The Haken–Kelso–Bunz (HKB) model: from matter to movement to mind

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Abstract

This article presents a brief retrospective on the Haken–Kelso–Bunz (HKB) model of certain dynamical properties of human movement. Though unanticipated, HKB introduced, and demonstrated the power of, a new vocabulary for understanding behavior, cognition and the brain, revealed through a visually compelling mathematical picture that accommodated highly reproducible experimental facts and predicted new ones. HKB stands as a harbinger of paradigm change in several scientific fields, the effects of which are still being felt. In particular, HKB constitutes the foundation of a mechanistic science of coordination called Coordination Dynamics that extends from matter to movement to mind, and beyond.

Keywords HKB model · Behavior · Brain · Symmetry breaking · Fluctuations · Intrinsic dynamics · Functional information · Complementarity · Coordination Dynamics

1 Introduction

On receiving the Editor’s invitation to write a retrospective on, as he said, “what came to be known as the Haken–Kelso–Bunz model of human hand movements” I wondered about the word “retrospective.” The first thing that came to mind was Proust’s “À la recherche du temps perdu” translated as “Remembrance of things past.” I have told the story of how the Haken–Kelso–Bunz model (known as HKB in the literature) came to be on a couple of occasions (e.g., Kelso 1995, Ch.3), most recently at a celebration of Hermann Haken’s 90th birthday (see Kriz and

Tschacher 2017). A fine biography by Bernd Kröger (2015) relates how Haken’s theory of the laser and our scientific collaboration came about, as well as some of its fruits, e.g., in brain research and pattern recognition (pp. 197–208). Early developments of HKB are summarized in a *Science* article by Gregor Schöner and myself which introduced the concepts of Synergetics and the mathematical methods and tools of nonlinear dynamical systems to a broader audience (Schöner and Kelso 1988a; see also Kelso et al. 1987). A further volume edited by two of Haken’s former students, the late Armin Fuchs and Viktor Jirsa—both theoretical physicists and key players themselves in the development of HKB—gathers together some of the main contributors to its development in the neural, behavioral and social sciences (Fuchs and Jirsa 2008; see also earlier contributions in Jirsa and Kelso 2004). Many of the methods to analyze behavior that emerged as a result of HKB’s quantitative approach are described in a book edited by Huys and Jirsa (2010). Over the years, several encyclopedia articles have appeared spanning the disciplines from the social and behavioral sciences (Kelso 2001) to sport and exercise science (Kelso 2014a) to a trilogy of articles in the *Encyclopedia of Complexity and System Science* edited by R.A. Meyers (Fuchs and Kelso 2009/2013; Kelso 2009/2013; Oullier and Kelso 2009), and of course there have been numerous excellent reviews (e.g., Beek et al. 1995, 2002; Swinnen 2002; Park and Turvey 2008). On the topic of rhythmic interlimb coordination

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To highlight the scientific impact of our Journal over the last decades, we asked authors of highly influential papers to reflect on the history of their study, the long-term effect it had, and future perspectives of their research. We trust the reader will enjoy these first-person accounts of the history of big ideas in Biological Cybernetics.

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alone, the summary of Dagmar Sternad (2008)—already nearly 15 years old—still seems apropos:

Over 20 years of research on interlimb coordination has produced a remarkably extensive and consistent body of literature on this topic. Held together by the HKB modeling framework...the variety of specific issues is striking: from the asymmetry between coordinated limbs, interpersonal coordination, to intentional switching; from synchronization with an external timer, cognitive influences, to learning new coordinative patterns; from polyrhythmic coordination, attentional effects, to perceptual anchoring and multi-sensory influences. This list is by no means complete and the appropriate references would overwhelm the writing. Importantly, the research direction also spread to other labs and investigators... (p. 106).

So what was HKB all about? An empirical research program on interlimb coordination? In part, yes, but it is quite clear even from the above statement that the HKB model infiltrated many other domains of inquiry, and affected the approaches taken to many topics and activities beyond human hand movements. The empirical and theoretical motivation for HKB and its mathematical and physical realization turned out to have unforeseen consequences. Indeed, HKB has been touted as a harbinger of paradigm change in the behavioral, brain, developmental, cognitive, sports and exercise sciences (e.g., Balagué, et al. 2013, 2017; Beer 1995; Chemero 2001; Port and Van Gelder 1995; Thelen, et al. 1987; Thelen and Smith 1994)—to name a few—as well as engaging the attention of philosophers. The reason is that HKB introduced, and demonstrated the power of, a new set of concepts, methods and tools for understanding behavior, the nervous system and their relation. This was revealed through a visually compelling and easily understood picture (see Box 1) that nevertheless precisely accommodated experimental facts and predicted new ones of very different kinds for many different situations—from “low level” to “high level” coordinative processes. HKB, as they say, “changed the furniture in the room”: it offered strategies for transcending levels of organization, all the way from neurons to the movements of people, interacting with each other and their environments.

For readers not entirely familiar with the background to HKB, it may be helpful to say a few words about what motivated it and what it led to. Following the guidance of the Editor, the question of “where it might be leading us” means one is obliged to address “the long-term impact of this paper” and the “future developments we may see as a result.” The reader may well question whether one of the authors of HKB is the best person to make such speculations. The scientific research surrounding the HKB model, however, stands above any single individual. Thus, as far

as is reasonable, and to the greatest extent possible, this article will rely on the work and opinions of others as they have been expressed since the paper appeared in *Biological Cybernetics* all those years ago.

2 Brief background

HKB was (and is) about getting a handle on the problem of coordination in living things. The eminent theoretical biologist Howard Pattee (1976) said it succinctly: “*I do not see any way to avoid the problem of coordination and still understand the physical basis of life*” (p. 157). Pattee realized that the origins problem of coordination in living things is not solved by calling upon natural selection. Some minimal level of coordination is needed in the first place for natural selection to act upon, as well as other faster-acting processes such as development and learning. “*At no other level of biological organization,*” says Pattee, “*does the concept of coordination take on such profound meaning or display such elegant examples as in the sensorimotor activities which integrate the acquisition of data through the sense organs, the classification of this data by the nervous system, and the decisive muscular responses of the organism based on this classification...* It is an area where experiments can be done to test theories of the **origin** of coordination” (emphasis mine, p. 156). But which experiments and which theories?

