W14X68'	'W8X48'	'W21X62'
5–7	CG1	'W36X245'	'W24X162'	'W36X245'	'W40X249'	'W27X217'	'W14X145'	'W40X235'	'W18X258'	'W30X235'
	CG2	'W12X136'	'W12X170'	'W18X106'	'W27X146'	'W14X109'	'W24X117'	'W14X176'	'W12X120'	'W12X96'
	CG3	'W36X170'	'W27X94'	'W30X116'	'W24X76'	'W12X106'	'W27X161'	'W30X99'	'W30X148'	'W33X118'
	CG4	'W36X135'	'W40X199'	'W27X94'	'W30X261'	'W44X230'	'W24X103'	'W18X119'	'W18X86'	'W18X86'
	CG5	'W10X112'	'W30X191'	'W12X190'	'W24X76'	'W24X94'	'W14X159'	'W36X150'	'W33X169'	'W40X174'
	IB	'W18X40'	'W18X55'	'W21X44'	'W21X57'	'W18X65'	'W27X84'	'W21X62'	'W24X55'	'W21X44'
	OB	'W40X149'	'W30X99'	'W24X68'	'W30X99'	'W24X62'	'W21X57'	'W27X102'	'W21X68'	'W24X84'
	BR	'W10X68'	'W12X50'	'W12X87'	'W10X39'	'W8X48'	'W8X58'	'W14X68'	'W14X68'	'W16X77'
8–10	CG1	'W30X132'	'W27X258'	'W18X76'	'W16X77'	'W21X73'	'W33X130'	'W10X68'	'W12X87'	'W24X76'
	CG2	'W27X178'	'W12X152'	'W21X57'	'W14X99'	'W12X79'	'W14X193'	'W24X104'	'W12X40'	'W18X50'
	CG3	'W14X53'	'W27X146'	'W30X124'	'W18X192'	'W12X50'	'W16X57'	'W21X44'	'W27X114'	'W30X124'
	CG4	'W14X90'	'W30X108'	'W33X201'	'W12X45'	'W44X230'	'W18X76'	'W24X68'	'W33X201'	'W40X199'
	CG5	'W12X152'	'W14X193'	'W21X101'	'W24X207'	'W33X201'	'W18X143'	'W14X99'	'W21X122'	'W27X114'
	IB	'W18X55'	'W18X35'	'W18X55'	'W10X26'	'W24X62'	'W16X26'	'W18X46'	'W21X44'	'W21X50'
	OB	'W18X40'	'W14X30'	'W16X26'	'W27X84'	'W8X18'	'W21X83'	'W10X22'	'W8X18'	'W10X22'
	BR	'W14X38'	'W12X50'	'W27X84'	'W16X45'	'W16X57'	'W16X40'	'W21X62'	'W14X61'	'W21X83'
Weight (ton)		581.05	578.96	576.45	598.97	568.48	543.02	559.32	550.61	560.26
Max drift index		0.95	0.99	0.99	0.98	0.99	1.00	0.99	0.99	0.98
Max capacity index		0.89	0.90	0.99	0.98	0.92	0.96	0.94	0.95	0.91
Practical weight		581.05	578.96	576.45	598.97	568.48	543.02	559.32	550.61	560.26

Table 7 10-story frame results; section properties, literature report, the total weight of converged designs, maximum DI and CI, weights in practice resulted utilizing penalty along with 9 meta-heuristics

Stories	Groups	Azad et al. 2010	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
1	CG1*	'W33X201'	'W18X192'	'W18X143'	'W21X147'	'W30X211'	'W14X455'	'W40X211'	'W18X283'	'W36X359'	'W33X318'
	CG2	'W24X146'	'W27X146'	'W24X131'	'W27X161'	'W24X146'	'W21X122'	'W30X124'	'W40X199'	'W40X278'	'W24X192'
	CG3	'W24X104'	'W16X100'	'W40X199'	'W18X175'	'W10X112'	'W21X147'	'W30X211'	'W30X108'	'W40X431'	'W30X116'
	CG4	'W40X174'	'W14X159'	'W30X191'	'W30X191'	'W14X176'	'W24X162'	'W36X150'	'W24X176'	'W24X117'	'W40X174'
	CG5	'W40X321'	'W14X342'	'W36X160'	'W18X158'	'W14X311'	'W33X130'	'W14X159'	'W36X135'	'W30X211'	'W21X201'
	IB*	'W27X84'	'W24X84'	'W33X118'	'W24X94'	'W18X86'	'W24X55'	'W10X30'	'W21X50'	'W8X21'	'W8X24'
	OB*	'W27X84'	'W14X90'	'W12X58'	'W14X43'	'W24X76'	'W18X60'	'W27X84'	'W24X84'	'W21X73'	'W33X118'
	BR*	'W18X76'	'W14X74'	'W33X152'	'W12X65'	'W10X68'	'W14X90'	'W12X65'	'W21X50'	'W18X76'	'W8X24'
2–4	CG1	'W36X328'	'W33X318'	'W12X152'	'W27X129'	'W27X307'	'W40X167'	'W44X230'	'W40X215'	'W40X249'	'W40X278'
	CG2	'W36X245'	'W36X232'	'W14X176'	'W21X166'	'W36X260'	'W30X211'	'W33X118'	'W40X199'	'W33X169'	'W24X207'
	CG3	'W36X135'	'W33X141'	'W30X148'	'W21X201'	'W27X146'	'W36X150'	'W40X149'	'W27X178'	'W30X99'	'W40X183'
	CG4	'W33X118'	'W27X129'	'W21X111'	'W12X106'	'W33X118'	'W30X124'	'W27X146'	'W33X169'	'W30X99'	'W24X146'
	CG5	'W44X262'	'W40X278'	'W21X111'	'W30X124'	'W40X278'	'W30X99'	'W12X120'	'W24X176'	'W21X182'	'W27X146'
	IB	'W16X26'	'W16X31'	'W24X55'	'W24X68'	'W10X26'	'W16X40'	'W24X94'	'W24X55'	'W27X84'	'W14X26'
	OB	'W36X135'	'W40X149'	'W24X84'	'W33X118'	'W33X130'	'W33X130'	'W12X26'	'W14X61'	'W14X26'	'W27X129'
	BR	'W21X62'	'W8X67'	'W24X76'	'W24X76'	'W18X65'	'W10X54'	'W10X49'	'W14X74'	'W10X60'	'W10X68'
5–7	CG1	'W27X258'	'W33X241'	'W40X183'	'W30X148'	'W18X258'	'W40X174'	'W18X158'	'W36X182'	'W30X148'	'W30X124'
	CG2	'W18X106'	'W14X109'	'W14X109'	'W14X120'	'W21X111'	'W27X178'	'W40X199'	'W18X234'	'W21X122'	'W30X191'
	CG3	'W33X130'	'W33X118'	'W30X132'	'W27X114'	'W33X118'	'W27X161'	'W36X135'	'W30X108'	'W18X119'	'W27X129'
	CG4	'W27X94'	'W18X97'	'W21X83'	'W21X111'	'W27X102'	'W33X169'	'W24X131'	'W14X132'	'W27X161'	'W16X89'
	CG5	'W24X192'	'W24X176'	'W24X207'	'W24X131'	'W14X176'	'W33X141'	'W36X182'	'W27X146'	'W27X102'	'W27X84'
	IB	'W21X44'	'W18X46'	'W18X50'	'W12X65'	'W16X45'	'W24X62'	'W24X55'	'W24X68'	'W21X57'	'W21X68'
	OB	'W21X73'	'W24X76'	'W30X90'	'W30X116'	'W24X84'	'W24X103'	'W36X135'	'W24X76'	'W18X46'	'W30X99'
	BR	'W30X90'	'W14X90'	'W12X79'	'W18X71'	'W16X89'	'W16X31'	'W14X61'	'W10X49'	'W21X62'	'W30X90'

Table 7 (continued)

