An evidence combination approach based on fuzzy discounting

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Abstract

In evidence theory, Dempster's rule of combination is the most commonly applied method to aggregate bodies of evidence obtained from different sources to make a decision. However, when multiple independent bodies of evidence with conflict are aggregated by Dempster's rule of combination, the counterintuitive results can be generated. Evidence discounting is proved to be an efficient way to eliminate the counterintuitive combination results. Following the discounting ideas, a new combination approach based on fuzzy discounting is put forward. Both the conflict between bodies of evidence and the uncertainty of a body of evidence itself are taken into account to determine the discounting factors. Jousselme's evidence distance is used to represent conflict between bodies of evidence, and discriminability measure is defined to represent uncertainty of a body of evidence itself. Consider that both the evidence distance and the discriminability measure are semantically fuzzy. Thus, fuzzy membership functions are defined to describe both of them, and a fuzzy reasoning rule base is constructed to derive the discounting factors. Numerical examples indicate that this new combination approach proposed can achieve fast convergence speed and is robust to disturbing evidences, i.e., it is an effective method to process conflicting evidences combination.

Keywords Evidence theory · Evidence combination · Fuzzy discounting · Evidence distance · Discriminability measure

1 Introduction

Multisource information fusion is being more and more applied in many areas. Usually, the data acquired from various sources are uncertain or imprecise to some extent. How to properly represent and deal with the information with uncertainty is an important problem. Evidence theory (Dempster 1967; Shafer 1976), which is first proposed by Dempster and is extended to a systematic theory by Shafer, is an effective tool to model and process uncertain information. Evidence theory has become an important data fusion algorithm and is widely used in pattern recognition (Zhang et al. 2018; Yang et al. 2015; Zhang et al. 2017), fault diagnosis (Yan et al. 2019; Dong et al. 2019; Yuan et al. 2017), artificial intelligence (Han and Deng 2019;

Guo et al. 2017; Deng et al. Jan. 2019) and so on. To make a decision, multiple bodies of evidence obtained from different sources can be aggregated by combination rule. Being commutative and associative, Dempster's rule of combination plays an important role in evidence theory and is the most commonly used combination method in practice. When applying this rule, there is a prerequisite that each body of evidence has equal reliability, which is often not satisfied in real applications. Thus, undesirable results may be generated when aggregating conflicting bodies of evidence by Dempster's rule of combination.

To address this problem when applying Dempster's rule, there are two views. One ascribes it to the combination rule, and some alternative combination rules (Smets 2007; Florea et al. 2009; Lefevre and Elouedi 2013) are put forward. Although these new rules can resolve the problem to a certain degree, almost all of them do not possess the properties of commutativity or associativity which Dempster's rule has. Therefore, they are hard to be popularized in practice. The other view holds that the problem can be caused by evidence model rather than the combination rule, and then, a series of combination approaches based on revising evidence (Li et al. 2016a; Zhang and Mu 2016; Xue et al. 2018; Zhang and Deng 2019) are put forward.

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We prefer the latter view. Evidences from different sources usually have different reliabilities and should be preprocessed before being aggregated by Dempster's rule. In general, there are two main thoughts on revising evidence. One is weighted average operation (Murphy 2000). The other is discounting operation. Although weighted average operation can usually achieve better focusing degree, it destroys the specificity of the bodies of evidence, which is not what we desire. Discounting operation first proposed by Shafer (1976) is proved an effective approach to eliminate counterintuitive combination results. When using discounting method, each body of evidence is discounted by a corresponding discounting factor before being combined. Discounting factor reflects the reliability of a body of evidence. The more reliable the evidence is, the bigger the corresponding discounting factor is. Generally, the reliability of a body of evidence is related to two aspects. One is the conflict between a body of evidence and others. Another is the uncertainty of a body of evidence itself. Nevertheless, most of the existing discounting approaches only take into account one aspect. Discounting factor is usually defined as a function of the conflict between a body of evidence and others (Lu et al. 2008) or the uncertainty of a body of evidence itself (Han et al. 2011). This is not enough.