Around the same time, but in an entirely different context, psychologists, neurophysiologists, computer scientists and engineers were inquiring into the significant units that the nervous system uses to control and coordinate movements (see, e.g., the contributions in Stelmach and Requin 1980). Powerful arguments coming largely from the Russian school led by Israel Gelfand (Gelfand 1971; see also Turvey et al. 1978, 1982) proposed that due to the large number of degrees of freedom involved, control decisions must refer to *functional* groupings of neurons, muscles and joints that are constrained to act as a single unit. In those days, evidence for such synergies or coordinative structures in voluntary movements was in short supply to say the least (Kelso et al. 1979). Much has changed, of course, in the intervening years. Entire books have been written about synergies in motor control (e.g., Latash 2008). Similar ideas regarding “motor primitives” (e.g., Mussa-Ivaldi et al. 1994) located in the spinal cord have been hypothesized to explain the modular organization of the motor system. However, with perhaps a single exception to be found in the writings of the philosopher–biologist Maxine Sheets-Johnstone (1999/2011), the fundamental question of how coordinated movements (and biological coordination in general) could arise in a complex system composed of a very large number of interacting elements operating on multiple timescales—Pattee’s origins

Box 1 Coordination potential includes the *original HKB model* at the level of the order parameter/collective variable, relative phase ϕ (top row) and its *extended, symmetry breaking* version (second and third rows). Italics are intended to convey some of the key concepts and strategies of the nonlinear paradigm that HKB brought to the behavioral and brain sciences. The *order parameter/collective variable* ϕ is an expression of the *information exchange* between the (nonlinearly) interacting parts and processes. Plotted is the *coupling strength* against a parameter, delta omega ($\Delta\omega$) which expresses the difference—hence termed heterogeneity or diversity—between the natural frequencies of the individual oscillatory elements (which could be cells, neurons, muscles, body parts, brain areas, people, virtual partners, visual, auditory, tactile stimuli, and combinations of these). Coupling strength (b/a) corresponds to a *control parameter*, frequency in the Kelso experiments though by no means restricted to such. Notice that when continuously varied, the control parameter promotes *instability* and the emergence of new coordination patterns. (A control parameter is not at all the same as a randomly assigned independent variable in traditional experiments.) The region around each local minima of the potential constitutes a basin of *attraction*, trapping the system into coordinated states or *attractors*. This attractive force is not mechanical. It acts like a causal force but is informationally based. Black (white) balls correspond to stable (unsta-

ble) collective states (fixed points) of this elementary *coordination dynamics*. The original HKB model (*top row*) was based on spatial and temporal symmetry and assumed identical components, viz. the index fingers of the two hands in the Kelso experiments, i.e., $\Delta\omega = 0$. Notice the system is initially *bistable* and its attractor layout does not change with the control parameter. However, at a *critical value* of the control parameter/coupling strength the initially stable anti-phase coordination state loses stability and the system switches to in-phase. Mathematically, this abrupt change takes the form of a *pitchfork bifurcation*. Further experiments and stochastic theory showed that switching takes the form of a *nonequilibrium phase transition*, thus demonstrating a basic hallmark of *self-organization*. The second and third rows show the effect of the delta omega term which breaks the symmetry of the dynamics causing a tilt in the coordination potential and shifts in the system's collective states. In this *extended HKB model*, switching now takes the mathematical form of a *saddle node bifurcation*, where stable and unstable fixed points kiss or collide. Notice also, e.g., bottom right, even though all the fixed points have disappeared, some curvature in the potential remains: the system still exhibits *coordination tendencies* to where the fixed-point states used to be. This is the *metastable* regime of the coordination dynamics which is characteristic of coordination on many levels (see text for more details)

question—remained unanswered. Then dominant views of movement control by means of closed-loop feedback loops, open-loop programming and even feedforward, internal comparator mechanisms seemed inadequate for the job, to wit, solving the problem of coordinating and controlling a system containing so many degrees of freedom (Fowler and Turvey 1978). Through the writings and insights of Michael Turvey, this came to be known as Bernstein's problem (Bernstein 1967; Turvey, et al. 1982) after the Soviet physiologist Nicolai Alexandrovitch Bernstein (1896–1966).

In a pair of papers emanating from an 11-day NATO Advanced Study Institute held at the Abbaye de Senanque

in June, 1979, “a tentative proposal” was put forward that inroads into Bernstein's problem might lie in physical theories of pattern formation in open, nonequilibrium dynamical systems (Kugler et al. 1980; Kelso et al. 1980; see also Kelso 1981a). There, the emergence of structure and pattern follows a particular path, namely that under certain conditions new forms arise spontaneously as a succession of instabilities. Near such instabilities, a system composed of uncountably many individual elements orders itself in new or different ways to accommodate current conditions. Specifically, in Haken's *Synergetics* (Haken 1977/1983), this compression of degrees of freedom near critical points is because the

faster relaxing individual elements become “enslaved” to the slower emergent collective variables and can be effectively eliminated. This means that all the information necessary to describe the system’s behavior rests on these collective variables or order parameters and their nonlinear dynamics. The question then was this: might a solution to Pattee’s origins problem and Bernstein’s degrees of freedom problem lie in how we understand the spontaneous emergence of pattern in systems that are open to matter, energy and information exchange with their surroundings, specifically the nonequilibrium phase transitions studied in physics?

Back then it seemed a long way from pattern forming instabilities in physics to the control and coordination of animate movement. How to put such a tentative proposal to test? One idea was that equine gait transitions, as when a pony spontaneously changes gait from a walk to a trot or a trot to a gallop, might be analogous to the phase transitions and cooperative phenomena seen in non-living physical systems, as when matter changes state. Only recently has this turned out to be true (Granatosky et al. 2018). Back then it was a far stretch of the imagination, and certainly beyond anything one could test experimentally—in the wild or in the laboratory. A “simpler” starting point was needed that could be solved in principle before more complex situations like the ordering seen in animal gaits could be tackled. Hence, emerged quite suddenly—a kind of “ideational phase transition” (Sheets-Johnstone 2015)—the idea of “let your fingers do the walking” in which spontaneous, self-organized transitions in finger and hand movements were observed at both kinematic (Kelso 1981b, 1984) and neuromuscular levels (Kelso and Scholz 1985) in bimanual coordination experiments. An order parameter equation, which formed the top tier of the HKB model and its stochastic equivalent (Schöner et al. 1986), was shown to accommodate all the experimental findings—in addition to predicting new effects. In particular, the hallmark features of nonequilibrium phase transitions including a strong enhancement of fluctuations and critical slowing down—“anticipatory signatures” of upcoming pattern change (cf. Kelso 2010; Scheffer et al. 2012)—were observed. This discovery of synergetic phase transitions in coordinated movement was not just a demonstration of the whole being greater than the sum of its parts. Rather, HKB showed explicitly that the emergent self-organization could be captured by a time-varying order parameter or collective variable whose dynamics was derived from a level below, namely nonlinearly coupled finger or hand movements. In a way, HKB demystified the notion of emergence.

3 The HKB model and beyond

The question of HKB’s impact can be posed with respect to the field of movement itself as well as to other fields that have been influenced by HKB (all the time respecting the caveat that it is impossible to be inclusive).¹ The order in which the topics below are addressed is an attempt at history but is likely more Proustian. In each case, there is an effort to draw out their theoretical significance, the insights that have arisen as well as paths to further research. Sprinkled in with HKB’s influence on research are the changes that it wrought on the promotion and practice of interdisciplinary science, implications for the training of future scientists and even the philosophy of science itself.

