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The efflux problem: how hydraulics became divorced from hydrodynamics

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Abstract
The efflux problem deals with the outflow of water through an orifice in a vessel, the flow over the crest of a weir and some other ways of discharge. The difficulties to account for such fluid motions in terms of a mathematical theory made it a notorious problem throughout the history of hydraulics and hydrodynamics. The treatment of the efflux problem, therefore, reflects the diverging routes along which hydraulics became an engineering science and hydrodynamics a theoretical science out of touch with applications. By the twentieth century, the presentation of the efflux problem in textbooks on hydraulics had almost nothing in common with that in textbooks on hydrodynamics.

1 Introduction

Hydraulics and hydrodynamics were considered for a long time synonyms for the science dealing with the flow of water. Johann Bernoulli and his son, Daniel Bernoulli, titled their famous treatises *Hydraulica* (Bernoulli 1742) and *Hydrodynamica* (Bernoulli 1738), respectively. They did not distinguish between hydraulics as a practical and hydrodynamics as a theoretical science. Beyond their rivalry about priority, they both represent the beginnings of rational fluid mechanics (Truesdell 1954). At the dawn of the twentieth century, the situation had changed. Hydraulics was an engineering science dealing with applied flow problems, whereas hydrodynamics addressed the mathematical and physical principles of fluid flow with little concern for applications. To quote a quip among modern fluid dynamicists, they were divided "into hydraulic engineers who observed things that could not be explained

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and mathematicians who explained things that could not be observed" (Gad-El-Hak 1998, 181).

How and when did hydraulics become divorced from hydrodynamics? In previous studies, I focused on turbulence as a phenomenon that eluded an analysis based on first principles and thus opened the door for empirical approaches. We encounter turbulence in a host of flow problems as the culprit that prevented closed solutions (Eckert 2019, 2021, 2022). Yet, turbulence has not been the only cause for the widening gulf between hydrodynamics and hydraulics since the time of the Bernoullis. Here, I focus on a problem where turbulence may not be blamed as the culprit: the efflux problem. Consider the simplest case: a circular hole (cross section a) in the bottom of a cylindrical bucket (cross section A) filled with water (height h). What is the rate of the water discharge? Until the mid-eighteenth century, the efflux problem may be regarded as *the* paradigmatic problem of hydrodynamics which sparked the development of "rational fluid mechanics". Yet, this breakthrough did not lay the efflux problem to rest. It rather shifted its character due to the difficulties to solve the hydrodynamic equations under the boundary conditions of the flow through an orifice or over a weir. Thus, the efflux problem may be regarded as a probe into the relationship of theory and practice in fluid mechanics. Even long after the Bernoullis, it was considered as deficient of a satisfactory solution for practical applications.

I start with "Torricelli's law" (1644) and briefly review the approaches of Johann and Daniel Bernoulli, Leonhard Euler and others who provided the theory for the discharge through an orifice that may be called classical in retrospect (Sect. 2). By the end of the eighteenth century, this theory was disputed in France and subjected to further elaboration (Sect. 3). The opening gap between theory and practice is further illustrated by German hydraulic treatises around 1800 which followed the role model of French hydraulics (Sect. 4). Then, I explore the semantics of the terms with which theoretical and applied investigations of fluid motions such as the efflux and discharge of water were labeled throughout the nineteenth century (Sect. 5). This sets the stage for the appropriation of the efflux problem in the nineteenth century by engineers who developed methods of measurement and determined the discharge in a variety of configurations for which theoretical solutions could be achieved only in terms of empirical coefficients (Sect. 6). I conclude this study with an outlook to the twentieth century when modern fluid mechanics served as an umbrella for hydraulics and hydrodynamics—without resolving their historic divorce (Sect. 7).

2 From Torricelli to Euler: the origins of the classical efflux theory

The efflux problem may be considered to have started in 1644 with Evangelista Torricelli's *Opera Geometrica* where in a chapter on the motion of water ("De Motu Aquarum"), he observed the outflow from an orifice at the bottom of a bucket: If the orifice is directed vertically upward, the jet of water would rise almost to the upper level of the water in the bucket. A droplet ejected from the orifice, Torricelli argued, would reach this height only if it had acquired at the bottom the free fall velocity V from the height h of the upper water surface in the bucket. In modern notation

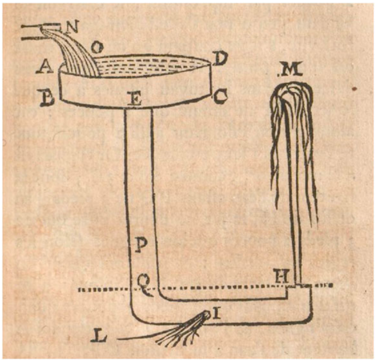

Fig. 1 Mariotte's representation of a jet rising to the height of the reservoir (Torricelli's law) if there is no loss in the conduit pipe (Mariotte 1686, 337)

$$V = \sqrt{2gh}, \tag{1}$$

where g is the gravitational acceleration. This formula has become known as "Torricelli's law".[1]

Torricelli's law became subject of further examinations.[2] In 1668, Christian Huygens formulated a research program for the newly founded Paris Academy which explicitely called attention to the moving force of running and falling water ("La force mouvante de l'eau, courante et tombante"). On July 25, 1668, the academicians stressed in particular the need to verify Torricelli's law ("d'éprouver ce que dit Torricelli") (Blay 1986, 91–93). Edme Mariotte, an academician of the first hour, dedicated great efforts to efflux problems in his *Traité du mouvement des eaux* by performing a number of outflow experiments (Mariotte 1686).[3] The contemporary constructions at Versailles and other Royal castles with splendid parks made the water supply from elevated reservoirs for the fountains a particular challenge. Three out of five parts of Mariotte's treatise accounted for experiments on the measurement of running and jumping water ("De la mesure des eaux courantes et jailissantes"), on the height of jets ("De la hauteur des jets"), and on the conduit of water ("De la conduite des eaux"). Mariotte presented the results in the form of drawings (Fig. 1) and empirical rules, such as a table on the height of jets that confirmed Torricelli's law as a limiting case (Mariotte 1686, 309).

