Correction to: Isogeometric hyperelastic shell analysis with out-of-plane deformation mapping

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Correction to:
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In the original publication [1], a term was missing in Eq. (99). The correct form is

$$\delta \bar{\mathbf{g}}^\gamma = (\bar{\mathbf{g}}^{\gamma\delta} \mathbf{nn} - \bar{\mathbf{g}}^\delta \bar{\mathbf{g}}^\gamma) \cdot \delta \bar{\mathbf{g}}_\delta. \quad (99)$$

Consequently, Eqs. (97) and (98) also had missing terms. The correct forms are

$$\begin{aligned} \delta_a \delta_b \bar{\kappa}_{\alpha\beta} &= \left(\delta_a \bar{\Gamma}_{\alpha\beta} - (\bar{\Gamma}_{\alpha\beta} \cdot \bar{\mathbf{g}}^\gamma) \frac{\partial \delta_a \bar{\mathbf{x}}}{\partial \xi^\gamma} \right) \cdot \bar{\mathbf{g}}^\gamma (\mathbf{n} \cdot \delta_b \bar{\mathbf{g}}_\gamma) \\ &\quad + \delta_a \bar{\mathbf{g}}_\gamma \cdot \mathbf{n} \left(\bar{\mathbf{g}}^\gamma \cdot \delta_b \bar{\Gamma}_{\alpha\beta} + \bar{\Gamma}_{\alpha\beta} \right. \\ &\quad \left. \cdot (\bar{\mathbf{g}}^{\gamma\delta} \mathbf{nn} - \bar{\mathbf{g}}^\delta \bar{\mathbf{g}}^\gamma) \cdot \delta_b \bar{\mathbf{g}}_\delta \right) \\ &= \left(\delta_a \bar{\Gamma}_{\alpha\beta} - (\bar{\Gamma}_{\alpha\beta} \cdot \bar{\mathbf{g}}^\gamma) \frac{\partial \delta_a \bar{\mathbf{x}}}{\partial \xi^\gamma} \right) \cdot \bar{\mathbf{g}}^\gamma (\mathbf{n} \cdot \delta_b \bar{\mathbf{g}}_\gamma) \\ &\quad + \left(\delta_b \bar{\Gamma}_{\alpha\beta} - (\bar{\Gamma}_{\alpha\beta} \cdot \bar{\mathbf{g}}^\delta) \frac{\partial \delta_b \bar{\mathbf{x}}}{\partial \xi^\delta} \right) \cdot \bar{\mathbf{g}}^\gamma (\mathbf{n} \cdot \delta_a \bar{\mathbf{g}}_\gamma) \\ &\quad + (\mathbf{n} \cdot \delta_a \bar{\mathbf{g}}_\gamma) \bar{b}_{\alpha\beta} \bar{\mathbf{g}}^{\gamma\delta} (\mathbf{n} \cdot \delta_b \bar{\mathbf{g}}_\delta). \end{aligned} \quad (98)$$

We note that the missing term is on the left side of the iteration equations, and the computations were performed without the

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term. However, we confirm that the inclusion of the term does not change the results presented in the article. We also note that in a subsequent publication [2], we had equations similar to Eq. (99), but in correct form.

Equation (99) had a missing term because Eq. (142) in Appendix C had a missing term. We now provide the corrected Appendix C in its entirety.

Appendix C: Variation of the contravariant basis vector

Here we show that $\delta \mathbf{g}^\gamma$ can be expressed as

$$\delta \mathbf{g}^\gamma = (g^{\gamma\delta} \mathbf{nn} - \mathbf{g}^\delta \mathbf{g}^\gamma) \cdot \delta \mathbf{g}_\delta. \quad (137)$$

We start with the transformation from the contravariant basis vectors to the covariant basis vectors:

$$\mathbf{g}_\alpha = g_{\alpha\delta} \mathbf{g}^\delta. \quad (138)$$

We take the variation of both sides:

$$\delta \mathbf{g}_\alpha = \delta g_{\alpha\delta} \mathbf{g}^\delta + g_{\alpha\delta} \delta \mathbf{g}^\delta, \quad (139)$$

and from that obtain

$$g_{\alpha\delta} \delta \mathbf{g}^\delta = \delta \mathbf{g}_\alpha - \delta g_{\alpha\delta} \mathbf{g}^\delta. \quad (140)$$

From that and Eq. (11), we obtain

$$g_{\alpha\delta} \delta \mathbf{g}^\delta = \delta \mathbf{g}_\alpha - (\delta \mathbf{g}_\alpha \cdot \mathbf{g}_\delta + \mathbf{g}_\alpha \cdot \delta \mathbf{g}_\delta) \mathbf{g}^\delta \quad (141)$$

$$= \delta \mathbf{g}_\alpha - \delta \mathbf{g}_\alpha \cdot \underbrace{\mathbf{g}_\delta \mathbf{g}^\delta}_{=\mathbf{I} - \mathbf{nn}} - \mathbf{g}_\alpha \cdot \delta \mathbf{g}_\delta \mathbf{g}^\delta \quad (142)$$

$$= \delta \mathbf{g}_\alpha - \delta \mathbf{g}_\alpha + \mathbf{nn} \cdot \delta \mathbf{g}_\alpha - \mathbf{g}_\alpha \cdot \delta \mathbf{g}_\delta \mathbf{g}^\delta \quad (143)$$

$$= \mathbf{nn} \cdot \delta \mathbf{g}_\alpha - \mathbf{g}_\alpha \cdot \delta \mathbf{g}_\delta \mathbf{g}^\delta. \quad (144)$$

Multiplying both sides with $g^{\gamma\alpha}$, we obtain

$$\underbrace{g^{\gamma\alpha} g_{\alpha\delta}}_{=\delta_\delta^\gamma} \delta \mathbf{g}^\delta = g^{\gamma\alpha} \mathbf{nn} \cdot \delta \mathbf{g}_\alpha - \underbrace{g^{\gamma\alpha} \mathbf{g}_\alpha}_{=\mathbf{g}^\gamma} \cdot \delta \mathbf{g}_\delta \mathbf{g}^\delta. \quad (145)$$

Thus,

$$\delta \mathbf{g}^\gamma = g^{\gamma\delta} \mathbf{nn} \cdot \delta \mathbf{g}_\delta - \mathbf{g}^\gamma \cdot \delta \mathbf{g}_\delta \mathbf{g}^\delta \quad (146)$$

$$= (g^{\gamma\delta} \mathbf{nn} - \mathbf{g}^\delta \mathbf{g}^\gamma) \cdot \delta \mathbf{g}_\delta. \quad (147)$$

References

1. Takizawa K, Tezduyar TE, Sasaki T (2019) Isogeometric hyperelastic shell analysis with out-of-plane deformation mapping. *Comput Mech* 63:681–700. <https://doi.org/10.1007/s00466-018-1616-3>
2. Sasaki T, Takizawa K, Tezduyar TE (2019) Medical-image-based aorta modeling with zero-stress-state estimation. *Comput Mech* 64:249–271. <https://doi.org/10.1007/s00466-019-01669-4>

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