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SPECIAL

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Effect of periodicity of railway track and wheel–rail interaction on wheelset–track dynamics

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Abstract The paper deals with the investigation of some phenomena which are essential for dynamic interaction of a wheelset or bogie with the railways track. The track is modeled as discrete–continuous system composed of rails, fastening systems and sleepers interacting with ballast by means of viscous–elastic elements. Transient and stationary problems are investigated. The stationary periodic problems are connected with wave propagation in stopping bound, passing bound and mistuning. In this paper, a review of selected problems and exemplary results of simulation and experimental investigations is given supplemented by a few results of analysis and simulation.

Keywords Moving load · Resonance · Periodicity · Passing band · Mistuning

1 Introduction

The safety, comfort and cost-efficiency of maintenance of railway operation depend strongly on the quality of the wheelset–track interaction. Design of the track for modern high-speed railway requires deep understanding of the behavior of wave propagation phenomena generated by rail vehicles, responsible for noise being emitted, generation of rail corrugation, wheel polygonization, damage of the ballast and wear of a wheel–rail system [1]. Some of these phenomena, which originate from the interaction between the vehicle and track for low as well for high speeds, are complicated and thereby not commonly known. The majority of investigations are focused on the effects of vertical dynamics, where only the vertical relative motion of wheel and rail in the contact are considered. Furthermore, the problem is often simplified assuming a constant stiffness of the contact, i.e., a linear relation $F_N = c_{\mathrm{lin}}\delta$ between the normal force and the relative vertical motion of wheel and rail, and constant velocity of the contact, which is equal to the running speed of vehicle. We will show later that such simplifications are not admissible in real cases. For a better understanding of fundamental phenomena, some

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new results will be supplemented by an overview of previous studies. The formation of corrugation as periodic shape of the rails' running surface will be discussed as one of an important kind of tracks damage. The study of the stationary periodic problems for tracks of classical or innovative construction will be complemented by an analysis dynamics of a periodic track in the transition zone as an example of connection between bridge and railway track possessing a double fastening system and thereby a stiffer coupling between the rails and the sleepers.

In the beginning, we recall the problem of an oscillating load moving along a beam supported by an elastic foundation as a model of wheel–rail interaction, in particular, when the sleeper spacing or corrugation are taken into account. Then, the results of measurement will be given which show that plastic deformation is a fundamental reason of initiation and development of corrugation.

2 Moving oscillating load

Let us consider a beam which is modeled according to the Bernoulli–Euler, Rayleigh or Timoshenko theory. The beam is supported by a Winkler foundation and subjected to a discrete oscillating load, which moves along the beam. This system will be used as the base for the track modeling.

Such problem was formulated for the case of Bernoulli–Euler beam model by Mathews [2] and after about 30 years properly solved in paper [3]. The scheme of the system is given in Fig. 1. Generalization of the solution for the Rayleigh and the Timoshenko models can be found in [4]. Selected cases of the problem of a beam on an elastic foundation subjected to concentrated, moving loads has been studied by several authors, but [4] is the first paper where the problem of the Timoshenko beam model was generally discussed and solved.

The equations of motion for a Timoshenko beam, which is supported by viscoelastic foundation, are as follows:

$$EI\varphi_{,xx} + k'AG(w_{,x} - \varphi) - (m/A)I\varphi_{,tt} = 0 \tag{2.1}$$

$$k'AG(w_{,xx} - \varphi_{,x}) - mAw_{,tt} - hw_{,t} - cw = -(F_0 + F\cos\omega t)\delta(x - Vt) \tag{2.2}$$

where E is Young's modulus, I is the equatorial moment of inertia of cross-section, w is the vertical displacement, w_x denote derivative in respect to x, φ is the rotation angle, φ, xx denote second derivative in respect to x, k' is the shear coefficient, A is the cross-section area, G is the shear modulus, m is the linear mass density, h is the damping coefficient of the foundation, c is the elasticity of the foundation, V is the velocity of load motion, F_0 is the constant part of the external force, and F is the amplitude of the oscillating part of external force.

Describing the set of equation of motion (2.1), (2.2) in the moving system of coordinates connected with moving force, we obtain a system of equations with V as parameter which is convenient to solve using Fourier transformation. Looking for the solution in the form of traveling wave determined by wave number k, angular frequency ω and speed v in the set of coordinates $(\xi,\ t)$ moving with the velocity V, $\xi = x - Vt$, which has the form:

$$w(\xi, t) = W \exp \mathrm{i}(k\xi - \omega t), \quad kv = \omega, \quad v = v_x - V, \tag{2.3}$$

where v_x is the wave velocity in the inertially fixed system (x, t).

By applying a proper set of boundary conditions given in [3,4], a configuration of regions representing qualitatively similar solution is obtained. The configuration depends strongly on relation between longitudinal V_E and transversal V_G wave in the beam. Exemplary configuration for the special case of Timoshenko beam $(V_G = 0.5)$ is shown in Fig. 2.