Mariotte's treatise illustrated that the efflux of water from an orifice involved a number of aspects (such as the shape of the orifice, pipe friction and air resistance) that seemed to preclude a basic formula beyond Torricelli's law—which was rather an unproven statement than the result of a theory of flow. Yet, other contemporary and

[1] It should be noted that Torricelli's law was originally expressed verbally in terms of proportionalities (without g). The treatise is available online at http://echo.mpiwg-berlin.mpg.de/MPIWG:WHZEF9W9; see here pp. 191–192. For a historical discussion of Torricelli's law, see Dugas (1988, 145–148), Rouse and Ince (1957, 61–63), Calero (2008, 271–272), and Bistafa (2015, 174–176).

[2] Blay (2007, Chapter 2); for a review from the perspective of Daniel Bernoulli's early studies, see Mikhailov (1996).

[3] For a discussion of Mariotte's discharge measurements, see Calero (2008, 279–282).

subsequent treatises like Domenico Guglielmini's *Aquarum fluentium mensura nova methodo inquistia* and Giovanni Poleni's *De motu aquae mixto* published in 1690 and 1717, respectively, did not diminish the quest for more theoretical approaches.[4] The most famous was became Isaac Newton's concept for the efflux through a circular hole in the bottom of a bucket by dividing the water in the bucket in a central part that converged toward the hole in the bottom, and an outer part that was attached to the wall of the bucket. Newton argued that the shape of this central part ("cataract") entailed a contraction of the efflux, so that the ratio of the contracted section of the jet to the area of the hole was $1/\sqrt{2}$. "Newton's introduction of a contraction coefficient was surely a stride forward," historians of hydraulics commented on Newton's efflux concept, "but his cataract theory was not."[5]

The subsequent theoretical strides forward, Daniel Bernoulli's *Hydrodynamica* and Johann Bernoulli's *Hydraulica* published in 1738 and 1742, respectively, laid the fundaments of classical fluid mechanics. Both treatises have become the subjects of critical historical analysis[6] and need no further elaboration—except some remarks with regard to the efflux problem. From the outset, both Daniel and Johann Bernoulli aimed at a law that relates the pressure against the inner wall of a vessel or pipe to the flow velocity and the discharge in a variety of efflux configurations (Fig. 2).

Daniel Bernoulli divided the topics of *Hydrodynamica* in 13 chapters, including hydraulic machines ("De motu fluidorum, quae non proprio pondere, sed potentia aliena eiiciuntur, ubi praesertim de machinis hydraulicis") and the motion of "elastic" fluids such as air ("De affectionibus atque motibus fluidorum elasticorum, praecipue autem aeris"). Specific efflux problems were given detailed treatment in chapter 3 ("De velocitatibus fluidorum ex vase utcunque formata per lumen qualecunque effluentium"), chapter 4 ("De variis temporibus, quae in effluxu aquarum desiderari possunt"), and chapter 5 ("De motu aquarum ex vasis constanter plenis").

Johann Bernoulli also envisioned fluid motion in pipes and channels. He added on the title page of his *Hydraulica* that he was concerned with the flow of water through pipes of any shape ("De Motu Aquarum per vasa aut per canales quamcunque figuram habentes fluentium"). He divided his treatise in two parts. The first part dealt with the motion of water in vessels and cylindrical pipes composed of several components ("Agens de motu aquarum per vasa et canales cylindricos, qui ex pluribus tubis cylindricis sibi invicem adataptis sunt conflati"). In the second part, he presented a "direct and general method" for solving hydraulic problems with the flow of water through channels of any shape and arrangement ("Continens methodum directam et universalem solvendi omnia problemata hydraulica quacunque de aquis per canales cujuscunque figura fluentibus formari ac proponi possunt.") The flow configurations in the first part served to illustrate Johann Bernoulli's approach to account for the

[4] On the "Italian School" of hydraulics (Rouse and Ince 1957, see Maffioli 1994).

[5] Rouse and Ince (1957, 85). The cataract concept was elaborated in 1713 in the second edition of Newton's *Principia*; for more detail, see Mikhailov (1996, 220–225) and (Calero 2008, 100–104, 282–283).

[6] Bernoulli (1738, 1742) and Bernoulli and Bernoulli (1968). Most attention was given to the dispute between father (Johann) and son (Daniel), which culminated in the accusation that the father was plagiarizing the son and claiming precedence by predating his treatise, see Truesdell (1954, XXII), the preface of Hunter Rouse in Bernoulli and Bernoulli (1968), Szabó (1979, chapter III.B), Calero (2008, chapter 7) and Bistafa (2015, 177–185).

