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ORIGINAL

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Control of the transport direction and velocity of the two-way reversible vibratory conveyor

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Abstract Vibratory conveyors are very important elements of production lines. Very often there is a need to change the feed transport velocity, especially in the case of feeders, or to alter the transport direction in the case of symmetric production lines. The transport possibilities of the new vibratory conveyor, allowing fast changes of the velocity and direction of transporting, solely by changes of the angular velocity of the excitation vibrator, and patented by the authors, are analysed in this paper. The innovatory control method allowing fast changes of the transport direction and good stabilisation of the given velocity, without the need to pass through its resonance frequencies, is presented. This new solution increases the safety and functionality of machine operations.

Keywords Vibratory conveyor · Resonance zone · Direction and velocity control · Feed transport

1 Introduction

The point of departure for this paper is the authors' previous research, concerning analysis of the dynamics of vibratory conveyors. This paper constitutes the development of ideas presented in the work [1], where the dynamics of the new conveyor solution patented by the author was analysed. Frahm's eliminators were applied in this conveyor for damping the trough vibrations in the determined direction. The Frahm's eliminator, in this case, is not the trough of the conveyor, giving the possibility of changing the direction of transport.

The problem of the control of such a machine is addressed in this paper. In particular, the application of a control system which would enable the feed transport in the reverse mode without the necessity for the system to pass through the resonance zone, related to the eliminator tuned for a lower frequency, was analysed. The influence of the change of the feed mass on the transport velocity and in consequence the reaction of the control system to external disturbances were also examined. Generally, the control law was derived from the specific properties of the dynamic characteristics of the machine.

The new contribution of this work is the development of a control system allowing the rapid change of a transport direction without passing the resonance zone. In addition, the proposed solution is resistant to such disturbances as changes of the feed mass, assuring constant transport velocity.

The feed model utilised in this work—elaborated in the work [2]—was applied in several papers by the authors, among others in [3], which presented the thorough analysis of the feed material transport velocity

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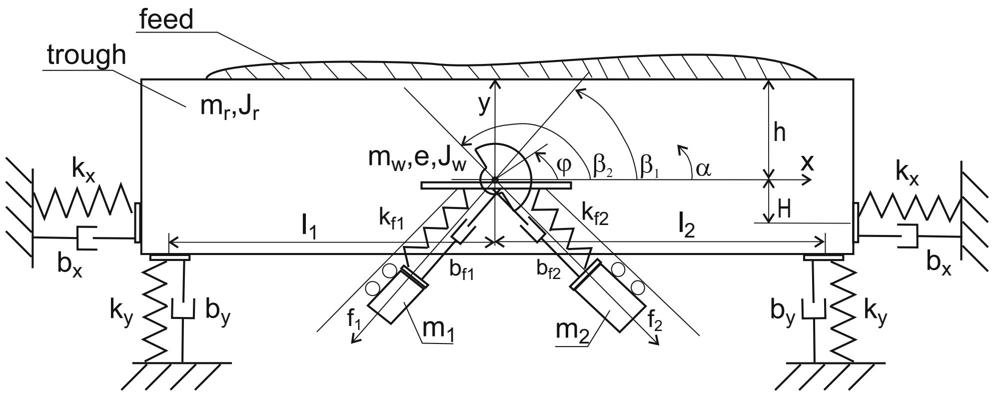

Fig. 1 Two-way vibratory conveyor according to the author's invention

distribution along the trough, due to rotational vibrations of the whole conveyor caused by non-central passing of the excitation force by the machine mass centre.

In the paper [4], frequently used conveyors—operating on the basis of the Frahm's eliminator—in which the trough of the vibrator constitutes the eliminator of frame vibrations causing a significant decrease in forces transmitted to the foundation, were analysed. The strategy of controlling the excitation frequency of such conveyors when they are loaded with a significant feed mass was outlined. The work presented in this paper was further developed in [5], in which control of excitation frequency (initiated by the step change of the feed mass) of two counter-rotating vibrators is provided in such way that the force transmitted to the foundations—at loading of the conveyor with the feed—was minimal.

2 Principle of operation of the two-way conveyor

The conveyor, according to the author's invention [6], has the trough open at both ends, supported flexibly on stiff bases, with the rotational vibratory drive suspended centrally to the trough (Fig. 1). The main purpose of the invention was the development of a new, improved two-way feeder or conveyor in which the transport direction can be changed very quickly only by changing the rotation frequency of the vibrator.

The principle of the conveyor operations is based on the fact that, when the excitation frequency of the vibrator satisfies a condition $\omega = \sqrt{\frac{k_{f1}}{m_1}}$, the mass m_1 vibrates with the amplitude, which triggers—in the spring k_{f1}—forces opposite to the excitation force of the vibrator in the direction f_1, causing—in accordance with the Frahm's eliminator—at small damping in spring k_{f1}, the excitation of the trough vibrations in this direction [7]. This does not change the vibrations in the perpendicular direction f_2, which enables the feed transport to the left. When the vibrator excitation frequency, in the steady state, satisfies the condition $\omega = \sqrt{\frac{k_{f2}}{m_2}}$, the mass m_2 vibrates with the amplitude, which triggers in the element k_2—forces opposite to the excitation force of the vibrator in the direction f_2 causing the extinction of the trough vibrations in this direction, but does not change the vibration in the perpendicular direction f_1. This enables the feed transport to the right. This conveyor can be equipped with two or more vibration eliminators, attached to the trough in pairs from each side of the shaft.

This construction allows the material transport direction to be changed by altering the rotational velocity of the vibrator motor, and the obtained vibrations of the trough are of nearly linear character. Until now, disadvantages of this construction type were the necessity of the system's passage through the resonance zone, related to the eliminator tuned to the lower excitation frequency, and the significant difference in yield in opposite directions [1]. Great differences are the result of the fact that the vibration amplitude—in the case of a lower excitation frequency—is smaller and that the throwing up frequency is lower. However, the authors of this paper present the possibility of controlling the conveyor frequency in such a manner that changing the transportation direction can occur without the system's passage through the resonance zone. This increases the stability and safety of machine operations. It is also possible to control the vibrator in such a way as to obtain the same (or very similar) transporting velocity in both directions.

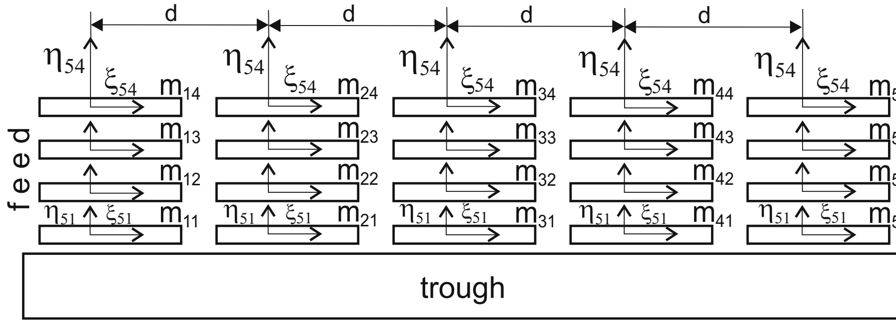

Fig. 2 Model of the feed

3 Simulation of the proposed conveyor structure

The system composed of the model of conveyor, shown in Fig. 1, and the model of feed, shown in Fig. 2, was assumed for the purposes of computer simulation [8]. The model consists of the inertial vibrator of an independent induction drive, described by means of the static characteristics, the machine body performing the flat movement of three degrees of freedom (described by x, y and α) and supported on a system of vertical springs with dampers of linear characteristic, and five, four-layer models of the loose feed [2], distributed in various points of the machine work surface [9].

