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Erratum

Correction to "Random perturbations of dynamical systems and diffusion processes with conservation laws"

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The proof of the form of the gluing conditions presented on pages 458–462 is not correct in general. We explain below how this proof can be adapted to cover the general case.

At page 459, starting with line 5, we consider processes $\hat{X}_t^\varepsilon$ in $D_k(\pm\delta)$ for some small $\delta > 0$ with normal reflection at the boundaries governed inside $D_k(\pm\delta)$ by operators L^ε, and we claim that the invariant measure M of the processes X_t^ε in R^d restricted to $D_k(\pm\delta)$ is invariant for $\hat{X}_t^\varepsilon$.

This is not true in general, but the general case can be reduced to a special one where this is true.

A gluing condition describes the local behavior of the limiting diffusion at an interior vertex O_k of the graph, and it depends only on the behavior of the processes X_t^ε in a small neighborhood $D_k(\pm\delta)$ of the critical trajectory $\Upsilon^{-1}(O_k)$ for some small $\delta > 0$. Therefore it is sufficient to consider the case that there is only one critical point x_k for H, and consequently, the graph Γ consists of just one interior vertex connected to two or three edges. Further, the gluing

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condition will not change if we change the data of the processes outside a small neighborhood of $\Upsilon^{-1}(O_k)$.

Let $\delta > 0$ be small and $\delta < \gamma$ if $\kappa > 0$ with γ from Assumption (3, i).

We choose a smooth function h in R^d that is equal to 1 in $D_k(\pm\delta/3)$ and equal to 0 outside $D_k(\pm\delta/2)$, and we replace the vector field b_1 in the operator L_1 by

$$\begin{array}{ll} h \cdot b_1, & \text{if } \kappa > 0\,, \\ h \cdot b_1 \; + (1-h) \cdot \bar{\nabla} H, & \text{if } d = 2, \kappa = 0\,, \end{array}$$

where

$$\bar{\nabla} H = \left(-\frac{\partial H}{\partial x_2}\,, \frac{\partial H}{\partial x_1} \right).$$

We denote the operator L_1 with the new b_1 again by L_1, as well as we denote the process governed by the new L_1 by $X_t^{(1)}$ again. We note that H is a first integral for this process.

If $d = 2$ and $\kappa = 0$ then $X_t^{(1)}$ has an invariant density by Proposition 2.1, and this density is constant outside $D_k(\pm\delta/2)$, and it is equal to the original one in $D_k(\pm\delta/3)$.

If $\kappa > 0$ then it follows from Lemma 2.1 and Lemma 2.3 that $X_t^{(1)}$ has an invariant density and that this density is constant outside $D_k(\pm\delta/2)$, as the operator L_1 is symmetric there. In $D_k(\pm\delta/3)$ it is equal to the original one (comp. Proposition 2.2).

After these changes, the proof is correct for the new processes, as now the invariant measure M restricted to $D_k(\pm\delta)$ is invariant for $\hat{X}_t^{\varepsilon}$.