3.1 Symmetry breaking and fluctuations

Significant extensions of the basic HKB model were necessary to accommodate new experimental findings. These included symmetry breaking (Kelso et al. 1990; Fuchs and Kelso 1994) and, as mentioned above, the explicit treatment of fluctuations. The introduction of symmetry breaking was to handle coordination among components that are not identical, such as coordinating different body parts, coordinating movements with periodic stimuli, and people coordinating with each other.² Both the original and symmetry breaking extension of the HKB model are illustrated as “The Coordination Potential” in Box 1 (adapted from Kelso 1995).

Symmetry breaking creates novel effects on coordination, especially shifts in coordination and metastability. Shifts in coordination have been extensively studied especially by Turvey and colleagues in their steady-state paradigm of handheld swinging pendulums. The range of results from this body of research is remarkable (Park and Turvey 2008 for review). Among the factors shown to affect coordination were mechanical and functional (handedness) asymmetries, as well as a variety of imperfection parameters (e.g., space-related pendulum orientation, axis of rotation) and other control parameters such as amplitude. The way these findings were accommodated by HKB was informative about which transformations are important to the coordination dynamics and which are equivalent. HKB used spatial

¹ This theme resonates with an important paper “Landscapes beyond the HKB model” by Newell et al. (2008) which addresses learning, a topic that will come up later.

² Of course there are many ways to break the symmetry of the coordination dynamics beyond simple eigenfrequency differences that have been experimentally investigated and incorporated into theoretical extensions of the HKB model. Among the factors are adaptation to specific environmental requirements, limb and hand dominance, hemispheric asymmetries, the influence of perception, intention, memory, learning and so forth (see text for some references).

and temporal symmetries to constrain the order parameter dynamics, promoting among other aspects a group-theoretic analysis of animal gaits (Schöner et al. 1990; see also Collins and Stewart 1993; Golubitzky et al. 1999) and investigations of the serial dynamics of human odometry (Abdolvahab et al. 2015; Turvey et al. 2012).

The significance of the explicit treatment of fluctuations in HKB (Schöner et al. 1986; Haken 1988 Ch. 11; see also Daffertshofer 2010) was not at all about data fitting (Chemero 2009, p. 89). While it is true that Schöner et al. were able to fit all the experimental data in the experiments of Kelso and colleagues (means, variances, relaxation times of the anti-phase and in-phase coordination modes under parametric change, switching times, etc.) with only a single free parameter, far more significant was what these results meant, namely the key role played by fluctuations for the emergence, persistence and exploration of (collective) states of coordination. Witness Haken's conclusion in his 1988/2006 book *Information and Self-Organization*:

In biology, in order to explain the high coordination between muscles, the idea of a motor program has quite often been proposed. According to this idea the brain acts more or less like a computer in which a specific program is stored, and which, after having been invoked, starts to steer individual motions. But in such a case it is hard to understand why fluctuations should occur at all. After all, a motor program is a fixed program which does not allow any fluctuations. On the other hand, critical fluctuations are quite typical for nonequilibrium phase transitions which occur when self-organization happens. For this reason, we believe that the fluctuations found in these experiments strongly support the idea that muscles and neurons form a self-organizing system (p. 151).

Not only did HKB promote the mechanism of self-organization as an explanation of how new patterns of biological coordination could arise, persist and change, it brought to the fore the crucial roles of stability and variability. Up to then, variability was often seen as just “noise,” something to be damped out or ignored.³ Now, experiments and theory were saying something else entirely, namely that variability is ubiquitous and essential for biological coordination. Among other aspects, variability allows the system to adapt and to change flexibly. Variability has also been shown to enhance the detection of weak sensory stimuli (Collins et al. 1997). Sources of variability may be both deterministic and

stochastic (Daffertshofer 2010). This has led to a great deal of ongoing research using various measures of variability (1/f spectra, fractal analysis, entropy, etc.) as indices of physiological and brain complexity. In short, the important role of fluctuations, quantitatively addressed in HKB drew attention to the significance of variability for both health and disease (Stergiou 2019; Tinker and Velazquez 2014; Van Emmerik et al. 2005), and the necessity to develop quantitative measures beyond the first and second moments of a Gaussian probability distribution.

3.2 Conceptual developments I: intrinsic dynamics and functional information

As shown in Box 1, HKB shed a new light on the study of behavioral and neural systems. With few exceptions, none of these concepts, methods and tools were much in evidence until HKB. As one might imagine, for most students in these fields, this was just not normal science. Whereas symmetry breaking and fluctuations were incorporated as early and quite fundamental additions to the HKB model, “real psychology” was quick to catch on. Many additional modeling modifications of HKB were incorporated in order to handle, e.g., handedness (e.g., DePoel 2007; Treffner and Turvey 1996), environmental (Schöner and Kelso 1988b,c), anchoring (e.g., Byblow et al. 1994; Fink et al. 2000a,b) attentional (e.g., Amazeen et al. 1997; Temprado et al. 1999), intentional (e.g., Deluca et al. 2010; Scholz and Kelso 1990), perceptual (e.g., Bingham 2004; Zaal et al. 2000), cognitive (e.g., Frank, et al. 2012, 2015), memory (e.g., Nordham, et al. 2018) and learning (e.g., Kostrubiec et al. 2012) contributions to the dynamics. These and other amplifications of the basic HKB model (e.g., to the excitator model of discrete movements, Jirsa and Kelso 2005; to incorporate crosstalk and time delays between effectors, Banerjee and Jirsa 2007; see also Daffertshofer et al. 2005; to multilimb movements, Jeka et al. 1993; to multijoint trajectory formation, Buchanan, et al. 1997a; to multifrequency coordination, DeGuzman and Kelso 1991; Haken et al. 1996; to the recruitment of degrees of freedom, Buchanan et al. 1997b, Fink et al. 2000b; parametric stabilization, Assisi et al. 2005; posture, Bardy et al. 2002; functional stabilization, Foo et al. 2000; and structured flows on manifolds, Huys et al. 2014; Pillai and Jirsa 2017) led to active research programs in their own right. For example, the extension of HKB into the entrainment of multifrequency N:M polyrhythms (the modeling domain of Arnol'd tongue structures and the like) has had a deep influence in the field of music perception and cognition (e.g., Large and Palmer 2002) and continues to grab the attention of mathematicians (e.g., Savinov et al. 2021). Particularly, impressive also were (and are) a number of excellent PhD theses stemming from universities around the world in which the HKB model has figured prominently

³ Variability refers here to the performance of movements themselves and the stability of coordination patterns. In the domain of learning simple discrete motor skills, variability of practice has long been a topic of importance (Schmidt 1975).

(e.g., Boonstra 2007; Cuijpers 2019; Houweling 2010; Park 2002).

From all this research emerged a fully fledged science of coordination called Coordination Dynamics, with new concepts of its own such as *intrinsic dynamics*, *functional information*, *complementary pairing* and so forth (e.g., Kelso 1995, 2009/2013; Kelso and Engström 2006) along with many new applications, e.g., in clinical and social psychology, psychotherapy, movement disorders and the sports sciences, etc. Intrinsic dynamics, as a concept that emerged directly from HKB, is important both conceptually and practically. In one context it was said, for example, to “*provide a new elan to the study of motor learning...helping to fill the large gap left by previous theories concerning the nature of pre-existing capabilities that performers bring with them when involved in the acquisition of new skills*” (Swinen et al. 1994, p. 23; see also Gooijers and Swinnen 2014).