The Lagrange function of the model presented in Fig. 1, assuming small vibrations of masses m_1 and m_2 and small moments of inertia of their centres of rotation versus the moment of inertia of the whole trough, is as follows:

$$\begin{aligned}L &= \frac{1}{2}m_r(\dot{x}^2+\dot{y}^2)+\frac{1}{2}m_w\left((\dot{x}-e\dot{\phi}\sin(\phi)^2+(\dot{y}-e\dot{\phi}\cos(\phi)^2\right)+\frac{1}{2}J_r\dot{\alpha}^2+J_w\dot{\phi}^2\\ &+\frac{1}{2}m_1\left((\dot{x}-\dot{f}_1\cos(\beta_1))^2+(\dot{y}-\dot{f}_1\sin(\beta_1))^2\right)+\frac{1}{2}m_2\left((\dot{x}-\dot{f}_2\cos(\beta_2))^2+(\dot{y}-\dot{f}_2\sin(\beta_2))^2\right)\\ &-\frac{1}{2}k_{f1}f_1^2-\frac{1}{2}k_{f2}f_2^2-k_x(x+h\alpha)-\frac{1}{2}k_y(y+l_1\alpha)^2-\frac{1}{2}k_y(y-l_2\alpha)^2.\end{aligned} \quad (1)$$

The power loss of the mechanical system equals:

$$N=-\frac{1}{2}b_{f1}\dot{f}_1{}^2-\frac{1}{2}b_{f2}\dot{f}_2{}^2-b_x(\dot{x}+h\dot{\alpha})-\frac{1}{2}b_y(y+l_1\dot{\alpha})^2-\frac{1}{2}b_y(y-l_2\dot{\alpha})^2. \quad (2)$$

By means of the Lagrange equation—for non-sustainable systems—it is possible to determine equations of motion of the system, which should be supplemented by equations related to the feed motion.

$$M\ddot{q}=Q, \quad (3)$$

$$M=\begin{bmatrix} m_r+m+m_1+m_2 & 0 & 0 & me\sin(\phi) & m_1\cos(\beta_1) & m_2\cos(\beta_2)\\ 0 & m_r+m+m_1+m_2 & 0 & me\cos(\phi) & m_1\sin(\beta_1) & m_2\sin(\beta_2)\\ 0 & 0 & J_r & 0 & 0 & 0\\ me\sin(\phi) & me\cos(\phi) & 0 & me^2+J_0 & 0 & 0\\ m_1\cos(\beta_1) & m_1\sin(\beta_1) & 0 & 0 & m_1 & 0\\ m_2\cos(\beta_2) & m_2\sin(\beta_2) & 0 & 0 & 0 & m_2 \end{bmatrix}, \quad (4)$$

$$\ddot{q}=\begin{bmatrix}\ddot{x} & \ddot{y} & \ddot{\alpha} & \ddot{\phi} & \ddot{f}_1 & \ddot{f}_2\end{bmatrix}^T, \quad (5)$$

$$Q=\begin{bmatrix} -m_we\dot{\phi}^2\cos(\phi)-2k_x(x+H\alpha)-2b_x(\dot{x}+H\dot{\alpha})-T_{101}-T_{102}-T_{103}-T_{104}-T_{105}\\ m_we\dot{\phi}^2\sin(\phi)-k_y(y+l_1\alpha)-k_y(y-l_2\alpha)-b_y(\dot{y}+l_1\dot{\alpha})-b_y(\dot{y}-l_2\dot{\alpha})-F_{101}-F_{102}-F_{103}-F_{104}-F_{105}\\ -2k_xH^2\alpha-2k_xHx-2b_xH\dot{x}-2b_xH^2\dot{\alpha}-k_y(y+l_1\alpha)l_1+k_y(y+l_2\alpha)l_2-b_y(\dot{y}+l_1\dot{\alpha})l_1+b_y(\dot{y}-l_2\dot{\alpha})l_2\\ +(T_{101}+T_{102}+T_{103}+T_{104}+T_{105})h+F_{101}2d+F_{102}d-F_{104}d-F_{105}2d\\ \mathcal{M}_{el1}\\ -k_{f1}f_1-b_{f1}\dot{f}_1\\ -k_{f2}f_2-b_{f2}\dot{f}_2 \end{bmatrix}. \quad (6)$$

The mathematical model of such a system consists of the matrix Eq. (3) describing the machine motion, Eq. (11) describing electromagnetic moments of drive motors, Eq. (10) used for determining movements of feed layers and Eqs. (8) and (9) describing normal and tangent interactions between feed layers and between the feed layer and the machine body, where

$F_{j,j-1,k}$ and $T_{j,j-j,k}$ are normal and tangent (respectively) components of the pressure of the jth layer on the $j-1$ layer in the kth column,
j—indicator of the material layer ($j=0$ refers to the machine body),
k—indicator of the column of the material layer.

When the successive feed layers jth and $j-1$ (in the given column) are not in contact, the contact forces in the normal direction $F_{j,j-1,k}$ and tangent direction $T_{j,j-1,k}$ between these layers is equal zero:

$$F_{j,j-1,k} = 0,\ T_{j,j-1,k} = 0 \quad \text{for} \quad \eta_{j,k} \geq \eta_{j-1,k}. \tag{7}$$

In the opposite case, the contact force occurs in the normal direction between j, k and $j-1, k$ feed layers (in the case of the first layer between the layer and the trough), the model of which is of a form described in [2]:

$$F_{j,j-1,k} = (\eta_{j-1,k} - \eta_{j,k})^p k_{\mathrm{s}} \left\{1 - \frac{1-R^2}{2}\left[1 - \operatorname{sgn}(\eta_{j-1,k} - \eta_{j,k}) \operatorname{sgn}(\dot{\eta}_{j-1,k} - \dot{\eta}_{j,k})\right]\right\} \tag{8}$$

and the force originated from friction in the tangent direction

$$T_{j,j-1,k} = -\mu F_{j,j-1,k} \operatorname{sgn}(\dot{\xi}_{j,k} - \dot{\xi}_{j-1,k}), \tag{9}$$

where k_{s}—feed stiffness coefficient, p—Herz–Stajerman constants, R—restitution rate of normal impulses at collision.

Equations of motion in ξ and η directions of the individual feed layers (Fig. 2), with taking into account influences of the conveyor on bottom feed layers, are of the form:

$$\begin{aligned} m_{nj,k}\ddot{\xi} &= T_{j,j-1,k} - T_{j+1,j,k}, \\ m_{nj,k}\ddot{\eta} &= -m_{nj,k}g + F_{j,j-1,k} - F_{j+1,j,k}. \end{aligned} \tag{10}$$

Electromagnetic moment developed by the motor, assumed in the form corresponding to the static characteristics of the motor, is

$$\mathcal{M}_{el} = \frac{2\mathcal{M}_{\max}(\omega_{ss} - \dot{\phi})(\omega_{ss} - \omega_{\max})}{(\omega_{ss} - \omega_{\max})^2 + (\omega_{ss} - \dot{\phi})^2}, \tag{11}$$

where

$\mathcal{M}_{\max}$—break-over torque of the drive motor,
ω_{ss}—synchronous frequency of the drive motor,
$\omega_{\max}$—frequency of break-over of the drive motor.

The simulation model developed for verification of analytical solutions takes into account the influence of the machine body collisions with the feed, as well as allowing the material transport velocity to be determined in both directions. The simulation was performed for the parameter values listed in Nomenclature. It is assumed that the vibrator is powered by an electric motor type Sg 132 M-4 (Celma Inducta) with rated output = 7.5 kW and rated speed = 1450 rev/min.

4 Results of simulations

The dependence of vertical vibrations on horizontal ones, for two different excitation frequencies, to which eliminators m_1 and m_2 were tuned, is presented in Figs. 3 and 4. It can be seen, that vibrations of the conveyor frame are correct (in dependence on the excitation frequency they are inclined either to the left or to the right), while amplitudes of these vibrations are different, which at various excitation frequencies cause great differences in the material transport velocity in opposite directions (Figs. 3b, 4b). Such differences also result from the fact that the coefficient of throw, in the case of a lower excitation frequency, is smaller than unity, which means that the material is not thrown up (Fig. 4c). Such control of the change of the transport direction also necessitates the system's passage through the resonance zone related to the eliminator tuned to a lower frequency.

