Correction to: Metric Properties of Semialgebraic Mappings

Krzysztof Kurdyka¹ · Stanisław Spodzieja² · Anna Szlachcińska²

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In some formulas in the original article we have omitted in the exponent the number of functions (or equivalently the dimension of the target) which is denoted by m . This shortcoming could lead to misunderstandings in applications.

1. In Corollary 2.2, inequality (2.6), the exponent $N + r - 1$ should be replaced by $N + m + r - 1$. Namely it should be

$$\mathcal{L}_0^{\mathbb{R}}(F|X) \leq d(6d - 3)^{N+m+r-1} \quad (2.6)$$

instead of

$$\mathcal{L}_0^{\mathbb{R}}(F|X) \leq d(6d - 3)^{N+r-1}. \quad (2.6)$$

Indeed, let $F: X \rightarrow \mathbb{R}^m$ be a continuous semialgebraic mapping, where $X \subset \mathbb{R}^N$ is a closed semialgebraic set. Assume that $0 \in X$ and $F(0) = 0$. Set $r = r(X) + r(\text{graph } F)$ and $d = \max\{\kappa(X), \kappa(\text{graph } F)\}$. Clearly

$$\text{graph } F \subset \mathbb{R}^{N+m}.$$

The original article can be found online at <https://doi.org/10.1007/s00454-016-9776-4>.

Krzysztof Kurdyka
Krzysztof.Kurdyka@univ-savoie.fr

Stanisław Spodzieja
spodziej@math.uni.lodz.pl

Anna Szlachcińska
anna_loch@wp.pl

¹ Laboratoire de Mathématiques (LAMA), Université Savoie Mont Blanc, UMR-5127 de CNRS, 73-376 Le Bourget-du-Lac Cedex, France

² Faculty of Mathematics and Computer Science, University of Łódź, S. Banacha 22, 90-238 Łódź, Poland

Therefore, it follows from (2.4) in Theorem 2.1 that

$$|F(x)| \geq \text{dist}((x, 0), \text{graph } F) \geq C \text{dist}((x, 0), [X \cap F^{-1}(0)] \times \{0\})^{d(6d-3)^{N+m+r-1}}$$

for $(x, 0) \in U \cap [X \times \{0\}]$, where U is a neighborhood $0 \in \mathbb{R}^{N+m}$. This gives the correct version of inequality (2.6).

2. Analogously, in Corollary 2.2, inequality (2.7), the exponent $N + r$ should be replaced by $N + m + r$. Namely it should be

$$\mathcal{L}_0^{\mathbb{R}}(F|X) \leq \frac{(2d-1)^{N+m+r} + 1}{2} \quad (2.7)$$

instead of

$$\mathcal{L}_0^{\mathbb{R}}(F|X) \leq \frac{(2d-1)^{N+r} + 1}{2}. \quad (2.7)$$

3. Since formula (2.6) was corrected, in the last paragraph of Remark 2.4 the statement “... with our estimate $\mathcal{L}_0^{\mathbb{R}}(F|X) \leq d(6d-3)^{N+r-1}$ ...” should be replaced by “... with our estimate $\mathcal{L}_0^{\mathbb{R}}(F|X) \leq d(6d-3)^{N+r+m-1}$ ...”.

4. In Corollary 3.3 (the first centered inequality), m should be added in the exponent. Namely it should be

$$|F(x)| \geq C \left(\frac{\text{dist}(x, F^{-1}(0) \cap X)}{1 + |x|^d} \right)^{d(6d-3)^{N+m+r-1}} \quad \text{for } x \in X$$

instead of

$$|F(x)| \geq C \left(\frac{\text{dist}(x, F^{-1}(0) \cap X)}{1 + |x|^d} \right)^{d(6d-3)^{N+r-1}} \quad \text{for } x \in X.$$

5. Finally, in Corollary 3.3 (the second centered inequality), it should be

$$\mathcal{L}_\infty^{\mathbb{R}}(F|X) \geq (1-d)d(6d-3)^{N+m+r-1}$$

instead of

$$\mathcal{L}_\infty^{\mathbb{R}}(F|X) \geq (1-d)d(6d-3)^{N+r-1}.$$

These corrections do not affect the asymptotics of the Łojasiewicz exponent.

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