Practically speaking, the concept of intrinsic dynamics champions the individual learner—coupled, as all learners are, to their environments—as the significant unit of analysis for the study of learning (Zanone and Kelso 1992, 1997; Kostrubiec et al. 2012). This focus on identifying (typically spontaneous) individual predispositions prior to learning contrasts with (or at least complements) regular experimental designs in which independent variables are applied to groups of subjects followed by standard statistical tests such as analysis of variance. Recognition of preexisting capacities and individual variability turns out to be crucial not just for the study of motor learning but in the treatment of brain injury or disease where the same structural damage can have entirely different functional effects depending on individual predispositions *qua* intrinsic dynamics. As in the studies of bimanual coordination modeled by HKB, preference for symmetric and antisymmetric movements turns out to be a predominant feature of natural movements (Howard et al. 2009). Assessing such individual predispositions is at the core of understanding adaptation and change in general and designing beneficial treatment regimes (e.g., Roerdink 2009; Schalow 2002). The concept of intrinsic dynamics goes beyond the context of learning new motor skills and rehabilitation. For example, in social settings, individuals display natural tendencies to coordinate with each other even when that is not the goal (e.g., Richardson et al. 2007; Tehran 2016). Studies show that even at the level of competitive individual (e.g., Palut and Zanone 2005) and team sports settings (e.g., McGarry et al. 2002) tendencies for states of in-phase and anti-phase coordination still predominate. This is important quite generally because it is the intrinsic dynamics that is subject to modification.

A complementary aspect of intrinsic dynamics is the concept of *functional information*. Whereas in HKB control parameters are sources of non-specific information that simply lead the system through its repertoire of

coordination states and tendencies (Box 1; for an experimental example in the area of deep brain stimulation, see e.g., Synofzik et al. 2012), the idea is that for cognitive influences on coordination to matter, *information must be meaningful and specific to the system’s intrinsic dynamics*. Formally, functional information acts as an attractor in the same space as the intrinsic dynamics attracting them to the perceived, memorized, desired or to be learned coordination pattern. This means that each of the various influences on coordination can be interpreted (and modeled) in terms of the very same collective variable that expresses the coordination pattern. Recent realizations of this mechanism are evident, for example, in the experiments of Kovacs and colleagues which used Lissajous feedback to learn complicated patterns of coordination (e.g., Kovacs et al. 2009, 2010; see also Mechsner et al. 2001).

HKB modeled not just the intrinsic or preferred coordination modes of the system, but also led to a whole new way to incorporate the influences of intention, learning and memory—viewed as functional information—on the coordination dynamics. Intentional information, for example, was shown to destabilize one pattern and stabilize another—even under circumstances in which the latter would typically be unstable. The neural structures involved in intentional stabilization and switching were identified using fMRI (see, e.g., DeLuca et al. 2010). Obviously, much more can be done to reveal the neural coordination dynamics underlying intentional stability and change, and in general to understand the interplay between intrinsic dynamics and functional information.

Nowhere does “the soft assembly of nonlinearly and informationally coupled modes of coordination” (a nice way to say the HKB model) appear so unexpectedly than in language itself. In language, the association between abstract discrete categories and continuous speech poses a significant challenge. My former colleagues at Haskins Laboratories have exploited the HKB model to explain linguistically defined patterns of coordination (Pouplier 2021 for review). In the case of syllable structure, for example, there are just two ways to coordinate a consonant (C) and a vowel (V) gesture: in-phase and anti-phase. Selecting in-phase produces the coordination underlying CV structures, while selecting anti-phase produces VC structures (see Tuller and Kelso 1991 for early evidence). To say the word “mad,” for instance, lip closure and velum opening are coupled in-phase with the V gesture (wide pharyngeal constriction of the tongue body), and the coda gesture (tongue tip closure) is coupled anti-phase with the V. Of the two modes, in-phase is more stable and accessible than anti-phase, a fact that seems to fit so-called universal patterns of syllable structure (where here “universal” means across all languages, cf. Chomsky 1965). Apparently, as in studies of adult learning, infants come into the language

learning process with these basic predispositions, and it is these that are modified according to the specific language environment. Good to know if you are trying to learn a second language.

The fact that the same basic HKB coordination dynamics—with its multistability, instability, transitions and metastability—has been shown to govern coordination between body parts (including speech articulators) within a person; between body parts and tactile, auditory and visual stimuli (Kelso et al. 1990; Lagarde and Kelso 2006; Wimmers et al. 1992); between two people (Schmidt et al. 1990; Schmidt and Richardson 2008); between humans and other species (Lagarde et al. 2005); and between humans and virtual partners or machines (Kelso et al. 2009; Dumas et al. 2014; Tognoli et al. 2020), strongly attests not only to its universality, but its *fundamentally informational nature* (Kelso 1994; Kelso et al. 2001; Kovacs et al. 2009; Mechsner 2004; Wilson et al. 2005). The drama of between-person coordination echoing within person and sensorimotor coordination shows that coupling is by means of information exchange among parts and processes not (or not only) by mechanical interactions.⁴

3.3 Conceptual developments II: the few and the many

Albeit with some exceptions, much of the research supporting the extended HKB model is dyadic in nature, involving the coordination of two interacting components or processes. An entirely independent approach aimed at capturing statistical features of large-scale coordination among very many elements was developed by Kuramoto (1984). (An historical aside: Both the extended HKB and Kuramoto models, though developed independently, share a common conceptual origin in Hermann Haken's Synergetics (Haken 1977, 1984). This connection is evident in the *Preface* of Kuramoto's magnum opus (1984) which was completed in Haken's Institute for Theoretical Physics in Stuttgart, some three weeks before the author arrived to work on what became the HKB model.) Kuramoto's model was initially aimed at explaining the onset of chemical oscillations, but soon became a paradigm for large-scale coordination in complex biological systems ranging from the flashing of fireflies, the firing of heart cells and neurons, to the clapping of concert audiences (Pikovsky et al. 2001; Strogatz 2003 for reviews). The fundamental idea of the Kuramoto model is that when

coupled, a very large population of diverse elements undergoes a phase transition from incoherent, essentially random behavior to highly coherent behavior in which the entire ensemble behaves as a synchronized synergy. In the context of understanding biological coordination, synchrony is well-known: though largely unacknowledged, it corresponds to what the great behavioral physiologist Eric von Holst called *absolute coordination* (von Holst 1937/1973).

