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Maximal regularity for stochastic convolutions driven by Lévy processes

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Abstract We generalize the maximal regularity result from Da Prato and Lunardi (Atti Accad Naz Lincei Cl Sci Fis Mat Natur Rend Lincei (9) Mat Appl 9(1):25–29, 1998) to stochastic convolutions driven by time homogenous Poisson random measures and cylindrical infinite dimensional Wiener processes.

Keywords Stochastic convolution · Time homogeneous Poisson random measure and maximal regularity · Martingale type p Banach spaces

Mathematics Subject Classification (2000) 60H15 · 60J75 · 60G57 · 60H05

1 Introduction

The aim of the article is to investigate the maximal regularity of Ornstein–Uhlenbeck type processes driven by purely discontinuous noise. In particular we will generalize a result due to Da Prato and Lunardi [12] about maximal regularity of a stochastic convolution process driven by one-dimensional Wiener process. In order to put the reader into a framework, we briefly present here [12, Theorem 2.2].

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Theorem (Da Prato and Lunardi) *Assume that E is a separable martingale type 2 Banach space, see the Appendix for a definition, and a linear operator $-A$ in E (with the domain $D(A)$) is an infinitesimal generator of an analytic semigroup $\{e^{-tA}\}_{t\geq 0}$ on E. Suppose that $W = (W(t))_{t\geq 0}$ is a real valued Wiener process defined on some filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. Assume that $q \in [2, \infty)$, $\vartheta \in (0, 1)$ and $T > 0$. Then there exists a constant $K > 0$ such that for any progressively measurable $D_A(\theta, q)$-valued process φ the following inequality holds.*

$$\mathbb{E}\int_0^T |u(t)|^q_{D_A(\theta+\frac{1}{2},q)}\,dt \leq K^q\mathbb{E}\int_0^T |\varphi(t)|^q_{D_A(\theta,q)}\,dt, \tag{1.1}$$

where $u = (u(t))_{t\in[0,T]}$ is the stochastic convolution process defined by

$$u(t) := \int_0^t e^{-(t-s)A}\varphi(s)\,dW(s), \quad t \in [0, T]. \tag{1.2}$$

In the above $D_A(\theta, q)$ stands for the real interpolation space between $D(A)$ and E, defined by

$$\begin{aligned} D_A(\theta, q) &:= \{x \in E : |x|_{D_A(\theta,q)} < \infty\}, \\ |x|^q_{D_A(\theta,q)} &= |x|^q_E + \int_0^1 \left|t^{1-\theta}Ae^{-tA}x\right|^q \frac{dt}{t}, \quad x \in E. \end{aligned} \tag{1.3}$$

For $\theta \notin (0, 1)$, one has to modify the above definition, see Sect. 2 for details.

The process u defined by the equality (1.2) can be viewed, see e.g. [8] for a discussion of this subject, as a solution to the following E–valued Langevin equation written in the Itô-form

$$\begin{aligned} du(t) + Au(t)\,dt &= \varphi(t)dW(t), \quad t \geq 0, \\ u(0) &= 0. \end{aligned} \tag{1.4}$$

When the process φ is both deterministic and time independent, the process u is called an Ornstein–Uhlenbeck process with drift $-A$.

Let us now present the main result of the current paper. Let us assume that $(S, \mathcal{S})$ is a measurable space, ν a non negative measure on $(S, \mathcal{S})$, and $\tilde{\eta}$ is a time homogeneous compensated Poisson random measure defined on a filtered probability space $(\Omega; \mathcal{F}; (\mathcal{F}_t)_{0\leq t<\infty}; \mathbb{P})$ with intensity measure ν on S, to be specified later.

Let us further assume that $1 < p \leq 2$, $1 \leq q \leq p$, E is a Banach space of martingale type p, see the Appendix A for a definition, and that $-A$ is an infinitesimal generator of an analytic semigroup $(e^{-tA})_{0\leq t<\infty}$ in E. We consider the following

SPDE written in the Itô-form

$$\begin{cases} du(t) = Au(t)\,dt + \int_S \xi(t;x)\tilde{\eta}(dx;dt), \\ u(0) = 0, \end{cases} \tag{1.5}$$

where $\xi : [0,T] \times \Omega \times S \to E$ is a progressively measurable process satisfying certain integrability conditions also specified later.

We define the solution to (1.5) to be the following stochastic convolution process with respect to $\tilde{\eta}$.

$$u(t) := \int_0^t \int_S e^{-A(t-r)} \xi(r,x)\, \tilde{\eta}(dx;dr), \quad t > 0. \tag{1.6}$$

Our main result will be the following inequality

$$\boxed{\mathbb{E}\int_0^T |u(t)|^p_{D_A(\theta+\frac{1}{p},p)}\, \nu(dz)dt \le C\mathbb{E}\int_0^T\int_S |\xi(t,z)|^p_{D_A(\theta,p)}\, \nu(dz)dt.} \tag{1.7}$$

As mentioned in the beginning, in the case of Stochastic Evolution Equations (briefly SEEs) driven by a scalar Wiener process, the question of optimal regularity was investigated by Da Prato in [11] or Da Prato and Lunardi [12].

We generalize their's results to equations driven by discontinuous noise.

Notation 1 *By $\mathbb{N}$ we denote the set of natural numbers, i.e. $\mathbb{N} = \{0, 1, 2, \ldots\}$ and by $\bar{\mathbb{N}}$ we denote the set $\mathbb{N} \cup \{+\infty\}$. Whenever we speak about $\mathbb{N}$ (or $\bar{\mathbb{N}}$)-valued measurable functions we implicitly assume that this set is equipped with the trivial σ-field $2^{\mathbb{N}}$ (or $2^{\bar{\mathbb{N}}}$). By $\mathbb{R}_+$ we will denote the interval $[0,\infty)$. If X is a topological space, then by $\mathcal{B}(X)$ we will denote the Borel σ-field on X. By λ we will denote the Lebesgue measure on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$. For a measurable space $(S, \mathcal{S})$ let M^+_S be the set of all non negative measures on $(S, \mathcal{S})$.*

2 Main results

Let us suppose that $1 < p \le 2$, $1 \le q \le p$ and E is a Banach space of martingale type p, see the Appendix A for the definition. Let us assume that $(S, \mathcal{S})$ is a measurable space and $\nu \in M^+_S$. Suppose that $\mathfrak{P} = (\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\ge 0}, \mathbb{P})$ is a filtered probability space, $\eta : \mathcal{S} \times \mathcal{B}(\mathbb{R}_+) \to \bar{\mathbb{N}}$ is time homogeneous Poisson random measure with intensity measure ν defined over $(\Omega, \mathcal{F}, \mathbb{P})$ and adapted to the filtration $(\mathcal{F}_t)_{t\ge 0}$. We will denote by $\tilde{\eta} = \eta - \gamma$ the compensated Poisson random measure associated to η, where the compensator γ is defined by

$$\gamma : \mathcal{S} \times \mathcal{B}(\mathbb{R}_+) \ni (A, I) \mapsto \nu(A)\lambda(I) \in \mathbb{R}_+.$$

We will prove in Appendix C, see also [17] for a different approach to this question, that there exists a unique continuous linear operator which associates with each progressively measurable process $\xi : \mathbb{R}_+ \times S \times \Omega \to E$ such that

$$\mathbb{E}\int_0^T\int_S |\xi(r,x)|^p\,\nu(dx)\,dr < \infty, \quad T > 0, \tag{2.1}$$

an adapted cádlág E-valued process, denoted by $\int_0^t\int_S \xi(r,x)\tilde{\eta}(dx,dr)$, $t \geq 0$, such that if a process ξ satisfying the above condition (2.1) is a random step process with representation

$$\xi(r) = \sum_{j=1}^n 1_{(t_{j-1},t_j]}(r)\xi_j, \quad r \geq 0,$$

where $\{t_0 = 0 < t_1 < \cdots < t_n < \infty\}$ is a finite partition of $[0,\infty)$ and for all j, ξ_j is an E-valued $\mathcal{F}_{t_{j-1}}$ measurable, p-summable random variable, then

$$\int_0^t\int_S \xi(r,x)\tilde{\eta}(dx,dr) = \sum_{j=1}^n \int_S \xi_j(x)\eta\left(dx,(t_{j-1}\wedge t, t_j\wedge t]\right). \tag{2.2}$$

The continuity mentioned above means that there exists a constant $C = C(E)$ independent of ξ such that

$$\mathbb{E}\left|\int_0^t\int_S \xi(r,x)\tilde{\eta}(dx,dr)\right|^p \leq C\mathbb{E}\int_0^t\int_S |\xi(r,x)|^p\,\nu(dx)\,dr, \quad t \geq 0. \tag{2.3}$$

One can prove,[1] see e.g. the proof of Proposition 3.3 in [17], Theorem 3.1 in [4] for the case $q < p$, or Corollary C.2 in Appendix C, that for any $q \in [1,p]$ there exists a constant $C = C_q(E)$ such that for each process ξ as above and for all $t \geq 0$,

$$\boxed{\mathbb{E}\left|\int_0^t\int_S \xi(r,x)\tilde{\eta}(dx,dr)\right|^q \leq C\mathbb{E}\left(\int_0^t\int_S |\xi(r,x)|^p\,\nu(dx)\,dr\right)^{q/p}.} \tag{2.4}$$

[1] The case $q \in (p,\infty)$ is different and will be discussed later.