Stories	Groups	Azad et al. 2010	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
8–10	CG1	'W18X86'	'W24X76'	'W24X62'	'W36X160'	'W16X77'	'W24X84'	'W18X40'	'W16X77'	'W14X53'	'W24X84'
	CG2	'W21X50'	'W10X54'	'W30X173'	'W14X99'	'W18X55'	'W30X99'	'W18X106'	'W14X90'	'W14X120'	'W14X145'
	CG3	'W36X135'	'W27X129'	'W21X122'	'W30X116'	'W24X131'	'W18X46'	'W18X86'	'W30X132'	'W12X120'	'W8X40'
	CG4	'W33X201'	'W14X211'	'W14X74'	'W21X111'	'W40X215'	'W12X87'	'W24X68'	'W18X130'	'W33X141'	'W24X76'
	CG5	'W30X108'	'W14X99'	'W12X120'	'W21X73'	'W24X117'	'W14X68'	'W12X120'	'W8X35'	'W16X77'	'W14X109'
	IB	'W21X57'	'W24X55'	'W21X50'	'W14X43'	'W24X55'	'W14X43'	'W14X34'	'W14X34'	'W16X36'	'W16X45'
	OB	'W16X26'	'W8X21'	'W24X76'	'W21X44'	'W14X26'	'W24X55'	'W21X50'	'W21X50'	'W21X73'	'W10X22'
	BR	'W18X76'	'W24X68'	'W12X58'	'W30X90'	'W18X76'	'W14X48'	'W12X45'	'W14X38'	'W18X55'	'W18X71'
Weight (ton)		584.93	591.36	591.53	595.71	591.34	571.03	549.24	563.19	549.51	558.05
Max drift index		–	1.00	0.97	1.00	1.04	1.00	1.02	1.01	1.03	1.03
Max capacity index		–	0.89	0.90	0.88	0.92	0.99	0.93	0.99	0.97	1.03
Practical weight		584.93	591.36	591.53	595.71	612.08	571.03	561.13	585.74	558.23	569.55

braces apply to frames one in a between; namely, grids 1,3,5, and 7 are braced parallel to x-direction and grids b, d, f, and h are braced parallel to y-direction as shown in Fig. 9. Considering practical construction easiness, members are divided into 73 groups, in every two stories; columns are divided into five groups, beams into two inner and outer groups and braces solely into one group for both stories. The effective length factor and unbraced length factor are the same as in the first example. This example was optimized by Azad et al. (2014) applied gravity loads, dead, and live loads, assigned as distributed loads on the beams of each floor. For the floors, 14 KN/m and 10 KN/m were applied as dead load and live load, respectively; also, 12 KN/m and 7 KN/m were applied on the roof story as dead and live loads, respectively. As a reminder, the self-weight of the structure is also added to the dead load. Like the first example, the base shear is equal to 10 percent of the effective weight of the structure; C_T is taken equal to 0.0488; and H is the height of the structure equal to 70 m for this example. The fundamental period of this structure, T, is equal to 1.181 s, and the K factor for this value of the fundamental period is equal to 1.341. Load combinations are the same as in the first example.

5.4 60-story Mega-braced tubed-frame

This high-rise building is made up of four rectangular independent tubed frames. As shown in Fig. 10, tube A has 4 bays in x-direction and 2 bays in the y direction; tube B has 6 bays in x-direction and 4 bays in the y-direction. Both tubes A and B continue to the top story. Tube C has 8 bays in the x-direction and 6 bays in the y-direction, and tube D has 10 bays in the x-direction and 8 bays in the y-direction in the plan. Tubes C and D continue to stories 42 and 24, respectively. The outer tube D has mega-braces on each of its four elevation views, which are lozenge-shaped braces repeated every 6 stories. As depicted in Fig. 8, every lozenge brace consists of 4 independent elements. However, tubes B and C have just bracings in the extreme right and extreme left bays at each of their four elevation views. This structure contains 8272 frame members, 3960 beams, 3960 columns, and 352 braces. It is noteworthy that tubes connect by in-plane beam elements, hinged to the joints of the two tubes; therefore, the gravity loads of the floor system apply to these elements. However, since the floor joints are rigid-body diaphragms, these elements use determined sections and do not play any role in structural optimization and are therefore excluded from structural members and optimization process. All the members are divided into 103 groups; each group covers every six succeeding stories; in the plan of a story, any tube has its own beam group and columns are dedicated to the corner and side groups. Every 6 adjacent stories have their own

Table 8 X-braced 20-story frame results; section properties, the total weight of converged designs, maximum DI and CI, weights in practice resulted utilizing SMF method along with 9 meta-heuristics

Story	Group	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
1–2	*CG1	'W36X256'	'W33X221'	'W12X210'	'W12X45'	'W18X106'	'W40X466'	'W14X211'	'W14X283'	'W33X291'
	CG2	'W33X291'	'W36X280'	'W21X147'	'W14X233'	'W30X235'	'W18X234'	'W14X145'	'W18X311'	'W30X235'
	CG3	'W30X477'	'W30X235'	'W40X264'	'W36X300'	'W36X230'	'W12X190'	'W33X201'	'W40X264'	'W33X221'
	CG4	'W18X234'	'W36X194'	'W24X492'	'W40X297'	'W33X263'	'W30X391'	'W24X207'	'W12X305'	'W14X500'
	CG5	'W14X257'	'W40X215'	'W12X106'	'W24X207'	'W14X120'	'W21X101'	'W21X147'	'W36X170'	'W24X117'
	*IB	'W18X55'	'W30X116'	'W21X68'	'W18X86'	'W30X108'	'W16X67'	'W27X84'	'W18X55'	'W21X73'
	*OB	'W8X58'	'W24X62'	'W16X67'	'W12X96'	'W8X58'	'W14X120'	'W12X50'	'W21X68'	'W12X106'
	*BR	'W24X94'	'W12X53'	'W8X40'	'W14X68'	'W8X40'	'W8X48'	'W8X58'	'W10X68'	'W14X61'
3–4	CG1	'W12X336'	'W40X277'	'W40X235'	'W27X194'	'W24X176'	'W27X102'	'W33X118'	'W27X307'	'W14X90'
	CG2	'W40X167'	'W14X398'	'W18X143'	'W33X152'	'W36X182'	'W36X300'	'W18X130'	'W12X305'	'W30X326'
	CG3	'W12X305'	'W36X300'	'W40X215'	'W40X297'	'W12X336'	'W14X211'	'W36X232'	'W21X147'	'W24X250'
	CG4	'W12X210'	'W36X280'	'W40X321'	'W36X182'	'W44X262'	'W40X503'	'W27X217'	'W40X392'	'W14X342'
	CG5	'W14X176'	'W14X193'	'W30X148'	'W36X230'	'W36X160'	'W14X90'	'W21X132'	'W40X264'	'W24X207'
	IB	'W24X62'	'W18X60'	'W21X101'	'W14X99'	'W18X55'	'W24X131'	'W21X62'	'W30X116'	'W24X68'
	OB	'W18X71'	'W21X101'	'W21X101'	'W18X119'	'W16X67'	'W12X58'	'W16X89'	'W21X68'	'W27X102'
	BR	'W10X54'	'W18X76'	'W24X68'	'W18X76'	'W21X111'	'W14X61'	'W14X68'	'W8X40'	'W8X40'
5–6	CG1	'W10X26'	'W16X40'	'W33X201'	'W12X120'	'W36X182'	'W16X31'	'W12X58'	'W18X143'	'W30X90'
	CG2	'W12X210'	'W30X132'	'W21X93'	'W24X94'	'W24X117'	'W36X210'	'W14X211'	'W30X132'	'W18X119'
	CG3	'W36X210'	'W33X354'	'W12X230'	'W27X235'	'W40X331'	'W18X119'	'W33X263'	'W33X221'	'W33X241'
	CG4	'W12X152'	'W12X136'	'W40X235'	'W14X605'	'W27X307'	'W36X328'	'W24X408'	'W14X283'	'W14X311'
	CG5	'W30X235'	'W27X161'	'W24X94'	'W36X439'	'W30X124'	'W12X120'	'W30X148'	'W40X593'	'W16X100'
	IB	'W16X57'	'W30X90'	'W12X58'	'W16X77'	'W21X73'	'W24X68'	'W27X114'	'W24X76'	'W21X83'
	OB	'W18X130'	'W30X148'	'W14X68'	'W36X182'	'W24X94'	'W18X86'	'W14X53'	'W12X65'	'W21X68'
	BR	'W14X61'	'W10X49'	'W21X73'	'W30X116'	'W14X82'	'W14X61'	'W10X39'	'W14X48'	'W12X50'
7–8	CG2	'W12X120'	'W18X211'	'W18X76'	'W14X99'	'W30X173'	'W44X335'	'W24X131'	'W27X129'	'W21X166'
	CG3	'W40X297'	'W27X194'	'W40X372'	'W36X150'	'W27X368'	'W14X257'	'W21X201'	'W24X492'	'W24X192'
	CG4	'W14X90'	'W30X261'	'W30X235'	'W27X217'	'W14X257'	'W30X391'	'W21X132'	'W24X229'	'W24X279'
	CG5	'W30X477'	'W40X249'	'W14X68'	'W44X230'	'W18X192'	'W12X72'	'W12X106'	'W14X145'	'W12X152'
	IB	'W30X108'	'W27X102'	'W18X60'	'W21X62'	'W27X114'	'W24X94'	'W24X62'	'W16X57'	'W24X62'
	OB	'W14X48'	'W21X73'	'W30X173'	'W12X152'	'W12X58'	'W30X99'	'W30X326'	'W21X73'	'W21X182'
	BR	'W12X58'	'W14X99'	'W10X49'	'W12X72'	'W8X58'	'W8X31'	'W14X61'	'W10X54'	'W8X48'

