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Experimental research on the influence of modal nonlinearities of paintings under mechanical loads

Abstract In the traditional transportation of paintings and the design of the packaging systems, paintings are usually assumed to behave like a linear system. In order to verify this hypothesis, in this contribution, by means of a hammer experiment and a sweep excitation experiment to simulate the shock and vibration during transportation, respectively, the modal nonlinearities of two real paintings and a dummy painting are experimentally studied. The experimental results show that paintings can be treated as a linear system only when being subjected to shock, but the modal nonlinearities of paintings cannot be ignored when being subjected to vibration. The general behaviour of the paintings modal nonlinearities is then summarised based on experimental results, and their consequences for painting transportation are discussed. First of all, the offset of the resonance frequency is the most important problem which will lead to failure of the original vibration isolation measures. Further, the decrease in the resonance peak amplitude will increase the probability of the eigenmode being excited. Besides, it is also necessary to attenuate the harmonic vibrations of paintings. Lastly, the different modal characteristics obtained by a sweep with increasing and decreasing frequency make the analysis of different excitation schemes more complicated. Therefore, the identification of the paintings modal nonlinearities is necessary and important.

1 Introduction

Paintings are inevitably subjected to shock and vibration during transportation. Excessive shock or vibration may cause irreversible changes in the complex materials of canvas and paint and accelerates their natural aging [1–7]. A transportation experiment has shown that once the eigenmodes of the painting are excited, the vibration intensity of the canvas can be much higher than the vibration intensity transferred to the frame through the crate [8]. Therefore, it is necessary to study the modal characteristics of paintings and then improve the packaging system accordingly, so as to prevent the eigenmodes of the painting from being excited during transportation. For many works on the transportation of paintings, there is the hypothesis that since the vibration displacement is small, paintings can be treated as a linear system, i.e. their modal parameters can be always considered constant [9–14]. However, other research has shown that the modal characteristics of paintings are not only affected by climate [15–17], but also vary with different vibration intensities [6,18]. This indicates that there are nonlinear aspects in the paintings modal characteristics and the linearity hypothesis remains to be verified. Therefore, in order to improve the packaging system of paintings, it is of great significance to study whether the modal nonlinearities of paintings will show during transportation and whether these modal nonlinearities

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need to be considered in the design of the packaging system. In this contribution, the modal nonlinearities of paintings under different shock and vibration intensities during transportation will be carefully studied.

There have been few researches on the modal nonlinearities of paintings under different vibration intensities in the last decades. In [6], research about the modal nonlinearities of paintings is carried out, and in [18] modal nonlinearities were discovered as well. In [6], through sweep experiments to simulate the vibration of the painting, the canvas showed to have strong progressive stiffness characteristics and degressive damping characteristics. Namely, with the increase in external excitation, the resonance frequency of the painting increases, and the amplitude of the resonance peak decreases. It is also mentioned in [6] that the resonance peaks are asymmetric, and the resonance peaks obtained by up-sweep and down-sweep, i.e. sweep with increasing and decreasing frequency, respectively, are different. These nonlinear phenomena are discussed by the Duffing oscillator model in [6]. However, whether these nonlinear phenomena will occur during transportation and whether the design of painting packaging systems needs to consider these nonlinear factors still need to be studied. Moreover, paintings are not only subjected to vibration but also to shock during transportation, which means that the general conclusion about the modal nonlinearities of paintings under different types of excitations deserves further study.

Based on the above motivation, in order to simulate the state of a painting during transportation, hammer experiments and sweep experiments are conducted in this contribution. The hammer experiment on the frame is realised by an automatic impulse hammer which is used to simulate a shock on the painting during transportation. The sweep experiment is realised by an electrodynamical shaker, as well as a loudspeaker and is used to simulate the vibration on the painting during transportation. Both the hammer and sweep excitation differ from the undeterministic excitation occurring during real transportation. However, they represent limit cases that are uniquely defined and can easily be compared for different measurements. Broadband excitation, e.g. with noise, would have been possible but was not conducted in the measurements presented here. Since the aim is the evaluation of nonlinear behaviour, a simple axial excitation is chosen instead of a multiaxial excitation. This allows to directly relate excitation amplitude with (possibly nonlinear) response. The excitation is chosen perpendicular to the painting for nonlinear effects are more likely to occur in this direction. In order to avoid excessive excitation amplitudes, these are chosen in the range which was recorded during a real transport, [8].

The sweep experiment with shaker is a common experiment which focuses on the modal nonlinearities of a painting under different vibration intensities during transportation. The sweep experiment with loudspeaker is a contactless measurement method, more suitable for real precious paintings that do not allow use the above mentioned contact measurement methods. In this contribution, the feasibility of this contactless method to measure the modal nonlinearities of paintings under different vibration intensities will be verified.

In order to draw a general conclusion about the modal nonlinearities of paintings, two real paintings with natural aging and a recently manufactured dummy painting are tested in this contribution. The two real paintings are of negligible artistic value, and thus, they can be tested under lab conditions. The dummy painting was made by an experienced art conservator, with uniform tension and uniform and homogeneous distribution of primer and paint. It does not yet show any deterioration in terms of aging. In all experiments, the vibration acceleration on the three tested painting frames as well as the vibration velocity and vibration displacement at selected measurement points on the canvases is collected. The vibration velocity of the canvas is combined with the corresponding excitation force to calculate the frequency response function (FRF) and analyse its modal nonlinearities. The vibration displacement of the canvas can intuitively express the vibration intensity of the tested paintings. The vibration acceleration of the frame is compared with those recorded during a real transportation in [8], which is then used to assess whether the modal nonlinearities will occur during the transportation. Based on all the experimental results for the three tested paintings, the general behaviour of the modal nonlinearities of paintings under different shock and vibration intensities are summarised. Then, a conclusion of whether the modal nonlinearities of paintings should be considered during the transportation is derived.

This contribution focuses on experimental research on the modal nonlinearities of paintings. The novelty of this contribution is the investigation to what extent these nonlinearities occur for excitation amplitudes of typical art transports, and consequently, a discussion about whether the modal nonlinearities of paintings should or must be considered during the transportation as well as the related consequences of the modal nonlinearities for painting transportation. Moreover, the most suitable experimental methods for measuring the modal nonlinearities of paintings are also summarised.

This article is divided as follows. Section 2 contains the description of the three tested paintings and the experimental setup. Section 3 contains a discussion whether paintings can be treated as a linear system during transportation through a short introduction to the results of a hammer experiment and a sweep experiment. In
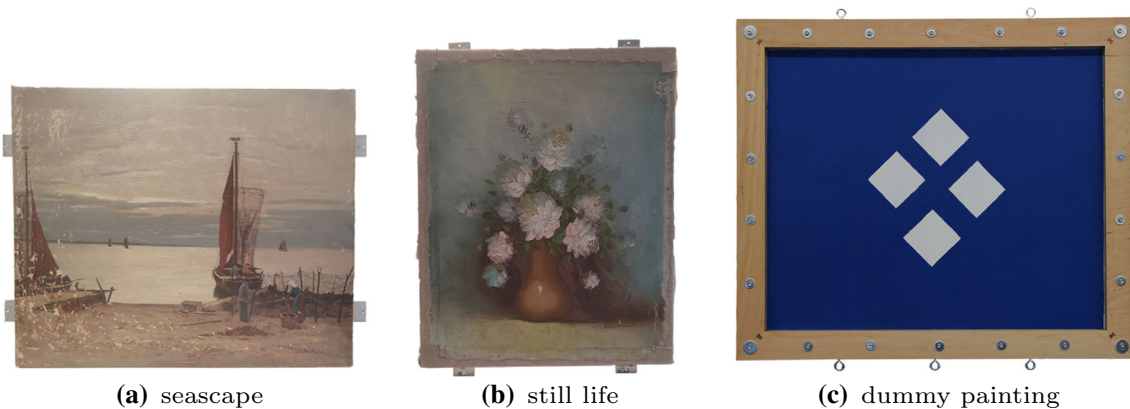
(a) seascape **(b)** still life **(c)** dummy painting

Fig. 1 Three paintings used for experimental research

Sect. 4, the general behaviour of the paintings modal nonlinearities is summarised and their consequences for painting transportation are discussed. Section 5 gives a brief summary.