Remark 2.1 Let us denote

$$I(\xi)(t) := \int_0^t \int_S \xi(r,x)\tilde{\eta}(dx,dr), \quad t \geq 0$$

$$\|\xi\| := \left(\int_S |\xi(x)|^p \, \nu(dx) \right)^{1/p}, \quad \xi \in L^p(S,\nu;E).$$

Then the inequality (2.4) takes the following form

$$\mathbb{E}|I(\xi)(t)|^q \leq C_q(E)\mathbb{E}\left[\left(\int_0^t \|\xi(r)\|^p \, dr \right)^{q/p} \right].$$

This should be (and will be) compared with the Gaussian case. Note that in the present case $\|\xi\|$ is simply the $L^p(S,\nu,E)$ norm of ξ. In the Gaussian case the situation is different.

Let us also point out that the inequality (2.4) for $q < p$ follows from the same inequality for $q = p$. In fact, using Proposition IV.4.7 from [30], see the proof of Theorem 3.1 in [4], one can prove a stronger result. Namely that if inequality (2.4) holds true for $q = p$, then for $q \in [1,p)$ there exists a constant $K_q > 0$ such that for each accessible[2] stopping time $\tau > 0$,

$$\mathbb{E} \sup_{0 \leq t \leq \tau} |I(\xi)(t)|^q \leq K_q \, \mathbb{E} \left(\int_0^\tau \|\xi(t)\|^p \, dt \right)^{q/p}. \tag{2.5}$$

Assume further that $-A$ is an infinitesimal generator of an analytic semigroup denoted by $(e^{-tA})_{t \geq 0}$ on E. By $D(A)$ we denote the domain of A.

Define the stochastic convolution of the semigroup $(e^{-tA})_{t \geq 0}$ and an E-valued process ξ as above by the following formula

$$SC(\xi)(t) = \int_0^t \int_S e^{(t-r)A}\xi(r,x)\tilde{\eta}(dx,dr), \quad t \geq 0. \tag{2.6}$$

Let us recall that there exist constants C_0 and ω_0 such that

$$\|e^{-tA}\| \leq C_0 e^{t\omega_0}, \quad t \geq 0. \tag{2.7}$$

[2] Following [6,15,20], a stopping time τ is called *accessible* if there exists an increasing sequence $\{\tau_n\}_{n\in\mathbb{N}}$ of stopping times such that $\tau_n < \tau$ a.s. and $\lim_{n\to\infty} \tau_n = \tau$.

Without loss of generality, we will assume from now on that $\omega_0 < 0$. Let us also recall the following characterization of the real interpolation[3] spaces $(E, D(A^m))_{\theta,q} = (D(A^m), E)_{1-\theta,q}$, where $m \in \mathbb{N}$, between $D(A^m)$ and E with parameters $\theta \in (0, 1)$ and $q \in [1, \infty)$, see section 1.14.5 in [32] or [11]. If $\delta \in (0, \infty]$ then

$$(D(A^m), E)_{1-\theta,q} = \left\{ x \in E : \int_0^\delta \left| t^{m(1-\theta)} A^m e^{-tA} x \right|^q \frac{dt}{t} < \infty \right\};$$

$$(E, D(A^m))_{\vartheta,q} = \left\{ x \in E : \int_0^\delta \left| t^{m(1-\vartheta)} A^m e^{-tA} x \right|^q \frac{dt}{t} < \infty \right\}. \tag{2.8}$$

The norms defined by the equality (2.8) for different values of δ are equivalent. The spaces $(D(A^m), E)_{1-\theta,q} = (E, D(A^m))_{\theta,q}$ depend on m, θ and q but for special choices of these parameters they are equal, e.g. see the identity (2.12) below.

The space $(D(A^m), E)_{1-\theta,q} = (E, D(A^m))_{\theta,q}$ is often denoted by $D_{A^m}(\theta, p)$ and we will use the following notation

$$\boxed{|x|^q_{D_{A^m}(\theta,q);\delta} = \int_0^\delta \left| t^{m(1-\theta)} A^m e^{-tA} x \right|^q \frac{dt}{t}}\ . \tag{2.9}$$

In the general case, i.e. when the semigroup $\{e^{-tA}\}_{t\geq 0}$ satisfies the condition (2.7) with possibly positive ω_0, one has the following equality but only for $\delta \in (0, \infty)$:

$$(E, D(A^m))_{\theta,q} = \left\{ x \in E : \int_0^\delta \left| t^{m(1-\theta)} (\omega_0 I + A)^m e^{-t(\omega_0 + A)} x \right|^p \frac{dt}{t} < \infty \right\}. \tag{2.10}$$

In this case, the norm defined in the formula (2.9) has to be redefined in the following way

$$|x|^q_{D_{A^m}(\theta,q);\delta} = \int_0^\delta \left| t^{m(1-\theta)} A^m e^{-tA} x \right|^q \frac{dt}{t} + |x|^q. \tag{2.11}$$

Let us finally recall that if $0 < k < m \in \mathbb{N}$, $p \in [1, \infty)$ and $\theta \in (0, 1)$, then $(E, D(A^k))_{\theta,p} = (E, D(A^m))_{\frac{k}{m}\theta,p}$, see [32, Theorem 1.15.2 (f)]. Therefore, if

[3] In order to fix the notation let us point out that the interpolation functor $(X_0, X_1)_{\theta,q}$, $\theta \in (0, 1)$, $q \in [1, \infty]$, between two Banach spaces X_1 and X_0 such that both are continuously embedded into a common topological Hausdorff vector space, satisfies the following properties: (i)$(X_1, X_0)_{\theta,q} = (X_0, X_1)_{1-\theta,q}$, (ii) if $X_0 \subset X_1$, $0 < \theta_1 < \theta_2 < 1$ and $p, q \in [1, \infty]$, then $(X_0, X_1)_{\theta_1,p} \subset (X_0, X_1)_{\theta_2,q}$. Roughly speaking, (ii) implies that, if $X_0 \subset X_1$, then $(X_0, X_1)_{\vartheta,p} \searrow X_0$ as $\vartheta \searrow 0$ and $(X_0, X_1)_{\theta,p} \nearrow X_1$ as $\vartheta \nearrow 0$. Or equivalently, if $X_0 \subset X_1$, then $(X_1, X_0)_{\theta,p} \searrow X_0$ as $\theta \nearrow 1$ and $(X_1, X_0)_{\theta,p} \nearrow X_1$ as $\theta \searrow 1$. See Proposition 1.1.4 in [23] and section 1.3.3 in [32].

$p \in [1, \infty)$ and $\theta \in [0, 1 - \frac{1}{p})$, then

$$D_A\left(\theta + \frac{1}{p}, q\right) = D_{A^2}\left(\frac{\theta}{2} + \frac{1}{2p}, q\right) \tag{2.12}$$

with equivalent norms.

Our main result in this note is the following:

Theorem 2.1 *Let us assume that* $1 < p \leq 2$, *E is a Banach space of martingale type p,* $(S, \mathcal{S})$ *be a measurable space and* $\nu \in M_S^+$. *Suppose that* $\mathfrak{P} = (\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ *is a filtered probability space,* $\eta : \mathcal{S} \times \mathcal{B}(\mathbb{R}_+) \to \bar{\mathbb{N}}$ *is time homogeneous* $(\mathcal{F}_t)_{t\geq 0}$*-adapted Poisson random measure with intensity measure* ν *defined over* $(\Omega, \mathcal{F}, \mathbb{P})$, *a linear operator* $-A$ *in E is an infinitesimal generator of an analytic semigroup* $\{e^{-tA}\}_{t\geq 0}$ *on E. Then for all* $\theta \in (0, 1 - \frac{1}{p})$, *there exists a constant* $C = \hat{C}_\theta(E)$ *such that for any progressively measurable* $D_A(\theta, q)$*-valued process* ξ *and all* $T \geq 0$, *the following inequality holds*

$$\boxed{\mathbb{E}\int_0^T |SC(\xi)(t)|^p_{D_A(\theta+\frac{1}{p},p)}\, dt \leq C\mathbb{E}\int_0^T\int_S |\xi(t,z)|^p_{D_A(\theta,p)}\,\nu(dz)\,dt}. \tag{2.13}$$

In the Gaussian case, with $q = p = 2$ and E being a Hilbert space, a version of the above result was proved by Da Prato in [11]. This result was then generalized to a class of so called Banach spaces of martingale type 2 in [2], see also [3], for nuclear Wiener process and in [7], to the case of cylindrical Wiener process. Finally, Da Prato and Lunardi studied in [12] the case when $q \geq p = 2$ with a one dimensional Wiener process. However, a generalization of the last result to a cylindrical Wiener process does not cause any serious problems, see Theorem 5.1 in Sect. 5 at the end of this Note.