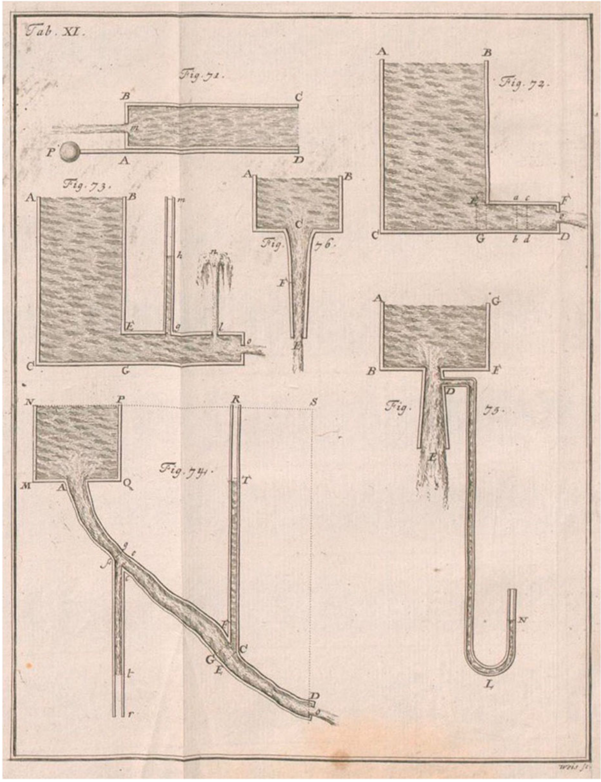

Fig. 2 Efflux configurations in Daniel Bernoulli's *Hydrodynamica* (Bernoulli 1738, Tab. XI)

change of velocity at junctions of pipes with different diameter in terms of a moving force visualized by a whirl ("gurges"). In the second part, he generalized the method for calculating the efflux—stationary as well as non-stationary—from a continuously shaped pipe. What we know as "Bernoulli's theorem" was presented in Daniel and Johann Bernoulli's treatises in a form which requires considerable effort to translate into the formulae presented in modern textbooks.[7] For our purposes, it is sufficient to conclude that this theorem arose from considerations about the efflux from a vessel such as in Fig. 72 (Fig. 2). Daniel Bernoulli's analysis of this configuration amounted to a formula for the outflow velocity in modern terminology

$$V = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{a^2}{A^2}}}, \tag{2}$$

where a and A are the areas of cross sections of the orifice and the vessel, respectively. In the limit of small orifices, this formula yields Torricelli's law.

[7] Johann Bernoulli's concept of "gurges" requires careful historical interpretation. For a detailed analysis, see Szabó (1979, 176–181).

A third treatise deserves to be mentioned in this regard, although its scope exceeded that of the Bernoullis (questioning the theoretical pillars upon which their results were based), d'Alembert's *Traité de l'equilibre et du mouvement des fluides* (D'Alembert 1744). To cut a complicated history short: Daniel Bernoulli employed the principle of energy conservation, while Johann Bernoulli resorted to the calculation of forces. Both the Bernoullis and d'Alembert derived formulae for the discharge from one-dimensional calculations (involving integrations along the centerline of the flow through an orifice).[8]

Another effort to account for the discharge of water was made by Leonhard Euler. He is famous for his treatise on the *Principes généraux du mouvement des fluides* where Euler established the general three-dimensional equations of motion for the mechanics of ideal, i.e., frictionless fluids (Euler 1755).[9] Despite its fundamental character, this achievement was also rooted in practical problems. Before Euler arrived at the general equations of motion he had dealt with a specific efflux problem, the discharge of water into an elevated reservoir, set in motion by a pump at the other end of a conduit (Euler 1752) (Fig. 3).

Euler's efflux problem referred to the contemporary design of fountains in the Royal Garden of Frederick the Great at Sanssouci which Euler considered as doomed to failure. He presented the King with a numerical calculation: For a desired discharge at a vertical height of 60 feet, pumped through a 3000 feet long pipeline, the pressure at the lower end of the pipeline corresponded to an equivalent height by far in excess of the vertical height of 60 feet. If the pipeline would have been designed to withstand only the hydrostatic pressure corresponding to 60 feet, it would inevitably have burst. The example was meant to show that the design at Sanssouci had to be changed, for example, by moving the pumps closer to the reservoir to avoid such a long pipe line (Eckert 2002, 2008).

3 The crisis of hydrodynamics in late-eighteenth century France

Despite the variety of outflow configurations from vessels through pipes and orifices, the formulae derived for the efflux velocity and discharge according to the theoretical concepts of the Bernoullis, d'Alembert and Euler were based on the assumption that the fluid approached an orifice in parallel slices and could be calculated onedimensionally as a motion along the centerline through the orifice. A critical evaluation of this concept, together with experimental verification, was presented in 1766 by Jean-Charles Borda at the Paris Academy of Science in a "Mémoire sur l'écoulement des fluides par les orifices des vases" (Borda 1766). By taking into account the contraction of the efflux at sharp-edged orifices, he corrected the flawed application of the principle of

[8] For a brief review see Darrigol (2005, Chapter 1), Calero (2008, Chapter 7) and Bistafa (Bistafa 2015, 185–190). Instead of giving preference to either Daniel or Johann by the designation "Bernoulli's theorem", Calero suggested to call it 'the Bernoullis' theorem', "as it is due as much to the father as to the son" (Calero 2008, 270).

[9] This and related treatises are reprinted in Euler's Opera Omnia, Series 2, Volume 12, introduced and commented in Truesdell (1954, LXXXIV-XCI). See also "The Euler Archive", https://scholarlycommons.pacific.edu/euler-works/226/ (Szabó 1979, Chapter III.D; Darrigol and Frisch 2008).