2 Experimental setup

In this contribution, three paintings are investigated as shown in Fig. 1. The first painting shows a seascape with the dimensions $50 \times 60.7 \times 2$ cm. The second painting is a still life painting with dimensions $57 \times 46 \times 2$ cm. Both of them are real paintings, though of negligible artistic value, and have become fragile and massively damaged after natural aging. They would not have been transported in this condition as part of the museum loan system without consolidation measures. The purpose of studying two real paintings simultaneously is to discover the similarities and the differences in modal nonlinearities between real paintings. The last painting is a dummy painting with the logo of the authors' institute, which measures $60 \times 70 \times 4$ cm.

In contrast with the previously described paintings which were built, primed, painted etc. by the artist and were also left in the pre-tension state as of purchase/loan, the purpose of the dummy painting is to set up an intensionally simplified dummy model in order to assess whether some of the observed properties change with those simplifications. The production process of the dummy painting is similar to that in [18]. The canvas of the dummy painting is decatised and then uniformly tensioned on a biaxial stretcher. Next, the stretched canvas is treated with a so called pre-gluing to prevent the primer from permeating into the canvas in the next step. The primer is heated and then applied on the canvas in three layers with a brush from different directions. The blue oil paint layer and the white paint layer are also applied sequentially on the primer layer with a brush. Lastly, the canvas is sandwiched between two wooden frames to form the final dummy painting. Because the dummy painting has only been dry for less than a month and there is almost no deterioration in terms of aging, through in the research related to this dummy painting, the similarities and the differences in modal nonlinearities between the real paintings after natural aging and the young painting can be discovered. Each painting is equipped with 4 perforated metal plates which are used to simulate the state of the painting being fixed on a transportation plate through the perforated metal plate, and it is also convenient to hang the painting during the experiment.

Figure 2 depicts the experimental setup used to study the modal nonlinearities of paintings. It consists of three different paintings, two different excitation mechanisms, a Polytec VibroFlex laser Doppler vibrometer (LDV), three triaxial accelerometers and a climate box. The investigated painting is mounted with rubber bands on an aluminium support, forming a very soft suspension which can be approximately considered as a free suspension. Although the described suspension differs from typical boundary conditions during transportation, the vibration of the painting (and the paint layer) could still be generated to be essentially the same as for a transportation, if multidimensional excitation forces were produced such that the multidimensional accelerations measured in the laboratory were the same as measured during the actual transport. As long as the desired motion of the painting is generated, the difference in boundary conditions has no negative impact on the measurements. Three triaxial accelerometers are attached on perforated metal plates in order to monitor the acceleration of the tested painting and compare it with the vibration acceleration during the transportation experiment in [8]. The model of the triaxial ICP accelerometers is PCB-356A03/NC.

Fig. 2 Experimental setup for the tested painting in a climate box with approximately free suspension

The vibration velocity of the canvas and the excitation force are collected to study the modal nonlinearities of tested paintings, i.e. so-called mobilities are measured. To rule out misinterpretations by leakage effects due to finite measurement time, different window functions were applied to the time domain signal. Among others, a Bartlett and a Von Hann window was used for the sweep experiments and an exponential window was used for the impact experiments. In the frequency range of interest, apart from the most subtle differences, the mobilities were identical. Also, coherence was checked to evaluate the quality of the measurement. No detrimental effects were found.

In principal, a mobility measurement would have to be performed for many points to fully capture the spacially distributed motion of the paint layer and to also fully evaluate the spatial distribution of nonlinearities. However, measurements conducted at different points on the canvas revealed similar results as to the extent of nonlinear behaviour. Therefore, for simplicity, the green point labelled MP on the tested painting shown in Fig. 2 is selected as a representative point to present the experimental results in this contribution. In order to avoid the loss of modal information during the experiment due to the measurement point being located on one of the nodal lines of the first eight eigenmodes, this measurement point is selected at one fourth of the width and height. The seascape painting is less damaged at this point, while the still life painting has obvious cracks at this point. The vibration velocity signals at the measurement point are measured with the Polytec VibroFlex LDV. The VibroFlex LDV has the advantage of using a CO_2-based Laser source and a multiple channel interferometer which virtually avoids laser drop outs even when measuring uncooperative surfaces. Rigorous inspection of the measurement signals in time domain revealed no laser drop outs. Velocity and displacement were measured simultaneously.

In this contribution, two different excitation mechanisms are adopted for experimental research. The first type of excitation is an impulse excitation generated by an automatic impulse hammer as shown in Fig. 3a. The automatic impulse hammer is composed of an electrodynamical shaker and a modified tip mechanism that generates a very short hit and avoids double hits. The impulse excitation directly acts on a metal plate on the tested painting from the back, which is used to simulate the shock of the painting during transportation. The excitation point is shown as the yellow point labelled EP1 in Fig. 2. The contact force between the excitation point of the tested painting and the tip of the automatic impulse hammer is measured by an ICP force sensor PCB 086E80. The excitation is chosen such that measured acceleration does not exceed the acceleration measured for shock events in [8].

The second type of excitation is a sweep excitation generated by an electrodynamical shaker or loudspeaker. The sweep excitation is used to simulate the vibration of the painting during transportation. When the excitation is generated by the electrodynamical shaker, the excitation position is the same as for the hammer experiment on the frame, but the brass stinger of the shaker is firmly connected to the perforated metal plate with two nuts as shown in Fig. 3b. The input force of the shaker is measured by an ICP force sensor PCB 208C01. In order to obtain a realistic excitation intensity, root mean square values of the painting frame response as measured in [8] are used. When the excitation is generated by the loudspeaker, the loudspeaker simply faces to the painting in a distance of round about 1 cm, as shown in Fig. 3c. This contactless experimental method is particularly suitable for measuring the modal nonlinearities of real precious paintings. However, because the input pressure generated by acoustic excitation is difficult to measure, the modal nonlinearities can only be analysed and evaluated based on the velocity response of the canvas instead of the mobility which would require a force or pressure measurement. The impulse signals or sweep signals of the above three excitation mechanisms are provided by a function generator, and the intensity of shock and vibration are adjusted by
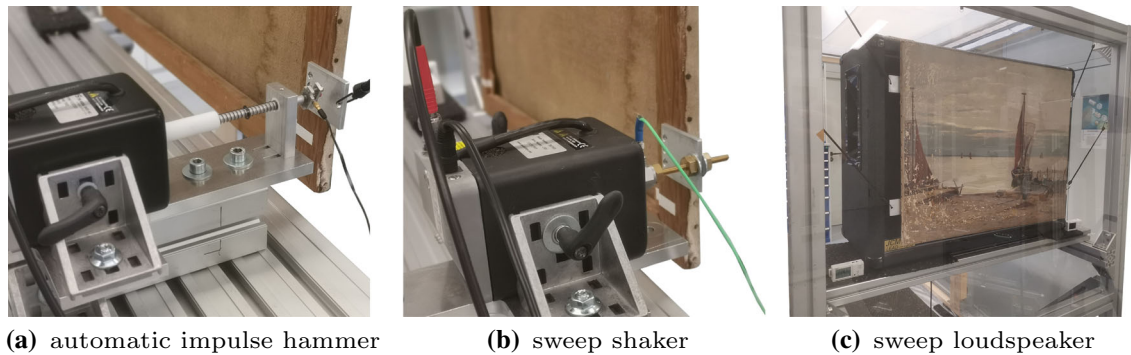
(a) automatic impulse hammer   **(b)** sweep shaker   **(c)** sweep loudspeaker