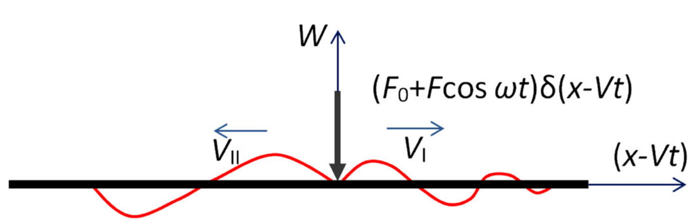

Fig. 1 Scheme of the beam system subjected to moving and oscillating, concentrated force

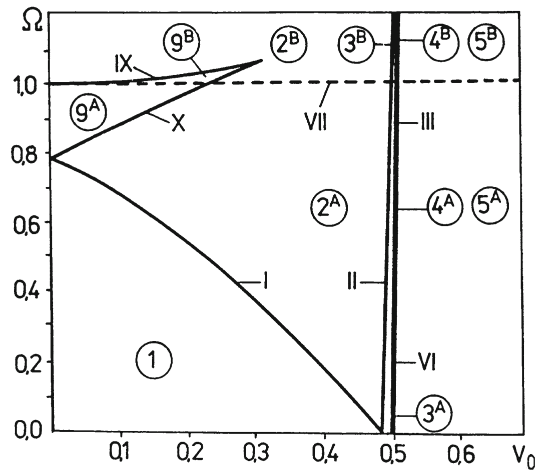

Fig. 2 Configuration of regions with qualitatively similar solution in the speed–frequency plane for the Timoshenko beam, $V_G = 0.5$

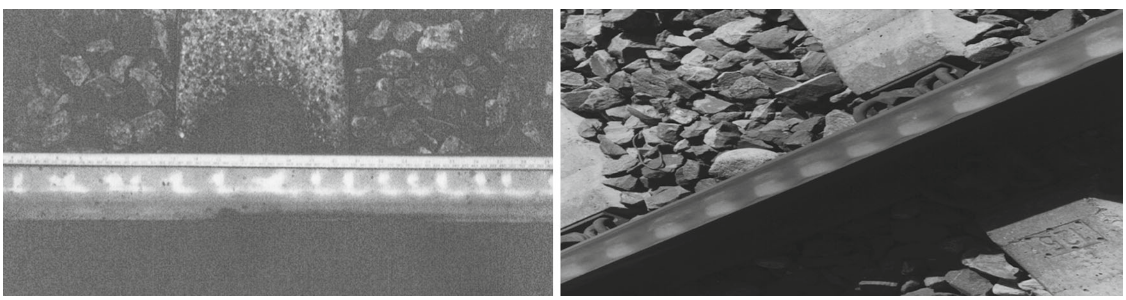

Fig. 3 The rail corrugation (*left*) and sleep waves on rail (*right*)

Here, V_0, V_G and Ω are normalized velocities and angular frequency, which are defined in the following way:

$$\Omega = \omega\sqrt{\frac{m}{c}}, \quad Vo = V\sqrt[4]{\frac{(m)^2}{4cEI}}, \quad V_G^2 = k'GA^2/I \cdot c. \tag{2.4}$$

We can see that the configuration of boundaries between different regions is relatively complicated. All boundaries of regions are related to resonances. Taking into account that the running speed of $V = 100 - 300$ km/h corresponds to dimensionless speed value of about $V_0 = 0.1 - 0.3$, we can state that there are three resonance frequencies and four qualitatively different solutions. In the pure elastic case, the solution consists of four decaying waves only in first region. It is easy to point out that in the case of corrugation due to high frequency of the waves generated by the wheel–rail interaction do not decay.

3 Corrugation of rails: measurement and simulation of initiation and development

Rolling surfaces of rails shows a number of different shapes irregularities. There are a few hypotheses of initiation and development of various irregularities. Some of them which are periodic and appear on straight rails are called the corrugation. The majority of researchers has developed simulation procedures based on frictional wear [1,5]. There exist various dynamical phenomena of wheel–rail (train–track) interaction caused by influence of the periodicity of track, Fig. 3. The rail corrugation growth is influenced among others by dynamical stiffness of the fastening system, passing bands in the track as periodic structure or braking zones and residual stress distribution in the railhead.

Braking zones and curved track shape are associated with rail roughness which is called slip waves, as shown in the right image of Fig. 3. The problem of wavy shape of the wear of rails is very important; therefore, it is investigated in many scientific and technological centers.