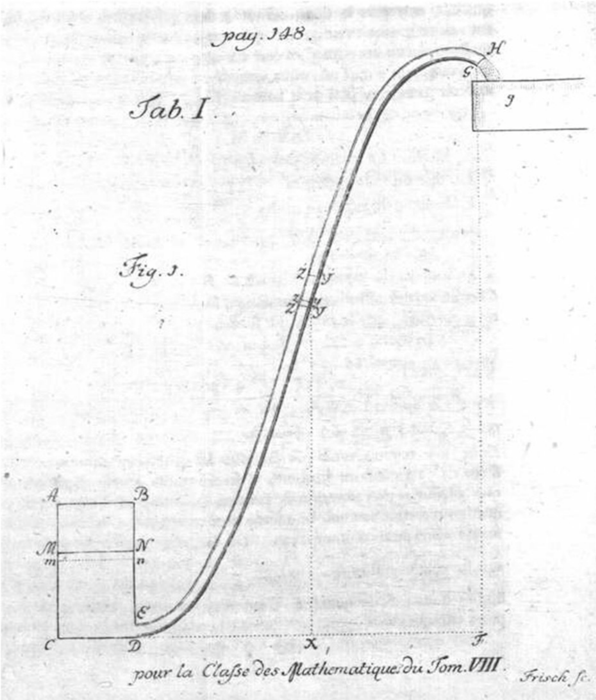

Fig. 3 Euler's efflux problem: What pressure has to be exerted against the piston of a pump to yield a desired discharge into an elevated reservoir through a pipeline (Euler 1752, Tab. I)

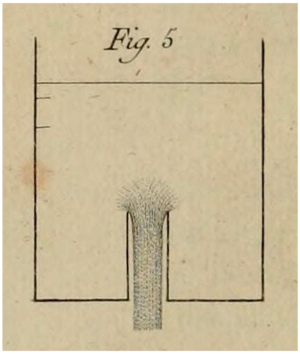

Fig. 4 "Borda's mouthpiece" is an orifice that protrudes inwardly into a vessel; it yields a maximal coefficient of contraction (ratio of jet area to aperture area) close to 1/2 (Borda 1766, 607)

energy conservation in some cases in the treatises of Daniel Bernoulli and d'Alembert. Borda's efflux experiments through a sharp-edged circular aperture in the thin wall of a cylindrical vessel yielded a jet contraction (ratio of jet area to aperture area) of 100:160 = 0.625. A maximal contraction ratio of 100:194 1/5 = 0.514 was achieved for an inwardly protruding orifice. The latter was close to 1/2, a ratio which Borda derived theoretically from a balance of momentum (Borda 1766, 587–589). In honor of this achievement, such an orifice is called in modern hydraulic textbooks "Borda's mouthpiece" (Fig. 4).

In France, Borda’s memoir aroused strong reactions. In 1770, d’Alembert published a new edition of his *Traité des Fluides* which contained implicit answers to Borda’s criticisms and continued to preoccupy his further work on hydrodynamics. He called on Joseph–Louis Lagrange to join the debate and claimed that his new theory explained the experiments in a more satisfactory manner. Borda’s memoir seemed to him “to be full of bad reasoning, some of which I have already refuted and the rest of which I hope to refute when I publish my new research on this subject.”[10]

The dispute launched a “crisis of hydrodynamics”, because it pointed to a gulf between theory and experiment in this discipline. D’Alembert’s close colleague, Charles Bossut, had just published a two-volume treatise on hydrodynamics of which the first volume was dedicated to theory and the second to experiments. The detailed treatment of the efflux problem was reserved to a chapter on “Experimental researches on the direction of fluid particles in the interior of a vessel where they are mouving, and on the contraction of the fluid jet at the exit of the orifice” in the second volume.[11] Bossut’s efflux experiments provided amunition for d’Alembert in the polemic against Borda which further pointed to a stronger inclusion of experiments in hydrodynamics.[12]

Within few years, the crisis of the 1770s entailed a surge of treatises that expressed a turn toward practice. In 1779, Pierre Louis George Du Buat, a military engineer, published *Principes d’Hydraulique* with the subtitle *Ouvrage dans lequel on traite du mouvement de l’ eau dans les rivières, les canaux, et les tuyaux de conduite* (Du Buat 1779); 7 years later, a new edition appeared with the title *Principes d’Hydraulique, vérifiés par un grand nombre d’éxpériences faites par ordre du gouvernement* (Du Buat 1786). One year later, Pons-Joseph Bernard, a scholar from the Royal Naval Observatory at Marseille, published *Nouveaux Principes d’hydraulique appliqués à tous les objets d’utilité et particulièrement aux rivières* (Bernard 1787). In the following decade, Gaspard de Prony, a professor at the École Polytechnique in Paris, made himself a name with his two-volume *Nouvelle Architecture Hydraulique* (Prony 1790, 1796). Despite a considerable diversity with regard to the content of these treatises, there was a common denominator: the principles of hydraulics required justification by experiments—and the efflux problem, in the form of the discharge from vessels or the flow over a weir, served as a gauge to what extent practice diverged from theory. Du Buat, for example, summarized his efflux measurements with the following conclusion:[13]

[10] D’Alembert to Lagrange, 14 June 1771: “A propos de cela, je vous serais très-obligé de lire à votre loisir le Mémoire du chevalier de Borda, qui est dans notre Volume de 1766, sur le mouvement des fluides dans des vases; il me paraît plein de mauvais raisonnements, dont j’ai déjà réfuté quelques-uns et dont j’espère réfuter le reste quand je donnerai mes nouvelles recherches sur ce sujet.” Quoted in Guilbaud (2007, 393).