Fig. 3 Three different excitation mechanisms used for experimental research

the input voltage. In order to avoid the influence of changing climate on the modal characteristics of the tested paintings as much as possible, all experiments are carried out in a climate box. The potential relative humidity adjustment range of the climate box is 35–80%, while 52.5%±0.4% is used for the experiments if not separately mentioned, and the temperature is kept constant at 22 °C. In this range, the changes in modal characteristics caused by changing climate have been checked to be negligible.

3 Identification of dynamical behaviour

In the following, by means of the hammer experiment and the sweep experiment, the dynamical behaviour of the three considered paintings under different shock or vibration intensities are identified. By comparing the shock and vibration intensity of the seascape painting in the experiment with the transportation in [8], it is clarified that the dynamic behaviour of the three tested paintings observed in the experiments covers the dynamic range that occurs during transportation. This way, the conclusion of whether the painting can be treated as a linear system during transportation can later be derived.

3.1 Hammer experiment with excitation on the frame

The first experiment is realised by an automatic impulse hammer exciting the frame of the three tested paintings. This simulates the shock during transportation, so as to study the influence of different shock intensities on the modal characteristics. During the experiment, the mobility

$$Y(\omega) = \frac{V(\omega)}{F(\omega)} \tag{1}$$

at the measurement point MP is calculated with the Fourier transformed velocity $V(\omega)$ at the measurement point and the Fourier transformed excitation force $F(\omega)$ at the excitation point.

Figure 4 depicts the experimental results of the mobilities at the measurement point for different hammering forces on the frame of the seascape painting. The identified first resonance frequency of the seascape painting is about 7.83 Hz. All the peaks below 7.83 Hz are identified as the rigid body modes of the painting, i.e. the undeformed painting vibrates relative to the aluminium support. With respect to the structural eigenmodes of the painting, it can also be clearly observed that the corresponding resonance peaks do not change with the changing hammering force intensities. Moreover, despite their different modal characteristics, the still life painting and the dummy painting have also shown the same general behaviour, i.e. the modal characteristics did not change for different shock intensities.

Limited by the maximum input voltage of the automatic impulse hammer, the maximum hammering force that can be achieved on the frame of the seascape painting is about 22 N in a contact time of 10 ms. Within this range of hammering force, the maximum acceleration measured on the frame is about 13 g (1 g ≈ 9.8 m/s^2). Since during the real transportation [8], the maximum acceleration of the frame caused by the shock does not exceed 1 g, the maximum shock intensity of the seascape painting during the hammer experiment is much higher than that during the transportation. For the still life painting and the dummy painting real transportation
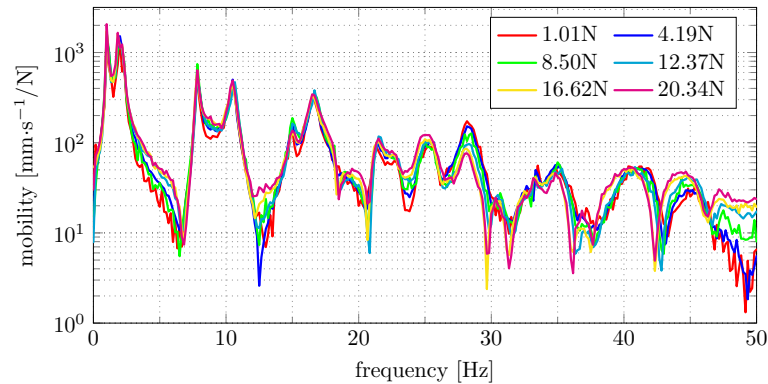
Fig. 4 Mobilities of the measurement point under different hammering forces on the frame of the seascape painting

data are not available. However, similar accelerations were measured for the still life painting and the dummy painting in the laboratory experiments. Since the weight of the still life painting is similar to the seascape painting, and the weight of the dummy painting is roughly twice as much as the seascape painting, yet the measured acceleration in the experiment exceeds those of the transportation of the seascape painting by a factor of 10, it can be considered that the modal characteristics of the three tested paintings observed in this hammer experiment cover the modal characteristics when being subjected to shock during a real transportation.

In summary, the modal parameters of the paintings remain similar under shock much higher than that during transportation. The hammer experiment on the frame supports the hypothesis that the painting can be treated as a linear system when being subjected to shock.

3.2 Sweep experiment with excitation on the frame

Next, the sweep experiment on the frame with an electrodynamical shaker is conducted which aims to simulate different vibration intensities during transportation. The sweep excitation range extents from 1 to 55 Hz, and was performend with a constant sweep rate in a sweep time of 100 s. In order to guarantee that this can be considered a quasi-static excitation with no eminent transient effects, experiments with up to five times slower sweep rates were conducted with no apparent changes in the measurement results. To avoid misinterpretation in terms of nonlinearity, the input force was evaluated in terms of harmonics. The second harmonic turned out to be suppressed by more than 30 dB and higher harmonics were suppressed at least 36 dB for the full frequency range of interest.

Similar to the hammer experiment on the frame, the mobilities at the measurement point under different vibration intensities are analysed. The measured mobilities appear to be very noisy and littered with additional peaks. A thorough analysis turned out that this is not caused by an improper measurement setup (bad signal-to-noise ratio, etc.), but was found to be caused by harmonics and their influence on the mobility when calculated according to Eq. (1). Those effects are discussed in detail in Sect. 4. In Fig. 5, the mobility spectrum of the seascape painting under different sweep intensities in the entire sweep range is shown, but a moving average filter with a window length of 10 samples was used to smooth the mobilities. Different sweep intensities are denoted by $L_1, L_2, \ldots, L_{12}$, where L_1 represents the lowest sweep intensity of 0.0664 N (peak to peak value) at 50 Hz and L_{12} represents the highest sweep intensity of 4.69 N (peak to peak value) at 50 Hz.