Beyond the number of interacting elements, per se, there are a few important differences between the HKB and Kuramoto models. In Kuramoto, the nature of the phase transition is always from disorder to order, whereas the nature of the phase transition in HKB is from one ordered coordination state to another (as in the anti-phase to in-phase transition in the original Kelso experiments; Box 1 top row) or the transition from an ordered state to a partially ordered metastable form of coordination (as in later experiments of coordinating movements with visual and auditory stimuli; Box 1 second and third rows). In the latter symmetry breaking version of HKB, there are no states at all, but only the trapping~wrapping⁵ tendencies characteristic of metastable coordination dynamics. Metastability constitutes a theoretical account of what von Holst (1937/1973) called *relative coordination* (Kelso 1995). Unlike HKB, fluctuations do not receive a rigorous treatment in Kuramoto's model (but see Breakspear et al. 2010). A further difference is that in Kuramoto, the limit cycle nature of the components is assumed to be, for obvious reasons, a phase oscillator. In contrast, in HKB, the functional form of the oscillators and the nature of the coupling was and continues to be an active area of research. For example, the HKB, so-called hybrid oscillator, has attracted the attention of mathematicians and engineers. Their analysis has led to a host of new predictions (Avitabile et al. 2016; Alderisio et al. 2016; Cass and Hogan 2021; Slowinski et al. 2018, 2020; see also Beek et al. 2002; and Leise and Cohen 2007) that are especially relevant to those situations, quite common one suspects, where stable solutions of the dynamics converge on phase relations that are not in-phase and anti-phase.

Though both extended HKB and Kuramoto models deal with basic forms of coordination, an important question comes up: How might the large-scale dynamics of the Kuramoto model be reconciled with the small-scale dyadic dynamics of HKB? Recent experimental work on intermediate-sized ensembles (in-between the few and the many) proves to be the key to uniting large- and small-scale theories of coordination (Zhang et al. 2018). In this research,

⁴ This is not to say that mechanical influences and neuromuscular-skeletal factors (Carson 1996) cannot play a significant coupling role. For an excellent example of the former, see the work of Laura Cuijpers (2019) on crew rowing where mechanical coupling (likely mediated proprioceptively) can stabilize anti-phase rowing and minimize velocity fluctuations in the boat.

⁵ The tilde or squiggle (~) symbol expresses the idea that the members of a given pair are complementary. Both tendencies, e.g., trapping and wrapping coexist at the same time; the interplay of both is required for understanding (Kelso & Engström, 2006).

disorder-to-order transitions, multistability, order-to-order phase transitions, and especially metastability are shown to figure prominently on multiple levels of description, suggestive of a basic coordination dynamics that operates on all scales. The latter turns out to be a marriage of Kuramoto and HKB models (Zhang et al. 2019; see “Appendix” for a summary of the mathematical models). This *Generalized HKB model* (Kelso 2021) handles how a diverse group of complex (human) agents interact with each other when their actions influence—and are influenced by—the rest of the ensemble in unpredictable ways, and appears to be a key step toward embracing coordinative complexity. By going beyond synchrony, per se it promises a deeper understanding of collective behavior in all kinds of settings, from groups of neurons to groups of animals and people (e.g., Bardy et al. 2020; Couzin 2018). Viewed in a positive light, much research shows that coordinating our actions with others enhances the social bond on dimensions such as trust and affiliation, at both individual and group levels (e.g., Alderisio et al. 2017; Tunçgenç et al. 2021).

4 HKB and the brain: phase transitions, rhythms and metastability

Once upon a time it was argued that “*proponents of the dynamical approach are predominantly concerned with developing equations of motion that capture the coordination dynamics, whereas neuroscientists focus at unraveling the architecture of the neural networks and pathways that give rise to these pre-existing preferred coordination modes*” (Swinnen et al. 1994, p. 17). However, in a long series of studies beginning in the early 90’s, the nonlinear paradigm of HKB (varying a control parameter to identify transition-revealing collective variables or order parameters and their dynamics) was used to discover signature features of self-organizing nonequilibrium phase transitions in human brain activity—as recorded using large SQuID (e.g., Frank et al. 2000; Fuchs et al. 1992, 2000a, b, c; Kelso et al. 1991, 1992; Kelso and Fuchs 1995; Mayville et al. 2001), and multielectrode arrays (e.g., Banerjee et al. 2008, 2012; Mayville et al. 1999; Wallenstein et al. 1995).

Follow-up research used fMRI to identify the neural structures and circuitry involved in behavioral stability and instability (e.g., Aramaki et al. 2006; Mayville et al. 2002; Oullier et al. 2005). A remarkable study by Meyer-Lindenberg et al. (2002) demonstrated that a transition between bistable coordination patterns can be elicited in the human brain by transient transcranial magnetic stimulation (TMS). As predicted by the HKB model, TMS perturbations of relevant brain regions such as premotor and supplementary motor cortices caused a behavioral transition from the less

stable anti-phase state to the more stable in-phase state, but not *vice versa*. Near criticality, tickling task-relevant parts of the brain at the right time caused behavior to switch. Fluctuations matter.

Following the tenets of HKB, Coordination Dynamics predicts that cortical circuitry should be extremely sensitive to both the *stability* and *instability* of behavioral patterns. One of the lessons of HKB is that if we did not know how behavioral patterns form and change in the first place, such predictions would be moot (see also Jirsa 2021). Jantzen et al. (2009) demonstrated a clear dissociation between the neural circuitry that is activated when a control parameter (rate) changes and that related to a pattern’s stability (see also Oullier et al. 2005). Activity in sensory and motor cortices increased linearly with rate, independent of coordination pattern. The key result, however, was that the activation of cortical regions supporting coordination (e.g., left and right ventral premotor cortex, insula, pre-SMA and cerebellum) scaled directly with the *stability* of the coordination pattern. As the anti-phase pattern became increasingly less stable and more variable, so too did the level of activation increase in these areas. Importantly, for identical control parameter values, the same brain regions did not change their activation at all for the more stable (less variable) in-phase pattern. Apparently, these parts of the brain—which form a functional circuit—need to work significantly harder to hold coordination together. Task difficulty, often described in terms of “information processing load,” is captured—after HKB—by a dynamic measure of stability that is directly and lawfully related to the amount of energy used by the brain, as measured by BOLD activity (in a different context, see also Othayoth et al. 2020).

Together with the Meyer-Lindenberg et al., and Aramaki et al. studies, the Jantzen et al. (2009) work demonstrates that as in HKB, dynamic stability and instability are major determinants of the recruitment and dissolution of brain networks, providing system flexibility in response to control parameter changes (see also Houweling et al. 2010). Multistability confers a tremendous selective advantage to the brain and to nervous systems in general: it means that the brain has multiple patterns at its disposal in its “restless state” and can switch among them to meet environmental or internal demands (Deco et al. 2015; Kelso 2012; Kelso 2014b).

Theoretical modeling work at the neural level has complemented experiments and theory at the behavioral level. Neurobiologically grounded accounts based on known cellular and neural ensemble properties of the cerebral cortex (motivated by the original works of Wilson, Cowan, Ermentrout, Nunez and others) have been proffered for both unimanual and bimanual coordination (see Kelso et al. 2013 for review). An important step was to extend neural field theory to include the heterogeneous connectivity between neural

ensembles in the cortex (Jirsa and Kelso 2000; see also Deco et al. 2012). The latter describes the connectivity of the neocortical sheet and incorporates both continuous properties of the neural network as well as discrete long-distance projections between neural populations. Analysis of such heterogeneously connected systems shows that local changes in connectivity can alter the timing relationships between neuronal populations destabilizing neural activity patterns giving rise to macroscopic phase transitions. Beyond providing a mechanism for flexibility and rapid switching among cortical states, (nonequilibrium) phase transitions in the brain—experimentally and theoretically—reveal that the brain’s dynamic patterns are due to the self-organized collective action of very many cortical neurons.