Theorem 2.1 will be deduced from a more general result whose idea can be traced back to Remark 2.1.

Theorem 2.2 *Let* $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ *be a filtered probability space,* $p \in (1, 2]$ *and* $q \in [p, \infty)$. *Let us denote by* $\mathcal{E}_{p,q}$ *a class of separable Banach spaces satisfying the following assumptions:*

(R1) *With each space E belonging to the class* $\mathcal{E}_{p,q}$ *we associate a separable Banach space* $R = R(E)$, *such that there is a family* $(I_t)_{t\geq 0}$ *of linear operators from the class* $\mathcal{M}^p_{loc}(R(E))$ *of all progressively measurable* $R(E)$*-valued processes to* $L^p(\Omega, \mathcal{F}_t, \mathbb{P}; E)$ *such that for some constant* $C_p > 0$

$$\mathbb{E}|I_t(\xi)|^p_E \leq C_p\mathbb{E}\int_0^t \|\xi(r)\|^p_{R(E)}\,dr. \tag{2.14}$$

(R2) *If* $E \in \mathcal{E}_{p,q}$ *and* E_1 *isomorphic to E, then* E_1 *belongs to* $\mathcal{E}_{p,q}$ *as well.*

(R3) *If* $E_1, E_2 \in \mathcal{E}_{p,q}$ *and* $\Phi : E_1 \to E_2$ *is a bounded linear operator, then*

$$\|\Phi\xi\|_{R(E_2)} \le |\Phi| \|\xi\|_{R(E_1)}, \quad \xi \in R(E_1).$$

(R4) *If* $E_1, E_2 \in \mathcal{E}_{p,q}$ *are such that* $E_2 \subset E_1$ *densely and continuously, then the real interpolation spaces* $(E_2, E_1)_{\theta,p}$, $\theta \in (0,1)$*, belong to the class* $\mathcal{E}_{p,q}$ *as well.*

(R5) *There exists a constant* $\hat{C}_q > 0$ *such that for all* $t > 0$

$$\mathbb{E}|I_t(\xi)|_E^q \le \hat{C}_q \mathbb{E}\left(\int_0^t \|\xi(r)\|_{R(E)}^p \, dr\right)^{q/p}, \quad \xi \in \mathcal{M}_{loc}^p(R(E)). \tag{2.15}$$

For each $E \in \mathcal{E}_{p,q}$ *let* $\mathcal{A}(E)$ *denote the class of operators* A*, such that* $-A$ *is an infinitesimal generator of an analytic semigroup* $\{e^{-tA}\}_{t\ge 0}$ *on the space* E*. We further assume that*

(R6) *For each* $E \in \mathcal{E}_{p,q}$, $\theta \in (0,1)$ *and each* $\delta > 0$ *here exists a constant* $K > 0$ *such that*

$$\int_0^\delta \left\| r^{1-\theta} A e^{-rA} \xi \right\|_{R(E)}^q \frac{dr}{r} \le K^q \|\xi\|_{R(D_A(\theta,q))}^q, \quad \xi \in R(E). \tag{2.16}$$

For each pair (E, A) *such that* $E \in \mathcal{E}_{p,q}$ *and* $A \in \mathcal{A}(E)$*, let us define a family* $(SC_t)_{t\ge 0}$ *of linear operators from* $\mathcal{M}_{loc}^p(R(E))$ *to* $L^p(\Omega, \mathcal{F}_t, \mathbb{P}; E)$ *by the following formula*

$$SC_t(\xi) = I_t\left(e^{-(t-\cdot)A}\xi(\cdot)\right), \quad t \ge 0. \tag{2.17}$$

Then, for every pair (E, A) *such that* $E \in \mathcal{E}_{p,q}$ *and* $A \in \mathcal{A}(E)$ *and* $\theta \in (0, 1-\frac{1}{p})$*, there exists a constant* $\hat{C}$ *such that for all* $T > 0$ *the following inequality holds*

$$\mathbb{E}\int_0^T |SC_t(\xi)|_{D_A(\theta+\frac{1}{p},q)}^q \, dt \le \hat{C}\, \mathbb{E}\int_0^T \|\xi(s)\|_{R(D_A(\theta,q))}^q \, dt. \tag{2.18}$$

Remark 2.2 It follows from (R1) that if $\xi(r,\omega) = \eta(r,\omega)$ $\lambda \hat{\times} \mathbb{P}$-a.e. with respect to $(r,\omega) \in [0,T] \times \Omega$, then $I_t(\xi) = I_t(\eta)$.

Now we shall present two basic examples.

Example 2.1 Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\ge 0}, \mathbb{P})$ be a filtered probability space, $p = 2$ and $q \in (1, \infty)$. Let H be a separable Hilbert space and let $\mathcal{E}_{2,q}$ be a class of all 2-smoothable Banach spaces, see Appendix A for a definition. With $E \in \mathcal{E}_2 = \mathcal{E}_{2,q}$ we associate the space $R(E) := R(H, E)$ of all γ-radonifying operators from H to E. It is known, see

[25], that $R(H, E)$ is a separable Banach space endowed with the following norm[4]

$$\|\varphi\|^q_{R(H,E);q} := \mathbb{E}\left|\sum_k \beta_k \varphi\, e_k\right|^q_E, \quad \varphi \in R(H, E), \tag{2.19}$$

$\{e_k\}_k$ be an ONB of H and $\{\beta_k\}_k$ a sequence of i.i.d. Gaussian N(0,1) random variables.

Example 2.2 Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ be a filtered probability space, $p \in (1, 2]$. Let $(S, \mathcal{S})$ be a measurable space and $\eta : \mathcal{S}\hat{\times}\mathcal{B}(\mathbb{R}_+) \to \mathbb{N}^+$ be a time homogeneous, compensated Poisson random measure over $(\Omega; \mathcal{F}; \mathbb{P})$ adapted to filtration $(\mathcal{F}_t)_{t\geq 0}$ with intensity $\nu \in M^+_S$. Let $\mathcal{E}_{p,q} = \mathcal{E}_p$ be the set of all separable Banach spaces of martingale type p. With $E \in \mathcal{E}_p$ we associate a class of measurable functions $\xi : S \to E$ such that

$$\|\xi\|^p_{R(E)} := \int_S |\xi(x)|^p_E\, \nu(dx) < \infty.$$

An important case, often studied in the literature, see e.g. [31], is when the intensity measure ν induces a Lévy measure on a certain Banach space E.

3 Proof of Theorem 2.2

We begin with the case $q = p$. Without loss of generality the norm $|\cdot|_{D_A(\theta+\frac{1}{p},p);1}$, defined by formula (2.9), will be denoted by $|\cdot|_{D_A(\theta+\frac{1}{p},p)}$. Also, we may assume that A^{-1} exists and is bounded so that the graph norm in $D(A)$ is equivalent to the norm $|A\,\cdot|$.

By the equality (2.12), Definition (2.9), the Fubini Theorem and formula (2.17) we have

$$\mathbb{E}\int_0^T |SC_t(\xi)|^p_{D_A(\theta+\frac{1}{p},p)}\, dt \leq C\mathbb{E}\int_0^T |SC_t(\xi)|^p_{D_{A^2}(\frac{\theta}{2}+\frac{1}{2p},p)}\, dt$$

$$= C\int_0^T\int_0^1 \mathbb{E}\left|r^{2(1-\frac{\theta}{2}-\frac{1}{2p})} A^2 e^{-rA} SC_t(\xi)\right|^p \frac{dr}{r}\, dt$$

4 It follows from the Khinchin–Kahane inequality, see [19], that the norms $\|\cdot\|_{R(H,E);q}$ for various $q \in (1, \infty)$ are equivalent.

$$= C \int_0^T \int_0^1 r^{p(2-\theta)-1} \mathbb{E} \left| A^2 e^{-rA} I_t \left(e^{-(t-\cdot)A} \xi(\cdot) \right) \right|^p \frac{dr}{r} \, dt$$

$$= C \int_0^T \int_0^1 r^{p(2-\theta)-1} \, \mathbb{E} \left| I_t \left(A^2 e^{-rA} e^{-(t-\cdot)A} \xi(\cdot) \right) \right|^p \frac{dr}{r} \, dt \leq \cdots$$