[11] “Recherches expérimentales sur la direction des particules d’un fluide dans l’intérieur du vase où elles se meuvent, et sur la contraction de la veine fluide au sortir de l’orifice” (Bossut 1771, Part II, Chapter III).

[12] For more detail on “la crise des années 1770” in hydrodynamics in France, see Guilbaud (2008).

[13] “On voit, d’aprè:s ces observations, que nous sommes bien loin de pouvoir calculer avec précision les effets de la contraction dans tous les cas; car il y a une variété infinie dans la disposition des orifices, sans parler de la contraction occasionnée par les piles d’un pont, ou par une vanne à demi-levée, sous laquelle on force l’eau de passer avec une certaine charge” (Du Buat 1786, 18).

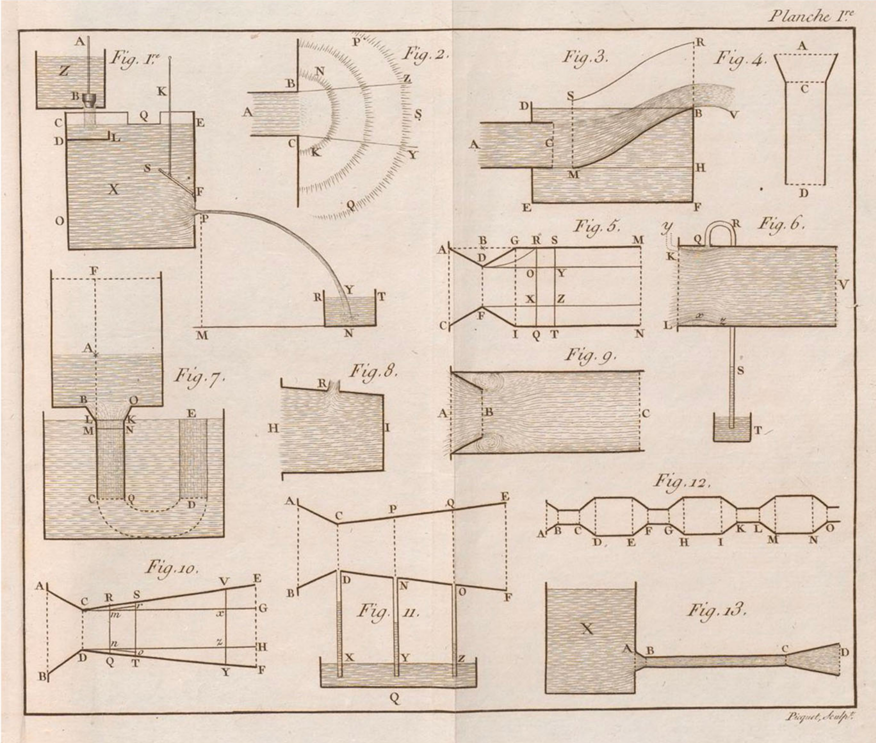

Fig. 5 Venturi's experimental apparatus (Fig. 1) used to measure efflux coefficients (Venturi 1797, Tab. I)

> It can be seen from these observations that we are far from being able to calculate with precision the effects of contraction in all cases; for there is an infinite variety in the arrangement of orifices, not to mention the contraction caused by the piers of a bridge, or by a half-raised valve, under which the water is forced to pass with a certain load.

Prony expressed the gap between theory and experiment in the form of tables where he calculated from Bossut's discharge measurements the ratio of "effective" to "theoretical" discharges. These ratios were found to be almost the same for different heights of water in the vessel. "Accordingly, there should be a fairly constant ratio between the effective and theoretical values for any given head of water and small orifice opening."[14] Thus, the notion of contraction and efflux coefficients was born which could be determined from experiments but eluded theory.

Another effort to determine efflux coefficients was published in 1797 by Giovanni Battista Venturi, professor at the military academy in Modena, then under french administration (Fig. 5). Unlike Prony, however, Venturi's *Recherches expérimentales*

[14] "D'après cela, il doit y avoir un rapport sensiblement constant entre les produits effectifs et les produits théoriques pour une hauteur d'eau et une ouverture de petit orifice quelconques" (Prony 1790, 369).

sur le principe de la communication latérale du mouvement dans les fluides, appliqué à l'explication de différens phénomènes hydrauliques largely abstained from theory:[15]

> The wisest physicists are wary of any abstract theory of fluid movement, and even the great geometers admit that the methods which have brought them such astonishing progress in the mechanics of solid bodies only give conclusions in hydraulics which are too general and uncertain for most particular cases. Penetrated by this truth, I did not concern myself with theory except insofar as it combined with the facts and was necessary to bring them together under a single point of view.