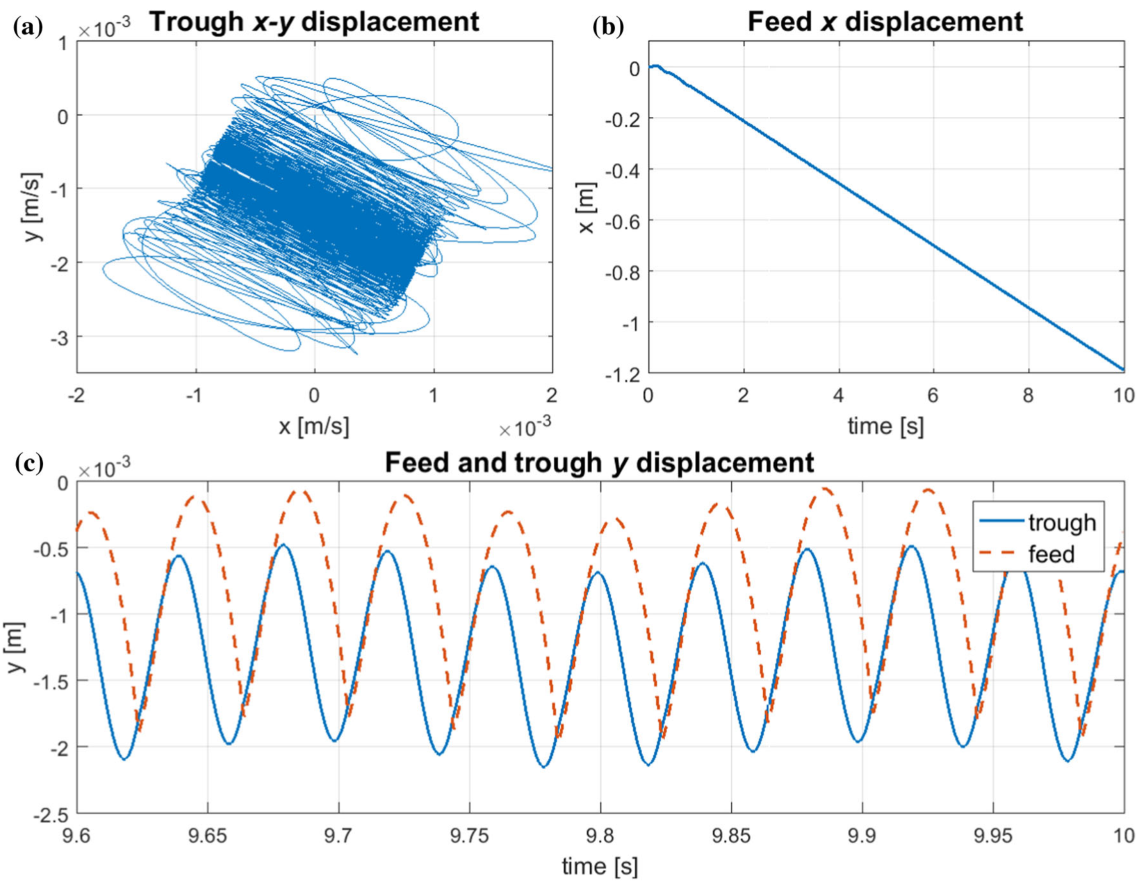

Fig. 3 **a** Upper left: dependence of the horizontal directions on the vertical ones for the excitation frequency $\omega = \sqrt{\frac{k_{f1}}{m_1}} = 157$ rad/s and the vibrator mass eccentricity of 0.0215 m; **b** upper right: feed displacement in the horizontal direction; **c** bottom: displacements of the vibroinsulating frame of the trough and feed in the vertical direction

4.1 New way of changing the transport direction

It can be noticed that the feed transportation direction in this system changes before it enters the resonance zone (Fig. 5). It should be emphasised that, in the further part of this study, the system of an increased vibrator unbalance $e = 0.086$ was tested, which—as shown in [1]—provides the possibility of high transporting velocities at low excitation frequencies of the two-way conveyor.

It can be seen in Fig. 5 that the highest transport velocities are obtained in the resonance zone, which—in practice—is not technically realisable. On the other hand, the transport direction change can be obtained before the system enters the resonance. Figure 6 presents the dependence of the frequency of the trough horizontal vibrations on vertical ones for the excitation frequency: $\omega_1 = 132.9$ rad/s and $\omega_2 = 137.9$ rad/s. As can be seen in these figures, the character of vibrations enables the feed transport in the opposite direction, while similar amplitudes of these vibrations, as well as similar excitation frequencies, enable the transport in both directions with similar velocities.

The effect of the change of vibration direction occurs due to the fact that the eliminator, tuned to a lower frequency, ceases fulfilling condition: $\omega = \sqrt{\frac{k_{f2}}{m_2}}$, and in consequence ceases to eliminate vibrations in its work direction f_2, while the other eliminator, even though it still does not fulfil condition $\omega = \sqrt{\frac{k_{f1}}{m_1}}$ (since the frequency is too low), is already in a large part—what results from the character of the Frahm's eliminator operations—eliminating vibrations in the direction f_1, enabling the possibility to obtain nearly rectilinear vibrations in the direction f_2. It should be noted that this is still not the resonance frequency related to the first eliminator, and thus, the vibration amplitudes are at the level of the amplitudes in the operating point.

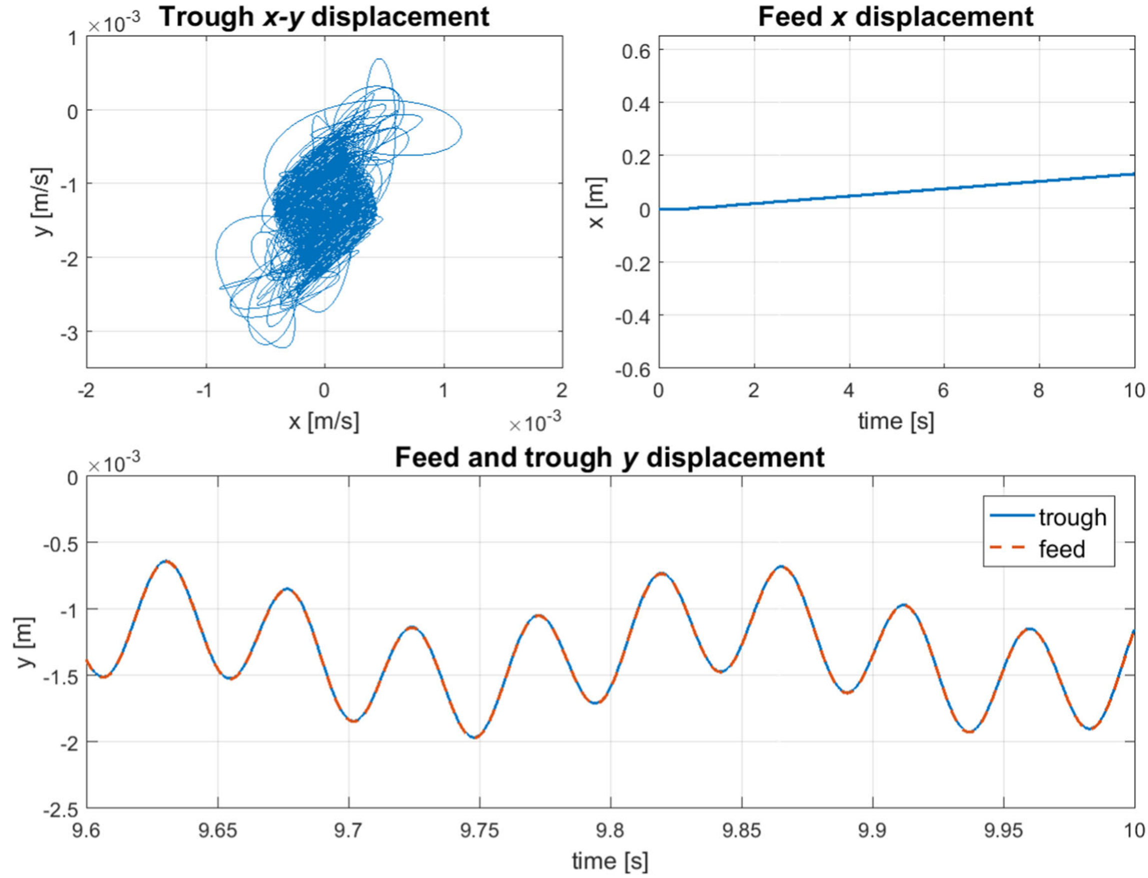

Fig. 4 Simulation results for the excitation frequency $\omega = \sqrt{\frac{k_{f2}}{m_2}} = 133.5$ rad/s and the vibrator mass eccentricity of 0.0215 m. Descriptions as in Fig. 3