In Fig. 5, the resonance peaks of the first eigenmode are identified around 7 Hz. Below 7 Hz, the resonance peaks do not change with different sweep intensities hence they are identified as rigid body modes. Above 7 Hz, the resonance peaks of each eigenmode are different under different sweep intensities, which clearly shows that the seascape painting shows considerable modal nonlinearities. For the reasons mentioned above, the analysis of modal nonlinearities above 20 Hz is almost impossible. Below 20 Hz four resonance frequencies are found which are used in the following to investigate the modal nonlinearities to more detail. Figure 6 shows the mobilities in the reduced frequency range in separate plots for smaller and stronger sweep intensities. The resonance frequencies first decrease slightly with increasing intensities, see the left side of Fig. 6, and then increase sharply with increasing intensities, see the right side of Fig. 6. In particular, the first and second resonance frequencies at higher sweep intensity even rise to the positions of the second and third resonance frequencies at lower sweep intensity, respectively. Another observation is that the amplitude of each resonance
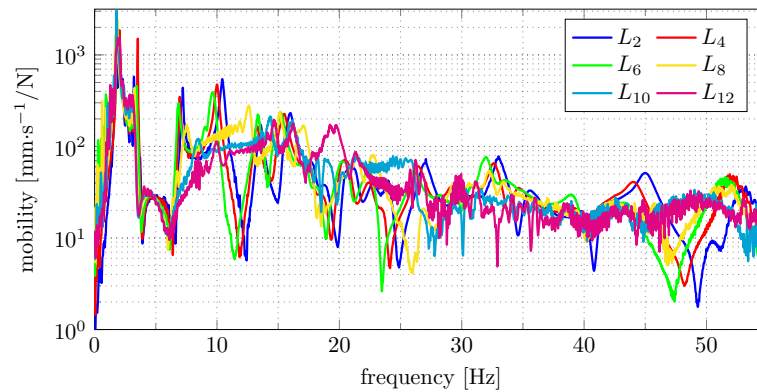
Fig. 5 Mobility spectrum at the measurement point under different sweep intensities when sweeping upwards on the frame of the seascape painting
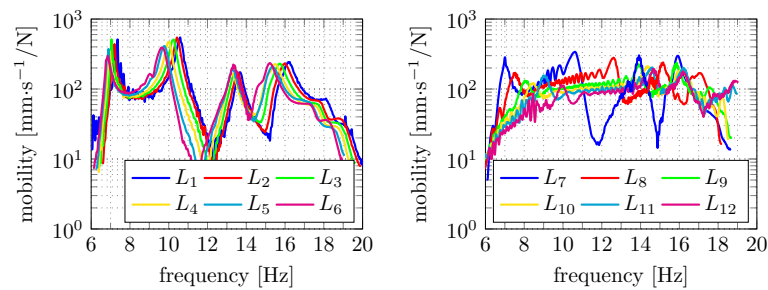

Fig. 6 Changes of the first four eigenmodes at the measurement point when sweeping upwards on the frame of the seascape painting

peak decreases with increasing sweep intensities. When the sweep intensity is high enough, the resonance peak even becomes relatively flat and difficult to identify. Qualitatively, similar behaviour was observed for the still life painting and the dummy painting. For the lack of fundamentally different insight, these results are not provided here.

Although it is known that there are modal nonlinearities in paintings, it is still unknown whether they will show effects during transportation. For this reason, a comparison of the vibration intensity of the seascape painting during the sweep experiment and during transportation is carried out. Table 1 lists the statistics of the vibration acceleration on the frame during the entire transportation experiment, as well as the lowest intensity sweep experiment (L_1) and the highest intensity sweep experiment (L_{12}). A direct comparison is difficult because the excitation during transportation is quasi-random in difference to the constant intensity sweeps in the laboratory. Also, the vibration during transportation is three dimensional. Therefore, only the acceleration component perpendicular to the painting, a_z, is considered and split into four categories: $a < 0.1\,g$, $0.1\,g \leq a < 0.5\,g$, and $a \geq 0.5\,g$. Table 1 summarises the statistics of the vibration acceleration on the frame during the entire transportation experiment as well as for the lowest and highest sweep intensities, L_1 and L_{12}, and also for all three acceleration sensors. Additionally, the root mean square values (rms) and the limit values of the acceleration are given. All values indicate that the entire range of accelerations measured during transportation is fully captured by the laboratory experiments. Furthermore, based on the mean square value, the vibration intensity of the seascape painting during transportation is roughly between the sweep intensities L_6 and L_7, where its modal characteristics have already undergone obvious nonlinearity. Therefore, it can be concluded that the seascape painting will produce non-negligible modal nonlinearities due to the changing vibration intensity during transportation. The same conclusion must be drawn for the still life painting and the dummy painting.

In short, the sweep experiment reveals that all three paintings produce apparent modal nonlinearities during transportation. Therefore, the paintings cannot be simply treated as a linear system when being subjected to vibration during transportation. This is contrary to the prevalent hypothesis. Even though this conclusion is only verified for the three paintings investigated in this work, a high probability is claimed that this is the case with many other paintings of comparable size and weight. It is even believed that nonlinear effects of a similar

Table 1 Comparison of the vibration acceleration on the frame of the seascape painting in a sweep experiment and the transportation experiment

Acceleration	$a < 0.1\,\mathrm{g}\,(\%)$	$0.1\,\mathrm{g} \leq a < 0.5\,\mathrm{g}\,(\%)$	$a \geq 0.5\,\mathrm{g}\,(\%)$	RMS	Min	Max
$a_{z,\,\text{transportation}}$ [g]	86.4	13.5	0.1	0.08	−0.83	0.83
$a_{z1,\,L_1}$ [g]	100	0	0	0.0065	−0.034	0.018
$a_{z2,\,L_1}$ [g]	100	0	0	0.0087	−0.047	0.039
$a_{z3,\,L_1}$ [g]	100	0	0	0.0078	−0.036	0.036
$a_{z1,\,L_{12}}$ [g]	68.3	27.1	4.6	0.28	−1.45	1.46
$a_{z2,\,L_{12}}$ [g]	69.6	20.6	9.8	0.40	−1.46	1.50
$a_{z3,\,L_{12}}$ [g]	63.1	24.2	12.6	0.41	−1.14	1.39

All values are given in g $\approx 9.8\,\mathrm{m/s}^2$
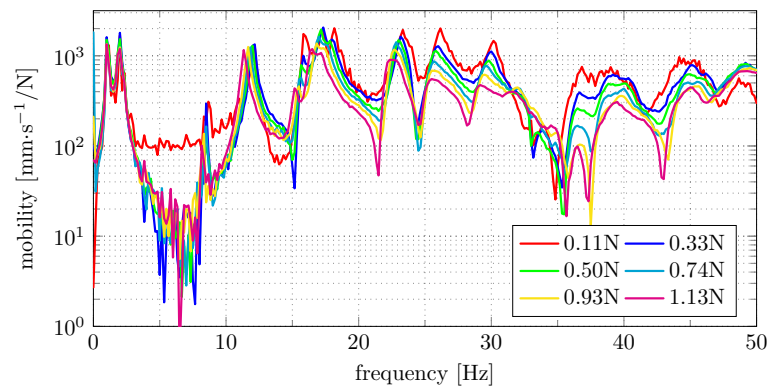
Fig. 7 Mobilities of the measurement point under different hammering forces on the canvas of the seascape painting

extent appear in paintings of other size and weight. The general behaviour of the nonlinearities for sweep excitation and their consequences for transportation will be further analysed and are discussed in Sect. 4.1.