By applying next the inequality (2.14), the property (R3), the Fubini Theorem, the fact that $|Ae^{-\frac{r}{2}A}| \leq Cr^{-1}$, $r > 0$, for some constant $C > 0$ (see e.g. [27], as well as by observing that $\frac{1}{t-u+r} \leq \frac{1}{r}$ for $t \in [u, T]$, $r > 0$), we infer that

$$\cdots \leq C_p \int_0^1 r^{p(2-\theta)-1} \int_0^T \mathbb{E} \int_0^t \left\| A^2 e^{-(t-u+r)A} \xi(u) \right\|^p_{R(E)} du \, dt \, \frac{dr}{r}$$

$$\leq C_p \int_0^1 r^{p(2-\theta)-1} \int_0^T \mathbb{E} \int_0^t \left| Ae^{-\frac{t-u+r}{2}A} \right|^p \left\| Ae^{-\frac{t-u+r}{2}A} \xi(u) \right\|^p_{R(E)} du \, dt \, \frac{dr}{r}$$

$$\leq C_p \mathbb{E} \int_0^1 r^{p(2-\theta)-1} \left[\sup_{0 \leq u \leq t} (t-u+r)^{-p} \right] \int_0^T \left[\int_u^T \left\| Ae^{-\frac{t-u+r}{2}A} \xi(u) \right\|^p_{R(E)} dt \right] du \, \frac{dr}{r}$$

$$\leq C_p \mathbb{E} \int_0^T \int_0^{T+1-\rho} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^p_{R(E)} \left[\int_{\rho \vee (\sigma+\rho-1)}^{\rho+\sigma} (\sigma + \rho - \tau)^{p(1-\theta)-2} \, d\tau \right] d\sigma \, d\rho$$

$$\leq C_p \mathbb{E} \int_0^T \int_0^{T+1-\rho} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^p_{R(E)} \left[\int_{\rho}^{\rho+\sigma} (\sigma + \rho - \tau)^{p(1-\theta)-2} \, d\tau \right] d\sigma \, d\rho$$

$$= C_p \mathbb{E} \int_0^T \int_0^{T+1-\rho} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^p_{R(E)} \left[\int_0^{\sigma} \tau^{p(1-\theta)-2} \, d\tau \right] d\sigma \, d\rho$$

$$= C'_p \mathbb{E} \int_0^T \int_0^{T+1-\rho} \sigma^{p(1-\theta)-1} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^p_{R(E)} d\sigma \, d\rho$$

$$\leq C''_p \mathbb{E} \int_0^T \int_0^{T/2} \left\| \sigma^{1-\theta} Ae^{-\sigma A} \xi(\rho) \right\|^p_{R(E)} \frac{d\sigma}{\sigma} \, d\rho$$

$$\leq \hat{C}'''_p K_T^p \mathbb{E} \int_0^T \|\xi(r)\|^p_{R(D_A(\theta,p))} \, dr,$$

where the last inequality is a consequence of the assumption (R6) with $q = p$.

Remark 3.1 Note that the assumption $\theta < 1 - \frac{1}{p}$, i.e. $p(1-\theta) - 2 > -1$, was used in one of the last inequalities.

The proof in the case $q > p$ follows the same ideas. Note also that the above proof resembles closely the proof from [12]. We give full details below.

We consider now the case $q > p$. We use the same notation as in the previous case. But we will make some (or the same) additional assumptions. By the equality (2.12), definition (2.9), the Fubini Theorem and formula (2.17) we have

$$\begin{aligned}
\mathbb{E}\int_0^T |SC_t(\xi)|^q_{D_A(\theta+\frac{1}{p},q)}\,dt &\le C\mathbb{E}\int_0^T |SC_t(\xi)|^q_{D_{A^2}(\frac{\theta}{2}+\frac{1}{2p},q)}\,dt\\
&= C\int_0^T\int_0^1 s^{q(2-\theta)-\frac{q}{p}}\mathbb{E}\left|A^2e^{-sA}SC_t(\xi)\right|^q\frac{ds}{s}\,dt\\
&= C\int_0^T\int_0^1 s^{q(2-\theta)-\frac{q}{p}}\mathbb{E}\left|A^2e^{-sA}I_t\left(e^{-(t-\cdot)A}\xi(\cdot)\right)\right|^q\frac{ds}{s}\,dt\\
&= C\int_0^T\int_0^1 s^{q(2-\theta)-\frac{q}{p}}\,\mathbb{E}\left|I_t\left(A^2e^{-sA}e^{-(t-\cdot)A}\xi(\cdot)\right)\right|^q\frac{ds}{s}\,dt \le \cdots
\end{aligned}$$

Before we continue, we formulate the following simple Lemma.

Lemma 3.1 *There exists a constant $C > 0$ such that for all $t > 0$, $s \in (0,1)$*

$$\left(\int_0^t \frac{1}{(t-r+s)^{\frac{pq}{q-p}}}\,dr\right)^{\frac{q}{p}-1} \le C\frac{1}{s^{q(1-\frac{1}{p})+1}}$$

Proof of Lemma 3.1 Denote $\alpha = \frac{pq}{q-p}$ and observe that $\alpha > 1$. Since $\int_0^t \frac{1}{(t-r+s)^\alpha}\,dr = \int_0^t \frac{1}{(r+s)^\alpha}\,dr \le \int_0^\infty \frac{1}{(r+s)^\alpha}\,dr = \frac{1}{\alpha-1}\frac{1}{s^{\alpha-1}}$ and $(\alpha-1)(\frac{q}{p}-1) = q(1-\frac{1}{p})+1$, the result follows.

As in the earlier case, by applying the inequality (2.14), the property (R3), the Fubini Theorem, the fact that $|Ae^{-\frac{s}{2}A}| \le Cs^{-1}$, $s > 0$, for some constant $C > 0$ as well as Hölder inequality and Lemma 3.1 we infer that

$$\begin{aligned}
\cdots &\le \hat{C}_q\int_0^1 s^{q(2-\theta)-\frac{q}{p}}\int_0^T\mathbb{E}\left[\int_0^t\left\|A^2e^{-(t-r+s)A}\xi(r)\right\|^p_{R(E)}\,dr\right]^{q/p}dt\,\frac{ds}{s}\\
&\le C\hat{C}_q\mathbb{E}\int_0^1 s^{q(2-\theta)-\frac{q}{p}}\int_0^T\left[\int_0^t\left|Ae^{-\frac{t-r+s}{2}A}\right|^p\left\|Ae^{-\frac{t-r+s}{2}A}\xi(r)\right\|^p_{R(E)}\,dr\right]^{q/p}dt\,\frac{ds}{s}
\end{aligned}$$

$$
\begin{aligned}
&\le C\hat{C}_q \mathbb{E} \int_0^1 s^{q(2-\theta)-\frac{q}{p}} \int_0^T \Bigg[\Bigg(\int_0^t \left| Ae^{-\frac{t-r+s}{2}A} \right|^{\frac{pq}{q-p}} dr \Bigg)^{\frac{q}{p}-1} \\
&\qquad\qquad \times \int_0^t \left\| Ae^{-\frac{t-r+s}{2}A} \xi(r) \right\|^q_{R(E)} dr \Bigg] dt \, \frac{ds}{s} \\
&\le C'\hat{C}_q \mathbb{E} \int_0^1 s^{q(2-\theta)-\frac{q}{p}} \int_0^T \frac{1}{s^{q(1-\frac{1}{p})+1}} \int_0^t \left\| Ae^{-\frac{t-r+s}{2}A} \xi(r) \right\|^q_{R(E)} dr \, dt \, \frac{ds}{s} \\
&= C'\hat{C}_q \mathbb{E} \int_0^1 s^{q(1-\theta)-1} \int_0^T \left[\int_r^T \left\| Ae^{-\frac{t-r+s}{2}A} \xi(r) \right\|^q_{R(E)} dt \right] dr \, \frac{ds}{s} \le \cdots \qquad (3.1)
\end{aligned}
$$

Let us now introduce new integration variables: $\rho = u$, $\tau = t$ and $\sigma = t - u + r$. Then the set over which the last integral is calculated is not larger than $\{(\rho, \sigma, \tau) : 0 < \rho < T, 0 < \sigma < T + 1 - \rho, \rho \vee (\sigma + \rho - 1) < \tau < \rho + \sigma\}$. Therefore, we have

$$
\begin{aligned}
\cdots &\le C'\hat{C}_q \mathbb{E} \int_0^T \int_0^{T+1-\rho} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^q_{R(E)} \left[\int_{\rho\vee(\sigma+\rho-1)}^{\rho+\sigma} (\sigma+\rho-\tau)^{q(1-\theta)-2} d\tau \right] d\sigma \, d\rho \\
&\le C'\hat{C}_q \mathbb{E} \int_0^T \int_0^{T+1-\rho} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^q_{R(E)} \left[\int_\rho^{\rho+\sigma} (\sigma + \rho - \tau)^{q(1-\theta)-2} \, d\tau \right] d\sigma \, d\rho \\
&= C'\hat{C}_q \mathbb{E} \int_0^T \int_0^{T+1-\rho} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^q_{R(E)} \left[\int_0^\sigma \tau^{q(1-\theta)-2} \, d\tau \right] d\sigma \, d\rho \\
&= \hat{C}'_q \mathbb{E} \int_0^T \int_0^{T+1-\rho} \sigma^{q(1-\theta)-1} \left\| Ae^{-\frac{\sigma}{2}A} \xi(\rho) \right\|^q_{R(E)} d\sigma \, d\rho \\
&\le \hat{C}''_q \mathbb{E} \int_0^T \int_0^{T/2} \left\| \sigma^{1-\theta} Ae^{-\sigma A} \xi(\rho) \right\|^q_{R(E)} \frac{d\sigma}{\sigma} \, d\rho \le \hat{C}''_q K^p_{T/2} \mathbb{E} \int_0^T \|\xi(r)\|^q_{R(D_A(\theta,q))} dr,
\end{aligned}
$$

where the last inequality follows from Assumption (R6). This completes the proof.