In this paper, we follow the discounting ideas. A combination approach based on fuzzy discounting is put forward. Both the conflict between a body of evidence and others and the uncertainty of a body of evidence itself are taken into account in this method. Considering that "the conflict degree is big or small" and "the uncertainty degree is big or small" do not have a definite bound, and either the conflict degree or the uncertainty degree varies gradually, the fuzzy sets are introduced to describe both of them. Moreover, according to intuitive experience, a fuzzy reasoning rule base is constructed for deducing the discounting factors. Finally, numerical examples are provided to illustrate that the new discounting approach proposed can effectively deal with combination of conflicting evidences.

2 Basis of theory and problem existing in evidence combination

2.1 A fundamental conception of evidence theory

In evidence theory, propositions are represented as a form of set. Let Θ be a finite set consisting of n mutually exclusive elements, then Θ is called the frame of discernment. The set composed of the subset of Θ is called the power set of Θ and is denoted as 2^Θ .

Definition 1 If $m: 2^\Theta \rightarrow [0,1]$ and satisfied that

$$\begin{cases} m(A) \geq 0, \forall A \in 2^\Theta \\ m(\emptyset) = 0 \\ \sum_{A \in 2^\Theta} m(A) = 1 \end{cases} \quad (1)$$

then m is called a basic probability assignment (BPA) or a mass function over Θ . If A is a subset of Θ and $m(A) > 0$, A is called a focal element.

Definition 2 Suppose that $m_1, m_2, \dots, m_n$ are n independent BPAs defined over Θ , then to aggregate them by Dempster's rule of combination can be described as follows:

$$m(A) = \begin{cases} 0, & \sum_{B_j \subseteq \Theta, \cap B_j = A} \prod_{i=1}^n m_i(B_j) \\ \frac{\sum_{B_j \subseteq \Theta, \cap B_j = A} \prod_{i=1}^n m_i(B_j)}{1 - k}, & A \neq \emptyset \end{cases} \quad (2)$$

where k is called conflicting coefficient, which reflects the conflict degree among bodies of evidence. Conflicting coefficient k is defined as follows:

$$k = \sum_{B_j \subseteq \Theta, \cap B_j = \emptyset} \prod_{i=1}^n m_i(B_j) \quad (3)$$

Definition 3 Suppose that m is a BPA defined on Θ and α is a discounting factor, then m is discounted by α which can be depicted as follows:

$$m^\alpha(X) = \begin{cases} \alpha m(X), & \forall X \subseteq U, X \neq \Theta \\ 1 - \alpha + \alpha m(X), & X = \Theta \end{cases} \quad (4)$$

where α is determined by the reliability of m and $\alpha \in [0, 1]$. When a body of evidence is reliable, the corresponding discounting factor is big. Otherwise, the corresponding discounting factor is small.

2.2 Problems in applying Dempster's rule of combination

In evidence theory, Dempster's rule is commonly applied to aggregate several bodies of evidence obtained to make the final decision. When using Dempster's rule of combination, each body of evidence is treated with the same reliability. It is not always reasonable in applications and can bring about irrational results as the famous example presented by Zadeh (Zadeh Feb. 1986) shows.

Example 1 Suppose that the frame of discernment $\Theta = \{a, b, c\}$ and m_1, m_2 is two BPAs defined over Θ . Two BPAs are listed as follows:

$$m_1(a) = 0.99, m_1(b) = 0.01, m_1(c) = 0;$$

$$m_2(a) = 0, m_2(b) = 0.01, m_2(c) = 0.99.$$

When using Dempster's rule to aggregate these two BPAs, the result acquired is that $m(a) = 0$, $m(b) = 1$, $m(c) = 0$. Although both m_1 and m_2 give relatively little support to b , the combination result supports b completely. It is obviously counterintuitive.