Table 8 (continued)

Story	Group	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
9–10	CG2	'W36X280'	'W36X280'	'W30X132'	'W14X61'	'W33X130'	'W21X101'	'W24X104'	'W18X106'	'W24X131'
	CG3	'W33X318'	'W36X280'	'W12X190'	'W27X448'	'W40X264'	'W16X100'	'W44X290'	'W30X116'	'W24X250'
	CG4	'W12X87'	'W27X307'	'W14X665'	'W10X112'	'W21X132'	'W24X162'	'W18X119'	'W33X152'	'W14X283'
	CG5	'W12X252'	'W24X117'	'W18X234'	'W12X120'	'W27X94'	'W24X84'	'W21X111'	'W40X149'	'W16X67'
	IB	'W12X79'	'W30X116'	'W36X160'	'W27X84'	'W30X124'	'W10X49'	'W16X57'	'W12X72'	'W18X86'
	OB	'W14X109'	'W18X106'	'W12X65'	'W24X62'	'W8X48'	'W14X48'	'W21X68'	'W21X101'	'W36X194'
	BR	'W12X65'	'W12X106'	'W21X62'	'W10X68'	'W8X48'	'W14X90'	'W18X97'	'W12X53'	'W24X104'
11–12	CG2	'W24X229'	'W18X106'	'W21X68'	'W18X86'	'W14X82'	'W14X99'	'W14X99'	'W16X67'	'W40X199'
	CG3	'W18X283'	'W10X112'	'W44X230'	'W36X245'	'W18X119'	'W18X192'	'W14X257'	'W40X331'	'W12X79'
	CG4	'W21X111'	'W36X210'	'W36X359'	'W27X539'	'W18X258'	'W24X492'	'W24X117'	'W40X199'	'W27X161'
	CG5	'W40X321'	'W24X335'	'W8X58'	'W12X279'	'W40X149'	'W14X99'	'W30X173'	'W14X90'	'W12X96'
	IB	'W18X60'	'W24X103'	'W30X132'	'W24X146'	'W21X68'	'W21X62'	'W18X86'	'W27X129'	'W30X108'
	OB	'W14X90'	'W14X145'	'W21X147'	'W14X109'	'W12X53'	'W36X135'	'W18X55'	'W16X100'	'W8X48'
	BR	'W18X76'	'W6X16'	'W16X67'	'W24X94'	'W16X77'	'W14X48'	'W14X61'	'W8X35'	'W8X40'
13–14	CG2	'W18X311'	'W24X104'	'W21X101'	'W30X235'	'W24X279'	'W40X183'	'W33X152'	'W14X99'	'W21X93'
	CG3	'W33X169'	'W12X136'	'W40X174'	'W12X120'	'W24X162'	'W27X102'	'W12X305'	'W10X100'	'W18X311'
	CG4	'W12X79'	'W36X256'	'W30X116'	'W10X88'	'W36X194'	'W30X326'	'W10X60'	'W14X99'	'W33X201'
	CG5	'W30X261'	'W18X258'	'W18X97'	'W36X160'	'W18X60'	'W10X68'	'W33X291'	'W14X99'	'W21X132'
	IB	'W21X68'	'W16X57'	'W24X68'	'W24X68'	'W8X48'	'W24X104'	'W18X106'	'W24X94'	'W12X79'
	OB	'W40X167'	'W21X147'	'W8X48'	'W10X112'	'W18X65'	'W10X88'	'W18X86'	'W27X84'	'W24X94'
	BR	'W24X104'	'W14X43'	'W8X48'	'W14X61'	'W14X43'	'W8X31'	'W10X60'	'W8X40'	'W10X60'
15–16	CG2	'W30X191'	'W10X88'	'W24X162'	'W12X58'	'W14X48'	'W30X148'	'W44X335'	'W21X101'	'W10X60'
	CG3	'W33X130'	'W40X167'	'W44X262'	'W14X370'	'W21X62'	'W12X120'	'W24X162'	'W24X192'	'W27X178'
	CG4	'W44X335'	'W10X88'	'W27X217'	'W21X93'	'W14X257'	'W24X103'	'W10X77'	'W44X262'	'W40X167'
	CG5	'W21X147'	'W33X318'	'W21X62'	'W18X60'	'W40X167'	'W14X68'	'W16X67'	'W18X97'	'W40X297'
	IB	'W18X76'	'W10X77'	'W16X57'	'W10X60'	'W27X129'	'W16X67'	'W14X48'	'W14X82'	'W24X103'
	OB	'W36X160'	'W24X62'	'W30X108'	'W12X190'	'W27X146'	'W14X48'	'W30X148'	'W18X97'	'W30X90'
	BR	'W8X31'	'W8X31'	'W18X50'	'W27X161'	'W12X53'	'W18X71'	'W8X31'	'W8X48'	'W10X33'
17–18	CG2	'W21X101'	'W27X114'	'W27X307'	'W8X48'	'W33X141'	'W14X48'	'W12X65'	'W21X50'	'W18X192'
	CG3	'W16X57'	'W14X53'	'W33X241'	'W40X211'	'W36X150'	'W27X84'	'W21X201'	'W18X130'	'W8X67'
	CG4	'W30X292'	'W16X57'	'W36X260'	'W36X300'	'W14X90'	'W18X60'	'W16X67'	'W36X182'	'W33X130'
	CG5	'W12X79'	'W24X229'	'W30X235'	'W36X393'	'W14X61'	'W30X99'	'W18X86'	'W18X71'	'W36X135'
	IB	'W18X55'	'W12X170'	'W30X116'	'W36X182'	'W18X60'	'W10X54'	'W30X116'	'W40X149'	'W24X104'
	OB	'W24X103'	'W12X53'	'W10X88'	'W18X71'	'W14X109'	'W12X79'	'W10X60'	'W8X48'	'W8X67'
	BR	'W10X60'	'W8X67'	'W8X28'	'W16X40'	'W10X77'	'W8X31'	'W8X31'	'W8X31'	'W24X68'

Table 8 (continued)

Story	Group	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
19–20	CG2	'W30X235'	'W14X120'	'W8X67'	'W30X116'	'W21X132'	'W14X30'	'W14X43'	'W14X90'	'W24X146'
	CG3	'W12X96'	'W12X30'	'W21X147'	'W21X132'	'W8X31'	'W16X50'	'W14X68'	'W14X30'	'W21X73'
	CG4	'W24X146'	'W14X38'	'W44X290'	'W18X211'	'W44X230'	'W14X48'	'W10X30'	'W36X160'	'W10X45'
	CG5	'W24X146'	'W12X136'	'W14X48'	'W24X335'	'W12X106'	'W12X96'	'W40X235'	'W21X68'	'W12X65'
	IB	'W12X136'	'W18X106'	'W24X76'	'W16X77'	'W12X120'	'W24X103'	'W12X58'	'W21X68'	'W16X57'
	OB	'W12X87'	'W18X192'	'W21X83'	'W40X199'	'W30X99'	'W18X65'	'W14X99'	'W12X53'	'W14X109'
	BR	'W6X15'	'W8X24'	'W6X20'	'W14X193'	'W6X20'	'W10X33'	'W24X104'	'W8X28'	'W12X79'
Weight (ton)		2993.51	3112.13	3036.29	3614.55	2913.79	2713.57	2817.29	2775.39	2921.28
Max drift index		0.85	0.8	0.69	0.74	0.91	0.97	0.85	0.92	0.92
Max capacity index		1.00	0.98	0.99	1.00	0.99	0.98	0.96	0.99	0.99
Practical weight		2993.51	3112.13	3036.29	3614.55	2913.79	2713.57	2817.29	2775.39	2921.28

bracing group, as well. The typical story height is equal to 3 m for all the stories. Like the former example, dead and live loads applied on story beams are equal to 14KN/m and 10KN/m, respectively; however, for roof story, these values are equal to 12KN/m and 7KN/m, respectively, because of the high-rise nature of this structure, the wind load is supposed to be the major lateral load. According to ASCE (2000), wind load may be computed as

$$P_w = (0.613 K_z K_{zt} K_d V^2 I)(GC_p) \tag{26}$$

in which P_w is the wind pressure on the structural surface in KN/m^2; Kz is the factor of velocity exposure; Kzt is the factor related to topography; Kd is the factor of winding direction; V is the wind speed; G denotes the gust factor; and C_p denotes the external pressure coefficient. In this example, wind speed was set to 85 mph and exposure type to B; Kzt is set to 1, Kd equals to 0.85, and G is 0.85; Cp equals to 0.8 and 0.45 for windward and leeward faces of the building, respectively. Wind loads are considered as the major lateral loading.