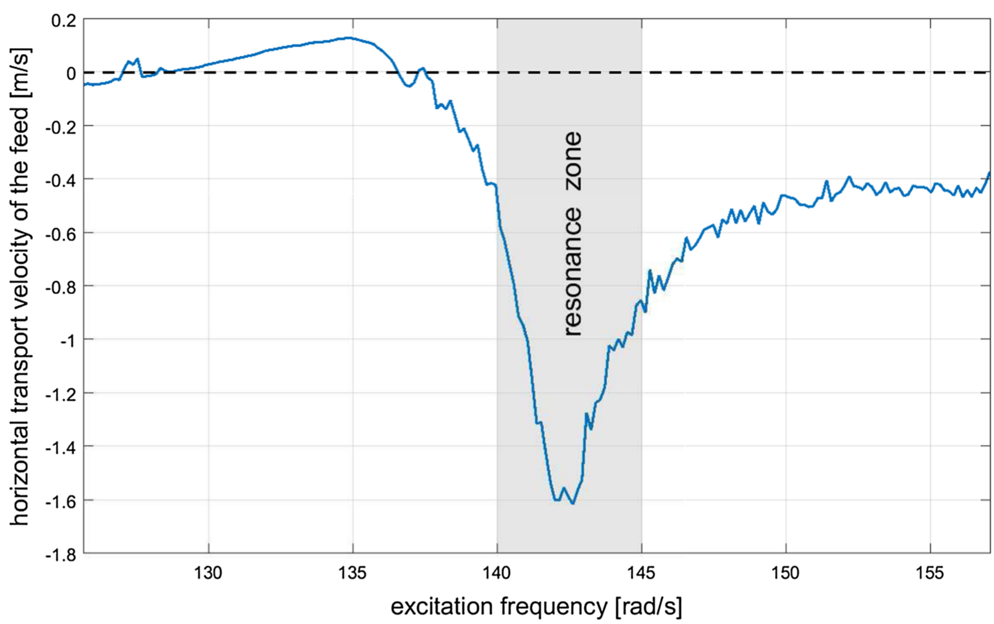

Fig. 5 Dependence of the transport velocity on the excitation frequency of the system for the vibrator mass eccentricity of $e = 0.086$ m and the feed mass of 20 kg

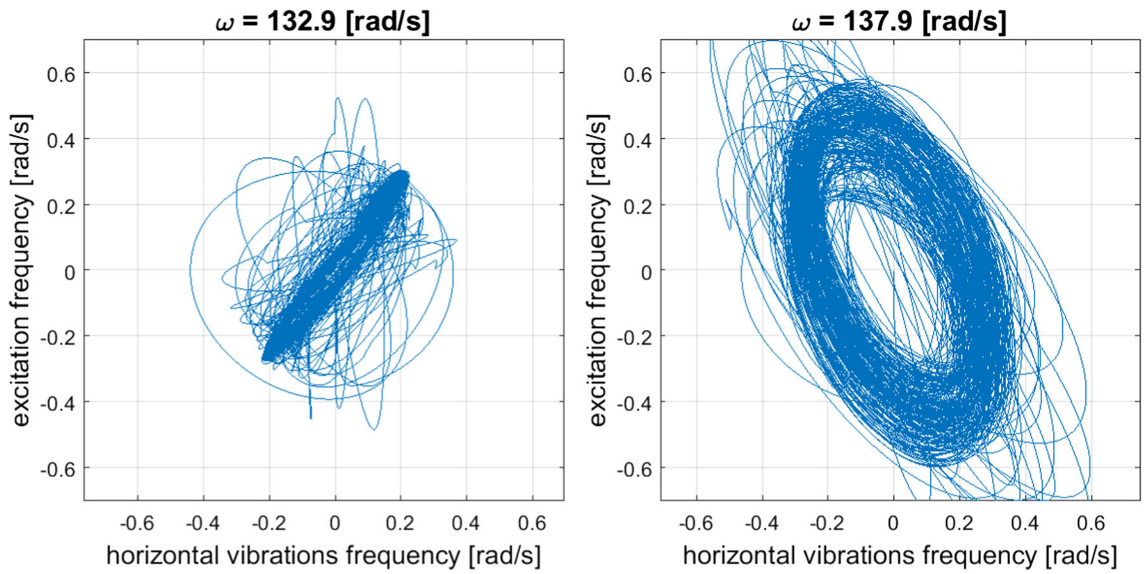

Fig. 6 Dependence of the machine horizontal vibrations frequency on the excitation frequency of 132.9 rad/s (left) and 137.9 rad/s (right)

4.2 Determination of the resonance frequencies of the system

To facilitate the analysis of the conveyor operations, its natural frequencies should be determined. The conveyor cannot operate near its natural frequencies, which are mainly dependent on the mass of the trough. Frequencies mainly dependent on the masses of eliminators should be determined since the conveyor operates in their vicinity. It should be mentioned that none of the frequencies—to which eliminators are tuned—are equal to the natural frequency of the system. The resonance frequencies of the system can be analytically determined for this type of the conveyor. However, since this is a system in which five natural frequencies occur—three related to the flat movement of the body and two related to eliminators in two perpendicular directions—the solution of such a system is very difficult. It is much easier to determine the approximate frequencies by the system decomposition. Assuming that masses of eliminators are point masses, the frequency related to the system rotation can be determined from equation (numbering of frequency results from their numerical values—beginning from the lowest)

$$\omega_3 = \sqrt{\frac{2k_x l^2 + 2k_y h^2}{J + m_1 f_1^2 + m_2 f_2^2}}. \tag{12}$$

The natural rotational frequency changes insignificantly during the machine operation due to variable distances f_1 and f_2, but since—in practice—values of these displacements are small and masses of eliminators are located near the system centre, the frequency of rotational vibrations can be approximately determined from the equation

$$\omega_3 = \sqrt{\frac{2k_x l^2 + 2k_y h^2}{J}}. \tag{13}$$

In order to determine the remaining frequencies, meaning those which are related to additional degrees of freedom in the form of eliminators, it is possible—for calculation reasons—to simplify the system.

Since the eliminators operate in perpendicular directions, they do not influence each other on the resonance frequencies of the system, and if the suspension of the main mass has the same stiffness coefficients in directions x and y ($k_x = k_y$), the diagrams for the determination of resonance frequencies related to eliminators can be simplified as shown in Fig. 7 [4].

Equations of motion of this system are as follows:

$$\begin{aligned} &m_1 \ddot{f}_1 + k_{f1}(f_1 - f_{11}) = 0, \\ &(m_r + m_2)\ddot{f}_{11} - k_{f1}(f_1 - f_{11}) + k f_{11} = 0. \end{aligned} \tag{14}$$

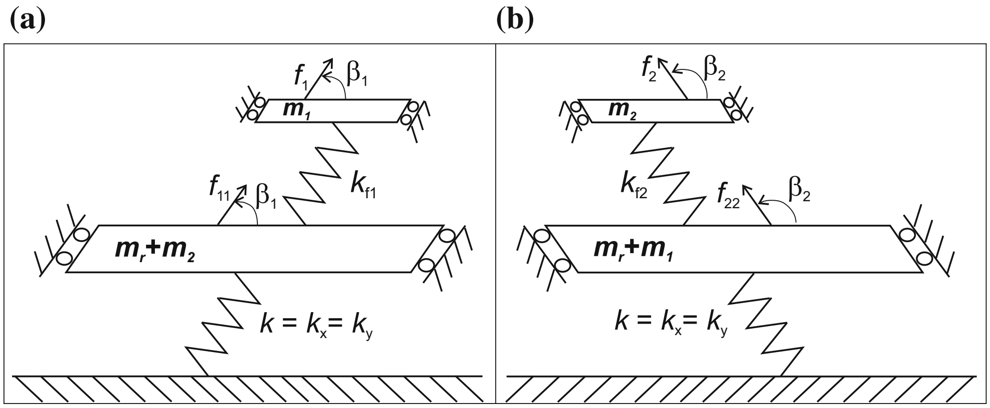

Fig. 7 Simplified diagrams of the conveyor allowing natural frequencies of the system to be determined in direction: **a** parallel to f_1 (left) and **b** parallel to f_2 (right)