The above example shows that Dempster's rule can yield an unreasonable result, especially when there is high conflict between bodies of evidence. As to this problem, some researchers impute it to the combination rule and some new combination rules are proposed. Although these new rules may solve the problem to some extent, almost all of them lose the good properties of Dempster's rule. Whereas most researchers agree that the unreasonable results may be caused by evidence model, this is not the fault of Dempster's rule. Therefore, series of new combination methods based on revising evidence are put forward.

3 Combination approaches based on revising evidence

3.1 Revising evidence method based on weighted average operation

When applying weighted average method, every body of evidence should be preprocessed before be aggregated by Dempster's rule.

Suppose that $m_1, m_2, \dots, m_n$ are n BPAs defined over Θ , then the preprocessing operation can be depicted as follows:

$$m_{\text{WAE}} = \sum_{i=1}^n (\beta_i \cdot m_i), \quad (5)$$

where β_i is the weight of corresponding BPA m_i and m_{WAE} is the weighted average of n BPAs. Then, by using Dempster's rule to aggregate m_{WAE} for $n - 1$ times, the final result can be obtained.

In this method, the weight reflects the credibility or quality of the corresponding body of evidence and can be generated based on conflict or uncertainty degree. For the revising evidence method based on weighted average operation excessively pursuing the convergence degree, the specificity of each body of evidence is destroyed. This is undesirable.

3.2 Revising evidence method based on discounting operation

In discounting method, every body of evidence should be discounted by corresponding discounting factor firstly, and then, the discounted evidences are aggregated by Dempster's rule of combination.

Suppose that $m_1, m_2, \dots, m_n$ are n BPAs defined over Θ , then the preprocessing of discounting operation can be illustrated as follows:

$$m_j^{\alpha}(X) = \begin{cases} \alpha_j m_j(X), & \forall X \subseteq U, X \neq \Theta \\ 1 - \alpha_j + \alpha_j m_j(X), & X = \Theta \end{cases} \quad (6)$$

where α_j is the discounting factor of BPA m_j and m_j^{α} is the discounted BPA of m_j . Then, the n discounted BPAs are aggregated by Dempster's rule to generate the final result.

Compared with the weighted average operation-based method, discounting operation-based method not only maintains the specificity of bodies of evidence, but also redistributes the unreliable belief to the frame of discernment. Therefore, the revising method based on discounting operation is more reasonable. In discounting operation, discounting factors are critical parts. How to select proper discounting factors is vital. Following the discounting operation ideas, a new evidence combination approach based on fuzzy discounting is proposed in the following sections.

4 Combination approach based on fuzzy discounting

The discounting factor is determined by the reliability of corresponding body of evidence. The reliability of a body of evidence is related to not only the conflict with others, but also the uncertainty of itself. The conflict reflects the consistency between the body of evidence and others. Under the assumption that the truth lies in the majority, the smaller the conflict degree is, the more reliable the evidence is. The uncertainty reflects the decision clarity of the body of evidence itself. The smaller the uncertainty degree is, the more helpful for decision making the evidence is. Both conflict degree and uncertainty degree vary gradually, and then, it is not appropriate to define a threshold to distinguish "big" from "small." Hence, fuzzy sets are introduced to represent them, and then, a set of fuzzy reasoning rule is built to induce discounting factors.

4.1 Conflict measure based on evidence distance

In evidence theory, conflicting coefficient k is the most usually applied method to measure the conflict between bodies of evidence. However, under some circumstances,

conflicting coefficient k cannot correctly reflect the conflict between bodies of evidence. To resolve this problem, some researchers put forth many new conflict measures, such as evidence distance (Jousselme et al. 2001), pignistic probability distance (Smets 2005), singular value (Ke et al. 2013), correlation coefficient (Song et al. 2014) and belief entropy (Liu et al. 2019). Besides, some researchers propose joining conflicting coefficient k with other conflict measures to represent conflict between bodies of evidence (Jiang et al. 2010; Li et al. 2016b).

