Correction to: Classification of non-local rings with genus two zero-divisor graphs

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The purpose of this note is to correct some errors in the article [1]. The notation here will follow that of the original article.

It was argued in [1] that the graph $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_6)$ or $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8)$ is (isomorphic to) a subgraph of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8)$ and $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2[x]/(x^3))$ is isomorphic to $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8)$. Using these facts it was proved that $\gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_6)) = \gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8)) = \gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2[x]/(x^3))) = 2$. Actually, the degree three vertices namely $(0, 1, 6)$, $(1, 0, 6)$ and $(1, 1, 4)$ of the graph $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8)$ have not been included in the S_2 -embedding of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8)$ provided in Fig. 5 of [1]. This mistake causes a change in the value the genus of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8)$. So in this note, we find the exact genus value of the following four rings:

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_6, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2[x]/(x^3)$.

First of all, a S_2 -embedding of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8)$ is given in Fig. 1 so that $\gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8)) = 2$.

Also, by Fig. 2, we get $\gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_6)) = 2$ and so $\gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3)) = 2$.

For the ring $R = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8$, the zero-divisor graph $\Gamma(R)$ contains $K_{3,7}$ with bipartite sets $X = \{(1, 0, 0),$

Fig. 1 An embedding of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8)$ in S_2

$(1, 1, 0), (0, 1, 0)\}$ and $Y = \{(0, 0, 1), (0, 0, 2), (0, 0, 3), (0, 0, 4), (0, 0, 5), (0, 0, 6), (0, 0, 7)\}$. Note that $\gamma(K_{3,7}) = 2$ and there are 6 rectangular faces and 3 hexagonal faces in any S_2 -embedding of $K_{3,7}$. Now, the vertices $(0, 1, 4)$ and $(1, 0, 4)$ would have to be embedded in a face of $K_{3,7}$. Since $((0, 1, 4), (1, 0, 4)) \in E(\Gamma(R))$, we observe that they are attached into a single face of S_2 -embedding of $K_{3,7}$. Further, in $\Gamma(R)$, the neighbor of $(0, 1, 4)$ contains the vertices $(0, 0, 2), (0, 0, 4), (0, 0, 6), (1, 0, 0)$ and the neighbor of $(1, 0, 4)$ contains the vertices $(0, 0, 2), (0, 0, 4), (0, 0, 6), (0, 1, 0)$. But none of the hexagonal face adopt such 9 edges together. Therefore $\gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8)) \geq 3$ and thus $\gamma(\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2[x]/(x^3))) \geq 3$.

From the above observation, we have to remove the two rings $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8$ and $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2[x]/(x^3)$ from the list given in Theorem 4 [1]. Hence the revised statement of Theorem 4 [1] is as follows:

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Fig. 2 The embedding of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_6)$ in S_2

Theorem 1 Let R be a non-local commutative ring. Then the genus of the zero-divisor graph is 2 if and only if R is isomorphic to one of the following 31 rings:

$$\begin{aligned} &\mathbb{F}_4 \times \mathbb{F}_8, \mathbb{F}_4 \times \mathbb{F}_9, \mathbb{F}_4 \times \mathbb{F}_{11}, \mathbb{Z}_5 \times \mathbb{Z}_7, \mathbb{Z}_4 \times \mathbb{Z}_8, \\ &\mathbb{Z}_2[x]/(x^2) \times \mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2[x]/(x^3), \\ &\mathbb{Z}_2[x]/(x^2) \times \mathbb{Z}_2[x]/(x^3), \mathbb{Z}_2[x]/(x^2) \times \mathbb{Z}_9, \\ &\mathbb{Z}_4 \times \mathbb{Z}_3[x]/(x^2), \mathbb{Z}_2[x]/(x^2) \times \mathbb{Z}_3[x]/(x^2), \\ &\mathbb{Z}_4 \times \mathbb{Z}_9, \mathbb{Z}_2 \times \mathbb{Z}_{25}, \mathbb{Z}_2 \times \mathbb{Z}_5[x]/(x^2), \mathbb{Z}_2 \times \mathbb{Z}_{16}, \\ &\mathbb{Z}_2 \times \mathbb{Z}_2[x]/(x^4), \mathbb{Z}_2 \times \mathbb{Z}_4[x]/(x^2 - 2, x^4), \\ &\mathbb{Z}_2 \times \mathbb{Z}_4[x]/(x^3 - 2, x^4), \mathbb{Z}_2 \times \mathbb{Z}_4[x]/(x^3 + x^2 - 2, x^4), \\ &\mathbb{F}_4 \times \mathbb{Z}_8, \mathbb{F}_4 \times \mathbb{Z}_2[x]/(x^3), \mathbb{F}_4 \times \mathbb{Z}_9, \mathbb{F}_4 \times \mathbb{Z}_3[x]/(x^2), \\ &\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_8, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{F}_9, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{11}, \\ &\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_2[x]/(x^2), \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5, \\ &\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{F}_4, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3. \end{aligned}$$

Moreover, in the diagram of the S_2 -embedding of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4)$ provided in Fig. 7 [1], the vertex $(2, 0, 2)$ was misprinted. There is no such vertex in $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4)$ and hence wrongly included vertex $(2, 0, 2)$ is deleted from the S_2 -embedding of $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4)$ given in Fig. 7 [1].

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Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interests regarding the publishing of this paper.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Reference

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