Not only are the language and tools of HKB appropriate at the behavioral level (e.g., in the patterns among muscles and kinematic events), but also, as we suggested early on, at the more microscopic scale of neurons and neuronal assemblies. Many of the dynamical features observed in behavior for example, synchronization, phase-locking, switching, metastability, etc., are observed in the neuronal behavior of even the lowliest creatures (Grillner 2006). Since even single neurons are endowed with the capacity to express multiple frequencies, it has been proposed to view neural oscillations as the “critical middle ground” connecting neurons to behavior (Buzsáki and Draguhn 2004). The long sought-for link between neuronal activities (microscopic events) and behavior (macroscopic events) may reside in the coupling of dynamics on different levels. This view stands as a fertile program of research aimed at linking levels of brain and behavioral organization. The fact that HKB has been derived not just from the level of the interacting behavioral components, but also from the population level of excitatory and inhibitory neural ensembles and their connections in the brain (Jirsa et al. 1998; Kelso et al. 2013) is encouraging and suggests that as Beek et al. (1995) envisaged: “*The strategy from which we expect the most...involv[ing] the identification of macroscopic variables and their dynamics at multiple levels of neural organization, and the attempt to link the identified level-specific collective variables across their corresponding levels of observation*” (p.600) is paying off.

The brain is a constellation of improvised rhythms (see Buzsáki 2006; Kelso 1995, Ch. 8). Though the brain is not a clock, it appears to be run physically: Coordination Dynamics shows, after HKB, that relative phase is a key quantity that captures the coupling between diverse regions of the brain whose neurons (the elements) exhibit tendencies to oscillate. The primitives of the brain, like the primitives of movement, appear to take the form of oscillations or rhythms and the natural language of the brain is the way these oscillatory ensembles or rhythms are coupled. Rhythmic modulation of neuronal activity and task-dependent modulation of oscillation frequencies and phases are thought to underlie

the nervous system’s ability to encode “relations.” Synchronization is hypothesized to be used to establish relations among distributed responses all the way from the retina to the neocortex (Singer 2010), though other possibilities exist as well (Fries 2005, 2015; see also Engel et al. 2010). A strong case can be made that theta and gamma oscillations are modified during memory tasks and that gamma activity phase-locked to theta predicts performance. Theta–gamma, so-called cross-frequency coupling during working memory, has also been demonstrated in humans both in cortex and hippocampus leading to the idea of a neural code (see Lisman and Jensen 2013 for review). Although the link between the patterned spatiotemporal activity of the brain and the information contained in such as a “code” is by no means certain (Laurent 2010), the former aspect seems suited to an HKB-like dynamics and its derivatives such as the phase-attractive circle map (e.g., Kelso and DeGuzman 1988; DeGuzman and Kelso 1991; for an interesting application see Dotov and Trainor 2021).

As shown in Box 1, the two “informational forcings” in the extended HKB model deal fundamentally with information exchange. One is the strength of coupling between the elements; this allows information to be distributed to all participating elements and is a key to integrative, collective action. The other is the ability of individual elements to express their autonomy, and thereby minimize the influence of others. Self-organization in this metastable regime is the interplay of both.

Metastable coordination dynamics has deep consequences not only for our understanding of sensorimotor coordination in larger ensembles of interacting elements (e.g., Zhang et al. 2019) but also for how the balance of integration and segregation is coordinated in the brain (e.g., Breakspear 2017; Bressler and Kelso 2001, 2016; Fingelkurts and Fingelkurts 2004; Freeman and Holmes 2005; Fries 2015; Friston 1997; Kelso 1995, 2012; Kozma 2014; Tinker and Velazquez 2014; Tognoli and Kelso 2014; Valencia and Froese 2020). Metastability expresses the joint tendency or disposition for neural areas to synchronize together as well as to oscillate independently (for recent review, see Tognoli and Kelso 2014). This subtle slow~fast, dwell~escape dynamic (see footnote 5) characteristic of extended (and now generalized) HKB captures coordination in a brain that is never still—a brain that realizes its complexity not in its most ordered (integration *qua* synchrony) or disordered form (segregation *qua* antisynchrony), but as a complementary blend of both (Kelso and Engstrøm 2006; see also Tononi et al. 1994).

5 Impact of HKB

Though open to ongoing debate, there seems to be consensus on the following aspects, some of which follow pretty directly from the above discussion: (1) by making wider connections with the wider world of nonlinear math, complex systems and interdisciplinary research, HKB brought the fairly arcane field of movement and its allied sub-disciplines of motor control, motor learning and motor development into mainstream science; (2) whereas approaches such as information processing provided some techniques and methods, HKB's capability to describe, explain and predict changes of coordination in a quantitative manner was a huge step forward; (3) HKB stimulated much empirical research as well as serving as a springboard for further theoretical modeling, some quite specific (such as the excitator model and the approach of structured flows on manifolds) and others more broadly conceptual, such as task dynamics, behavioral dynamics, articulatory phonology, ecological dynamics, etc. All of these approaches can trace their origins, in one way or another, to the theoretical modeling of HKB; (4) Relatedly, HKB emphasized the need to explore lawfulness on a given level of description (in common parlance "top-down") rather than hoping that answers will emerge magically through a strictly "bottom-up" approach. Contemporary neuroscience is still grappling with this issue; (5) some, mostly physicists, think that the main impact of HKB is that it led to the discovery of phase transitions in the human brain; (6) others (such as the late Esther Thelen) thought that the concepts, methods and tools that arose from HKB were a highly generative system for understanding human development in general; (7) by demonstrating self-organization in the movements of human beings HKB generalized a physical (inanimate) concept to (animate) intentional systems. This border between the animate and the inanimate seems worthy of further exploration; (8) the HKB model is not only the conceptual but also the operational foundation of an entirely new approach to the study of human behavior, aptly coined Coordination Dynamics; (9) HKB led to technical innovations such as virtual partner interaction (VPI) or the human dynamic clamp (HDC) that has proven a useful tool to parametrically explore the neural underpinnings of social interaction as well as social cognition, e.g., in autism (Baillin, et al., 2020); (10) whereas it was often assumed that cognition is a computational process, HKB offered cognitive scientists a different picture leading many to suggest that dynamical systems are a better way to account for real-time cognitive performance than purely computational/representational approaches (see also below); (11) the mix of an intriguing empirical phenomenon in the form of a phase transition and the conceptual and formal machinery of synergetics and nonlinear dynamics is a, if not *the* main reason for the impact of the HKB model; and (12) science plus sociological reasons will influence the long-term impact

of HKB, such as standing practices of academic education and the interdisciplinary training of young scientists.