Remark 3.2 Note that the assumption $q > p$ was used in (3.1). In the first part of the proof we considered the case $q = p$. We do not know if our result is valid in the case $q \in (1, p)$.

4 Proof of Theorem 2.1

We will deduce Theorem 2.1 from Theorem 2.2. For this we fix a number $p \in (1, 2]$, put $q = p$, and consider a measurable space $(S, \mathcal{S})$, $\nu \in M_S^+$, a filtered probability space $\mathfrak{P} = (\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ and a time homogeneous $(\mathcal{F}_t)_{t\geq 0}$-adapted Poisson random measure $\eta : \mathcal{S} \times \mathcal{B}(\mathbb{R}_+) \to \bar{\mathbb{N}}$ with intensity measure ν defined over $(\Omega, \mathcal{F}, \mathbb{P})$. We set $\mathcal{E}_{p,q} = \mathcal{E}_p$ to be the class of all martingale type p Banach spaces. Note that we impose here an additional assumption that $q = p$. For any $E \in \mathcal{E}_{p,q}$ we put $R(E) := L^p(S, \nu, E)$.

We will verify now that this class satisfies the conditions (R1)–(R5). In Appendix C, we prove that conditions (R1) and (R5) are satisfied. It follows directly from the definition of martingale type p Banach spaces, see Appendix A, that condition (R2) is also satisfied. Moreover, condition (R3) is obviously satisfied. Validity of condition (R4) follows from Theorem A.4 in [3]. Hence, we only need to verify condition (R6). We have

Proposition 4.1 *Let us assume that $\theta \in (0, 1)$, $p \in (1, 2]$ and $T > 0$. Then there exists a constant $K_T > 0$ such that for each $\varphi \in L^p(S, \nu, E) =: R(E)$ the following inequality holds*

$$\begin{aligned} K_T^{-1} \|\varphi\|^p_{R((E,D(A))_{\theta,p})} &\leq \int_0^T t^{(1-\theta)p} \left\| Ae^{-tA}\varphi \right\|^p_{R(E)} \frac{dt}{t} \\ &\leq K_T \|\varphi\|^p_{R((E,D(A))_{\theta,p})}. \end{aligned} \tag{4.1}$$

In particular, $\varphi \in R((D(A), E)_{\theta,p})$ iff (for some and/or all $T > 0$) the integral $\int_0^T t^{(1-\theta)p} \|Ae^{-tA}\varphi\|^p_{R(E)} \frac{dt}{t}$ is finite.

Proof of Proposition 4.1 Follows by applying the Fubini Theorem.

5 Stochastic convolution in the cylindrical Gaussian case

Assume now that H is separable Hilbert space and $W(t)$, $t \geq 0$, is an H- cylindrical Wiener process defined on some complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, see e.g. [4].

Theorem 5.1 *Assume that E is a martingale type 2 Banach space, $q \in [2, \infty)$ and a linear operator $-A$ in E (with the domain $D(A)$) is an infinitesimal generator of an analytic semigroup $\{e^{-tA}\}_{t\geq 0}$ on E. Then under the above assumptions there exists a constant $\hat{C}_q(E)$ such that for any process ξ described above the following inequality holds*

$$\mathbb{E} \int_0^T |SC(\xi)(t)|^q_{D_A(\theta+\frac{1}{p},q)}\, dt \leq \hat{C}_q(E)\mathbb{E} \int_0^T \|\xi(t)\|^q_{R(H,D_A(\theta,q))}\, dt, \ \ T \geq 0. \tag{5.1}$$

The proof of Theorem 5.1 will be preceded by the following useful result.

Proposition 5.1 *Let us assume that $\theta \in (0,1)$, $q \geq 1$ and $T > 0$. Then there exists a constant $K_T > 0$ such that for each bounded linear map $\varphi : H \to E$ the following inequality holds*

$$K_T^{-1} \|\varphi\|^q_{R(H,(E,D(A))_{\theta,q}} \leq \int_0^T t^{(1-\theta)q} \left\| Ae^{-tA}\varphi \right\|^q_{R(H,E)} \frac{dt}{t} \leq K_T \|\varphi\|^q_{R(H,(E,D(A))_{\theta,q})}. \tag{5.2}$$

In particular, $\varphi \in R(H,(D(A),E)_{\theta,q})$ iff (for some and/or all $T>0$) the integral $\int_0^T t^{(1-\theta)q} \|Ae^{-tA}\varphi\|^q_{R(H,E)} \frac{dt}{t}$ is finite.

Proof of Proposition 5.1 Let $\{e_k\}_k$ be an ONB of H and $\{\beta_k\}_k$ a sequence of i.i.d. Gaussian N(0,1) random variables. It is known, see e.g. [21] that there exists a constant $C_p(E)$ such that for each linear operator $\varphi : H \to E$ the following inequality holds.

$$C_p(E)^{-1}\mathbb{E}\left|\sum_j \beta_j \varphi e_j\right|^p_E \leq \|\varphi\|^p_{R(H,E)} \leq C_p(E)\mathbb{E}\left|\sum_j \beta_j \varphi e_j\right|^p_E \tag{5.3}$$

We have

$$\begin{aligned}
&\int_0^T t^{(1-\theta)q} \left\| Ae^{-tA}\varphi \right\|^q_{R(H,E)} \frac{dt}{t} \leq C_q(E) \int_0^T t^{(1-\theta)q}\mathbb{E}\left|\sum_k \beta_k Ae^{-tA}\varphi e_k\right|^q_E \frac{dt}{t} \\
&= C_q(E)\mathbb{E}\int_0^T t^{(1-\theta)q}\left|\sum_k \beta_k Ae^{-tA}\varphi e_k\right|^q_E \frac{dt}{t} \\
&= C_q(E)\mathbb{E}\left|\sum_k \beta_k Ae^{-tA}\varphi e_k\right|^q_{D_A(\theta,q);T} \leq C(T)C_q(E)\|\varphi\|^q_{R(H,D_A(\theta,q))}.
\end{aligned}$$

Since $D_A(\vartheta,q) = (E,D(A))_{\theta,q}$ with equivalent norms, this proves the second inequality in (5.2). The first inequality follows the same lines.

Proof of Theorem 5.1 We set $\mathcal{E}_{2,q}$ to be the class of all martingale type p Banach spaces. Note that we impose here an additional assumption that $q \leq p$. Let us recall that the space $R(H,E)$ of γ-radonifying operators is described in Appendix B. We will verify now that this class satisfies the conditions (R1)–(R5). Conditions (R1) follows from [25] while condition (R5) follows from e.g. Theorem 2.12 in [5], see also [4] and [26, section 5], where the Burkholder inequality is proved for all $q \in (0,\infty)$. Condition (R2) follows directly from the definition of martingale type 2 Banach spaces. Moreover, condition (R3) follows from the Definition B.1 of the γ-radonifying norm.

Validity of condition (R4) follows from Theorem A.4 in [3]. Finally, the condition (R6) has just been proved in Proposition 5.1. Hence the proof is complete.

Appendix

A: Martingale type p, $p \in [1, 2]$, Banach spaces

In this section, we collect some basic information about the martingale type p, $p \in [1, 2]$, Banach spaces.

Assume also that $p \in [1, 2]$ is fixed. A Banach space E is of martingale type p iff there exists a constant $L_p(E) > 0$ such that for all E-valued finite martingale $\{M_n\}_{n=0}^N$ the following inequality holds

$$\sup_n \mathbb{E}|M_n|^p \leq L_p(E) \sum_{n=0}^N \mathbb{E}|M_n - M_{n-1}|^p, \tag{A.1}$$

where as usually, we put $M_{-1} = 0$.