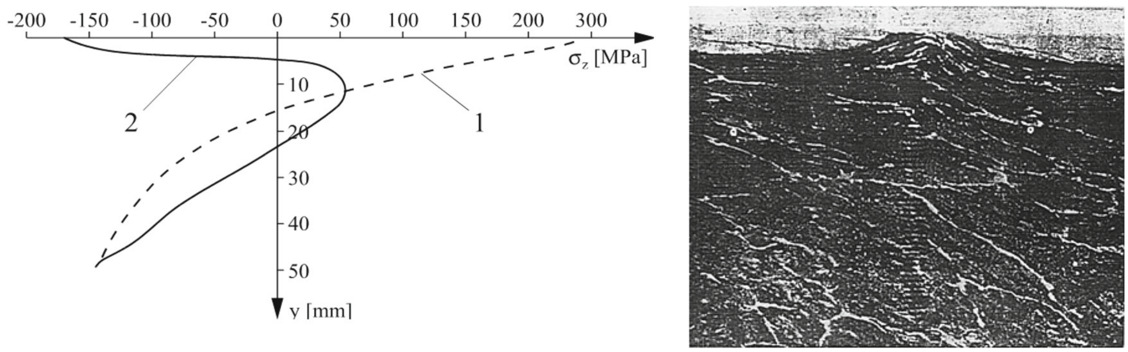

Fig. 4 Distribution of residual stress in the railhead in new rail (1), in rail after few years in service (2) and flow of week material to the surface (as initiation of corrugation)

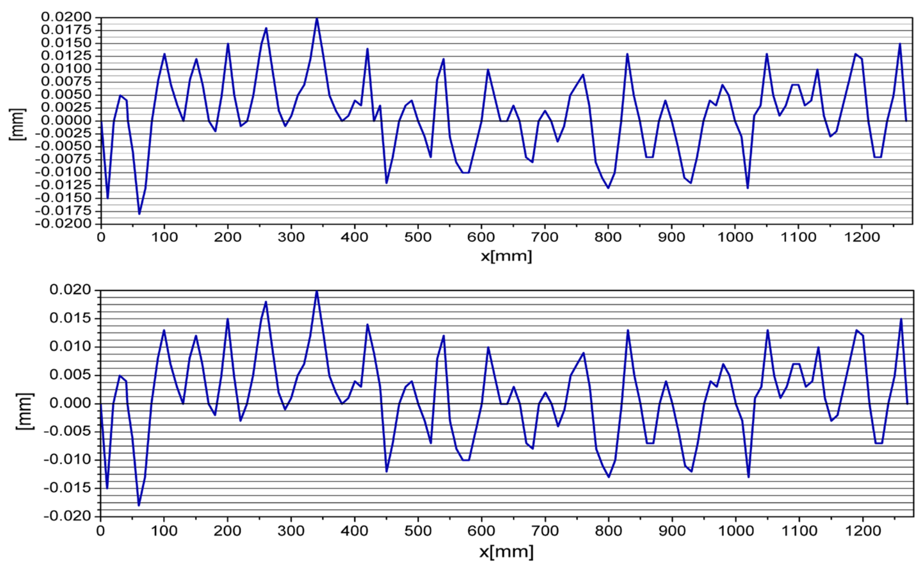

Fig. 5 Comparison of the corrugation shape and corresponding time of traveling ultrasonic wave (adequate to residual stress level)

Due to straightening, a residual stress is present in the rail. Its distribution is described by the curve (1)—on left diagram of Fig. 4. During service, this initial distribution changes to a distribution described by the curve (2) which is typical for a rail after several years in service. This is related to material flow, as shown in the right image of Fig. 4. After sufficient level of rolling load carried by the rail, the level of compressive stress reaches a critical value, which leads to an instability of the stress distribution and induces the flow of the weak material to the surface as is shown in Fig. 4 (on the right hand side), initiating formation of corrugation.

The measured residual stress distribution in the rail and the rail roughness are compared in Fig. 5. A correlation can clearly be seen confirming the hypothesis that plastic deformation lead to the wavy shape. Using the above confirmation, we can simulate the wheel–rail contact forces for verification the presented hypothesis of the corrugation formation (Fig. 6).

The rate of rail corrugation growth according to the investigations known from the literature [7] is independent on different sets of wheel irregularities what support the study using completely round wheel, [8]. The simplified theory of wheel–rail interaction shows great variation of contact force strongly dependent on vehicle speed also when longitudinal oscillation of contact point is neglected. For the typical value of contact stiffness the resonance appear at the speed of 40–60 km/h. In Fig. 7, simulation results for $V = 10$ km/h, $V = 50$km/h and $V = 100$km/h are shown.

We can state that for the running speed $V = 10$ km/h the fluctuation of the contact force around the static value is very small and can be neglected. At the speed of $V = 50$ km/h, the resonance frequency is reached and loss of wheel-rail contact occurs (bouncing with the max. force of about 3 times the static load). The