We expect that the solution will take a form:

$$\begin{aligned} f_1 &= a\cos(\omega t - \psi), \\ f_{11} &= b\cos(\omega t - \psi), \\ \psi &= \text{const.} \end{aligned} \tag{15}$$

After insertion of functions (15) and their derivatives into equations of motion (14), the system of homogeneous equations, on account of amplitudes a and b as unknowns, is obtained. With regard to the physical sense of amplitudes, this system should have a nonzero solution. The condition of nonzero solutions of the system of homogeneous equations constitutes the zero value of the determinant formed of coefficients at unknowns. Equating the main determinant of coefficients of amplitudes to zero, it is possible to determine two frequencies of natural vibrations of the system, which, in the whole system, will equal ω_1 and ω_4 (due to growing values of successive frequencies). The main determinant of the Eq. (14) will be of the form:

$$\begin{bmatrix} -\omega^2 m_1 + k_{f1} & -k_{f1} \\ -k_{f1} & -\omega^2(m_r + m_2) + k_{f1} + k \end{bmatrix} = 0. \tag{16}$$

This determinant after evolving takes the form:

$$\omega^4 m_1(m_r + m_2) - \omega^2 m_1 k_{f1} - \omega^2 m_1 k - k_1 \omega^2 m_2 + k_{f1} k = 0. \tag{17}$$

Equation (17) has two positive roots:

$$\omega_{1,4}^2 = \frac{1}{2}\left(\frac{k + k_{f1}}{m_r + m_2} + \frac{k_{f1}}{m_1}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k + k_{f1}}{m_r + m_2} + \frac{k_{f1}}{m_1}\right)^2 - \frac{k k_{f1}}{(m_r + m_2)m_1}}. \tag{18}$$

In an analogous way, there is a possibility of determining frequencies related to the second eliminator (Fig. 7b), and these frequencies are:

$$\omega_{2,5}^2 = \frac{1}{2}\left(\frac{k + k_{f2}}{m_r + m_1} + \frac{k_{f2}}{m_2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k + k_{f2}}{m_r + m_1} + \frac{k_{f2}}{m_2}\right)^2 - \frac{k k_{f2}}{(m_r + m_1)m_2}}. \tag{19}$$

Introducing the parameters of the conveyor shown in Nomenclature, the following is obtained:

$$\omega_1 = 21.89, \quad \omega_2 = 21.90, \quad \omega_3 = 65.46, \quad \omega_4 = 142.18, \quad \omega_5 = 167.20. \tag{20}$$

The simulation results confirm the correctness of the above assumption. An example of the amplitude–frequency diagram for eliminators is shown in Fig. 8. Frequencies determined from simplified systems (Fig. 7) on the basis of Eqs. (18, 19) are of a very high accuracy in the case of higher frequencies (ω_4, ω_5) and coincide with the simulated values. Also, frequency ω_3, related to the whole system rotation—determined from (13)—is

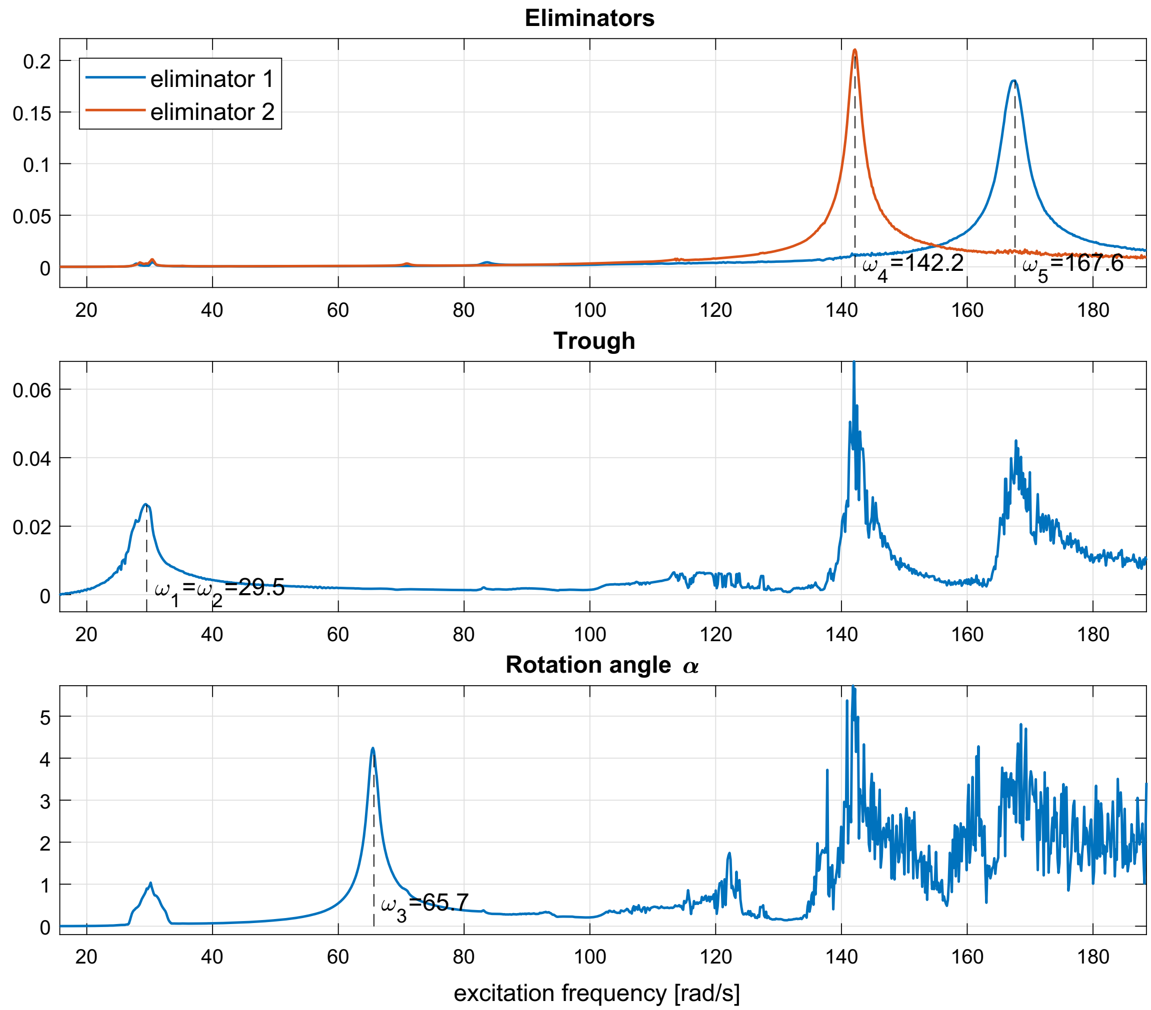

Fig. 8 Amplitudes of the resonance frequencies calculated by simulation for the eliminators, trough and rotation angle of the main body

the same as the simulated value. However, divergences occur in the case of lower frequencies (ω_1, ω_2). They are the result of the fact that the simulated trough system, together with eliminators, has five degrees of freedom (not two as in the simplified model) and in consequence vibrations in the other direction influence frequencies of the whole system. However, for efficient conveyor operation, the frequencies ω_4 and ω_5 are important, and the simplified model provides simple equations, easy for engineering applications. The accurate determination of ω_1 to ω_3 values is, in practice, not essential since these frequencies are usually significantly below the machine operation frequency.

4.3 Control of the transport direction and velocity

The control system must ensure the possibility of changing the feed transport direction as well as the stability of its velocity. The velocity of the feed displacement depends on its mass. Under normal working conditions, the feed mass can change in time, which would cause the transport velocity to change (see Fig. 9a). In order to prevent such a situation, the control system must adjust the excitation frequency appropriately, to stabilise the velocity.