A Banach space E is called p-smoothable, see the celebrated paper [28] by Pisier, iff there exists an equivalent norm $|\cdot|$ on E such that for some $k > 0$ the modulus of smoothness ρ_E of $(E, |\cdot|)$ defined by $\rho_E(t) := \sup_{|x|=|y|=1} \frac{1}{2}(|x+ty|+|x-ty|) - 1$, satisfies

$$\rho_E(t) \leq kt^p, \quad t \in (0, 1]. \tag{A.2}$$

It is now well known, see e.g. [28] and [29], that a Banach space E is of martingale type p iff it is p-smoothable. In particular, all spaces L^q for $q \geq p$ and $q > 1$, are of martingale type p.

Let us also recall that a Banach space E is of type p iff there exists a constant $K_p(E) > 0$ for any finite sequence $\varepsilon_1, \ldots, \varepsilon_n : \Omega \to \{-1, 1\}$ of symmetric i.i.d. random variables and for any finite sequence $x_1, \ldots, x_n$ of elements of E, the following inequality holds

$$\mathbb{E}\left|\sum_{i=1}^n \varepsilon_i x_i\right|^p \leq K_p(E) \sum_{i=1}^n |x_i|^p. \tag{A.3}$$

It is known, see e.g. [22, Theorem 3.5.2], that a Banach space E is of type p iff E is of Gaussian type p, i.e. there exists a constant $\tilde{K}_p(E) > 0$ such that for any finite sequence $\xi_1, \ldots, \xi_n$ of i.i.d. $N(0, 1)$ random variables and for any finite sequence $x_1, \ldots, x_n$ of elements of E, the following inequality holds

$$\mathbb{E}\left|\sum_{i=1}^n \xi_i x_i\right|^p \leq \tilde{K}_p(E) \sum_{i=1}^n |x_i|^p, \tag{A.4}$$

It is also known, see [18, Theorem 2.1], that a Banach space E is of type p iff there exists $K_p(X)$ such that for any finite sequence $f_1, \ldots, f_n$ of independent E-valued random variables mean zero with finite p-th moments,

$$\mathbb{E}\left|\sum_{i=1}^{n} f_i\right|^p \le K_p(E)\sum_{i=1}^{n}\mathbb{E}|f_i|^p. \tag{A.5}$$

Let us finally recall that a Banach space E is an UMD space (i.e. E has the unconditional martingale difference property) iff for any $p \in (1,\infty)$ there exists a constant $\beta_p(E) > 0$ such that for any E-valued martingale difference $\{\xi_j\}$ (i.e.: $\sum_{j=1}^{n}\xi_j$ is a martingale), for any $\epsilon : \mathbb{N} \to \{-1, 1\}$ and for any $n \in \mathbb{N}$

$$\mathbb{E}\left|\sum_{j=1}^{n}\epsilon_j\xi_j\right|^p \le \beta_p(E)\mathbb{E}\left|\sum_{j=1}^{n}\xi_j\right|^p. \tag{A.6}$$

It is known, see [9] and references therein, that for a Banach space E the following conditions are equivalent: (1) E is an UMD space, (2) E is ζ convex, (3) the Hilbert transform for E-valued functions is bounded in $L^p(\mathbb{R}, E)$ for any (or some) $p > 1$.

Finally, it is known, see e.g. [28, Proposition 2.4], that if a Banach space E is both UMD and of type p, then E is of martingale type p.

B: γ-Radonifying operators

Assume that H and E are real separable Hilbert and resp. Banach spaces and γ_H is the canonical Gaussian cylindrical distribution on H. A bounded linear operator $L : H \to E$ is called γ-radonifying iff $L(\gamma_H)$ is σ-additive. If this is the case, $L(\gamma_H)$ has a unique extension to a σ-additive Borel probability measure ν_L on E. The set of all γ-radonifying operators from H to E we will denote by $R(H, E)$. For $L \in R(H, E)$ one puts

$$\|L\|_{R(H,E)} := \left\{\int_E |x|^2 d\nu_L(x)\right\}^{\frac{1}{2}}. \tag{B.1}$$

Neidhardt in [25] proved that $\|\cdot\|$ is a norm on $R(H, E)$, that $R(H, E)$ with that norm is a separable Banach space and that the set $\mathcal{L}_{\text{fin}}(H, E)$ of bounded linear operators $L : H \to E$ with finite dimensional range, is a dense subspace of $R(H, E)$. It follows from Baxendale [1] that $R(H, E)$ is an operator ideal, i.e. if $L \in R(H, E)$, $A \in \mathcal{L}(G, H)$ and $B \in \mathcal{L}(E, Y)$ (where G and Y is another separable Hilbert, resp. Banach space) then also $BLA \in R(G, Y)$ and $\|BLA\|_{R(G,Y)} \le C|B|_{\mathcal{L}(E,Y)}\|L\|_{R(H,E)}|A|_{\mathcal{L}(G,H)}$ for some constant C independent of A, B and L.

Let us fix an orthonormal basis (ONB) $\{e_k\}_k$ of H and let us denote by Π_n the projection onto the space spanned by $e_1, \dots, e_n$. Let us choose and fix an i.i.d. sequence of standard centered real valued Gaussian random variables β_k, $k \in \mathbb{N}$. It follows from the Itô-Nisio Theorem, see e.g. [21] then $L \in R(H, E)$ iff $\left(\mathbb{E}|\sum_k \beta_k L e_k|_E^2\right)^{1/2} < \infty$. Moreover, $\|L\| = \left(\mathbb{E}|\sum_k \beta_k L e_k|_E^2\right)^{1/2}$. One can also show that the exponent 2 above can be replaced by any $p \in (1,\infty)$.

C: Proof of inequality (2.3)

In this appendix we formulate and prove inequality (2.3). Our approach is a sense similar to the approach used in the Gaussian case by Neidhard [25] and Brzeźniak [3] or in the Poisson random measure in Madrekar and Rüdiger [24]. In fact, our main result below can be seen a generalization of Theorem 3.6 from [24] to the case of martingale type p Banach spaces, with $p \in (1, 2]$.

Assume that E is a real separable Banach space of martingale type p.

Notation 2 *By $M^{\bar{\mathbb{N}}}_{S\times\mathbb{R}_+}$ we denote the family of all $\bar{\mathbb{N}}$-valued measures on $(S\times\mathbb{R}_+, \mathcal{S}\otimes\mathcal{B}(\mathbb{R}_+))$ and $\mathcal{M}^{\bar{\mathbb{N}}}_{S\times\mathbb{R}_+}$ is the σ-field on $M^{\bar{\mathbb{N}}}_{S\times\mathbb{R}_+}$ generated by functions $i_B : M \ni \mu \mapsto \mu(B) \in \bar{\mathbb{N}}$, $B \in \mathcal{S}\otimes\mathcal{B}(\mathbb{R}_+)$.*

Let us assume that $(S, \mathcal{S})$ is a measurable space, $\nu \in M^+_S$ is a non-negative measure on $(S, \mathcal{S})$ and $\mathfrak{P} = (\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ is a filtered probability space. We also assume that η is time homogeneous Poisson random measure over $\mathfrak{P}$, with the intensity measure ν, i.e. $\eta : (\Omega, \mathcal{F}) \to (M^{\bar{\mathbb{N}}}_{S\times\mathbb{R}_+}, \mathcal{M}^{\bar{\mathbb{N}}}_{S\times\mathbb{R}_+})$ is a measurable function satisfying the following conditions:

(i) for each $B \in \mathcal{S}\otimes\mathcal{B}(\mathbb{R}_+)$, $\eta(B) := i_B \circ \eta : \Omega \to \bar{\mathbb{N}}$ is a Poisson random variable with parameter[5] $\mathbb{E}\eta(B)$;
(ii) η is independently scattered, i.e. if the sets $B_j \in \mathcal{S}\otimes\mathcal{B}(\mathbb{R}_+)$, $j = 1, \dots, n$ are pair-wise disjoint, then the random variables $\eta(B_j)$, $j = 1, \dots, n$ are pair-wise independent;
(iii) for all $B \in \mathcal{S}$ and $I \in \mathcal{B}(\mathbb{R}_+)$, $\mathbb{E}[\eta(B\times I)] = \lambda(I)\nu(B)$, where λ is the Lebesgue measure;
(iv) for each $U \in \mathcal{S}$, the $\bar{\mathbb{N}}$-valued process $(N(t, U))_{t>0}$ defined by

$$N(t, U) := \eta(U \times (0, t]), \quad t > 0$$

is $(\mathcal{F}_t)_{t\geq 0}$-adapted and its increments are independent of the past, i.e. if $t > s \geq 0$, then $N(t, U) - N(s, U) = \eta(U \times (s, t])$ is independent of $\mathcal{F}_s$.

By $\tilde{\eta}$ we will denote the compensated Poisson random measure, i.e. a function defined by $\tilde{\eta}(B) = \eta(B) - \mathbb{E}(\eta(B))$, with the convention that $\infty - \infty = 0$.