3.3 Hammer experiment with excitation on the canvas

In the above hammer experiment and sweep experiment, completely different dynamical behaviours of paintings were identified under different excitation intensities. The shock signal has a short action time, and its energy is distributed over a wide frequency band. Thus, the energy allocated to each eigenmode is very small, resulting in a weak response of each eigenmode. On the other hand, the vibration signal has a long action time, and the energy is concentrated in a small frequency range. As long as the vibration frequency is close to the resonance frequency of a certain eigenmode, a strong response of this mode will be excited. From the sweep experiment, it can be seen that the stronger the eigenmode response, the more obvious the modal nonlinearity. Therefore, it can be reasonably assumed that as long as the energy of the shock was large enough to cause a stronger response of each eigenmode of the canvas, the modal nonlinearities would be similar to that in the sweep experiment and hence would also be observed in the hammer experiment.

In order to verify this assumption, an exaggerated experiment was designed and then conducted. Different from the hammer experiment with excitation on the frame, this experiment selects the excitation point directly on the canvas. This way, the shock energy consumption on the frame is avoided, and thus, the canvas can obtain excessive shock. The location of the excitation point on the canvas is shown as the yellow point labelled EP2 in Fig. 2 and is centrally symmetrical with the measurement point. The experimentally obtained mobilities at the measurement point under different hammering forces directly on the canvas of the seascape painting are shown in Fig. 7. Due to the soft material of the canvas, the contact force between the hammer and the canvas is much smaller than the contact force between the hammer and the frame. Figure 7 shows that the resonance frequency of the seascape painting tends to decrease as the hammering force increases, and the amplitude of the resonance peak also decreases accordingly, which is fully consistent with the trend of modal nonlinearity under lower sweep intensities. This indicates that the excessive shock on the canvas at this time has made the response of each eigenmode reach the same level of that under the lower sweep intensities. The hammer
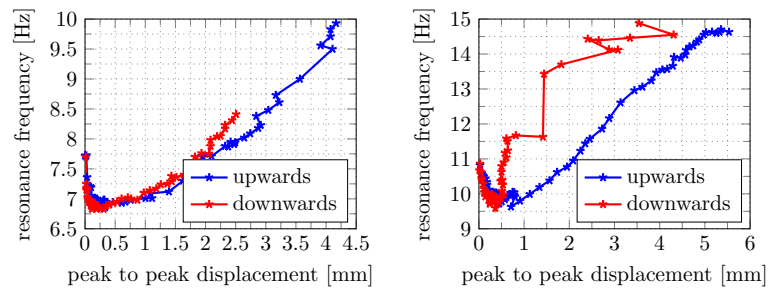
Fig. 8 Relationship between the first two resonance frequencies of the seascape painting and their corresponding vibration displacements at the measurement point

experiments on the canvases of the still life painting and the dummy painting also show similar dynamical behaviour, but are again not presented here.

In conclusion, with the help of the hammer experiment on the canvas, it is shown that no matter whether the painting is subjected to shock or vibration during transportation, as long as the excitation energy is high enough, the painting will produce modal nonlinearities. Although this experiment method is not practically allowed for real paintings of artistic value, and has only been conducted under laboratory conditions for paintings of negligible artistic value, it has still served being valuable for comparison and evaluation.

4 Modal nonlinearities during vibration

After the observation that nonlinearities occur for sufficiently large excitation energies in the previous section, here, the sweep experiment is further analysed for the three tested paintings. The general behaviour of the paintings modal nonlinearities is discussed and some consequences of the modal nonlinearities for transportation are deduced.

4.1 Analysis

As shown in the experimental results in Sect. 3, only the sweep experiment can fully identify the modal nonlinearities of the tested paintings. Thus, the analysis of the modal nonlinearities presented here is only based on the sweep experiment results. Furthermore, it is known that the modal nonlinearities of the paintings are directly related to the vibration intensity. The peak vibration displacement of each vibration mode can most intuitively reflect its vibration intensity. Even in the nonlinear case the frequency for peak amplitude is sometimes referred to as resonance frequency and the peak is referred to as resonance peak, [19]. We adopt this convention for conciseness, for it represents the same concept. However, it should be noted that in the nonlinear case both terms have a different bearing than in the linear case.

The relationship between the frequency and the peak vibration displacement of each vibration mode under different vibration intensities is established here, in order to facilitate the analysis of the modal nonlinearities of the tested paintings. Among them, sweeping with increasing and decreasing frequency, the relationship between the frequency of the first two vibration modes of the three tested paintings and their vibration displacements at the measurement point are shown in Figs. 8, 9 and 10 for the seascape painting, the still life painting and the dummy painting, respectively. In these figures, the peak to peak values of displacements are plotted. It should be noted that the intention is to express the change in frequency for peak amplitude under different vibration intensities which are characterised by vibration displacement.

First of all, the most obvious observation is the nonlinearity of the resonance frequency. With respect to the two real paintings, as the vibration displacement increases, their resonance frequency first drops slightly and then rises sharply. The only difference is that the resonance frequency of the still life painting changes less than that of the seascape painting. In contrast, for the dummy painting, its resonance frequency always increases with the increase of vibration displacement. Although fluctuations in the resonance frequency of the dummy painting are observed at lower vibration displacements, more detailed investigation has shown that those fluctuations are caused by very small climate changes. Even though a climate box was used, very small
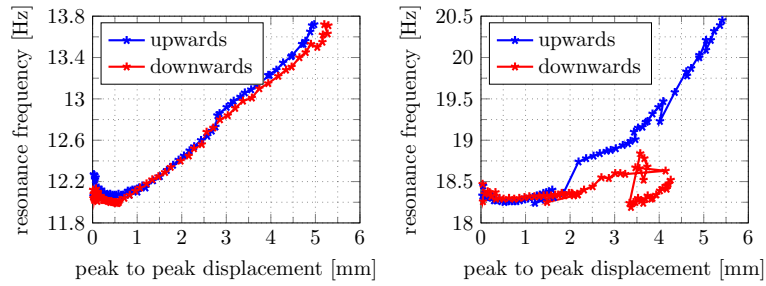
Fig. 9 Relationship between the first two resonance frequencies of the still life painting and their corresponding vibration displacements at the measurement point
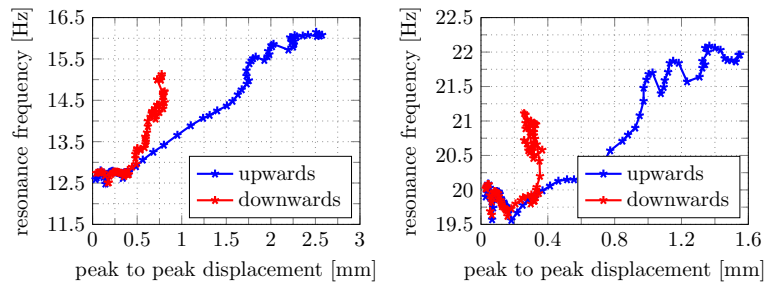

Fig. 10 Relationship between the first two resonance frequencies of the dummy painting and their corresponding vibration displacements at the measurement point

climate changes are unavoidable. Furthermore, it turned out that the freshly prepared dummy painting is more sensitive to climate changes as the real paintings after natural aging.