However, in this case, the classic feedback control in the closed loop would be difficult to apply due to the highly oscillatory character of the machine motion causing its destabilisation. On account of this, the implementation of a simple (and the most popular in industry) PID controller does not find application here. In

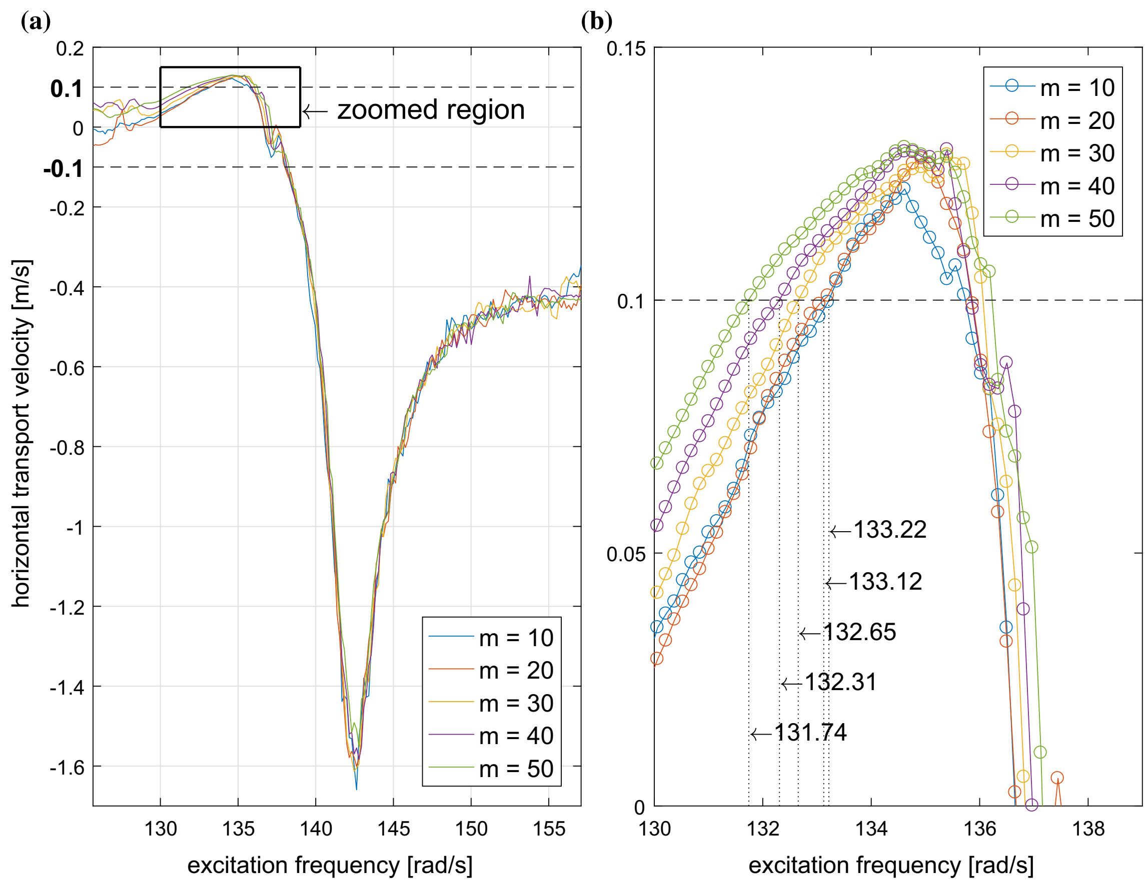

Fig. 9 **a** Left: dependence of the feed displacement velocity in dependence of the excitation frequency for various feed masses; **b** right: magnified fragment of the left figure

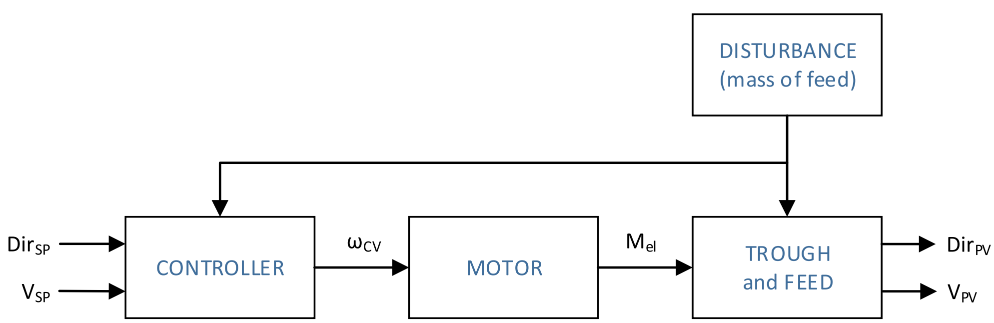

Fig. 10 Control system diagram

return, the application of the open loop feed-forward control, in which the feed mass is treated as a measurable disturbance, directly influencing the actual control, seems sensible.

A proposition of such a system is presented in Fig. 10. The feed-forward controller takes two setpoints as inputs: velocity of conveying V_{SP} and direction of conveying Dir_{SP} (left or right). Based on the setpoints and the disturbance value, the controller computes the control value ω_{CV} (frequency of the drive motor), which provides desired process values: velocity of conveying V_{PV} and direction of conveying Dir_{PV}.

The magnified fragment of diagrams showing the dependence of the feed transportation velocity in function of the excitation frequency for various feed masses is shown in Fig. 9b. It is clearly seen that—in this frequency range—the larger the feed mass, the higher the transport velocity. If the required velocity equals e.g. 0.1 m/s (marked in the figure by a dashed line), the coordinates of points of intersection of the velocity trajectory with this line can be read for various feed masses. It is also possible to read the frequency values necessary to ensure

Table 1 Values of the excitation frequency ensuring the displacement velocity of 0.1 m/s (movement to the right) and of − 0.1 m/s (movement to the left) for individual mass values

Mass (kg)	Frequency (rad/s) (move left)	Frequency (rad/s) (move right)
10	138.0102	133.2192
20	**138.0416**	**133.1250**
30	138.1358	132.6537
40	138.1515	132.3082
50	138.2615	131.7427

Bolded values were taken for simulations presented in Fig. 12

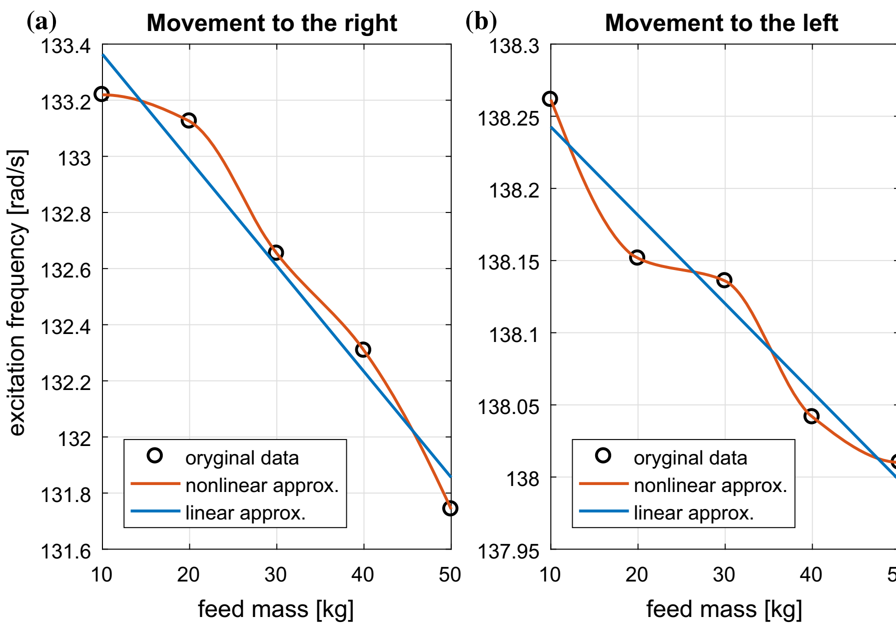

Fig. 11 Dependence between the feed mass values and the excitation frequency causing the constant velocity of the feed displacement and its nonlinear interpolation. **a** Feed movement to the right (with the velocity of 0.1 m/s). **b** Feed movement to the left (with the velocity of − 0.1 m/s)

the needed displacement velocities for individual masses. In a similar fashion, the displacement in the opposite direction—for the velocity − 0.1 m/s—can be found.