5.5 Discussion on results

5.5.1 Convergence results

Tables 6 and 7 present results for the 10-story frame, utilizing the SMF and a penalty method to handle design criteria, respectively. All nine algorithms with both constraint handling methods are used. The best weight among tables for the CSS algorithm, as shown in Table 6, is equal to 543.02 (ton) found via applying the SMF, which is 8% lighter than the result reported in the literature (i.e., in Azad et al. (2013) equal to 584.93 (ton)) as mentioned in Table 7. Since some results have some violations, to have a fair comparison, total weight is modified to present feasible weight, mentioned as practical weight. It means that we made some modification on the final sections reported by the algorithm to exchange an infeasible result for a feasible one. This modification is not necessary for feasible reported results. The reason is that results handled by penalties in structural design optimization usually are not feasible solutions, as seen in our results. Thus, manual manipulation in found final sections is necessary to gain a feasible solution in engineering practice. This process usually increases the total weight of the structure. As seen in Table 7, five of nine reported solutions obtained by GA, CSS, ISA, SOS, and WOA with the weight of 591.34, 549.24, 563.19, 549.51, 558.05 tons are infeasible, and some modifications are needed upon their sections. Authors reported new total weights as GA: 612.08 (tons), CSS: 561.13 (ton), ISA: 585.74(ton), SOS: 558.23 (ton), WOA: 569.55 (ton). All these manipulated designs experienced an increase in the total weight with a minimum value of 2%

Table 9 X-braced 20-story frame results; section properties, literature report, the total weight of converged designs, maximum DI and CI, weights in practice resulted utilizing penalty along with 9 meta-heuristics

Story	Group	Hasançebi and Erbatur 2000	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
1–2	*CG1	W24X94	W27X178	W24X207	W18X55	W14X109	W33X241	W21X83	W10X88	W18X192	W12X305
	CG2	W27X368	W21X166	W12X190	W12X136	W12X79	W36X256	W18X175	W27X235	W24X146	W21X182
	CG3	W30X477	W30X326	W12X230	W36X300	W33X221	W14X455	W36X182	W33X201	W12X230	W14X257
	CG4	W40X199	W44X335	W36X328	W40X199	W36X527	W18X106	W14X455	W24X492	W24X492	W24X192
	CG5	W30X191	W33X241	W36X160	W18X143	W36X160	W12X190	W40X167	W21X166	W36X170	W33X221
	*IB	W12X152	W24X76	W18X50	W24X68	W30X148	W27X84	W18X71	W24X62	W12X50	W27X84
	*OB	W10X30	W12X65	W18X50	W18X76	W24X117	W18X50	W12X53	W30X99	W16X57	W18X65
	*BR	W14X82	W14X26	W12X58	W12X53	W21X73	W24X84	W12X72	W12X40	W14X68	W12X53
3–4	CG1	W18X65	W18X211	W27X368	W10X49	W36X230	W12X190	W10X54	W21X111	W24X94	W14X43
	CG2	W36X439	W21X122	W40X215	W36X160	W10X77	W40X249	W27X161	W40X174	W18X130	W30X235
	CG3	W44X335	W40X264	W24X279	W40X297	W27X368	W18X258	W27X146	W24X162	W12X252	W33X291
	CG4	W36X170	W27X161	W27X448	W21X201	W33X354	W21X111	W24X335	W18X283	W33X291	W40X174
	CG5	W30X191	W36X260	W14X283	W33X241	W18X106	W12X336	W30X191	W12X136	W33X141	W27X258
	IB	W14X176	W14X53	W24X76	W24X103	W14X74	W14X48	W24X68	W27X84	W30X90	W21X83
	OB	W12X40	W18X71	W24X104	W10X45	W33X291	W27X84	W14X68	W24X104	W24X94	W24X117
	BR	W18X119	W27X146	W10X17	W24X68	W14X74	W10X112	W18X158	W8X67	W12X50	W30X108
5–6	CG1	W18X71	W21X111	W18X97	W24X94	W12X87	W30X148	W30X173	W12X50	W14X176	W27X178
	CG2	W44X335	W36X232	W24X192	W27X114	W18X234	W33X152	W18X158	W14X311	W40X199	W24X103
	CG3	W40X297	W33X141	W27X161	W27X161	W30X191	W14X500	W36X135	W14X455	W27X307	W30X326
	CG4	W36X160	W18X234	W36X328	W33X169	W36X300	W14X90	W30X292	W18X258	W27X235	W33X169
	CG5	W27X161	W36X150	W40X466	W18X119	W14X120	W18X158	W21X147	W12X210	W24X104	W36X256
	IB	W18X71	W18X71	W21X93	W14X61	W33X130	W21X73	W14X90	W24X68	W18X55	W24X103
	OB	W27X114	W12X53	W30X235	W18X86	W44X335	W27X129	W30X116	W30X132	W18X60	W8X48
	BR	W33X152	W10X88	W12X79	W16X89	W14X43	W10X19	W14X61	W14X53	W16X67	W14X74
7–8	CG2	W40X235	W18X234	W18X106	W24X104	W12X58	W12X96	W40X167	W24X207	W14X99	W10X100
	CG3	W24X207	W40X215	W36X280	W40X167	W14X550	W40X215	W40X392	W27X217	W12X190	W18X258
	CG4	W14X159	W27X368	W30X173	W33X152	W30X326	W14X61	W44X335	W40X593	W14X176	W14X193
	CG5	W12X210	W21X182	W12X136	W18X143	W27X235	W40X392	W27X539	W8X67	W21X101	W14X176
	IB	W30X99	W21X93	W14X90	W12X72	W14X82	W12X50	W24X76	W14X90	W12X58	W21X83
	OB	W8X35	W24X76	W36X135	W24X117	W21X73	W12X136	W12X53	W14X109	W10X88	W18X86
	BR	W12X53	W24X68	W8X35	W10X60	W12X22	W30X124	W8X35	W18X60	W8X67	W6X16

Table 9 (continued)