4 Hydraulics in Germany in the early nineteenth century

The surge of experimentally oriented treatises on hydrodynamics and hydraulics—both designations were still used almost synonymously (see Sect. 5)—spilled over to Germany. Johann Friedrich Lempe, professor of mathematics and physics at the mining academy in Freiberg, translated Du Buat's *Principes d'Hydraulique* (Du Buat 1796). The editor of the *Annalen der Physik* made Venturi's experiments in his journal available in German translation.[16] Another prolific translator was Karl Christian Langsdorf. In 1790, Langsdorf translated Bernard's *Nouveaux Principes d'Hydraulique* with the title *Neue Grundlehren der Hydraulik, mit ihrer Anwendung auf die wichtigsten Theile der Hydrotechnik*, praising it for its wealth of "most useful practical lessons" instead of "unnecessary mathematical subtleties".[17] Next, he chose Bossut's *Traité élémentaire d'hydrodynamique* as subject of another translation that appeared in 1792 with the German title *Lehrbegriff der Hydrodynamik nach Theorie und Erfahrung*. "When I praise Mr. Bossut's great merit for hydrodynamics, this is with respect to the extensive comparison of theory with experiments to which he has dedicated the second volume," Langsdorf revealed what motivated him for this chore.[18] Even more challenging was the translation of Prony's *Nouvelle Architecture Hydraulique* which impressed Langsdorf because of Prony's "great theoretical knowledges" and "rare practical insights based on his own many years of experiences".[19]

[15] "Les physiciens les plus sages sont en dé:fiance contre toute théorie abstraite sur le mouvement des fluides, et les grands géomètres même avouent que les méthodes, qui leur ont procuré des progrès si surprenans du côté de la mécanique des corps solides, ne donnent, du côté de l'hydraulique, que des conclusions trop générales et incertaines pour la plupart des cas particuliers. Pénétré de cette vérité, je ne me suis occupé de la théorie qu'autant qu'elle se combinoit avec les faits, et qu'elle étoit nécessaire pour les réunir sous un seul point de vue" (Venturi 1797, 5). For a brief review of Venturi's experiments, see Rouse and Ince (1957, 136–138).

[16] Annalen der Physik, 2, 1799, 418–465; 3, 1800, 35–47.

[17] "Des Hrn. Verfassers Werk empfiehlt sich einmal dadurch, daß es mit nichts weniger als unnützen mathematischen Subtilitäten, sondern wirklich mit den nützlichsten praktischen Lehren angefüllt ist" (Bernard 1790, VIII).

[18] "Wenn ich von des Hrn. Bossut's großem Verdienst um die Hydrodynamik rede, so geschieht es bloß in Rücksicht auf die ausführliche Vergleichung der Theorie mit der Erfahrung, wozu derselbe den ganzen zweiten Band, oder hier den anderen Teil dieses Bandes, bestimmt hat" (Bossut 1792, IX).

[19] "Die großen theoretischen Kenntnisse, die seltenen praktischen auf vieljährige eigene Erfahrung gegründeten Einsichten, die genaueste Bekanntschaft mit allen neuern Erfindungen, die vorzügliche Gabe

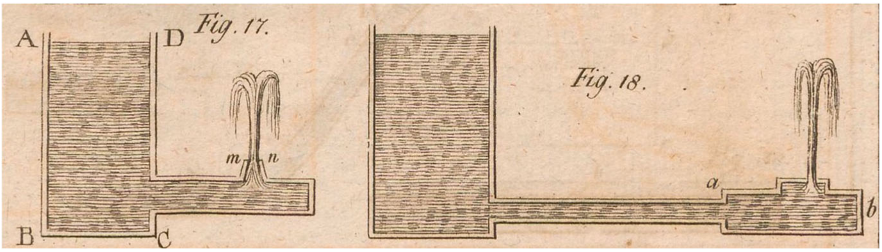

Fig. 6 One chapter of Langsdorf's textbook was titled "Von Springwerken" and dealt with the discharge for fountain jets (Langsdorf 1794, 126–127, Tafel II)

Langsdorf did not content himself with translations. Trained as a mathematician and employed as an inspector for salt works, he made the combination of theory and practice his own ideal. From this perspective, he added extensive commentaries to many translated passages. Based on his experience from these translations about the conflict between theory and practice, he authored in 1794 a textbook on hydraulics that aimed to bridge "the eternal quarrel of both parties", although his preference was on the side of the practicioners: "But I cannot suppress the confession that, with all my respect for the theory, I myself would have had as little use for the great Euler in any machine installation, nor would I still dare to suggest for this the profound author of the Mechanique analytique [i. e. Lagrange]." Ten of the 30 chapters of Langsdorf's textbook considered efflux problems, such as the discharge in the form of fountain jets ("Springwerke") (Fig. 6). Langsdorf's overall lesson from these and other examples was:[20]

> Without theory no hydraulic engineer can be formed, but in presenting the theory one must not unnoticed pass over into empty speculation, must not build on arbitrary hypotheses, must everywhere take experience as one's assistant, and, guided by it, must rather do without many a demonstration than weave untested presuppositions into the calculation and disregard important physical circumstances.

In the following years, Langsdorf authored a number of books that would be attributed to the category of mechanical engineering a few decades later (Langsdorf

Footnote 19 continued
der Deutlichkeit, die Trefflichkeit der Kupfer und der ganze Plan des Werks waren mir Bürge, dass diese Pronysche Archit. Hydr. mit allgemeinem Beifall ausgenommen werden müsse..." (Prony 1795, Vorerinnerung).