The frequency values found in the described way are listed in Tab. 1. Based on these measurements, one can find a continuous relationship between the feed mass value and the excitation frequency. This dependence can be approximated by simple linear regression or, for more accurate results, by nonlinear relation. The results of such approximations, using linear regression and nonlinear shape-preserving piecewise cubic Hermite interpolation, are shown in Fig. 11. Such defined dependency can be directly applied as a control law for the feed-forward controller. It should be emphasised that, in the further part of this study, nonlinear approximation was applied for simulations.

5 Results of the simulation of the control system

5.1 Change of the transport direction

The results of the system simulation in the case of changing the conveying direction (of feed mass 20 kg) from the right to the left are presented in the left column of Fig. 12. In the fourth second of the simulation, the control changes linearly from the initial value of $\omega_1 = 133.1$ rad/s to $\omega_2 = 138.04$ rad/s in 1 s (Fig. 12b). The control change is accompanied by the change of the conveying direction (seen in Fig. 12b). Before the direction change, the average feed velocity was app. 0.1 m/s. After this change, the average velocity oscillated around

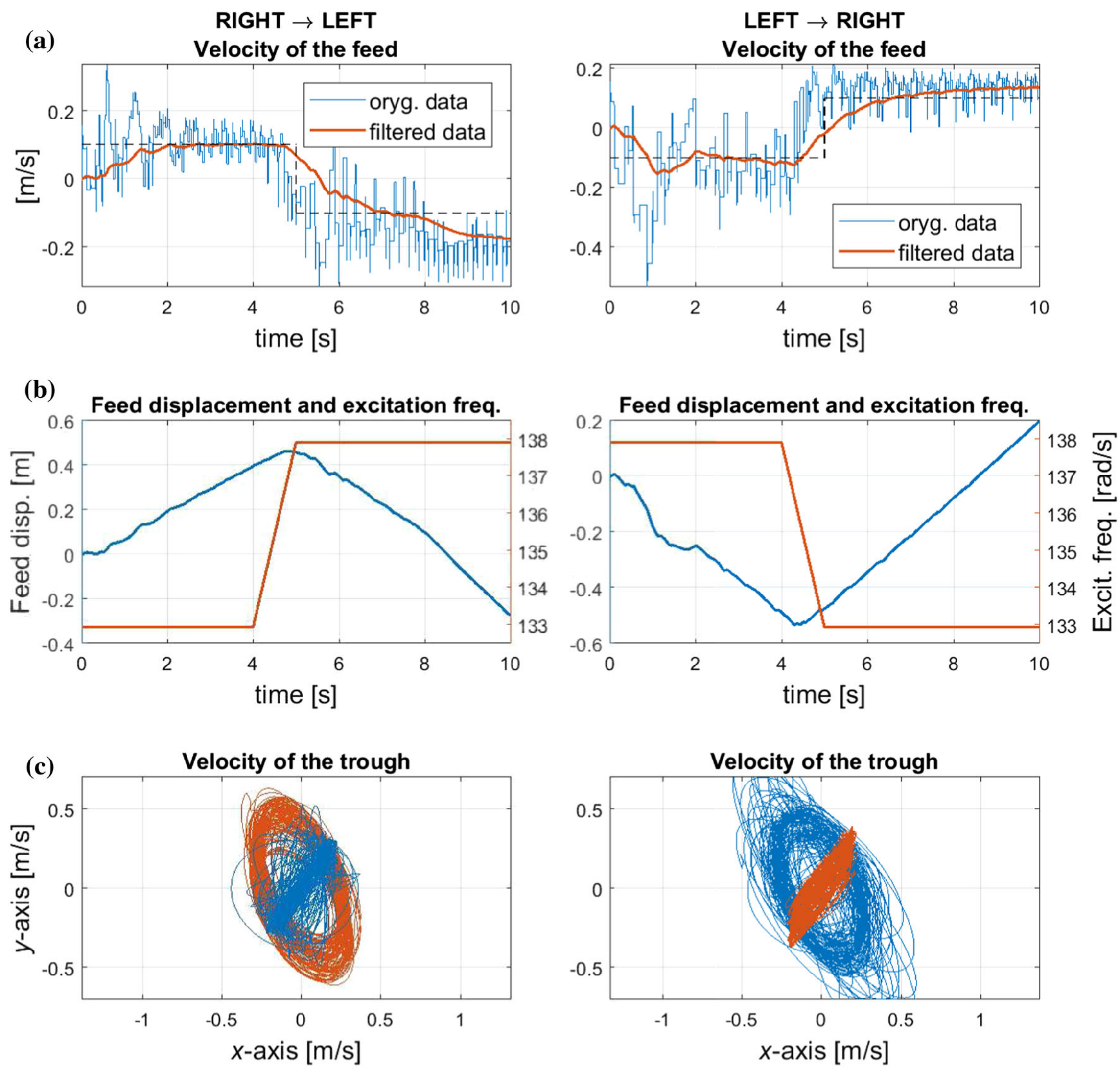

Fig. 12 Results of the system simulation in the case of changing the transport direction from the right to the left (left column) and from the left to the right (right column). **a** Upper: feed velocity in the horizontal direction and its average value; **b** middle: excitation frequency and displacement of the feed in the horizontal direction; **c** bottom: trough velocity in the horizontal versus vertical directions

-0.1 m/s value (Fig. 12a). The average velocity values were obtained by means of the first order recursive filter.

In the plot of the trough velocity (Fig. 12c), the pathway of the first half of the simulation time (i.e. from 0 to 5 s)—before the transport direction changes—was drawn in blue, and the pathway of the second half of the simulation time (i.e. from 5 to 10 s)—after the transport direction changes—in orange. Due to that, both vibration directions are clearly visible.

The results of the system simulation in the case of changing the feed transport direction from the left to the right are shown in the right column of Fig. 12.

5.2 Stabilisation of the transport velocity

A significantly different control problem is the stabilisation of the transport velocity in the event of outside disturbances, which in this case are mainly changes of the feed mass load.

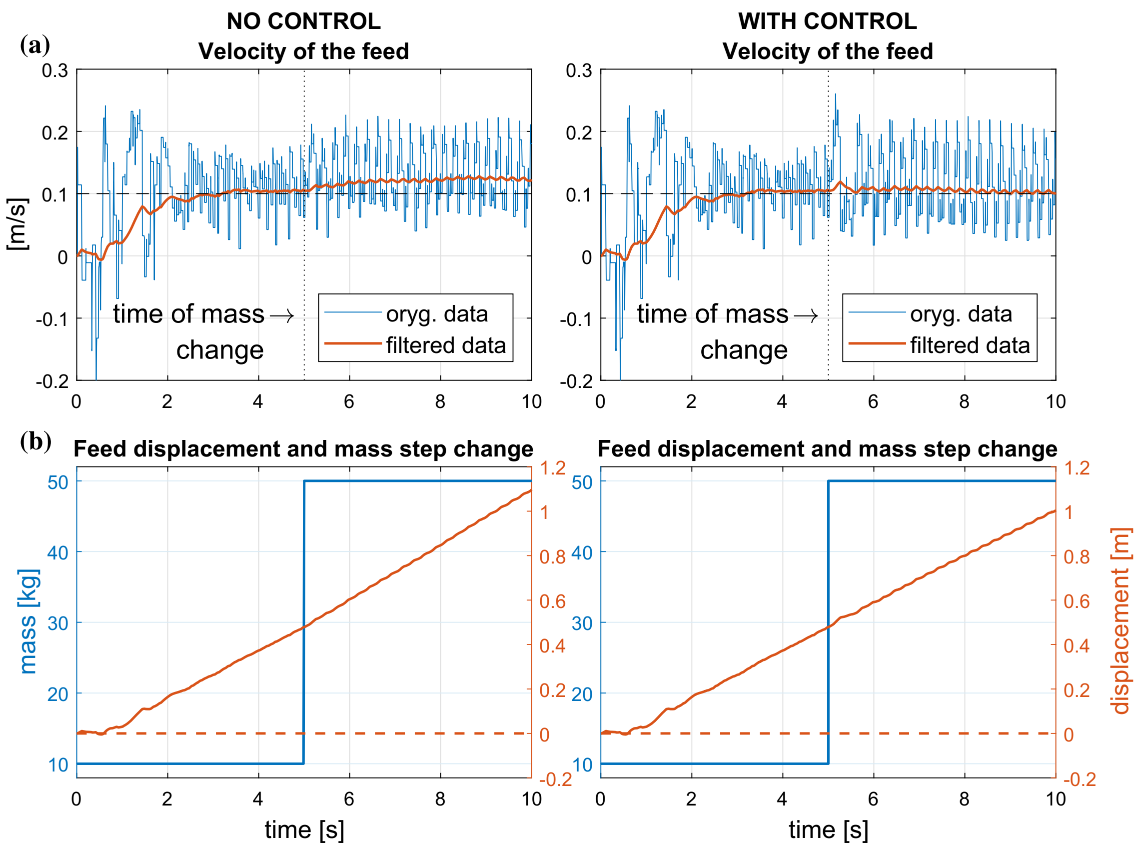

Fig. 13 Simulation results for the step change of the feed mass without (left) and with (right) the control system. **a** Upper: feed velocity in the horizontal direction and its average value. **b** Bottom: displacement of the feed in the horizontal direction and its mass