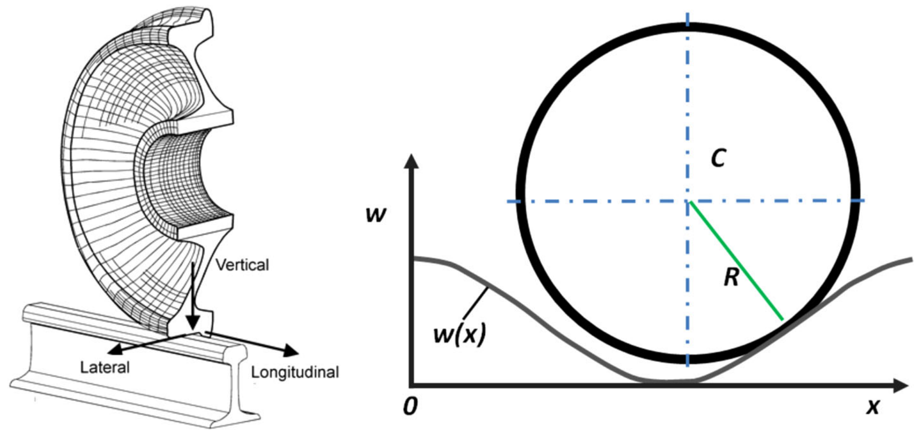

Fig. 6 The wheel–rail contact and unproportional scheme making evident longitudinal fluctuation of the contact point

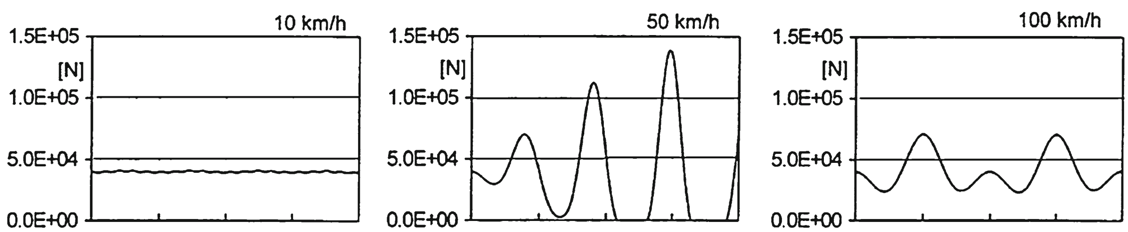

Fig. 7 Contact forces at various speed in the case of wheel without imperfections and sinusoidal rail corrugation with amplitude $a = 0.010$ mm and wavelength $\lambda = 50$ mm

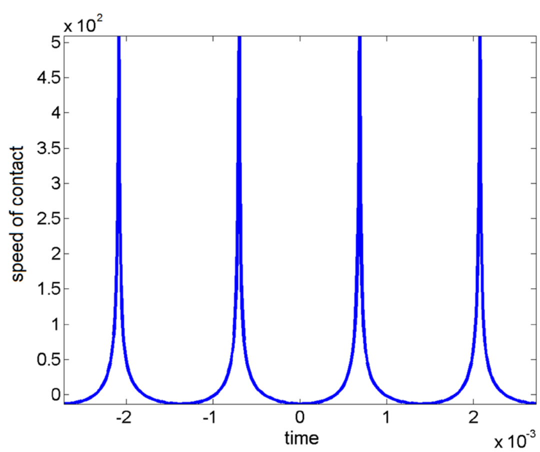

Fig. 8 Actual speed of contact point [m/s] versus time [s] $\times 10^{-3}$ during rolling of a wheel along a corrugated rail ($a = 0.03$ mm) at the running speed $V = 36$ m/s

fluctuation of contact force decreases with further increasing of the running speed. At the speed $V = 100$ km/h, the maximum dynamical force reaches a value of about 1.7 times the static force. For more precise evaluation of the contact force, the wheel plate stiffness and deflection of the rim ought to be taken into account, see [9]. In the further approximation, lateral and longitudinal displacements of the contact point could be included [10, 11]. For a constant running speed of the vehicle, extreme vertical or horizontal accelerations and very high velocities of contact point motion can be observed, also for moderate value of speed, i.e., $V = 36$ m/s see Fig. 8. It is interesting and important, particularly, from the view point of wave generation that the horizontal speed of the moving contact point is comparatively high, cf. Fig. 2.

In the case of load described by expression (2.2), a few resonance boundaries will be crossed. Response of beam subjected to a force moving with periodically fluctuating velocity is not easy to determine, even numerically. The difficulties are connected with non-stationary motion due to varying speed. Calculations concerning short-term dynamics are very sensitive to small perturbations of the surface geometry. As wear proceeds, i.e., the contact surfaces evolve, we observe considerable changes in normal forces, tangential forces and relative tangential speeds. It turns out that certain patterns may amplify themselves—depending on traveling speed and other parameters describing vehicle and track. Unfortunately, the problem is very complex, so that a direct numerical approach is expensive. For that reason, several simplifications were introduced in simulations. Some approximation of solution of the problem will be given in next paper.