20 "Aber das Gestä:ndnis kann ich doch nicht unterdrücken, dass ich bei aller meiner Achtung für die Theorie doch selbsten den großen Euler so wenig zu irgend einer Maschinenanlage hätte gebrauchen mögen, als ich noch jetzt den tiefsinnigen Verfasser der Mechanique analytique [Lagrange] dazu vorzuschlagen getraute. [...] In der Tat fallen die Anlagen eines bloßen Empirikers nicht so oft ins offenbar Lächerliche, als die eines bloßen Theoretikers—Daher der ewige Streit zwischen beiden Parteien. Das Resultat hiervon ist kurz dieses, daß ohne Theorie kein Hydrauliker gebildet werden kann, daß man aber beim Vortrag der Theorie nicht unvermerkt in leere Spekulationen übergehen, nicht auf willkürliche Hypothesen bauen, überall die Erfahrung zur Gehülfin nehmen, und, durch solche geleitet, lieber auf manche Demonstration Verzicht thun müsse, als daß man ungeprüfte Voraussetzungen mit in den Kalkül verwebt und wichtige physische Umstände außer Acht lässt" (Langsdorf 1794, VII).

1796, 1797, 1799, 1802). As corresponding member of the Göttingen Academy of Sciences and professor of mechanical engineering (“Maschinenkunde”) at the (then Prussian) University Erlangen, he disposed also of the credentials as member of an academic community. The efflux problem continued to be a subject of particular concern in his work, such as in his *Grundlehren der mechanischen Wissenschaften* in several chapters in the part on hydraulics.[21]

Langsdorf was not alone with this concern. Joseph Baader, principal of machines in the electoral administration in Munich and member of the Bavarian Academy of Science, had published in 1797 a treatise on pumps in which he declared the classical theory of fluid motion by Bernoulli, Euler, and their followers as wrong, because he could not apply it in practical engineering. Their efflux formula (2) appeared to him absurd, because it yields an infinite velocity in the limit $a \to A$. He derived an alternative “fundamental formula”

$$V = \frac{\sqrt{2gh}}{\sqrt{1 + (\frac{a}{A})^2 - (\frac{a}{A})^3}}, \tag{3}$$

for which he claimed better agreement with experiments. Thus, he conceived his own theory—with the focus on the efflux problem. In four chapters, he derived formulae for the outflow of water that appeared suitable for application in a theory of pumps (Baader 1797).

The increasing attention to the efflux problem with regard to the tension between theory and practice is also evident from the contemporary handbooks and encyclopedia. Langsdorf started the second volume of his *Handbuch der Maschinenlehre für Praktiker und akademische Lehrer* with a critical examination of Baader’s new theory. He arrived at the conclusion that Baader’s “Fundamentalformel” (3) for the outflow is wrong, and therefore, the application to the theory of pumps had to be rejected (Langsdorf 1799, 3–45). The author of the *Handbuch der Mechanik fester Körper und der Hydraulik*, Johann Albert Eytelwein, rejected theory from the outset and gave preference to experiments. “The lecture on the movement of water is based on the experiments of the most distinguished hydraulics experts, insofar as these were sufficient, and my own experiments, conducted with all possible care, have also been added in several places.”[22] Eytelwein was director of the Berlin Bauakademie and involved in a number of engineering projects in Prussia. In the first chapter, he reviewed “The movement of water as it flows out of containers and the contraction of the water jet”. The second chapter focused on “The outflow of water through horizontal and small side openings, of a vessel that is constantly kept full”, the third on “The outflow through open rectangular openings at the top, in the side walls of a container”, the fourth on “The outflow from containers with side openings of considerable size, with unchanged pressure head”, the fifth on “The outflow from containers that do not

[21] “Vom Ausflusse des Wassers durch Öffnungen aus Behältnissen, die beständig gleich voll erhalten werden”, “Von natürlichen Springwerken” (Langsdorf 1802, Inhaltsverzeichnis).

[22] “Der Vortrag über die Bewegung des Wassers, ist auf die Versuche der vorzüglichsten Hydrauliker, so weit solche hinreichend waren, gegründet, auch sind an mehrern Orten meine eigenen mit aller möglichen Sorgfalt angestellten Versuche beigefügt worden.” (Eytelwein 1801, VII)

receive an inflow", and the sixth on "The outflow from containers which are assembled or divided by partitions".[23] Rarely before has the outflow of water as a basic problem of hydraulics been dealt with in such detail. Subsequent encyclopediae and handbooks on mechanical engineering, such as the *Encyclopädie des gesammten Maschinenwesens* edited by Johann Heinrich Moritz Poppe, a professor of mathematics and founder of a polytechnic society at Frankfurt, paid due tribute to Eytelwein's account in entries on the "Outflow of water from vessels and pipes" ("Ausfluss des Wassers aus Gefäßen und Röhren") (Poppe 1803, 225–235).

5 Semantics

The fact that the outflow of water was often dealt with in monographs under the title "hydraulics" suggests that this designation implied a prevalence of applications, in contrast to "hydrodynamics" as a label for the fundamental principles of fluid mechanics. That distinction, however, emerged only gradually. By the 1730s, Johann Bernoulli's *Hydraulica* and Daniel Bernoulli's *Hydrodynamica* did not distinguish between applied and fundamental issues. "Hydrodynamics is therefore no different in purpose from the science formerly known as Hydraulics, and still very often referred to as Hydraulics." This is how Diderot's and d'Alembert's *Encyclopédie* declared in 1766 hydrodynamics and hydraulics as almost synonymous terms.[24] In Borda's *Mémoire sur l'écoulement des fluides par les orifices des vases*, the efflux problem is not presented under the label of hydraulics but with reference to Daniel Bernoulli's *Hydrodynamica* and d'Alembert's *Traité des Fluides* (Borda 1766). In the 1770s and 1780s, Bossut's *Traité élémentaire d'hydrodynamique* (Bossut 1771) and Du Buat's *Principes d'Hydraulique* (Du Buat 1779, 1786) still do not reveal a preference for one or the other term.