6 A way forward

Perhaps a caveat first. Nowadays, notions like attractors (of which quantities, one asks?), control parameters (such as, one asks?), collective variables (pray identify?), self-organization (how do you know that?) and so forth run freely in the literature and are often assumed to be relevant. The problem is they are not always established as such. They are often simply encapsulated in terms like "Dynamical Systems Theory" (DST). As an established acronym, DST is comfortably presented as if it is a theory, rather than a set of mathematical concepts and tools that needs to be turned into a specific research program (for an early example of such, see Kelso and Schöner 1987). As a result of HKB, it became readily—perhaps too readily—apparent that under the convenient banner of DST, the ideas introduced in HKB could be applied to all kinds of different settings. As Lopez-Felip and Turvey (2017) note, a limitation of DST approaches is that they make no direct claims about the characteristics of the system under inquiry, nor do they provide a theoretical basis for understanding the principles and mechanisms governing a given system's behavior on any level of description. DST has to be clothed with content. Then, via a modeling step akin to the HKB strategy, key variables and parameters can be mapped onto a dynamical system, opening the possibility of making and testing predictions.

As a prototypical example of dynamical modeling in cognitive science, cognitive neuroscience and "the special sciences" at large, the question of whether HKB is explanatory has revolved around whether it is truly mechanistic or merely descriptive (see, e.g. Kaplan & Craver, 2011). Some feel that explanations require not just a description of phenomenon but also an account of their causes. Others see HKB in terms of a covering-law mode of explanation. Still others suggest that dynamical explanation should concern itself with "invariant generalizations" (Meyer 2020; see also Chemero 2009). The issues range from whether the mind represents and computes as (once) typically assumed in cognitive science to whether dynamical models can constitute explanations of cognition. Here is what we said many years ago at a conference on the Physics of Structure and Complexity about what we mean by "understanding":

"Understanding" is sought, in the present approach, not through some privileged scale of analysis, but within the abstract level of the essential (collective) variables and their dynamics, regardless of scale or material substrate. Unlike certain physical systems, the path from the microscopic dynamics to the collec-

tive order parameters is not readily accessible. Understanding of biological order may still be possible... via an approach in which the nature and dynamics of the (low dimensional) order parameters are first empirically determined, particularly near nonequilibrium phase transitions (or bifurcations). Then the relevant subsystems and their dynamics can be identified... Not only may the language of dynamics be appropriate at the behavioral level (e.g., in the patterns among muscles and kinematic events), but also, we hypothesize, at the more microscopic scale of neurons and neuronal assemblies. Many of the dynamical features we have observed and modeled in our experiments, for example, can also be observed in the neuronal behavior of even the lowliest creatures... Thus, the long sought-for link between neuronal activities (microscopic events) and behavior (macroscopic events) may actually reside in the coupling of dynamics on different levels. Relatedly, the classical dichotomy in biology between structure and function may be one of appearance only. The present theory promotes a unified treatment, with the two processes separated only by the time scales on which they live" (Kelso et al. 1987).

This view still stands as a bourne. The empirical facts support HKB (its broken symmetry and generalized version) as a law of coordination instantiated in matter and movement albeit conditioned by the properties of specific structural features. The fact that HKB has been derived not just from the level of the interacting behavioral components, but also from lower levels such as the population level of excitatory and inhibitory neural ensembles and their connections in the human brain, would seem to put philosophical questions of mechanism to bed.

That said.....and here the author's own intrinsic dynamics come into play. In the light of HKB and beyond, the old trope that cells that wire together fire together seems in need of further elucidation. A single astrocyte, for example, envelops many synapses called the astrocyte's domain of synaptic influence. When enough presynaptic terminals release neurotransmitter simultaneously the astrocyte initiates a calcium wave. Astrocyte to astrocyte coupling occurs via gap junctions: A group of astrocytes acting as a single cell or syncytium may, through the generation of a pervasive calcium wave, control and coordinate the firing patterns of synapses of a great many neurons (see contributions in Wade et al. 2014). What is needed, it seems, is a move from synapses to synergies.

Instabilities of coordination can be sensed (e.g., Granatovsky et al. 2018), and multiple mechanisms may be involved depending on the situation. In the case of coordination between body parts, a massive number of receptors in skin, muscles and joints underlies the so-called 'forgotten sense'

of kinesthesia. It is not well-understood how kinesthetic couplings contribute to states of coordination and their changes, only that they do. The same may be said when the coupling appears to be visually or sound (and likely smell-) based. Even a "sense of effort" may be involved, because clearly to maintain a pattern of coordination stable under conditions in which it would otherwise become unstable demands not only greater attention, but greater cortical involvement and activity levels in specific brain regions. Time delays and neural connectivity play a key role here as well. The dynamic sensorimotor coupling between seeing, hearing, feeling and doing still seems an area wide open for empirical and theoretical study at both brain and behavioral levels for both individuals and groups. New work suggests it may be facilitated using the human dynamic clamp in individual and social settings (Dumas et al. 2014). As for "future developments," we are beginning to see how the same basic coordination dynamics plays out at several scales, from (tripartite) synapses to synergies of brain, cognition, emotion and social behavior.

Where else might one put one's money? It seems that all the necessary information needed for a system to function is carried in *collective variables*. Collective variables are relational quantities that are created by the cooperation among the individual parts of a system. They in turn may be seen to govern the behavior of the system's individual parts, in what has been termed "circular causality." It is in these collective variables and their dynamics, on a given level of observation, that the solution to Bernstein's problem is hypothesized to reside (see Kelso 2009/2013 for a more complete treatment of the issue). The Schöner–Kelso Conjecture (Turvey 2004) is that a system's behavior on any level of description is explained by identifying one or a few collective variables that capture the qualitatively distinct patterns exhibited by a system as it unfolds over time and by describing these patterns (their emergence and dissolution) in the precise terms of attractors, bifurcations, multistability and related notions. The strategy of folding together all aspects within the dynamics of collective variables is an effort to embrace the full complexity of biological perception–action without a proliferation of arbitrary divisions. Why is that important? Because collective variables span, fundamentally, the duality of sensory and motor, perception and action, organism and environment. In simple cases covered by HKB, the relative phase, a dimensionless quantity, captures the coordinative relation between stimuli and responses in such a way that the latter predicts or anticipates the former. Stimuli and responses exist, as John Dewey said not as separate things but inside a coordination. Active organisms are perceivers engaged in dynamical transactions with their environment. If this is so, much further work will be needed to reveal the collective variables (and their dynamics) that characterize the reciprocal and mutual relationship between organism and

environment. It is these collective variables in the theory of Coordination Dynamics that carry functional information. As someone who cares about movement, the embracing of HKB and Coordination Dynamics by the sports and exercise sciences—which possesses sophisticated measurement and analysis tools—promises novel discoveries (see Newell and Liu 2020; Passos et al. 2009; Balagué et al. 2013; see also, Fuchs & Kelso, 2018). On the more molecular side, neuroscience’s realization of the nonlinear, experiential effects of deep brain stimulation fits (and could be enhanced by) the present perspective (Frank 2018; Synofzik et al. 2012), though—speaking to future training in the fields of neuroscience and psychiatry—they may not realize it yet.