Among these new methods to measure conflict, evidence distance is the most widely used one in real applications. Evidence distance, which is originally put forward by Jousselme, has some good properties, such as satisfying three elements of distance and varying from 0 to 1.

Definition 4 Suppose that m_1, m_2 are two BPAs defined over Θ , then the evidence distance between m_1 and m_2 is defined as follows:

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)^T \cdot \mathbf{D} \cdot (\mathbf{m}_1 - \mathbf{m}_2)} \quad (7)$$

where $\mathbf{m}_1$ and $\mathbf{m}_2$ are 2^n column vectors corresponding to m_1 and m_2 , respectively, and $\mathbf{D}$ is a $2^n \times 2^n$ matrix whose elements are defined as

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad A, B \subseteq \Theta \quad (8)$$

For its good performance, evidence distance is used to measure conflict between bodies of evidence in fuzzy discounting approach proposed in this paper.

4.2 Uncertainty measure based on discriminability

In evidence theory, the approaches to measure the uncertainty degree of a body of evidence usually include (Jousselme et al. 2006) nonspecificity measure (NS), aggregated uncertainty measure (AU), total uncertainty measure (TU), ambiguity measure (AM), etc. The properties or differences of these uncertainty measures are discussed in detail in (Jousselme et al. 2006). Of these methods, AM is the relatively often applied one for it is sensitive to the change of evidence and is easy to be calculated.

Definition 5 Suppose that the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ and m is a BPA defined over Θ , then AM is defined as follows:

$$AM(m) = - \sum_{\theta_i \in \Theta} BetP_m(\theta_i) \log_2(BetP_m(\theta_i)) \quad (9)$$

where $BetP_m$ is the pignistic probability transformation (Xue et al. 2018) of m and that

$$BetP_m(\theta_i) = \sum_{\theta_j \in B \subseteq \Theta} \frac{1}{|B|} \frac{m(B)}{1 - m(\emptyset)} \quad (10)$$

AM reflects the comprehensive uncertainty of a body of evidence. Nevertheless, from decision-making view, sometimes we do not want the comprehensive uncertainty, but the discriminability of a body of evidence itself.

Example 2 Suppose that $\Theta = \{a, b, c\}$ and m_1, m_2, m_3 are three BPAs defined over Θ . Three BPAs are listed as follows:

$$\begin{aligned} m_1(a) &= 0.9, m_1(b) = 0.1, m_1(c) = 0; \\ m_2(a) &= 0.8, m_2(b) = 0.1, m_2(c) = 0.1; \\ m_3(a) &= 0.6, m_3(b) = 0.4, m_3(c) = 0. \end{aligned}$$

In this example, both m_1 and m_2 have high degree of support for a . However, m_3 has very close degree of support either for a or for b . From decision-making view, the uncertainty degree of m_1 and m_2 should be small and close, while the uncertainty degree of m_3 should be the biggest one among these three BPAs. By calculating, the AMs of three BPAs are that $AM(m_1) = 0.469$, $AM(m_2) = 0.922$ and $AM(m_3) = 0.971$. Obviously, the result does not accord with the above analysis. Here, what we want is the discriminability not the comprehensive uncertainty of evidence. AM cannot effectively indicate the discriminability of a body of evidence in some conditions. To solve such a problem, we introduce the discriminability measure (DM) to represent the uncertainty degree of a body of evidence here.

Definition 6 Suppose that the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ and m is a BPA defined over Θ , then DM is defined as follows:

$$DM(m) = \max_{\theta_i \in \Theta} (BetP_m(\theta_i)) - \max_{\theta_j \in \Theta, j \neq i} (BetP_m(\theta_j)) \quad (11)$$

According to calculation, the DMs of three BPAs in Example 2 are that $DM(m_1) = 0.8$, $DM(m_2) = 0.7$ and $DM(m_3) = 0.2$. This result is in accordance with what we analyze above. Therefore, the discriminability measure (DM) is applied to measure the uncertainty of a body of evidence in fuzzy discounting approach proposed in this paper.