4 Dynamics of track with periodic sleeper spacing

The basic qualitative feature of the classic railway track is the periodicity of sleeper spacing. The sleeper spacing influences the periodicity of viscoelastic supports coefficients and additional mass of sleepers with rotational inertia. In the case of classic periodically supporting sleepers, one can observe passing bands in the frequency of moving and oscillating forces. The solution method which allows determining the stopping and passing bands in the case of track, proposed in [12], is based on direct application of the Floquet's theorem. The steady-state system response is determined for a moving excitation by a force consisting of a constant and a periodic part (2.2), but the equation of motion we assume in simplified form using Bernoulli–Euler beam model

$$EI\, w_{,xxxx} + m w_{,tt} = (F_0 + F\cos\omega t)\delta(x - Vt) \tag{4.1}$$

The equation of motion is completed by interface conditions at the supports which depend on the model assumed, e.g., for the railway track condition of continuity (4.2) and equilibrium of vertical forces (4.3) are required:

$$w\,(nl+, t) = w\,(nl-, t)\,; \quad w_{,x}\,(nl+, t) = w_{,x}\,(nl-, t)\,; \quad w_{,xx}\,(nl+, t) = w_{,xx}\,(nl-, t)\,; \tag{4.2}$$

$$w_{,xxx}\,(nl-, t) - w_{,xxx}\,(nl+, t) = R\,(nl, t)\,/EI \tag{4.3}$$

where n denote the number of support, $l+$ and $l-$ denote the left and right end of the cell, R is the reaction of the fastening system.

To overcome the negative propriety of passing bands occurring in the ballasted track with classic sleepers due to the rotation of the rails in the classic fastening system, we will apply a new type of sleepers possessing double fasteners of each rail.

The main advantage of sleepers shown in Fig. 9 in comparison with the classic type is their higher mass and moment of inertia besides the greater stiffness of the fasteners system. Above features are suitable for application in the transition zones, when the foundation stiffness change rapidly or adaptation in the high-speed lines on ballast (Figs. 10, 11).

For the comparison of the behavior of a few sleepers in a ballasted track in the transient zone just behind a rigid base, the bogie is used moving with the speed $V = 72$ km/h. The sleepers are spaced at distance 0.65 m, the bogie possessing two wheelsets located at distance 2.5 m. It is visible that the largest displacement occurs at the first sleeper in both cases. The difference is significant. The displacement of sleeper with double fasteners (Fig. 12) is about 35 % smaller than the displacement of the classic sleeper but the maximal angle of rotation is three times greater, while the rotation of rail is considerably smaller.

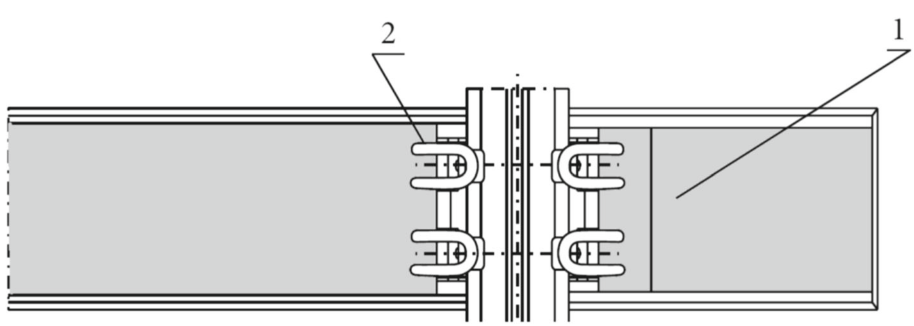

Fig. 9 The stiff sleeper (*1*) – possessing double fastening system of rail (*2*)

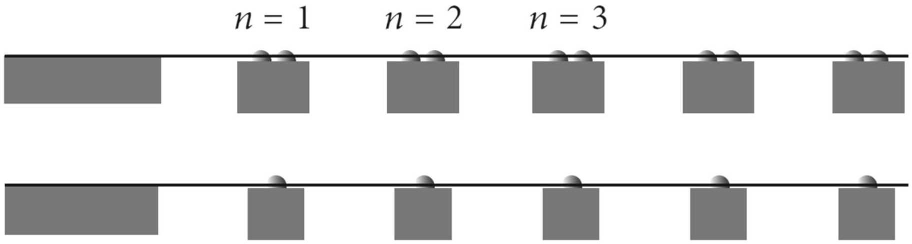

Fig. 10 Scheme of track in transition zone with two types of sleepers and fastening systems