The next obvious modal nonlinearity that can be observed is the difference in modal characteristics obtained by sweeping upwards and downwards. When the vibration displacement is lower, under the same vibration intensity, the resonance frequency obtained by up sweep is basically the same as that obtained by down sweep. However, as the vibration displacement increases, the difference between the modal characteristics obtained by up sweep and down sweep becomes larger and larger. Under the same vibration displacement, for the seascape painting and the dummy painting, the resonance frequency obtained by up sweep is higher than that obtained by down sweep, while for the still life painting, the opposite is true. Moreover, in the sweep experiment, the input voltage range of the shaker for the up sweep and down sweep is the same, but the maximum vibration displacement achieved by down sweep is smaller than that be achieved by up sweep. The only exception is that for the first eigenmode of the still life painting, its maximum vibration displacement achieved by down sweep is slightly higher than that achieved by up sweep.

The last modal nonlinearity relates to the decrease in the amplitude of the resonance peak in the mobility curve. Although the vibration displacement of the painting increases gradually with increasing input voltage of the shaker, the amplitude of each resonance peak in the mobility curve decreases with vibration intensity, as shown in Fig. 6. As for the reason of this behaviour, two assumptions can be given. Firstly, it may be that with increasing excitation intensity, hence increasing vibration amplitude, also damping due to moving air increases. Secondly, it may also be possible that as vibration intensity increases, the intensity of the harmonics gradually increase, thereby reducing the proportion of energy occupied by the primary resonance. In order to verify, or negate those assumptions, more detailed research is necessary.

4.2 Consequences

Since it is known that modal nonlinearities are inevitable during transportation, some possible consequences of modal nonlinearities for painting transportation are discussed in this subsection.

Table 2 Variation in the first three resonance frequencies of the three tested paintings and their maximum vibration peak to peak displacement in the up sweep experiment

Painting	Δf_1 [Hz]	Δd_1 [mm]	Δf_2 [Hz]	Δd_2[mm]	Δf_3 [Hz]	Δd_3 [mm]
Seascape	3.09 (41.8%)	4.16	4.07 (38.4%)	5.53	3.02 (22.1%)	4.28
Still life	1.65 (13.5%)	4.98	2.20 (12.0%)	5.42	2.21 (10.1%)	5.39
Dummy painting	3.67 (29.1%)	2.58	2.53 (7.8%)	1.55	2.00 (8.0%)	1.25

4.2.1 Resonance frequency

The change of the resonance frequency with vibration intensity is the most typical feature of the modal nonlinearities of paintings. For a linear system, the resonance frequency would not depend on excitation amplitudes. If the painting is only subjected to shock during transportation, the painting can be directly treated as a linear system, simply because the energy delivered to the painting is too small to drive the response into a region of considerable nonlinearity. Once the painting is subjected to vibration during transportation, the change of resonance frequency caused by its modal nonlinearity will become a problem that cannot be ignored. Table 2 lists the variation in the first three resonance frequencies of the three tested paintings and the peak to peak value of the corresponding maximum vibration displacement, which have shown to possibly occur during transportation in Sect. 3.2. Most of the resonance frequencies have a variation in more than 2 Hz when the amplitude of the vibration displacement is only 2 mm. Such a wide variation range will greatly increase the probability of the eigenmodes being excited during transportation, which will make the painting more susceptible to damage. Besides, if the packaging system only takes vibration isolation measures against the eigenfrequency of the painting, then when the painting is subjected to stronger vibrations, these measures will not help due to the change of the resonance frequency. Therefore, in order to prevent the eigenmodes of the painting from being excited during transportation, it is necessary to consider the variation range of the resonance frequency.

4.2.2 Amplitude of the resonance peak

As mentioned in Sect. 4.1, because of the modal nonlinearity, the change of the resonance frequency is accompanied by the change of the amplitude of the resonance peak. Generally, in the frequency response function curve, the amplitude of the resonance peak decreases with the increase in the vibration intensity. At a lower vibration intensity, the resonance peak is sharp and the amplification is higher, while at a higher vibration intensity, the resonance peak is broad or even flat, and the amplification is lower, as shown in Fig. 6. That does not mean that the actual vibration amplitude decreases with excitation intensity, but rather that a certain excitation is amplified to a lower extent. In fact, as the vibration intensity increases, the response of the canvas continues to increase, but to a smaller proportion than the increase in excitation intensity. The frequency range of a sharper resonance peak that can cause a strong vibration is smaller than that of a broader resonance peak. In summary, for higher excitation intensities large canvas displacements will likely be excited over a broad frequency bandwidth (broad and flat peak). For smaller excitation intensities strong canvas displacements will only occur if a resonance frequency is almost exactly hit, but if that happens, even small excitation intensities lead to large displacements (sharp and high peak).

4.2.3 Harmonics

It is a general property of nonlinearity that harmonics can be observed in sinusoidal sweep experiments and it is inevitable to observe them to some extent in modal testing. However, the excessively strong harmonics observed in the investigated paintings make the harmonic phenomenon a problem that cannot be ignored during transportation. Figure 11 shows the response of the seascape painting for a harmonic excitation with 21.8 Hz with an intensity of magnitude L_7. The intensity of the subharmonic at approximate 11 Hz observed in the seascape painting is of the same order of magnitude as the fundamental vibration. Besides, it has also been observed in the seascape painting that the intensity of the superharmonics for a fundamental frequency of 7.95 Hz is even higher than that of the fundamental vibration. Excessive harmonics will also damage paintings and accelerate their aging. Therefore, it is necessary not only to prevent the eigenmodes from being excited during the transportation of paintings, but also avoiding the excitation of harmonic vibrations as much as
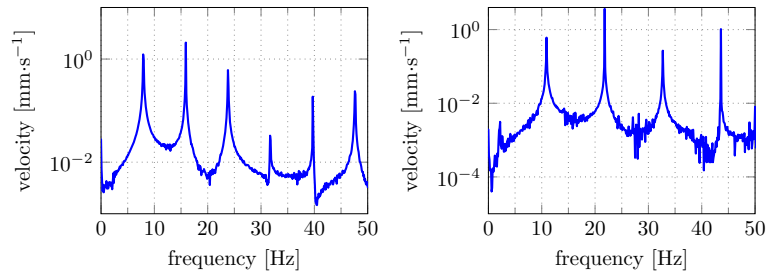
Fig. 11 Superharmonics with a fundamental frequency of 7.95 Hz (left) and the subharmonic with a fundamental frequency of 21.8 Hz (right) for the seascape painting
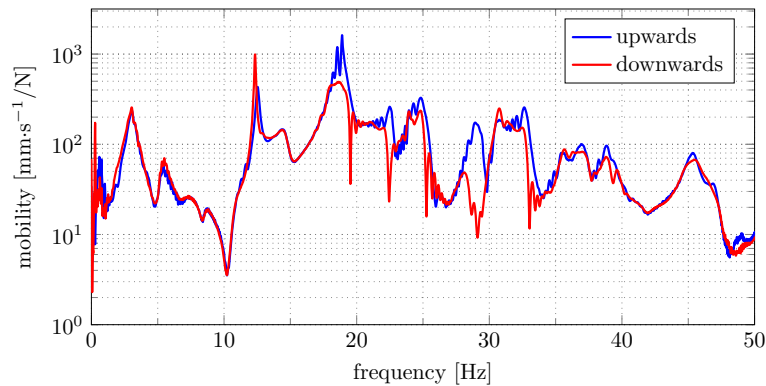

Fig. 12 Mobility difference between up and down sweep for the still life painting for an excitation intensity L_8

possible. This makes the dynamic design of isolation devices for transportation crates much more difficult because not only must the regions of fundamental resonance frequencies (referred to as eigenfrequencies from a linear perspective) be avoided, but also their sub- and superharmonics. This leaves very little room for harmless frequency ranges to which dynamic isolation devices for a safer transportation can be tuned.