4.3 Fuzzification of input variables

In fuzzy discounting approach, evidence distance d_J and discriminability measure (DM) are used as fuzzy input variables, while discounting factors are fuzzy output variables.

Let fuzzy sets “ d_J is big,” “ d_J is middle” and “ d_J is small” be denoted as “B,” “M” and “S,” respectively, then the fuzzy membership functions of evidence distance d_J are defined as follows:

$$\begin{aligned}\mu_B^{d_J}(x) &= \begin{cases} 0, & x \in [0, 0.4) \\ 2(x - 0.4), & x \in [0.4, 0.9) \\ 1, & x \in [0.9, 1] \end{cases} \\ \mu_M^{d_J}(x) &= \begin{cases} 2.5x, & x \in [0, 0.4) \\ -2(x - 0.9), & x \in [0.4, 0.9) \\ 0, & x \in [0.9, 1] \end{cases} \\ \mu_S^{d_J}(x) &= \begin{cases} -2.5(x - 0.4), & x \in [0, 0.4) \\ 0, & x \in [0.4, 1] \end{cases}\end{aligned}\quad (12)$$

Let fuzzy sets “DM is high,” “DM is middle” and “DM is low” be denoted as “H,” “M” and “L,” respectively, then the fuzzy membership functions of DM are defined as follows:

$$\begin{aligned}\mu_H^{DM}(x) &= \begin{cases} 0, & x \in [0, 0.3) \\ 2(x - 0.3), & x \in [0.3, 0.8) \\ 1, & x \in [0.8, 1] \end{cases} \\ \mu_M^{DM}(x) &= \begin{cases} \frac{10}{3}x, & x \in [0, 0.3) \\ -2(x - 0.8), & x \in [0.3, 0.8) \\ 0, & x \in [0.8, 1] \end{cases} \\ \mu_L^{DM}(x) &= \begin{cases} -\frac{10}{3}(x - 0.3), & x \in [0, 0.3) \\ 0, & x \in [0.3, 1] \end{cases}\end{aligned}\quad (13)$$

The curves of the fuzzy membership functions of evidence distance d_J and discriminability measure (DM) are shown in Fig. 1.

4.4 Fuzzy reasoning rule base

According to experience, the conflict degree between bodies of evidence influences the reasonability of the combination results and the uncertainty degree of the body of evidence influences the discriminability of the evidence combination results. Thus, conflict between bodies of evidence is the dominant factor to determine the discounting factors. When a body of evidence has a big conflict with others, it has low reliability and should be discounted. On the contrary, when a body of evidence has a small conflict with others, it has high reliability and should be

Fig. 1 The curves of the fuzzy membership functions of input variables

maintained. However, when a body of evidence has middle conflict with others, the lower the uncertainty degree of itself is, the more discriminable for making a decision the combination results are, i.e., when two bodies of evidence have the equal conflict with others, the one with low degree of uncertainty of itself is more reliable than the one with high degree of uncertainty of itself.

Let fuzzy sets “discounting factor is big,” “discounting factor is relatively big,” “discounting factor is relatively small” and “discounting factor is small” be denoted as “Bg,” “RB,” “RS” and “Sm,” respectively. From analysis above, a fuzzy reasoning rule base is constructed as follows:

- (1) IF $d_J = B$, THEN $y_i = Sm$;
- (2) IF $d_J = S$, THEN $y_i = Bg$;
- (3) IF $d_J = M \wedge DM = H$, THEN $y_i = Bg$;
- (4) IF $d_J = M \wedge DM = M$, THEN $y_i = RB$;
- (5) IF $d_J = M \wedge DM = L$, THEN $y_i = RS$.