Story	Group	Hasançebi and Erbatur 2000	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
9–10	CG2	'W36X194'	'W18X76'	'W14X145'	'W40X277'	'W14X82'	'W18X119'	'W18X106'	'W18X258'	'W27X84'	'W40X235'
	CG3	'W24X250'	'W36X150'	'W40X235'	'W40X211'	'W14X398'	'W18X175'	'W30X326'	'W14X283'	'W12X305'	'W36X245'
	CG4	'W40X183'	'W40X235'	'W40X331'	'W24X146'	'W14X233'	'W8X58'	'W44X262'	'W27X368'	'W30X391'	'W30X116'
	CG5	'W21X147'	'W40X211'	'W14X99'	'W21X93'	'W12X230'	'W12X230'	'W12X87'	'W12X58'	'W30X108'	'W33X263'
	IB	'W40X211'	'W30X99'	'W40X149'	'W16X57'	'W14X53'	'W14X53'	'W30X90'	'W16X57'	'W14X82'	'W12X96'
	OB	'W30X124'	'W18X192'	'W12X50'	'W18X175'	'W18X211'	'W18X65'	'W14X74'	'W21X73'	'W10X49'	'W24X68'
	BR	'W18X76'	'W8X31'	'W24X84'	'W14X82'	'W10X33'	'W14X74'	'W30X124'	'W12X45'	'W12X87'	'W12X45'
11–12	CG2	'W21X182'	'W40X211'	'W27X194'	'W14X193'	'W44X230'	'W24X84'	'W14X74'	'W10X60'	'W30X211'	'W8X67'
	CG3	'W12X210'	'W40X277'	'W24X192'	'W14X370'	'W21X122'	'W12X170'	'W33X141'	'W36X210'	'W36X245'	'W27X307'
	CG4	'W24X192'	'W33X141'	'W24X408'	'W24X229'	'W36X232'	'W14X61'	'W40X199'	'W40X278'	'W33X201'	'W14X145'
	CG5	'W24X162'	'W40X278'	'W10X100'	'W24X207'	'W30X108'	'W14X90'	'W36X300'	'W24X84'	'W24X117'	'W40X215'
	IB	'W40X149'	'W18X71'	'W18X60'	'W30X90'	'W27X94'	'W24X62'	'W30X90'	'W30X124'	'W21X68'	'W33X118'
	OB	'W10X54'	'W27X161'	'W10X45'	'W10X54'	'W21X101'	'W18X86'	'W18X71'	'W21X132'	'W36X232'	'W14X68'
	BR	'W16X67'	'W14X43'	'W8X15'	'W16X67'	'W33X141'	'W21X101'	'W12X22'	'W8X31'	'W6X12'	'W10X60'
13–14	CG2	'W14X159'	'W18X97'	'W24X229'	'W18X258'	'W12X58'	'W14X90'	'W30X90'	'W30X116'	'W21X101'	'W21X83'
	CG3	'W40X264'	'W40X277'	'W14X311'	'W40X297'	'W24X162'	'W40X277'	'W18X143'	'W40X167'	'W36X328'	'W33X263'
	CG4	'W27X102'	'W21X201'	'W44X230'	'W27X161'	'W18X76'	'W8X40'	'W24X192'	'W40X211'	'W18X143'	'W40X149'
	CG5	'W27X129'	'W36X182'	'W18X60'	'W24X62'	'W33X263'	'W12X252'	'W33X118'	'W18X175'	'W18X143'	'W12X72'
	IB	'W10X77'	'W16X77'	'W24X117'	'W40X149'	'W12X50'	'W33X241'	'W36X135'	'W27X84'	'W16X57'	'W30X90'
	OB	'W33X318'	'W12X72'	'W33X152'	'W10X77'	'W24X408'	'W14X68'	'W30X124'	'W8X58'	'W14X74'	'W12X79'
	BR	'W16X67'	'W12X72'	'W16X77'	'W8X58'	'W33X221'	'W14X120'	'W8X40'	'W8X48'	'W8X58'	'W16X100'
15–16	CG2	'W18X86'	'W12X79'	'W36X232'	'W12X106'	'W12X58'	'W30X132'	'W14X53'	'W14X61'	'W36X194'	'W12X79'
	CG3	'W36X232'	'W10X77'	'W24X131'	'W36X232'	'W27X178'	'W21X166'	'W10X68'	'W44X290'	'W18X71'	'W40X149'
	CG4	'W14X90'	'W24X94'	'W24X229'	'W40X277'	'W40X211'	'W24X250'	'W30X292'	'W40X321'	'W40X199'	'W30X292'
	CG5	'W12X79'	'W27X539'	'W14X132'	'W14X132'	'W12X45'	'W12X305'	'W10X68'	'W8X58'	'W14X311'	'W36X245'
	IB	'W27X129'	'W14X53'	'W12X87'	'W36X135'	'W21X132'	'W24X62'	'W24X68'	'W30X132'	'W10X60'	'W12X79'
	OB	'W10X26'	'W12X170'	'W33X130'	'W8X48'	'W12X53'	'W21X147'	'W33X169'	'W14X90'	'W14X233'	'W10X49'
	BR	'W21X83'	'W16X50'	'W10X54'	'W21X166'	'W8X40'	'W14X90'	'W8X13'	'W8X15'	'W14X53'	'W16X26'

Table 9 (continued)

Story	Group	Hasançebi and Erbatur 2000	PSO	ACO	FA	GA	ICA	CSS	ISA	SOS	WOA
17–18	CG2	'W21X83'	'W14X730'	'W12X79'	'W30X116'	'W36X280'	'W33X263'	'W36X210'	'W24X84'	'W10X30'	'W14X74'
	CG3	'W18X119'	'W30X292'	'W44X230'	'W33X241'	'W12X79'	'W30X173'	'W16X77'	'W10X77'	'W18X234'	'W14X132'
	CG4	'W16X67'	'W21X62'	'W24X192'	'W40X264'	'W10X49'	'W10X49'	'W16X77'	'W36X300'	'W10X39'	'W33X291'
	CG5	'W18X65'	'W14X68'	'W12X45'	'W14X211'	'W14X61'	'W36X160'	'W12X120'	'W36X328'	'W14X61'	'W30X116'
	IB	'W18X35'	'W36X150'	'W33X169'	'W18X55'	'W40X211'	'W30X116'	'W16X67'	'W8X67'	'W18X65'	'W12X96'
	OB	'W18X35'	'W21X83'	'W21X132'	'W27X84'	'W30X90'	'W27X102'	'W12X96'	'W14X48'	'W24X103'	'W36X160'
	BR	'W10X45'	'W12X72'	'W18X50'	'W21X73'	'W33X130'	'W8X28'	'W12X65'	'W10X45'	'W14X61'	'W16X89'
	CG2	'W10X30'	'W18X40'	'W14X90'	'W30X173'	'W18X86'	'W8X31'	'W10X30'	'W12X106'	'W12X35'	'W12X87'
	CG3	'W10X33'	'W8X48'	'W18X46'	'W30X90'	'W14X48'	'W14X53'	'W10X60'	'W24X94'	'W18X46'	'W10X112'
	CG4	'W36X210'	'W18X106'	'W14X34'	'W12X53'	'W36X260'	'W16X31'	'W21X166'	'W33X241'	'W36X439'	'W27X84'
	CG5	'W10X100'	'W30X191'	'W18X40'	'W14X211'	'W8X35'	'W30X235'	'W12X252'	'W12X210'	'W12X252'	'W40X278'
	IB	'W36X150'	'W14X68'	'W18X71'	'W10X77'	'W14X132'	'W18X60'	'W16X57'	'W18X76'	'W12X58'	'W10X88'
	OB	'W16X45'	'W14X132'	'W18X97'	'W12X170'	'W18X234'	'W14X53'	'W10X68'	'W16X77'	'W30X191'	'W12X152'
	BR	'W16X45'	'W14X82'	'W6X9'	'W10X39'	'W14X132'	'W21X101'	'W10X30'	'W12X40'	'W16X45'	'W6X9'
Weight (ton)		3539.83	3066.03	3155.36	3077.17	3985.43	3106.56	2822.70	2918.67	2818.76	2994.61
Max drift index		–	0.75	0.82	0.79	0.80	0.98	0.86	0.85	0.99	0.96
Max capacity index		–	1.04	1.03	1.03	1.02	1.02	1.01	1.05	1.05	1.03
Practical weight		3539.83	3290.15	3340.73	3077.17	4026.47	3211.41	2996.97	3052.41	3042.87	3100.23
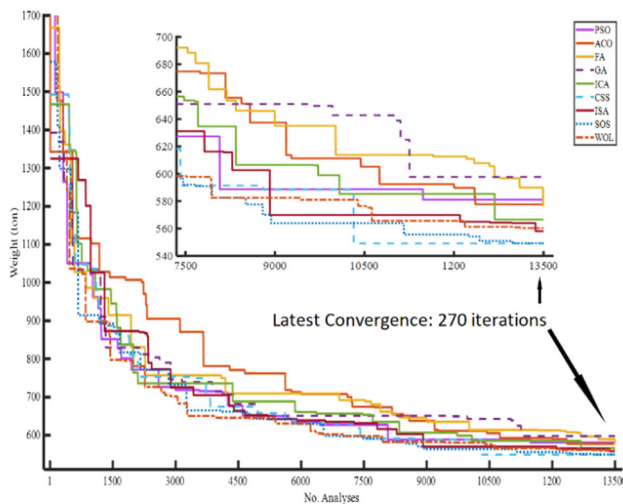
Fig. 11 Convergence curvature of the best result of first frame (SMF)
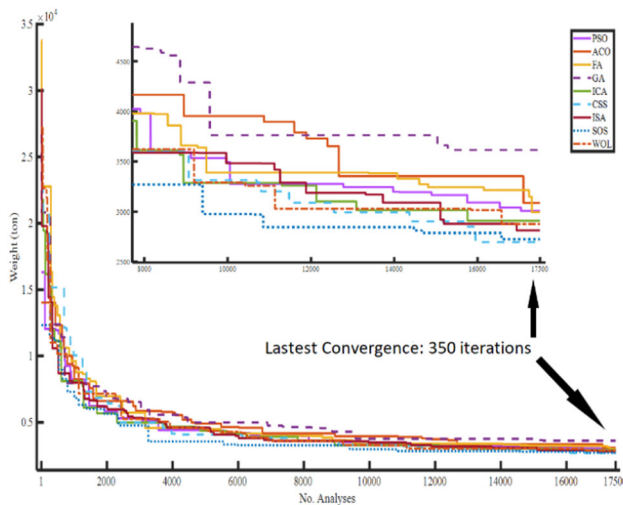
Fig. 12 Convergence curvature of the best result of second frame (SMF)