The excessive appearance of harmonics also makes the identification of the fundamental resonance frequencies more difficult. When calculating the mobility according to Eq. (1) the harmonics cause a very noisy mobility. The reason for this is that for a sinusoidal sweep the system responds with a considerable magnitude with a different spectral content (harmonics) than the excitation (fundamental). Hence, in Eq. (1) a response of considerable magnitude (harmonics) is divided by a very small excitation signal because the excitation frequency is the fundamental. For a real measurement, the response (harmonics) is essentially divided by noise and, therefore, the harmonic peak appears as an extremely noisy peak. This is the reason for the necessary filtering in Sect. 3.2.

4.2.4 Up and down sweep

From the observation that the same excitation intensity leads to different resonance amplitudes depending on whether the sweep was performed up or downward, another consequence for transportation arises which can be more readily be understood when presented as in Fig. 12 which shows the mobility for an up and down sweep.

Obviously for a frequency range between 17 and 35 Hz the response is different for whether the excitation frequency approaches a resonance frequency from above or from below. Not only does the resonance frequency change, but also the sharpness and width of the resonance peak are effected. This implies that besides the excitation intensity, the history of the excitation, too, has an influence on the vibration response which may lead to a wider band of undesired excitation frequencies. Therefore, it is not enough to only analyse the content of the excitation spectrum of the painting during transportation and propose vibration isolation measures accordingly. It is also necessary to analyse different excitation schemes, taking into account the situation that the excitation approaches the vibration mode from the forward and reverse direction.
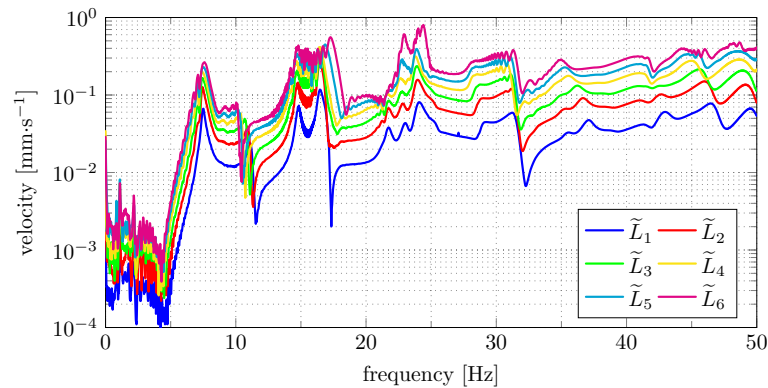
Fig. 13 Velocity response spectrum for the canvas of the seascape painting under different intensities of acoustic excitation when sweeping upwards

4.2.5 Experiment with loudspeaker

Although the sweep experiment with the shaker can fully measure the modal nonlinearities of the painting, there are some deviations between the observed modal characteristics of the painting and its actual modal characteristics, because the stinger of the shaker is firmly connected with the painting frame, which has changed the boundary conditions of the painting and consequently the dynamics, too. For example, in the results of the hammer experiment on the frame in Fig. 4 and the sweep experiment with shaker in Fig. 5, the first resonance frequencies of the seascape painting are identified to be 7.83 Hz and 7 Hz, respectively, which was precisely caused by the change of boundary conditions. For this reason, loudspeakers are used as excitation sources to measure the modal characteristics of the painting in some researches [13,14]. As a contactless measurement method, it is simpler to measure the modal characteristics of the painting by using loudspeakers. As shown in Fig. 3c, it is only necessary to face the loudspeaker to the painting and provide an excitation signal to the loudspeaker through the signal generator, then the vibration of the canvas can be excited. This is especially suitable for the study of the modal characteristics of real precious paintings. However, because it is difficult to obtain any knowledge about the energy distribution of the loudspeaker input to the canvas, all analysis of the modal characteristics of the painting can only be based on the response of the canvas. It is the purpose of this contribution to verify the feasibility of using loudspeakers to measure the modal nonlinearities of paintings.

The parameters of the sweep excitation with loudspeaker are same as the parameters of the sweep excitation with shaker. Figure 13 shows the velocity response spectrum for the canvas of the seascape painting under different acoustic excitation intensities when sweep upwards. In order to highlight that those intensities do not correspond to those of the shaker experiment, they are denoted by $\widetilde{L}_1$, $\widetilde{L}_2$, ..., $\widetilde{L}_6$. The loudspeaker used in this contribution is a woofer with an advertised frequency characteristic of less than 3 dB loss between 20 and 3000 Hz. But it can be seen from the velocity response spectrum that the eigenmodes below 20 Hz are also well excited, which proves a good applicability of this measurement method. It is observed that most of the resonance frequency first decreases and then increases with the increase in the acoustic excitation intensity, which is consistent with the trend observed in the sweep experiment with the shaker. Different resonance frequencies are also observed in the down sweep than in the up sweep. This proves the feasibility of using loudspeakers to measure the modal nonlinearities of the painting. However, unlike the shaker, due to the gap between the painting and the loudspeaker, the output energy of the loudspeaker does not entirely act on the painting, so the loudspeaker cannot excite the painting to reach the same high vibration intensities as in the sweep experiment with shaker, i.e. it is difficult for loudspeaker experiments to measure the modal nonlinearities of the painting to the full extend as observed under transportation. Moreover, because the acoustic excitation is of different nature and directly excites only the canvas and generally of smaller intensity, it is impossible to compare the state of the painting under the excitation of the loudspeaker with the state of the painting during transportation, and thus, it is difficult to evaluate the modal nonlinearities obtained by sweep experiment with loudspeaker.
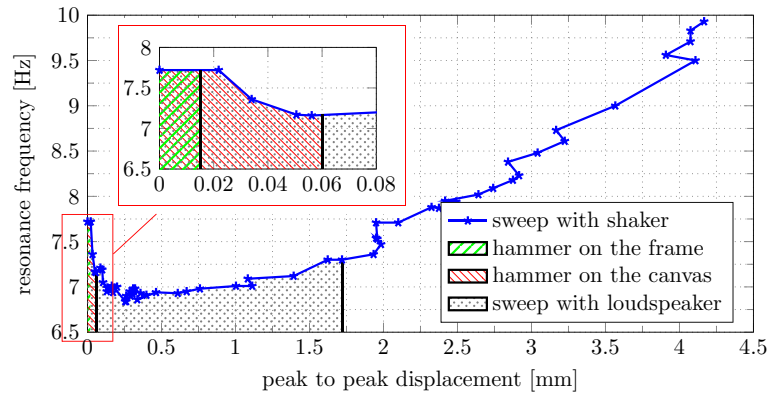
Fig. 14 Different experimental methods lead to different vibration intensities and different observation ranges of modal nonlinearities

4.2.6 Identification methods of modal nonlinearity

The above performed analysis demonstrates that the identification of the modal nonlinearities of the painting is absolutely necessary for improving transportation and the design of the packaging system. In this contribution, four experimental methods have been adopted to identify the modal nonlinearities of paintings, namely the hammer experiment on the frame, the hammer experiment on the canvas, the sweep experiment with shaker and the sweep experiment with the loudspeaker. Different experimental methods can excite different vibration intensities, resulting in different ranges of modal nonlinearities that can be observed. Taking the first resonance frequency of the seascape painting as an example, the results obtained by four different experimental methods are shown in Fig. 14.