Lemma C.1 *Let $p \in (1, 2]$ and assume that E is a Banach space of type p. If a finitely valued function f belongs to $L^p(\Omega\times S, \mathcal{F}_a\otimes\mathcal{S}; \mathbb{P}\otimes\nu; E)$ for some $a \in \mathbb{R}_+$, then for any $b > a$,*

$$\mathbb{E}\left|\int_S f(x)\tilde{\eta}(dx, (a, b])\right|_E^p \leq 2^{2-p} L_p^2(E)(b-a)\mathbb{E}\int_S |f(x)|_E^p \, \nu(dx). \tag{C.1}$$

The proof of this Lemma will be given later.

[5] If $\mathbb{E}\eta(B) = \infty$, then obviously $\eta(B) = \infty$ a.s..

Since the space of finitely valued functions is dense in $L^p(\Omega\times S, \mathcal{F}_a\otimes\mathcal{S}; \mathbb{P}\otimes\nu; E)$, see e.g. [10, Lemma 1.2.14], the previous result implies the following fundamental claim, whose proof is omitted.

Corollary C.1 *Assume that $p \in (1, 2]$ and that E is a p-smoothable Banach space. Then under the assumptions of Lemma C.1 there exists a unique bounded linear operator*

$$\tilde{I}_{(a,b)} : L^p(\Omega \times S, \mathcal{F}_a \otimes \mathcal{S}; \mathbb{P} \otimes \nu; E) \to L^p(\Omega, \mathcal{F}, E)$$

such that for a finitely-valued function f, we have

$$\tilde{I}_{(a,b)}(f) = \int_S f(x)\tilde{\eta}(dx, (a, b]).$$

In particular, for every $f \in L^p(\Omega \times S, \mathcal{F}_a \otimes \mathcal{S}; \mathbb{P} \otimes \nu; E)$,

$$\mathbb{E}\left|\tilde{I}_{(a,b)}(f)\right|_E^p \leq 2^{2-p} L_p^2(E)(b-a)\mathbb{E}\int_S |\xi(x)|_E^p\, \nu(dx). \tag{C.2}$$

In what follows, unless we are in danger of ambiguity, for every $L^p(\Omega \times S, \mathcal{F}_a \otimes \mathcal{S}; \mathbb{P} \otimes \nu; E)$ we will write $\int_S \xi(x)\tilde{\eta}(dx, (a, b])$ instead of $\tilde{I}_{(a,b)}(f)$.

Let X be any Banach space. Later on we will take X to be one of the spaces E, $R(H, E)$ or $L^p(S, \nu, E)$. For $a < b \in [0, \infty]$ let $\mathcal{N}(a, b; X)$ be the space of (equivalence classes of) progressively-measurable processes $\xi : (a, b] \times \Omega \to X$.

For $q \in (1, \infty)$ we set

$$\mathcal{N}^q(a, b; X) = \left\{\xi \in \mathcal{N}(a, b; X) : \int_a^b |\xi(t)|^q\, dt < \infty \text{ a.s.}\right\}, \tag{C.3}$$

$$\mathcal{M}^q(a, b; X) = \left\{\xi \in \mathcal{N}(a, b; X) : \mathbb{E}\int_a^b |\xi(t)|^q\, dt < \infty\right\}. \tag{C.4}$$

Let $\mathcal{N}_{\text{step}}(a, b; X)$ be the space of all $\xi \in \mathcal{N}(a, b; X)$ for which there exists a partition $a = t_0 < t_1 < \cdots < t_n < b$ such that for $k \in \{1, \ldots, n\}$, for $t \in (t_{k-1}, t_k]$, $\xi(t) = \xi(t_k)$ is $\mathcal{F}_{t_{k-1}}$-measurable and $\xi(t) = 0$ for $t \in (t_n, b)$. We put $\mathcal{M}^q_{\text{step}} = \mathcal{M}^q \cap \mathcal{N}_{\text{step}}$. Note that $\mathcal{M}^q(a, b; X)$ is a closed subspace of $L^q([a, b) \times \Omega; X) \cong L^q([a, b); L^q(\Omega; X))$.

In what follows we put $a = 0$ and $b = \infty$. For $\xi \in \mathcal{M}^p_{\text{step}}(0, \infty; L^p(S, \nu; E))$ we set

$$\tilde{I}(\xi) = \sum_{j=1}^n \int_S \xi(t_j, x)\tilde{\eta}(dx, (t_{j-1}, t_j]). \tag{C.5}$$

Obviously, $\tilde{I}(\xi)$ is a $\mathcal{F}$-measurable map from Ω with values in E.
We have the following auxiliary results.

Lemma C.2 *Let $p \in (1, 2]$ and assume that E is a Banach space of martingale type p. Then for any $\xi \in \mathcal{M}^p_{step}(0, \infty; L^p(S, \nu; E))$, $\tilde{I}(\xi) \in L^p(\Omega, E)$, $\mathbb{E}\tilde{I}(\xi) = 0$ and*

$$\mathbb{E}|\tilde{I}(\xi)|^p \le K_p^2(E) L_p(E) 2^{2-p} \int_0^\infty \mathbb{E} \int_S |\xi(t, x)|_E^p \nu(dx)\, dt \tag{C.6}$$

The proof of this Lemma will be given later.

Lemma C.3 *Suppose that $\xi \sim$ Poiss (λ), where $\lambda > 0$. Then, for all $p \in [1, 2]$,*

$$\mathbb{E}|\xi - \lambda|^p \le 2^{2-p}\lambda.$$

Also the proof of this Lemma will be given later.

Remark C.1 One can easily calculate that

$$\mathbb{E}(|\xi - \lambda|) = 2\lambda\, e^{-\lambda}, \quad \text{if } \lambda \in (0, 1).$$

Theorem C.1 *Assume that $p \in (1, 2]$ and E is a martingale type p Banach space. Then there exists a unique bounded linear operator*

$$\tilde{I} : \mathcal{M}^p(0, \infty, L^p(S, \nu; E)) \to L^p(\Omega, \mathcal{F}, E)$$

such that for $\xi \in \mathcal{M}^p_{\rm step}(0, \infty, L^p(S, \nu; E))$ we have $I(\xi) = \tilde{I}(\xi)$. In particular, for every $\xi \in \mathcal{M}^p(0, \infty, L^p(S, \nu; E))$,

$$\mathbb{E}|I(\xi)|_E^p \le 2^{2-p} L_p^2(E) L_p(E) \mathbb{E} \int_0^\infty \int_S |\xi(t, x)|_E^p\, \nu(dx)\, dt. \tag{C.7}$$

Moreover, for each $\xi \in \mathcal{M}^p(0, \infty, L^p(S, \nu; E))$, the process $I(1_{[0,t]}\xi)$, $t \ge 0$, is an E-valued p-integrable martingale. The process $1_{[0,t]}\xi$ is defined by $[1_{[0,t]}\xi](r, x; \omega) := 1_{[0,t]}(r)\xi(r, x, \omega)$, $t \ge 0$, $r \in \mathbb{R}_+$, $x \in S$ and $\omega \in \Omega$.

Proof of Theorem C.1 Follows from Lemma C.2 and the density of $\mathcal{M}^p_{\rm step}(0, \infty, L^p(S, \nu; E))$ in the space $\mathcal{M}^p(0, \infty, L^p(S, \nu; E))$.

In a natural way we can define spaces $\mathcal{M}^p_{\rm loc}(0, \infty, L^p(S, \nu; E))$ and $\mathcal{M}^p(0, T, L^p(S, \nu; E))$, where $T > 0$. Then for any $\xi \in \mathcal{M}^p_{\rm loc}(0, \infty, L^p(S, \nu; E))$ we can in a standard way define the integral $\int_0^t \int_S \xi(r, x)\tilde{\eta}(dx, dr)$, $t \ge 0$, as the cádlág modification of the process

$$I\left(1_{[0,t]}\xi\right), \quad t \ge 0. \tag{C.8}$$

In view of [14, Theorem 1, p. 181] the existence of the above mentioned cádlág modification follows from the last part of Theorem C.1.

Similarly, for a stopping time τ we can define a process $\xi \in \mathcal{M}^p_{\mathrm{loc}}(0,\infty, L^p(S,\nu;E))$ and the integral

$$\int_0^\tau \int_S \xi(r,x)\tilde{\eta}(dx,dr) := I(1_{[0,\tau]}\xi), \tag{C.9}$$

provided $1_{[0,\tau]}\xi \in \mathcal{M}^p(0,\infty, L^p(S,\nu;E))$. Theorem C.1 implies that in this case the following inequality holds.

$$\mathbb{E}\left|\int_0^\tau \int_S \xi(r,x)\tilde{\eta}(dx,dr)\right|_E^p \le C_p \mathbb{E}\int_0^\tau \int_S |\xi(r,x)|_E^p\, \nu(dx)\, dr. \tag{C.10}$$

with some constant $C_p > 0$ independent of ξ.