for SOS and WOA and the maximum of 4% for ISA. In addition to this, such manipulation is an excessive annoying process for designers. One superior feature demonstrated by the results in Table 6 is that the SMF converges to feasible solutions in almost all cases, unlike the penalty methods. All the solutions in Table 6 are feasible ones and even some of them have DI equal to or near to one (FA, ICA, CSS, ISA), which is the governing design criteria for this example. It means that SMF precisely detects infeasibility and performs better than penalty over the boundary.

For the second example, Tables 8 and 9 present results for an X-braced 20-story frame, utilizing the SMF and the penalty mothed, respectively. Again, the best weight in both tables belongs to the CSS with a weight of 2713.57 (tons) found via applying the SMF which is much better than results obtained by the same algorithms found by

penalty methods and 24% lighter than the result mentioned by Azad and Hasançebi (2015). Again, all the results of the SMF are feasible solutions, while many of the results of the penalty method needed handy manipulation. The increase in the weight after modification is presented in Table 9. Like the 10-story example, the SMF shows robust performance in discovering near/on the boundary solutions. Capacity, unlike the 10-story example, is the governing design criteria, and four of nine designs found by the SMF are exactly on the boundary with maximum CI near/equal to one.

Figures 11 and 12 are the convergence history of the best solutions found via the SMF for the two examples. As seen from the first example, after a minimum 13,500 analyses (270 iterations), all the algorithms show no more exploration while a minimum of 25,000 analyses (500 iterations) are required by most of the chosen penalty techniques. Corresponding numbers for the second example are17500 (350 iterations) and 25,000 (500 iterations), respectively. So, a tangible 46% and 30% reduction in the number of required analyses is achieved using SMF for the first and second examples, respectively.

The best results obtained for the first example using the SMF method, belonging to the CSS and SOS algorithms, are 543.02 and 550.61 (tons), respectively. ISA, WOA, ICA, FA, ACO, PSO, and GA are placed the third to ninth levels. The values are 559.32 (ton), 560.26 (ton), 568.48 (ton), 576.45 (ton), 578.96 (ton), 581.05 (ton), 598.27 (ton). The order for the results of the second example obtained by the SMF is mostly the same as the first example: CSS, SOS, ISA, WOA, ICA, FA, PSO, ACO, GA, with the weights equal to 2713.57 (ton), 2775.39 (ton), 2817.29 (ton), 2921.28 (ton), 2913.79 (ton), 3036.29 (ton), 2993.51 (ton), 3112.13 (ton), 3614.55 (ton), respectively. This consistency can be interpreted as the stability of the performance when applying the SMF. However, for the penalties, the order of the consequent weights is quite messy; for the 10-story example, the order is as CSS, SOS, WOA, ISA, ICA, GA, PSO, ACO, FA, while for the 20-story frame, it is as CSS, SOS, ISA, WOA, PSO, FA, ICA, ACO, GA. This disorder shows that the utilized penalty in this research needs tuning to assure high-quality convergence.

The all-stories DIs of the first example for the results of three superior algorithms, i.e., CSS, SOS, and ISA, are shown in Fig. 13, comparing the results by SMF and penalty methods. As it can be seen, the distribution of displacement among stories of the results reported by the SMF is smoother while the results obtained by the penalty method mostly show a harsh distribution that leads to slightly infeasible solutions. As this structure is not braced in the y-direction, the CI is not the governing criteria; therefore, as shown in Fig. 13, both the SMF and penalty
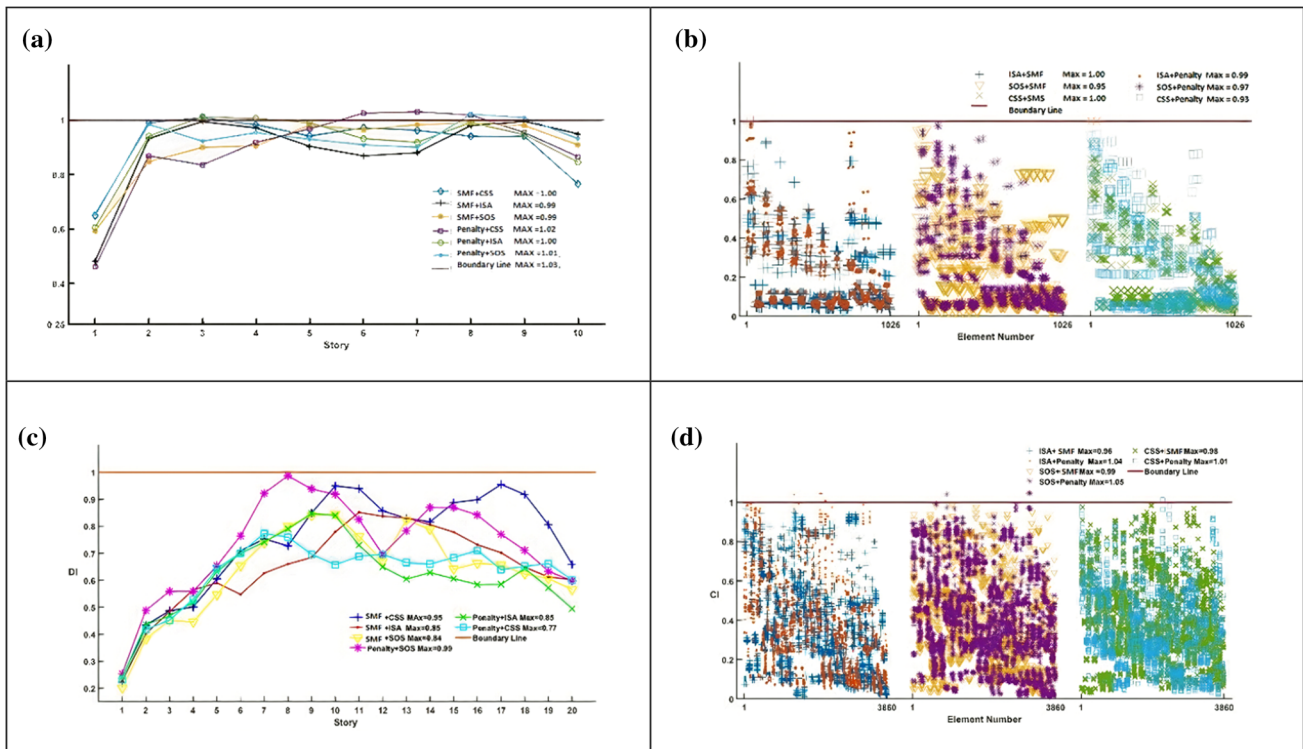
Fig. 13 **a** and **b** are the results of DIs and CIs of first example while applying SMF and Penalty with three optimizers ISA, SOS and CSS. **c** and **d** are the same for the second example

method show satisfied CI. The adverse situation is established for the 20-story frame; CI is the governing criteria since the structure is braced in both X and Y global directions. Figure 13 presents the value of DIs for the second example. All DIs are feasible. As shown in Fig. 13, CI for all the designs by the SMF methods is feasible and the result of SOS (Fig. 13d) is lying on the boundary where the CI equals to one for each element, while, like the 10-story example, two of three designs found by the penalty method have some violated CIs (Fig. 13).