The hammer experiment on the frame cannot excite a strong response of the eigenmode, so the modal characteristics obtained under different shock intensities are always the same. It is difficult to observe the modal nonlinearities of the painting conducting this experiment. But it is the best method to quickly measure the eigenfrequencies and the eigenmodes of the painting in the linear range. Because of excessive shock, the hammer experiment on the canvas can observe some slight modal nonlinearities of the painting, but this method is only suitable for lab conditions and for paintings of negligible artistic value or dummies. The sweep experiment is the best method to identify the modal nonlinearities of the painting. The modal nonlinearities observed in the sweep experiment with shaker are the most comprehensive, while the contactless sweep experiment with loudspeaker has a smaller range of observed modal nonlinearities.

5 Conclusion

In this contribution, through the hammer experiment and the sweep experiment, the modal nonlinearities of two real paintings and a dummy painting under different shock and vibration intensities have been studied experimentally. The experimental results show that the painting exhibits obvious modal nonlinearities only when being subjected to vibrations of different intensities. The comparison of the vibration acceleration of the seascape painting in the sweep experiment and the transportation experiment demonstrates that a painting can be treated as a linear system when being subjected to shock, but its modal nonlinearities are not negligible when being subjected to vibration. First of all, the most important problem caused by the modal nonlinearities is the significant resonance frequency offset, which is likely to directly lead to the failure of the original vibration isolation measures for the eigenmodes of the painting. Then, as the vibration intensity increases, the amplitude of the resonance peak decreases and the resonance peak becomes more flat, which increases the probability that the eigenmode is excited during transportation. Moreover, it was observed in the painting that the superharmonics and subharmonics may be of the same order of magnitude as the fundamental vibration or even higher. This indicates that it is also necessary to attenuate the harmonic vibration during the painting transportation. Lastly, it is worth noting that under the same vibration intensity, different modal characteristics exist for the paintings when sweeping with increasing and decreasing frequencies. This makes the analysis of different excitation schemes complicated. Therefore, the identification of the modal nonlinearities of the painting is absolutely necessary and important. The best identification method is the sweep experiment with shaker

or loudspeaker. The modal nonlinearity observed in the sweep experiment with shaker is most comprehensive, while the sweep experiment with loudspeaker is particularly suitable for real precious paintings.

So far, only the occurrence of nonlinear effects have been shown. It also shown that those nonlinearities are non-negligible for real paintings and excitation intensities observed for real transports. As to the physical origins of these non-linearities no conclusion can be drawn. In [6] some of the nonlinear phenomena were compared with the behaviour of a Duffing oscillator which is motivated by the cubic stiffness term that appears in the nonlinear description of membranes. This gives a hint about the origin of the general behaviour found in this contribution, e.g. the increase in the resonance frequency with excitation intensity. Other observed phenomena like the inverse relationship for small excitation intensities, different behaviour for different resonance frequencies an alike, are more involved and may be a nonlinearity of a different kind for which a manifold origins are conceivable. They remain sources for further research.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

1. Mecklenburg, M.F.: Art in transit: studies in the transport of paintings. In: Proceedings of International Conference on the Packing and Transportation of Paintings, London (1991)
2. Tsiranidou, E., Bernikola, E., Tornari, V., Fankhauser, T., Läuchli, M., Palmbach, C., Bäschlin, N.: Holographic monitoring of transportation effects on canvas paintings. In: SPIE Newsroom, pp. 1–3 (2011)
3. Hein, N.: Die materielle Veränderung von Kunst durch Transporte–Monitoring und Transportschadensbewertung an Gemälden durch das Streifenprojektionsverfahren. Ph.D. thesis, Staatliche Akademie der Bildenden Künste Stuttgart, Stuttgart (2015)
4. Krekel, C., Hein, N.: Kunsttransport: Gibt es eine Grenze zwischen Schaden und beschleunigter Alterung? In: Proceedings of ICOM International Council of Museums, Köln, vol. 4, pp. 12–17 (2014)
5. Krekel, C., Heinemann, C.: Wenn Kunstwerke auf Reisen gehen: Mikroschäden mithilfe hochauflösender 3D-Modelle finden und dokumentieren. Das Magazin der Deutschen Forschungsgemeinschaft **4**, 127–128 (2020)
6. Kracht, K.: Die Untersuchung des Schwingungsverhaltens von Ölgemälden in Abhängigkeit der Alterung. Ph.D. thesis, Technische Universität, Berlin (2011)
7. Gmach, A.: Erschütternde Umstände - Schwingungsbelastung von Kunst- und Bauwerken. Diplomarbeit, Technische Universität München, München (2010)
8. Heinemann, C., Ziegler, P., Hein, N., Krekel, C., Eberhard, P.: Objektiviertes Gemäldetransportmonitoring unter Berücksichtigung mechanischer Einflussfaktoren. Zeitschrift für Kunsttechnologie und Konservierung **1**(33), 178–198 (2019)
9. Läuchli, M., Bäschlin, N., Hoess, A., Fankhauser, T., Palmbach, C., Ryser, M.: Packing systems for paintings: damping capacity in relation to transport-induced shock and vibration. In: Proceedings of ICOM-CC 17th Trienniel Conference, Melbourne, vol. 15, p. 19 (2014)
10. Kracht, K., Kletschkowski, T.: From art to engineering: a technical review on the problem of vibrating canvas part I: excitation and efforts of vibration reduction. Facta Univ. Ser. Mech. Eng. **15**(1), 163–182 (2017)
11. Palmbach, C.: Messung Transportbedingter Schwingungen an Textilen Bildträgern. Hochschule der Künste Bern, Bern (2007)
12. Vivi, T., Tsiranidou, E., Bernikola, E.: Crack-growth on canvas paintings during transport simulation monitored with digital holographic speckle interferometry. Adv. Res. **2**, 967–986 (2014)
13. Wei, W., Kragt, W., Visser, A.: Non-contact measurement of vibrations in paintings using laser doppler vibrometry. In: Proceedings of 8th International Conference on Non-destructive Investigations and Microanalysis for the Diagnostics and Conservation of the Cultural and Environmental Heritage, Lecce (2005)
14. Chiriboga, A.P.G.: Finite element modeling of vibrations in canvas paintings. Ph.D. thesis, Delft University of Technology, Delft (2012)
15. Michalski, S.: Paintings: their response to temperature, relative humidity, shock, and vibration. In: Art in Transit: Studies in the Transport of Paintings, pp. 223–248 (1991)
16. Mecklenburg, M.F.: Some aspects of the mechanical behavior of fabric supported paintings. In: The Mechanics of Art Materials and Its Future in Heritage Science (1982)

17. Hagan, E.W., Charalambides, M.N., Young, C.T., Learner, T.J., Hackney, S.: Tensile properties of latex paint films with tio$_2$ pigment. Mech. Time Depend. Mater. **13**(2), 149–161 (2009)
18. Hartlieb, E.: Wie schwingt ein Gewebe? - Einfluss verschiedener Parameter auf die Schwingungseigenschaften von Gemälden auf textilen Trägern. Master thesis, Staatliche Akademie der bildenden Künste Stuttgart, Stuttgart (2021)
19. Nayfeh, A.H., Mook, D.T., Holmes, P.: Nonlinear Oscillations. Wiley, Hoboken (1980)