Proof of Lemma C.2 Let us observe that the sequence $(M_k)_{k=1}^n$ defined by $M_k = \sum_{j=1}^k \int_S \xi(t_j,x)\tilde{\eta}(dx,[t_{j-1},t_j))$, $k = 1,\ldots,n$, is an E-valued martingale (with respect to the filtration $(\mathcal{F}_{t_k})_{k=1}^n$). Therefore, by the martingale type p property of the space E and Lemma C.1 we have the following sequence of inequalities

$$\begin{aligned}\mathbb{E}|\tilde{I}(\xi)|_E^p = \mathbb{E}|M_n|_E^p &\le L_p(E)\sum_{k=1}^n \mathbb{E}\left|\int_S \xi(t_k,x)\tilde{\eta}(dx,[t_{k-1},t_k])\right|_E^p \\ &\le L_p(E)K_p^2(E)2^{2-p}\sum_{k=1}^n (t_k - t_{k-1})\,\mathbb{E}\int_S |\xi(t_k,x)|_E^p\,\nu(dx) \\ &= L_p(E)K_p^2(E)2^{2-p}\int_0^\infty \mathbb{E}\int_S |\xi(t,x)|_E^p\,\nu(dx)\,dt. \end{aligned} \tag{C.11}$$

This concludes the proof.

Proof of Lemma C.1 Put $I = (a,b]$. We may suppose that $f = \sum_{j,i} f_{ji} 1_{A_{ji}\times B_j}$ with $f_{ji} \in E$, $A_{ji} \in \mathcal{F}_a$ and $B_j \in \mathcal{S}$, the finite families of sets $(A_{ji} \times B_j)$ and (B_j) being pair-wise disjoint and $\nu(B_j) < \infty$. Let us notice that

$$\int_S f(x)\tilde{\eta}(dx,I) = \sum_{ji} 1_{A_{ji}}\tilde{\eta}(B_j \times I) f_{ji} = \sum_j \left(\sum_i 1_{A_{ji}} f_{ji}\right)\tilde{\eta}(B_j \times I).$$

Since the random variables $1_{A_{ji}} f_{ji}$ are $\mathcal{F}_a$-measurable and the random variables $\tilde{\eta}(B_j \times I)$ are $\mathcal{F}_a$-independent, we may suppose that these random variables are defined on a probability space $(\hat{\Omega}, \hat{\mathcal{F}}, \hat{\mathbb{P}})$ such that $\hat{\Omega} = \Omega_1 \times \Omega_2$, $\hat{\mathcal{F}} = \mathcal{F}_1 \otimes \mathcal{F}_2$,

$\hat{\mathbb{P}} = \mathbb{P}_1 \otimes \mathbb{P}_2$, where $\Omega_i, \mathcal{F}_i, \mathbb{P}_i)$, $i = 1, 2$ are probability spaces, and the first, resp. second, group of variables depend on $\omega = (\omega_1, \omega_2) \in \hat{\Omega}$ only via ω_1, resp. ω_2. Let us denote the integral with respect to probability measure $\mathbb{P}_i$ by $\mathbb{E}_i$. Since, for each fixed $\omega_1 \in \Omega_1$, the sequence $\sum_j \left(\sum_i 1_{A_{ji}} f_{ji}\right) \tilde{\eta}(B_j \times I)$, $j = 1, 2, \dots$ is an E-valued martingale difference, by the martingale type p property of the space E, Lemma C.3, we infer that

$$\begin{aligned}
&\mathbb{E}_2 \left| \sum_j \left(\sum_i 1_{A_{ji}} f_{ji} \right) \tilde{\eta}(B_j \times I) \right|_E^p \\
&\quad \le L_p(E) \sum_j \mathbb{E}_2 \left| \left(\sum_i 1_{A_{ji}} f_{ji} \right) \tilde{\eta}(B_j \times I) \right|_E^p \\
&\quad = L_p(E) \sum_j \left| \sum_i 1_{A_{ji}} f_{ji} \right|_E^p \mathbb{E}_2 \left| \tilde{\eta}(B_j \times I) \right|^p \\
&\quad = 2^{2-p} L_p(E) \sum_j \left| \sum_i 1_{A_{ji}} f_{ji} \right|_E^p \nu(B_j) \lambda(I)
\end{aligned}$$

For the same reasons

$$\mathbb{E}_1 \left| \sum_i 1_{A_{ji}} f_{ji} \right|_E^p \le L_p(E) \sum_i \mathbb{E}_1 |1_{A_{ji}} f_{ji}|_E^p = L_p(E) \sum_i \mathbb{P}(A_{ji}) |f_{ji}|_E^p$$

Therefore, by the Fubini Theorem,

$$\begin{aligned}
&\mathbb{E} \left| \int_S \xi(x) \tilde{\eta}(dx, I) \right|_E^p \\
&\quad = \mathbb{E}_1 \mathbb{E}_2 \left| \sum_j \left(\sum_i 1_{A_{ji}} f_{ji} \right) \tilde{\eta}(B_j \times I) \right|_E^p \\
&\quad \le 2^{2-p} L_p^2(E) \sum_j \sum_i \mathbb{P}(A_{ji}) |f_{ji}|_E^p \nu(B_j) \lambda(I) \\
&\quad = 2^{2-p} L_p^2(E) \lambda(I) \sum_j \sum_i |f_{ji}|_E^p (\mathbb{P} \otimes \nu)(A_{ji} \times B_j) \\
&\quad = 2^{2-p} L_p^2(E) \lambda(I) \int_{\Omega \times S} \left| \sum_{ji} 1_{A_{ji} \times B_j} f_{ji} \right|^p d(\mathbb{P} \otimes \nu) \\
&\quad = 2^{2-p} L_p^2(E) (b-a) \mathbb{E} \int_S |f(x)|_E^p \nu(dx).
\end{aligned}$$

The proof is complete.

Proof of Lemma C.3 The case $p = 2$ is well known. Since $\xi \geq 0$ and $\mathbb{E}(\xi) = \lambda$, the case $p = 1$ follows by the triangle inequality. The case $p \in (1, 2)$ follows then by applying the Hölder inequality. Indeed, with $\alpha = 2(p - 1)$ and $\beta = 2 - p$ we have the following sequence of inequalities, where $\eta := |\xi - \lambda|$.

$$\begin{aligned}\mathbb{E}(\eta^p) &= \mathbb{E}(\eta^\alpha \eta^\beta) \leq [\mathbb{E}((\eta^\alpha)^{2/\alpha})]^{\alpha/2}[\mathbb{E}((\eta^\beta)^{1/\beta})]^\beta \\ &= [\mathbb{E}(\eta^2)]^{\alpha/2}[\mathbb{E}(\eta)]^\beta \leq (\lambda)^{\alpha/2}(2\lambda)^\beta = 2^{2-p}\lambda.\end{aligned}$$

We conclude with a result corresponding to inequality (2.4).

Corollary C.2 *Assume that $1 \leq q \leq p < 2$ and E is a martingale type p Banach space. Then there exists a constant $C > 0$ such that for any process $\xi \in \mathcal{M}^p_{\mathrm{loc}}(0, \infty, L^p(S, \nu; E))$, and any $T > 0$,*

$$\mathbb{E}\left|\sup_{t\in[0,T]} \int_0^t \int_S \xi(r, x)\tilde{\eta}(dx, dr)\right|^q \leq C\mathbb{E}\left(\int_0^T \int_S |\xi(r, x)|^p\, \nu(dx)\, dr\right)^{q/p}. \tag{C.12}$$

The proof of the above result will be based on Proposition IV.4.7 from the monograph [30] by Revuz and Yor, which we recall here for the convenience of the reader.

Proposition C.1 *Suppose that a positive, adapted right-continuous process Z is dominated by an increasing process A, with $A_0 = 0$, i.e. there exists a constant $C > 0$ such that for every bounded stopping time τ, $\mathbb{E}Z_\tau \leq C\mathbb{E}A_\tau$. Then for any $k \in (0, 1)$,*

$$\mathbb{E} \sup_{0\leq t<\infty} Z_t^k \leq C^k \frac{2-k}{1-k}\mathbb{E}A_\infty^k.$$

Proof of Corollary C.2 Let now fix $q \in [1, p)$. Put $k = \frac{q}{p}$. We will apply Proposition C.1 to the processes $Z_t = |\int_0^t \int_S \xi(r, x)\tilde{\eta}(dx, dr)|_E^p$ and $A_t = \int_0^t \int_S |\xi(r, x)|_E^p\, \nu(dx)\, dr$, $t \in [0, T]$. Let us notice that in view of inequality (C.10), the process Z is dominated by the process A. Since Z is right continuous, $\sup_{0\leq t\leq T} Z_t^k = \sup_{0\leq t\leq T} |\int_0^t \int_S \xi(r, x)\tilde{\eta}(dx, dr)|_E^q$ and $A_\infty^k = \left(\int_0^T \int_S |\xi(r, x)|_E^p\, \nu(dx)\, dr\right)^{q/p}$, we get inequality (C.12). This completes the proof of Corollary C.2.

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