In this rule base, the “ y_i ” represents the deduced discounting factor according to corresponding reasoning rule. The corresponding values of “Bg,” “RB,” “RS” and “Sm” are 1.0, 0.8, 0.6 and 0.3, respectively. By fuzzy reasoning with “max-min” composition method, every rule can get a membership u_i corresponding to y_i , and then, the discounting factor α can be acquired as follows:

$$\alpha = \frac{\sum u_i y_i}{\sum u_i} \quad (14)$$

Each pair of d_J and DM can derive a discounting factor α , and then, the discounting factors surface can be obtained as Fig. 2 shows. From Fig. 2, when d_J is relatively small, the discounting factors are relatively big, i.e., the evidence is adjusted slightly. When d_J is middle, the discounting factors change with the DM of evidence itself. With

Fig. 2 Discounting factors surface

increase of d_j , the discounting factors decrease rapidly. It satisfied what we expect.

4.5 The procedure of combining evidences by combination approach based on fuzzy discounting

According to the above analysis, the procedure to aggregate evidences by combination approach based on fuzzy discounting can be concluded as follows:

- (1) Calculate evidence distance d_j between each body of evidence and others and discriminability measure (DM) of each body of evidence itself;
- (2) Calculate membership of d_j and DM;
- (3) Carry out fuzzy reasoning according to fuzzy rule base to derive discounting factors;
- (4) Discount bodies of evidence by discounting factors;
- (5) Aggregate discounted evidences by Dempster's rule of combination to obtain the final result.

5 Numerical examples and discussion

Example 3 Suppose that the frame of discernment $\Theta = \{a, b, c\}$ and m_1, m_2 is two BPAs defined over Θ . Two BPAs are listed as follows:

$$m_1(a) = 0.99 - \delta, m_1(b) = 0.01, m_1(c) = \delta;$$

$$m_2(a) = \delta, m_2(b) = 0.01, m_2(c) = 0.99 - \delta.$$

where δ vary from 0 to 0.1.

The fusion results of these two BPAs by Dempster's rule of combination and combination approach based on fuzzy discounting are illustrated in Fig. 3. In Fig. 3, the top row is the fusion result by fuzzy discounting method and the bottom row is the fusion result by Dempster's rule.

According to Fig. 3, when $\delta = 0$, the fusion result by Dempster's rule is counterintuitive. The fusion result by

Fig. 3 Fusion results by Dempster's rule and fuzzy discounting method

fuzzy discounting approach indicates that the degree of support for b is smaller than single evidence and the degree of support for a and c is relatively big. Furthermore, the degree of conflict between these two bodies of evidence is very high, and both of them have very low reliability degree. Then, the frame of discernment Θ is assigned some probabilities to show there exist some uncertainties in fusion result. Obviously, the fusion result by combination approach based on fuzzy discounting is reasonable.

Besides, from top row of Fig. 3, combination approach based on fuzzy discounting is very robust to the little change of evidence. With increase of δ , the conflict between two bodies of evidence decreases gradually, which lead to increase in the degree of support for a and c and decrease in the probabilities assigned to Θ . From bottom row of Fig. 3, the little change of evidence can cause the rapid varying of fusion result, which indicates that Dempster's rule is very sensitive to the change of evidence. This is not what we want.

Example 4 Suppose that the frame of discernment $\Theta = \{a, b, c\}$ and m_1, m_2, m_3, m_4 is four BPAs defined over Θ . Four BPAs are listed as follows:

$$m_1(a) = 0.5, m_1(b) = 0.2, m_1(c) = 0.3;$$

$$m_2(a) = 0.5, m_2(b) = 0.3, m_2(c) = 0.2;$$

$$m_3(a) = 0.6, m_3(b) = 0.1, m_3(c) = 0.3;$$

$$m_4(a) = 0.8, m_4(b) = 0.1, m_4(c) = 0.1.$$

In this example, all of these four bodies of evidence have big degree of support for a . Thus, they are evidences with consistency. According to intuitive experience, the aggregated result should have big degree of support for a . The fusion results of several different combination methods are illustrated in Table 1.