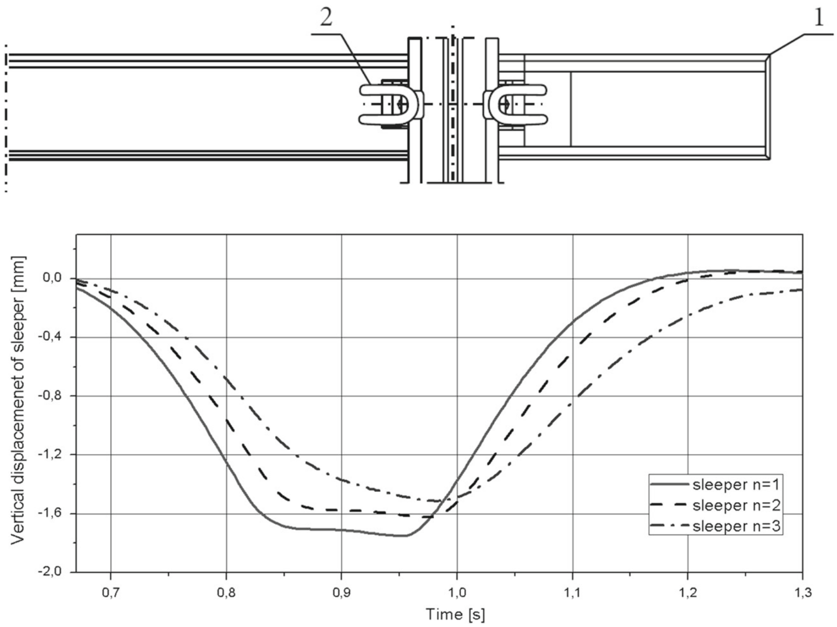

Fig. 11 Vertical displacement of the classic sleeper No. 1, 2 and 3 in the zone behind the rigid base (Fig. 10) during the passing of the first wheelset of bogie with a speed $V = 72\,\text{km/h}$

In the classic transient zone, a significant effect is settlement of the ballast. The parameter which is very important for the ballast deterioration [13] and track settlement [14] is the velocity of the sleeper motion, which is dependent on the elasticity of the rail pad and of the sleeper support, i.e., whether the sleeper is supported by an under-sleeper pad and, if so, how stiff this pad is. The sleeper velocity in the case of vehicle speed of $V = 72\,\text{km/h}$ is not of great importance, but in the case of vehicle speed of about $V = 300\,\text{km/h}$, it can be critical.

4.1 Changes of periodic sleeper spacing

Beside of the problem of transition zones in the ballasted track, there are other important problems in railway engineering connected with vibration resonances of rails known as the pinned–pinned mode and the phenomenon of wave propagation in the passing bands. In both cases, the periodicity of the track structure plays an important role. Therefore, it seems to be significant to modify this feature by changing the periodicity of the track. The application of the sleepers with double fasteners slightly changes the classic periodicity of supporting system. To strengthen the effect, the sleepers spacing can be changed. As an example, the interaction between a bogie and a track using three different types of sleeper spacing has been investigated. As the first step, let us consider the shape of the track which is shown in Fig. 13.

The results provided by this investigation providing results clearly show that an observable improvement of the dynamic property occurs at high running speed only. The effect of non-constant sleeper spacing can be achieved by arranging the sleepers non perpendicular to the rails. One of examples of such type of the track is shown on right hand side of Fig. 14. Due to much greater lateral resistance the track with "V-shaped"

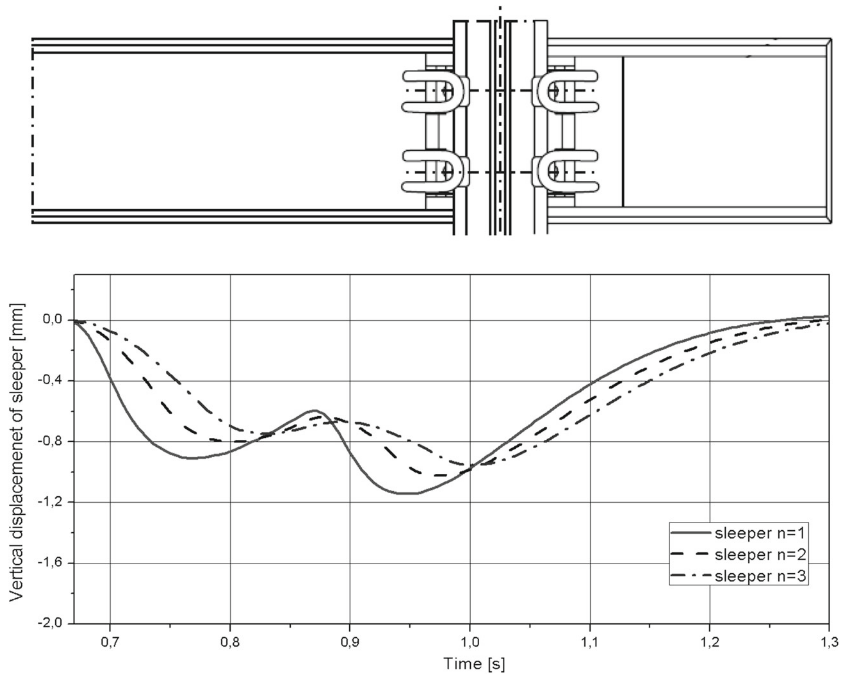

Fig. 12 Vertical displacement of sleepers the with double fasteners system No. 1, 2 and 3 in the zone behind the rigid base (Fig. 10) during the passing of the first wheelset of bogie with a speed $V = 72$ km/h