7 Final words

Many years ago, the author was invited to comment on Fitts’s Law, a famous empirical relationship between the amplitude of a movement and the time it takes, conditioned by accuracy requirements. My students and I had published a little paper showing that the “law” breaks down under certain, apparently quite natural conditions (Kelso et al. 1979). In my hubris then, I entitled the article “*Theoretical concepts and strategies for understanding perceptual-motor skill: From information capacity in closed systems to self-organization in open, nonequilibrium systems*” (Kelso 1992). It seems that the most significant aspect of HKB and beyond is that Coordination Dynamics deals in the currency of functional information, not Shannon’s notion which inspired Paul Fitts. That coordination between different functional and material components has been demonstrated to share a common coordination dynamics suggests a deep connection between matter, movement and mind (Kelso 2008; Sheets-Johnstone 2015). In light of over 40 years of research on coordination inspired and perspired by HKB, consciousness as ignition—Maxine Sheets-Johnstone’s “ideational phase transitions”—thinking in movement—sure has the right feel to it.

Many individuals and groups have contributed to the development of the HKB model and the research ideas surrounding it. The present article is not a review, and its bibliography is far from complete. All that is done here is to more or less vaguely mention a few integrative themes and highlights, drawing on the works of some of the people that have been inspired by the HKB model and what it means. The range of topics is broad and as hinted here stretches across many fields. This is only a thumbnail sketch of some of the main threads/contributions that hint at an emerging science of coordination in complex systems that cuts across many disciplines. Starting with “the primacy of movement” (Sheets-Johnstone 1999/2011) HKB was the beginning, offering concepts, methods and tools—a new language—for handling the multilevel coordinative complexity of living

things. HKB shows no sign of obsolescence (e.g., Avitabile et al. 2016; Cass and Hogan 2021). Nor should it, since it is the foundation for a science of coordination that in turn has inspired research and thinking in other fields.

HKB is still full of surprises, and no one really knows what the future might entail. One point remains: “*What makes coordination dynamics fascinating from our point of view, which is the perspective of two theoretical physicists, is that it can be put on a solid quantitative foundation and that the same basic laws govern on all levels: behavioral, neural or social dynamics*” (Fuchs and Jirsa 2008, p. 345). One might add that there is a hint here, perhaps heretical to some, that with respect to the principles that underlie them, the material systems that express the functional order of living things are as, if not more general than those that physics currently addresses. This viewpoint has been expressed and elaborated most fully by the late Robert Rosen (1991) and developed in the context of a physical psychology by Michael Turvey and Robert Shaw (1995) as well as the scholarly writings of Brian Josephson (2019). If it turns out to be true, this may be the longest lasting impact of HKB and Coordination Dynamics, with their roots in the physics of open, nonequilibrium systems.

Appendix: evolution of HKB model

In the HKB model, the dynamics of the order parameter φ is captured by an equation of motion of the form:

$$\dot{\varphi} = -a \sin \varphi - 2b \sin 2\varphi \quad (1)$$

The potential picture (top row in Box 1) corresponding to Eq. (1) takes the form:

$$V(\varphi) = -a \cos \varphi - b \cos 2\varphi \quad (2)$$

And the corresponding stochastic equation of motion is:

$$\dot{\varphi} = \frac{dV(\varphi)}{d\varphi} + \sqrt{Q}\xi_t \quad (3)$$

with $\langle \xi_t \rangle = 0$; $\langle \xi_t \xi_{t'} \rangle = \delta(t - t')$ where Q is the strength of delta correlated Gaussian noise.

The properties of Eq. (1) can be summarized as follows:

- For all values of $a > 0$ and $b > 0$, there are fixed points, corresponding to coordinated states at $\varphi = 0$ and $\varphi = \pi$
- In the parameter region $b/a > 1/4$ corresponding to low movement frequencies, both fixed points are stable, i.e., movements can be performed either in the in-phase or anti-phase coordination mode
- In the parameter region $b/a < 1/4$ corresponding to high cycling frequencies, the fixed point at $\varphi = \pi$ is unstable,

and the only remaining stable fixed point is at $\varphi=0$ corresponding to an in-phase pattern.

The interacting components are given as

$$\ddot{x}_1 + f(x_1, \dot{x}_1) = (\dot{x}_1 - \dot{x}_2)\{A + B(x_1 - x_2)^2\} \quad (4)$$

$$\ddot{x}_2 + f(x_2, \dot{x}_2) = (\dot{x}_2 - \dot{x}_1)\{A + B(x_2 - x_1)^2\} \quad (5)$$

where each so-called “hybrid” oscillator has the following functional form, again based on experimental evidence regarding the relationship between the amplitude, frequency and velocity of rhythmic movements:

$$f(x, \dot{x}) = \alpha\dot{x} + \beta\dot{x}^3 + \gamma\dot{x}x^2 + \omega^2x \quad (6)$$

The left-hand sides of Eqs. (4) and (5) describe the motions of the individual components and the right-hand sides the nonlinear coupling between them. The right-hand side of Eq. (6) consists of a linear self-excitation term, $\alpha\dot{x}$, a Van der Pol term, $\gamma\dot{x}x^2$, and a Rayleigh term, $\beta\dot{x}^3$, for saturation.

Mathematical analysis shows that the relative phase obeys the dynamics of Eq. (1) with the coupling terms given as $a = -(\alpha + 2\beta R^2)$ and $b = \frac{1}{2}\beta R^2$.

The extended HKB model, shown as a potential in Box 1 (middle and bottom rows), takes the form:

$$\dot{\varphi} = \Delta\omega - a \sin \varphi - 2b \sin 2\varphi \quad (7)$$

where the difference in natural frequencies between the oscillators is:

$$\Delta\omega = \frac{\omega_1^2 - \omega_2^2}{2\Omega} \approx \omega_1 - \omega_2 \quad (8)$$

The basic Kuramoto model is:

$$\dot{\theta}_i = \omega_i - \frac{K}{N} \sum_{j=1}^N \sin \phi_{ij} \quad (9)$$

where $\phi_{ij} = \theta_1 - \theta_2$ and ω_i corresponds to the dispersion of natural frequencies. Without the second-order coupling of HKB, Eq. 9 cannot handle the presence of both in-phase and anti-phase coordination and transitions between them.

The simplest model that captures all important observations in the mid-scale human experiment of Zhang et al. (2018) on multiple levels of description is a combination of Eqs. 7 and 9:

$$\dot{\varphi}_i = \omega_i - \sum_{j=1}^N a_{ij} \sin(\varphi_i - \varphi_j) - \sum_{j=1}^N b_{ij} \sin 2(\varphi_i - \varphi_j) \quad (10)$$

where φ_i is the phase of the i th oscillator, ω_i the natural frequency, $a_{ij} > 0$ and $b_{ij} > 0$ govern the coupling strength.

Equation (10) represents a marriage of Eqs. (7) and (9). Obviously, Eq. 10 becomes the basic Kuramoto model when the second-order coupling term b is equal to zero (see text for references and discussion).

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