Table 1 Fusion results of Example 4 by several different combination methods

Method	Fusion results		
	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4
Dempster's rule (Shafer 1976)	$m(a) = 0.6757$	$m(a) = 0.8621$	$m(a) = 0.9804$
	$m(b) = 0.1622$	$m(b) = 0.0345$	$m(b) = 0.0049$
	$m(c) = 0.622$	$m(c) = 0.1034$	$m(c) = 0.0147$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Murphy's method (Jousselme et al. 2001)	$m(a) = 0.6667$	$m(a) = 0.8491$	$m(a) = 0.9737$
	$m(b) = 0.1667$	$m(b) = 0.0448$	$m(b) = 0.0070$
	$m(c) = 0.1667$	$m(c) = 0.1061$	$m(c) = 0.0193$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Lu's method (Smets 2005)	$m(a) = 0.6757$	$m(a) = 0.6554$	$m(a) = 0.7846$
	$m(b) = 0.1622$	$m(b) = 0.1120$	$m(b) = 0.0518$
	$m(c) = 0.1622$	$m(c) = 0.2326$	$m(c) = 0.1636$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Han's method (Ke et al. 2013)	$m(a) = 0.6657$	$m(a) = 0.8496$	$m(a) = 0.9826$
	$m(b) = 0.1893$	$m(b) = 0.0576$	$m(b) = 0.0065$
	$m(c) = 0.1450$	$m(c) = 0.0928$	$m(c) = 0.0110$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Own method	$m(a) = 0.6757$	$m(a) = 0.8246$	$m(a) = 0.9692$
	$m(b) = 0.1622$	$m(b) = 0.0428$	$m(b) = 0.0085$
	$m(c) = 0.1622$	$m(c) = 0.1326$	$m(c) = 0.0223$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$

Of these combination methods, Murphy's method and Han's method are typical combination method based on weighted average operation. In Murphy's method, all the evidences are preprocessed with simple mathematical average operation before be aggregated. In Han's method, the weights are acquired by the use of the uncertainty of evidences. However, Lu's method is a typical combination method based on discounting operation. In Lu's method, the discounting factors are obtained by using the evidence distance.

According to Table 1, the final fusion result of each combination method has big degree of support for a and all of them have relatively fast convergence speed, i.e., when all the evidences are consistent, all of these combination methods can make right decisions. From Table 1, the fusion result by the combination approach proposed in this paper is very close to the result obtained by Dempster's rule of combination, which indicates that the combination approach proposed in this paper has little effect to the original bodies of evidence which have small conflict with others.

Example 5 Suppose that the frame of discernment $\Theta = \{a, b, c\}$ and m_1, m_2, m_3, m_4 is four BPAs defined over Θ . Four BPAs are listed as follows:

$$\begin{aligned}
 m_1(a) &= 0.5, m_1(b) = 0.2, m_1(c) = 0.3; \\
 m_2(a) &= 0, m_2(b) = 0.9, m_2(c) = 0.1; \\
 m_3(a) &= 0.6, m_3(b) = 0.1, m_3(c) = 0.3; \\
 m_4(a) &= 0.8, m_4(b) = 0.1, m_4(c) = 0.1.
 \end{aligned}$$

Of these four bodies of evidence, m_1, m_3 and m_4 have relatively big support degree for a and m_2 has relatively big support degree for b . Obviously, m_2 is a disturbance evidence. The fusion results by different combination methods are shown in Table 2.

According to Table 2, the fusion result by Dempster's rule is obviously unreasonable when m_2 occurs. Murphy's method can acquire the right result only when aggregating all of these four BPAs. Murphy's method is a combination method based on weighted average operation. In Murphy's method, all the evidences are only preprocessed with simple mathematical average operation before be aggregated and other useful information is not considered. Lu's method is a discounting method based on evidence distance. According to Lu's method, the sequence of the value of the discounting factors corresponding to four evidences is that $\alpha_1 > \alpha_3 > \alpha_4 > \alpha_2$. The relatively compromise evidence m_1 obtains the biggest discounting factor, and the evidence m_4 with relatively high DM gets relatively small discounting factor, which is not helpful for focusing degree