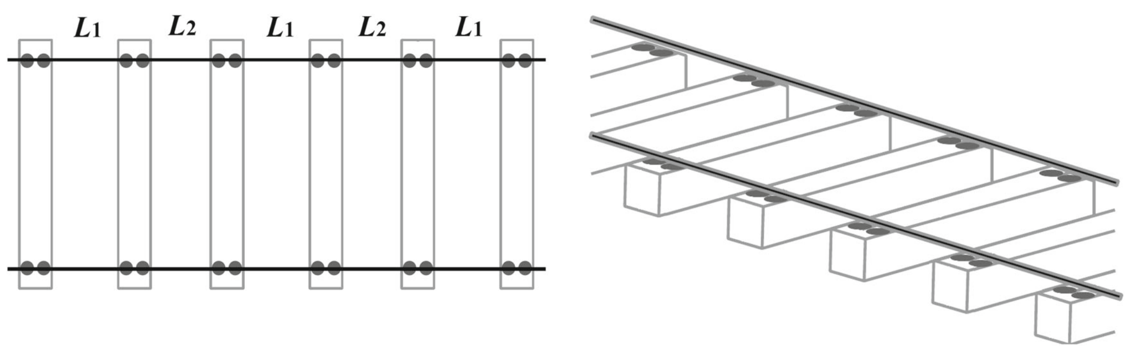

Fig. 13 The track with double fasteners spaced bi-periodically

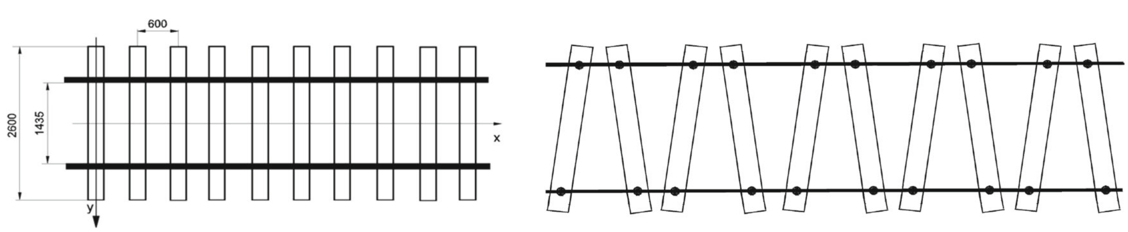

Fig. 14 The view of the track with classic spaced sleepers and the track with "V- spaced" sleepers

sleepers, they can be about 10 % shorter than sleepers the classic track shown of the left hand side of this figure. The simulation of wheelset–track dynamics for these two types of track, i.e., the classic track and V-sleeper track, shown in the Fig. 14, was done using the same parameters of the fastening system in both cases. The investigation was done for various running speeds. For comparison, the vertical displacement under both wheels for $V = 160$ km/h are illustrated in Fig. 15.

The results of simulation shown in Fig. 15 were obtained using the Simpack code supplemented with own programs developed for this simulation. For the vehicle, a single wheelset was assumed which is connected to

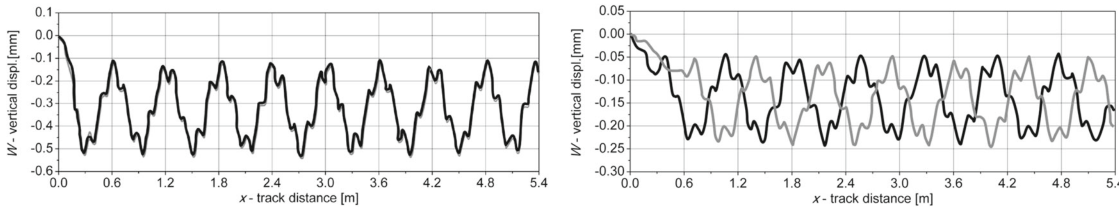

Fig. 15 Comparison of vertical displacement under wheel rolling along the track with classic and V-shaped sleepers at the speed 160 km/h

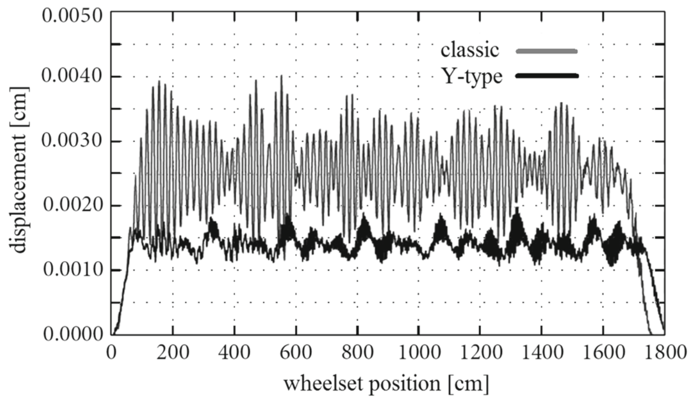

Fig. 16 Comparison of vertical displacement of rails 1.2 m in front of the first wheelset of the bogie for classic track and track with "Y-shaped" sleepers [16]

a bogie frame by springs and dampers representing the primary suspension. Furthermore, the stiffness of the wheel plate is taken into account by connecting the rigid wheel rims to the axle by a flexible element.