For the third example, some common penalty techniques (as described in Coello (2001)) are utilized. Table 10 shows the best result found by CSS for this example with a weight equal to 6779.56 (ton). The maximum DI and maximum CI of this example are 0.98 and 0.87 which are feasible and directly found by the algorithm with no handy modifications (no local search); the quality of the result shows that it is an on-the-boundary solution like the two other examples. Figure 14 shows the all-stories DIs and all-elements CIs for this structure, respectively. Figure 15 shows the convergence curvature of the mega-braced tubed frame.

5.5.2 Statistical results

Table 11 shows a statistical comparison among the SMF and the penalty method for the first two examples extracted

by 30 independent optimization runs for each algorithm. For the 20-story frame, all the best results for the SMF are lighter than the ones for the penalty method, while for the 10-story frame, GA (penalty method: 591.34 against SMF: 598.28), WOA (penalty method: 558.05 against SMF: 560.26) and SOS (penalty method: 549.51, against SMF: 550.61) have provided slightly better results; however, SMF is still competitive since five other algorithms have found lighter results compared to the penalties. To compare with the results of literature, only GA has found heavier results (i.e., 598.27 (ton) via the SMF) compared to Azad et al. (2013) with 584.93 (ton) for the first example and all others are smaller. Similarly, all the best results are lighter than the one reported by Azad & Hasançebi (2015) (3539.83 (ton)) for the second example, but the other ones are competitive. In most cases, the mean of the finally converged independent results by SMF shows an absolute superiority against the results of the utilized penalty method and only WOA has shown a slightly vice versa performance (about only 5 tons and just for the first example).

Table 12 presents the statistical comparison among the penalty techniques and the SMF using the CSS algorithm for all examples obtained from 30 independent runs. As it is clear from Table 12, small values of the best and standard deviation are asserted for the SMF. Unstable order of

Table 10 Sections of the best solution found for tubed-frame

Stories	Groups	Section	Stories	Groups	Section	Stories	Groups	Section
1–6	CC-A*	'W44X290'	7–12	CC-A	'W40X199'	13–18	CC-A	'W21X83'
	SC-A*	'W40X174'		SC-A	'W27X102'		SC-A	'W18X130'
	CC-B*	'W30X132'		CC-B	'W24X103'		CC-B	'W40X249'
	SC-B*	'W16X40'		SC-B	'W12X120'		SC-B	'W30X211'
	CC-C*	'W30X292'		CC-C	'W24X229'		CC-C	'W14X176'
	SC-C*	'W36X650'		SC-C	'W18X211'		SC-C	'W24X207'
	CC-D*	'W21X147'		CC-D	'W40X174'		CC-D	'W30X108'
	SC-D*	'W27X307'		SC-D	'W33X318'		SC-D	'W36X194'
	BM-A*	'W40X199'		BM-A	'W36X439'		BM-A	'W12X210'
	BM-B*	'W30X235'		BM-B	'W12X136'		BM-B	'W14X193'
	BM-C*	'W12X230'		BM-C	'W10X30'		BM-C	'W36X256'
	BM-D*	'W40X167'		BM-D	'W30X108'		BM-D	'W40X235'
	BR-D*	'W12X210'		BR-D	'W30X211'		BR-D	'W8X48'
19–24	CC-A	'W14X43'	25–30	CC-A	'W30X148'	31–36	CC-A	'W21X50'
	SC-A	'W30X108'		SC-A	'W33X291'		SC-A	'W24X162'
	CC-B	'W36X245'		CC-B	'W30X477'		CC-B	'W33X221'
	SC-B	'W12X210'		SC-B	'W14X120'		SC-B	'W24X176'
	CC-C	'W27X194'		CC-C	'W18X234'		CC-C	'W36X170'
	SC-C	'W27X235'		SC-C	'W30X108'		SC-C	'W16X100'
	CC-D	'W40X264'		BM-A	'W36X245'		BM-A	'W27X217'
	SC-D	'W21X101'		BM-B	'W8X58'		BM-B	'W18X65'
	BM-A	'W21X62'		BM-C	'W14X120'		BM-C	'W36X256'
	BM-B	'W36X328'		BR–C*	'W36X245'		BR–C	'W21X44'
	BM-C	'W18X65'		–			–	
	BM-D	'W16X50'		–			–	
	BR-D	'W14X82'		–			–	
37–42	CC-A	'W30X99'	43–48	CC-A	'W24X146'	49–54	CC-A	'W21X122'
	SC-A	'W27X217'		SC-A	'W18X175'		SC-A	'W12X65'
	CC-B	'W12X136'		CC-B	'W30X261'		CC-B	'W8X67'
	SC-B	'W12X152'		SC-B	'W14X74'		SC-B	'W18X258'
	CC-C	'W36X280'		BM-A	'W10X100'		BM-A	'W21X50'
	SC-C	'W33X130'		BM-B	'W24X62'		BM-B	'W33X118'
	BM-A	'W44X262'		BR-B*	'W30X108'		BR-B	'W12X50'
	BM-B	'W14X48'		–			–	
	BM-C	'W24X104'		–			–	
	BR–C	'W36X160'		–			–	
55–60	CC-A	'W14X53'						
	SC-A	'W10X39'						
	CC-B	'W40X249'						
	SC-B	'W18X55'	Total Weight (ton) = 6779.56					
	BM-A	'W27X114'						
	BM-B	'W14X99'						
	BR-B	'W40X321'						

a C-A, CC-B: CC-C, CC-D: Corner Columns in Tubes A, B, C and D, respectively. SC-A, SC-B, SC-C, SC-D: side columns in tubes A, B, C and D, respectively, BM-A, BM-B, BM-C, BM-D: beam members in tubes A, B, C and D, respectively, BR-B, BR–C, BR-D: bracing members in tubes B, C and D, respectively
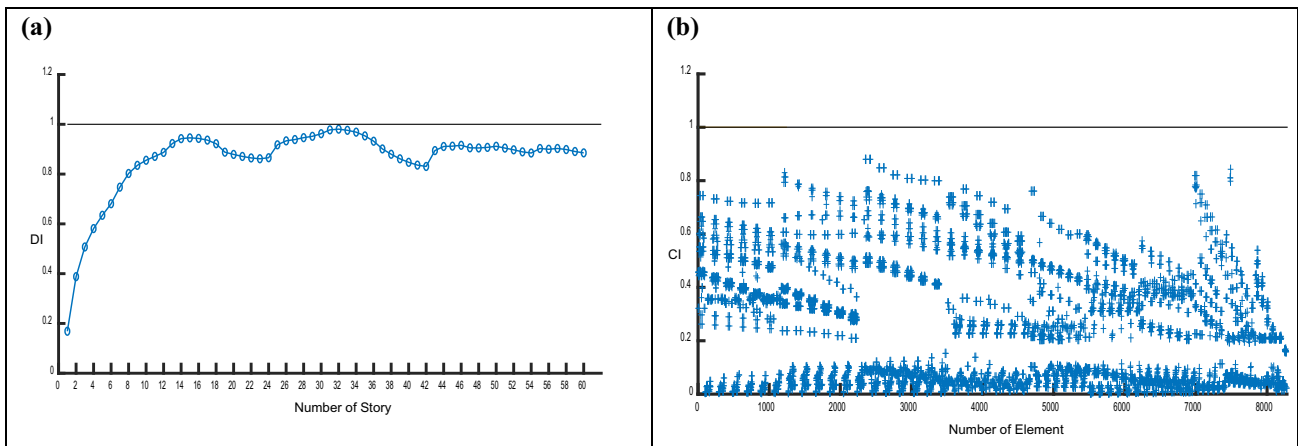
Fig. 14 **a** Drift Indices of the final design solution found via the SMF for the mega-braced 60-story frame; **b** Capacity Indices of the final design solution found via the SMF for the mega-braced 60-story frame
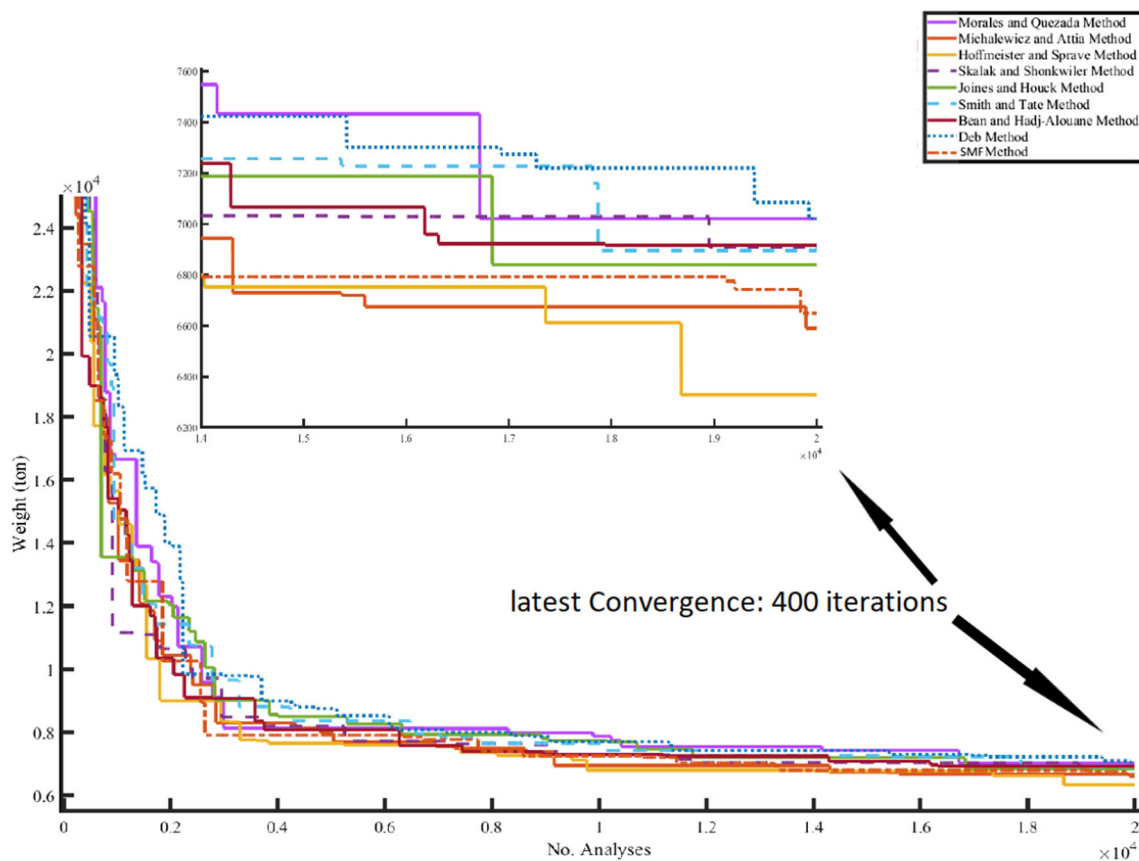