Simulation experiments based on the step change of the mass of the feed during the system operation were performed. In the first case, the influence of the step change of the feed mass (from 10 to 50 kg) on its velocity without the control system was shown. The excitation frequency was constant (133.22 rad/s). A drift of the average conveying velocity from the given value (0.1 m/s) starting in the fifth second of the simulation, i.e. in the moment of the step change, is shown in Fig. 13a (left column).

In the second case, the control system, by changing the excitation frequency (based on the dependence between the feed mass values and the excitation frequency found in the previous section), stabilises the velocity at the given level. It is seen in Fig. 13a (right column) that, due to the controller intervention (i.e. step change of the excitation frequency from 133.22 to 131.74 rad/s in the fifth second of the simulation), the average feed velocity is maintained at the given velocity 0.1 m/s, regardless of the mass step change.

The force in the eliminator 2 suspension and the force originating from the electro-vibrator projected on direction f_2, for the case when the eliminator 2 is operating at partial frequency are presented in Fig. 14. These forces are in the reverse phase, which is due to the character of the eliminator operation. In the case of a small amount of feed or lack thereof, these forces should be equal (Fig. 14b). However, in the case of a large amount of feed (which occurs after 5 s of the simulation—Fig. 14c), the force in the eliminator should be lower due to the energy dissipation in the feed. In order to obtain this, the control system changes the excitation frequency, adjusting it according to the rule described in Sect. 4.3. Diagrams for the other eliminator are analogous.

6 Conclusions

In the classic constructions of vibratory conveyors, the change of the transport direction is related to the necessity of stopping the vibrator and starting it in the opposite direction. Such operation requires passing the

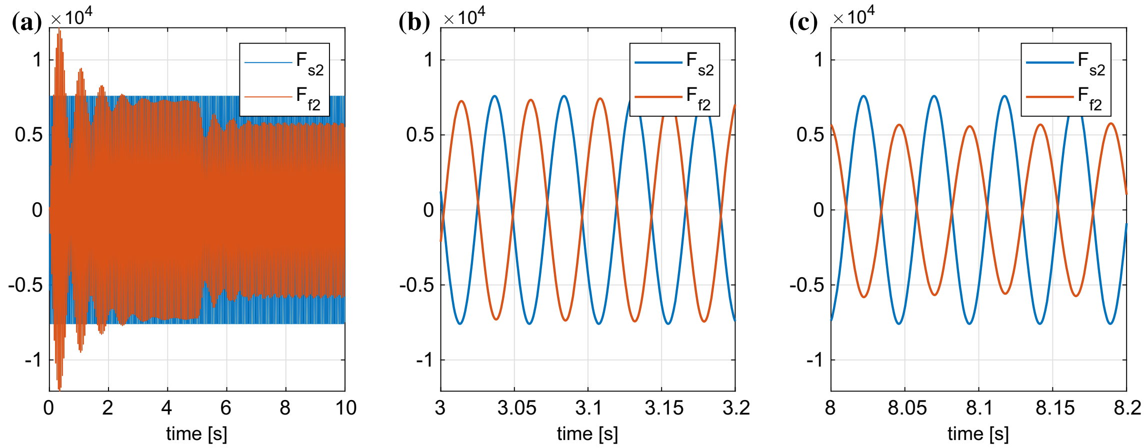

Fig. 14 **a** Left: simulation time series of force in the eliminator 2 suspension (F_{s2}) and force from the eliminator 2 projected onto the direction f_2 (F_{f2}); **b** middle: zoomed fragment—-between 3 and 3.2 s. of the simulation; **c** right: zoomed fragment—between 8 and 8.2 s. (after step mass change)

whole system through the resonance zone and takes a lot of time. It is also very disadvantageous, since it is always related to the occurrence of vibrations of an amplitude several times higher than in normal operation.

The concept of the authors' vibratory conveyor of a simple and reliable construction, presented in this paper, offers a solution to this problem. In this conveyor, the fast change of the transportation direction of the material is possible only by the change of the excitation frequency of the vibrator without the need of changing the direction of its rotation. Moreover, during the change of the direction there is no need to pass through any resonance zone, which improves the safety and reliability of the machine operation and significantly accelerates its reaction time for the change of direction.

A separate problem is the influence of the feed mass on its transport velocity. In the classic conveyor, the mass increase raises the transport velocity, which—from the point of view of industrial applications—is highly disadvantageous. On account of this, the control system and the stabilisation of the feed transport velocity are realised by the simple feed-forward controller utilising nonlinear dependency between the feed mass and excitation frequency, as proposed in this work. This controller—on the basis of continuously measuring the feed mass—adjusts the excitation frequency to assure the constant transport velocity of the material in both directions. For industrial use, the digital filtration of the mass measurements should be applied, and relevant dead zones of the controller insensitiveness should be introduced to avoid its excessive reaction to measurement disturbances.

Finally, it seems worth adding that, for the correct operation of the controller, continuous and reliable measuring of the feed mass is necessary, which in some industrial applications can be difficult. However, instead of measuring the mass, measurement of the force transmitted to the foundation by the machine can be carried out, on the basis of which the mass can be calculated, which in some cases can be simpler.

7 Nomenclature

The values of the simulation parameters.

$l_1 = l_2 = 0.5$ (m)	Distances from the mass centre to the flexible suspension
$h = 0$ (m)	Distance from centre of main mass to trough surface
$H = 0$ (m)	Distance from centre of main mass to the suspension hitch
$b_x = b_y = 50$ (Ns/m)	Coefficients of suspension damping in X and Y directions
$b_{f1} = 60$ (Ns/m)	Coefficient of suspension damping in eliminator 1
$b_{f2} = 50$ (Ns/m)	Coefficient of suspension damping in eliminator 2
$k_x = k_y = 75{,}000$ (N/m)	Coefficients of suspension stiffness in X and Y directions
$k_{f1} = 444{,}132$ (N/m)	Coefficient of suspension stiffness of eliminator 1 on its operation direction
$k_{f2} = 320885$ (N/m)	Coefficient of suspension stiffness of eliminator 2 on its operation direction
$m_1 = m_2 = 18$ (kg)	Masses of eliminators
$m_r = 120$ (kg)	Mass of the conveyor trough
$m_w = 5$ (kg)	Mass of the inertial vibrator
$J_r = 25$ (kg/m^2)	Mass moment of the trough inertia
$J_w = 0.1$ (kg/m^2)	Mass moment of the vibrator inertia
$\phi = variable$	Rotation angle of inertial vibrator
$\alpha = variable$	Rotation angle of the main mass
$\beta_1 = 45°$	Angle at which eliminator 1 is suspended
$\beta_2 = 135°$	Angle at which eliminator 2 is suspended
$R = 0.05$	Restitution rate
$\mu = 0.4$	Friction coefficient between feed elements and the trough
$p = 1$	Hertz–Stajerman coefficient for flat surfaces
$k_s = 10^8$ (N/m)	Feed stiffness coefficient
$e = 0.086$ (m)	Eccentricity of the inertial vibrator
$\omega_{ss} = variable$ (rad/s)	Synchronous frequency of the drive motor
$\omega_{\max} = variable$ (rad/s)	Frequency of break-over of the drive motor
$M_{\max} = 49.4$ (Nm)	Break-over torque of the drive motor

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