As follows from results obtained for a running of speed $V = 160$ km/h the maximal vertical displacement under the wheel rolling along the track with classic sleepers is about 20 % greater as in the case of wheelset rolling along the track with "V-shaped" sleepers. It is visible that the variation of displacements in case of "V-shaped" sleepers is considerable smaller than in the case of classic track. For the qualitative comparison of an exemplary result obtained in [16] for the case of classic track and the track with "Y-shaped" sleepers, we illustrated vertical displacements of rails in front of bogie for both cases shown in Fig. 16.

The results obtained in the case of track with "V-shaped" sleepers are partly similar as in the case of track with "Y-shaped" sleepers made of steel, [16]. Indeed, the difference between the displacements for the classic track and the case of track with "Y-shaped" sleepers is greater than difference for the case of the classic track and track with "V-shaped" sleepers, but the cost of construction of the track with "V-shaped" sleepers seems to be considerably lower than cost of the track with "Y-shaped" sleepers.

5 Conclusions

The problems discussed in the paper are devoted to several phenomena of the track dynamics. The main problem is related to the periodicity of the track structure, which is responsible for the resonances of vehicle–track interaction. The other periodicity occurs as a phenomenon of the corrugation formation. The overcome of the first problem is possible possessing knowledge of the phenomena of wave propagation in a periodic structure, while the second problem—ought to be eliminated on the basis of some hypothesis of initiation and generation of corrugation. These phenomena were discussed in the paper. The dynamic behavior of a classic track was supplemented by some possible changes of the sleeper spacing and exemplary results of simulation and experimental investigations are given.

An alternative hypothesis of corrugation formation as given in [17] is discussed. A few problems important for understanding of some wave propagation phenomena are only signalized and refer to the source in the literature.

References

1. Popp, K., Werner, S. (eds.): System dynamics and long-term behaviour of railway vehicles, track and subgrade. vol. 6. Springer Science & Business (2013)
2. Mathews, P.M.: Vibration of beam on elastic foundation. Z. Angew. Math. Mech. **38**, 105–115 (1958)
3. Bogacz, R., Krzyżynski, T.: On the Bernoulli–Euler beam on viscoelastic foundation under moving oscillating load. In: IFTR Polish Academy of Sciences, vol. 38 (1986)
4. Bogacz, R., Krzyżynski, T., Popp, K.: On the generalization of Mathews problem of the vibration of a beam on elastic foundation. Z. Angew. Math. Mech. **69**(8), 243–252 (1989)
5. Hempelman, K.: Short pitch corrugation on railway rails—a linear model for prediction. In: VDI-Fortschritt-berichte, No. 231, Reihe 12. VDI-Verlag (1995)
6. Bogacz, R.: Residual stresses in high-speed wheel/rail system; Shakedown and corrugations. In: Koanadis, A.N., Balkema, A.A. (eds.) Proceedings of the 1-st European Conference on Steel Structures "EUROSTEEL'95", Athens, pp. 331–343 (1995)
7. Johansson, A., Nielsen, J.C.O.: Out-of-round railway wheels—wheel-rail contact forces and track response derived from field tests and numerical simulations. J. Rail. Rapid Transit. Part F **217**(2), 135–146 (2003)
8. Bogacz, R., Kowalska, Z.: Computer simulation of the interaction between a wheel and a corrugated rail. Eur. J. Mech. A Solids **20**, 673–684 (2001)
9. Bogacz, R., Konowrocki, R.: On the new effects of wheel–rail interaction. Arch. Appl. Mech. **82**, 1013–1023 (2012)
10. Bogacz, R., Frischmuth, K.: On some new aspects of contact dynamics with application in railway engineering. J. Theor. Appl. Mech. **50**(1), 119–130 (2012)
11. Konowrocki, R., Bajer, C.I.: Friction rolling with lateral slip in rail vehicles. J. Theor. Appl. Mech. **47**(2), 275–293 (2009)
12. Bogacz, R., Krzyżyński, T., Popp, K.: Application of Floquet's theorem to high-speed train/track dynamics. In: Advance Automotive Technologies, ASME Congress, pp. 55–61 (1995)
13. Sato, Y.: Japanese studies on deterioration of ballasted track. Veh. Syst. Dyn. **24**, 197–208 (1995)
14. Savidis, S.A. et al.: 3D Simulation of Dynamic Interaction Between Track and Layered Subgrade. pp. 431–450. Springer, Berlin (2003)
15. Bogacz, R., Czyczuła, W., Konowrocki, R.: Influence of sleepers shape and configuration on track–train dynamics. Shock Vib. **2014**, 393867-1-7 (2014). doi:10.1155/2014/393867
16. Bajer, C., Bogacz, R.: Dynamics of "Y-Type" Track, XXI Symposium—Vibration in Physical Systems—Poznań-Kiekrz, pp. 3–7 (2004)
17. Hempelmann, K., Hiss, F., Knothe, K., Ripke, B.: The formation of wear patterns on rail tread. Wear **144**, 179–195 (1991)