Fig. 15 Convergence curvature of the best result of the mega-braced 60-story frame

performance was seen in the first two examples, and poor statistical performance is also seen using different penalty techniques. For the first and second examples, standard deviations of results by penalties are too bad and just the penalty by Michalewicz and Attia (1994) has shown a better and reasonable performance. Also, just three and two answers among penalty results are feasible ones, for the

10-story and X-braced frame, respectively. For the third example, only three out of ten results in Table 9 have weights less than 6780 tons: SMF, Michalewicz and Attia (1994) and Hoffmeister and Sprave (1996), with weights equal to 6779.56 (ton), 6720.45 (ton), and 6453.09 (ton); all others have weights more than 6930 tons which are severely over-design. The penalty by Deb (2000), Skalak

Table 11 Statistical results for the first and second examples with different optimization methods

Methods	GA	PSO	ACO	SOS	ICA	WOA	ISA	FA	CSS	Literature
10-story frame										Kim et al. 2010
Best result										
Penalty method	591.34	591.36	591.35	549.51	571.03	558.05	563.19	595.71	549.24	584.93
SMF method	598.27	581.05	578.96	550.61	568.48	560.26	559.32	576.45	543.02	–
Mean of results										
Penalty method	832.09	866.11	862.81	654.03	808.24	766.50	733.15	765.13	645.81	–
SMF method	739.092	740.758	755.55	627.942	732.23	771.40	655.7629	711.74	618.11	–
X-Braced 20-story frame										Hasançebi and Erbatur 2000
Best result										
Penalty method	3985.43	3066.03	3155.36	2818.76	3106.56	2994.61	2918.67	3077.17	2822.70	3539.83
SMF method	3614.55	2993.51	3112.13	2775.39	2913.79	2921.28	2817.29	3036.29	2713.57	–
Mean of results										
Penalty method	5626.49	4683.780	4654.03	3259.86	4150.00	4125.60	3701.93	4116.53	3148.67	–
SMF method	4351.38	3983.34	4293.17	3003.12	3675.29	3963.89	3116.21	3465.44	2803.92	–

Table 12 shows the statistical results of comparisons for all three problems by different penalties

Methods	10-story Frame			
	Best Weight (ton)	Max RI	Max DI	Weight Std
Morales and Quezada	571.25	0.98	0.96	92.68
Michalewicz and Attia	549.24	1.02	0.93	16.29
Hoffmeister and Sprave	535.63	1.08	0.91	85.36
Skalak and Shonkwiler	556.33	0.98	0.90	63.56
Joines and Houck	541.92	1.04	0.89	48.99
Smith and Tate	561.32	1.01	0.92	68.96
Bean and Hadj-Alouane	555.68	1.05	0.95	62.96
Deb	568.37	0.98	0.92	93.61
SMF	543.02	1.00	0.96	22.36
	X-Braced Frame			
Morales and Quezada	2901.39	0.96	0.95	256.36
Michalewicz and Attia	2822.70	0.86	1.01	46.29
Hoffmeister and Sprave	2650.32	1.03	1.02	262.36
Skalak and Shonkwiler	2785.59	1.01	0.99	195.33
Joines and Houck	2765.36	1.01	1.00	165.73
Smith and Tate	2755.49	1.02	0.98	155.56
Bean and Hadj-Alouane	2786.66	0.98	1.01	120.91
Deb	2801.88	0.99	0.98	270.22
SMF	2713.57	0.97	0.98	56.35
	60-story frame			
Morales and Quezada	7158.23	0.95	0.92	930.35
Michalewicz and Attia	6720.45	0.98	1.02	108.30
Hoffmeister and Sprave	6453.09	1.07	1.01	871.51
Skalak and Shonkwiler	7044.32	0.97	0.89	577.81
Joines and Houck	6975.29	1.01	0.89	445.14
Smith and Tate	7031.45	0.98	1.00	649.09
Bean and Hadj-Alouane	7052.44	0.99	0.97	508.46
Deb	7158.43	0.97	0.95	818.38
SMF	6779.56	0.98	0.87	202.12

and Shonkwiler (1998), Bean and Hadj-Alouane (1992) has found an over-design structure among all. Although the result obtained by Smith and Tate (1993) is on the boundary, interestingly, it has again an over design weight. The best-converged results among 30 runs by some of the penalty methods (i.e., Michalewicz and Attia (1994), Hoffmeister and Sprave (1996) and Joines and Houck (1994)) are infeasible and need handy modifications with try-and-errors. For the third example, the standard deviations show that penalties have reported weak distribution of answers since the deviation is about 6% to 13% for Joines and Houck (1994) and Morales and Quezada (1998) methods, respectively. However, the deviation for SMF as well as Michalewicz and Attia (1994) is very satisfactory equal to 3% and 2%, respectively. However, the answer offered by SMF is direct feasible, unlike the one obtained by Michalewicz and Attia (1994).

6 Concluding remarks

A list of SMF advantages is as below:

(1) It is a new *merit function* specified for structural optimization for the first time.
(2) There is no necessity for tuning factors, and no annoying try and errors are required.
(3) No time-consuming programming is assigned to the practical use of this method.
(4) Simple versions of meta-heuristics are usable and straightforward, and there is no necessity for utilizing co-evolutionary versions.
(5) By utilizing SMF, it does not matter what the topography of the structure is, what type of the moment-resisting system is, and which type of design criteria is governing the design process; results remain stable and reliable.
(6) The final manual manipulation (or local search) is eliminated, which is an annoying struggle, and the final converged results are directly feasible.
(7) Last and foremost, a valuable expedition in convergence can be observed, which provides more facilitation in computations of the optimization process.

A prospect list for future works might be as follows: (a) applying this technique on large-scale truss and concrete structures since the type of design variables and design criteria are somehow different, (b) developing and investigating different forms of formulation for SMF technique, (c) apply this technique on continuous problems instead of skeletal structures.

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