Table 2 Fusion results of Example 5 by several different combination methods

Method	Fusion result		
	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4
Dempster's rule (Shafer 1976)	$m(a) = 0$	$m(a) = 0$	$m(a) = 0$
	$m(b) = 0.8571$	$m(b) = 0.6667$	$m(b) = 0.6667$
	$m(c) = 0.1429$	$m(c) = 0.3333$	$m(c) = 0.3333$
Murphy's method (Jousselme et al. 2001)	$m(a) = 0.1543$	$m(a) = 0.3912$	$m(a) = 0.7996$
	$m(b) = 0.7469$	$m(b) = 0.5079$	$m(b) = 0.1752$
	$m(c) = 0.0988$	$m(c) = 0.1008$	$m(c) = 0.0251$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Lu's method (Smets 2005)	$m(a) = 0$	$m(a) = 0.6605$	$m(a) = 0.7502$
	$m(b) = 0.8571$	$m(b) = 0.1083$	$m(b) = 0.0660$
	$m(c) = 0.1429$	$m(c) = 0.2312$	$m(c) = 0.1838$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Han's method (Ke et al. 2013)	$m(a) = 0.0158$	$m(a) = 0.0309$	$m(a) = 0.2062$
	$m(b) = 0.9524$	$m(b) = 0.9496$	$m(b) = 0.7864$
	$m(c) = 0.0318$	$m(c) = 0.0195$	$m(c) = 0.0074$
	$m(\Theta) = 0$	$m(\Theta) = 0$	$m(\Theta) = 0$
Own method	$m(a) = 0.1168$	$m(a) = 0.5074$	$m(a) = 0.8887$
	$m(b) = 0.6102$	$m(b) = 0.2381$	$m(b) = 0.0546$
	$m(c) = 0.1367$	$m(c) = 0.2545$	$m(c) = 0.0567$
	$m(\Theta) = 0.1363$	$m(\Theta) = 0$	$m(\Theta) = 0$

of combination result. Thus, although Lu's method can eliminate the influence on the combination result caused by disturbing evidence, the convergence speed is relatively slow. Han's method is also a combination based on weighted average operation. According to Han's method, the sequence of the value of the weights corresponding to four evidences is that $\beta_2 > \beta_4 > \beta_3 > \beta_1$. The disturbing evidence m_2 obtains the biggest weight, which is unreasonable and can directly lead to a false decision. According to the combination method presented in this paper, the sequence of the value of the discounting factors corresponding to four evidences is that $\alpha_4 > \alpha_3 > \alpha_1 > \alpha_2$, which is reasonable and satisfied what we expect. When aggregating the first three bodies of evidence, the method proposed in this paper can make a right decision. Compared with other combination method, the combination approach based on fuzzy discounting has better focusing degree.

6 Conclusion

Dempster's rule of combination is the most commonly applied combination approach in evidence theory. However, when aggregating conflicting bodies of evidence, it may yield counterintuitive results. As to this unsatisfactory behavior, a lot of new combination methods based on revising rule and revising evidence are put forward. Most

researchers agree that the revising evidence-based combination method is more rational. Among revising evidence-based combination methods, discounting operation has been proved an effective way to eliminate the unreasonable combination results.

Therefore, we follow the discounting ideas and a new combination approach based on fuzzy discounting is put forward in this paper. Evidence distance reflects the conflict between a body of evidence and others, and discriminability measure indicates the uncertainty of a body of evidence itself. Both the conflict and uncertainty are taken into account in this new combination approach. Considering that conflict and uncertainty have semantical fuzziness, fuzzy membership functions are defined to depict them and a fuzzy reasoning rule base is constructed to deduce the discounting factors. Numerical examples show that the new combination approach based on fuzzy discounting can efficiently deal with the fusion of conflicting bodies of evidence. Furthermore, it is robust to disturbance and can achieve fast convergence speed. Thus, applying this proposed new combination approach in real applications will be for future work.

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Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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