The German translations mirrored this use of both terms. The gap between theory and practice was addressed without associating theory with hydrodynamics and practice with hydraulics. In the preface to his translation of Bossut's *Traité élémentaire d'hydrodynamique*, Langsdorf discriminated "true" from "hypothetical" hydrodynamics, the latter represented by "the gentlemen Bernoulli, Euler, d'Alembert, Kästner, Karsten, de la Grange and others", the former by Bossut with his "extensive comparison of theory with experience".[25] Langsdorf had made a similar distinction,

[23] "Von der Bewegung des Wassers bei dem Ausflusse aus Behältern, und von der Zusammenziehung des Wasserstrahls", "Vom Ausflusse des Wassers durch horizontale und kleine Seitenöffnungen, eines beständig voll erhaltenen Gefäßes", "Vom Ausflusse durch oben offene rechtwinklige Öffnungen, in den Seitenwänden eines Behälters", "Vom Ausflusse aus Behältern mit Seitenöffnungen von beträchtlicher Größe, bei unveränderter Druckhöhe", "Vom Ausflusse aus Behältern die keinen Zufluß erhalten", "Vom Ausflusse aus Behältern welche zusammengesetzt, oder durch Scheidwände abgetheilt sind" (Eytelwein 1801, part 2, chapters 1–6).

[24] "Ainsi, on voit que l'Hydrodynamique ne differe point, quant à l'objet, de la science qu'on appelloit autrefois et qu'on appelle encore très-souvent Hydraulique" (Diderot and d'Alembert 1766, Hydrodynamique, 371–373).

[25] "Es haben uns zwar die Herren Bernoulli, Euler, d'Alembert, Kästner, Karsten, de la Grange, u. a. teils ganze Systeme, teils einzelne Aufsätze geliefert [...] aber wer braucht wohl noch die Erinnerung, dass alle diese Theorien kaum die Elementen von Wahrheit enthalten? [...] unternahm es Herr Abt Bossut, neues

this time under the label of hydraulics, in his translation of Bernard's *Nouveaux Principes d'hydraulique* which appeared under the German title *Neue Grundlehren der Hydraulik, mit ihrer Anwendung auf die wichtigsten Theile der Hydrotechnik* (Bernard 1790), as well as in his own *Lehrbuch der Hydraulik mit beständiger Rücksicht auf die Erfahrung* (Langsdorf 1794, Vorrede).

By the early nineteenth century, there was still no close association of hydraulics with practice versus hydrodynamics with basic theory. The part of Eytelwein's handbook that was dedicated to hydraulics started with the following definition:

> The mechanics of fluid bodies (Mechanica corporum fluidorum) teaches the movement and the effects of fluid masses arising from it. A special section is hydraulics (Hydraulica) or hydrodynamics (Hydrodynamica), in which the laws of the movement of water and the effects arising from the movement of water are investigated. Note: Hydraulics is usually distinguished from hydrodynamics in that the former deals with the movement of water alone, the latter with the forces of water, although this distinction is rarely observed.[26]

Poppe's *Encyclopädie des gesammten Maschinenwesens* defined both hydrodynamics and hydraulics in terms of the monographs published with the respective titles. However, the entry on hydrodynamics contained also Prony's *Nouvelle Architecture Hydraulique* and referred to hydraulics with the remark that one would find there more books that could be associated with hydrodynamics (Poppe 1803, 875–876). In 1805, a *Handwörterbuch der Naturlehre* regarded hydraulics as "the science of the laws of the motion of water and every fluid generally" and hydrodynamics as "the doctrine of the forces and motions of water, but also each other fluid bodies in general. Hydrodynamics is a branch of hydraulics."[27]

Two decades later, the successor of this encyclopedia combined hydraulics and hydrodynamics in a single entry under "hydraulics", which together with "hydrostatics" formed "the necessary basis of the whole hydrotechnology or hydraulic architecture" and made it "indispensible for the doctrine of machines".[28] In the same

Footnote 25 continued
Licht über diesen Teil der Mathematik zu verbreiten, den Gesetzen der Natur nachzuspüren, nicht ihr Gesetze vorzuschreiben, nicht hypothetische, sondern wirkliche Hydrodynamik zu lehren [...] Wenn ich von des Hrn. Bossut's großem Verdienst um die Hydrodynamik rede, so geschieht es bloß in Rücksicht auf die ausführliche Vergleichung der Theorie mit der Erfahrung... (Bossut 1792, VII-VIII).

26 "Die Mechanik flüssiger Körper (Mechanica corporum fluidorum) lehrt die Bewegung und die aus derselben entspringenden Wirkungen flüssiger Massen kennen. Eine besondere Abtheilung ist die Hydraulik (Hydraulica) oder Hydrodynamik (Hydrodynamica), in welcher die Gesetze der Bewegung des Wassers, und die aus derselben Bewegung desselben entstehenden Wirkungen untersucht werden. Anmerkung: Man unterscheidet sonst die Hydraulik von der Hydrodynamik dadurch, dass erstere von der Bewegung des Wassers allein, letztere aber von den Kräften desselben handelt, ob gleich diese Abgrenzung selten beobachtet wird." (Eytelwein 1801, 93)

27 "Hydraulik. Die Wissenschaft von den Gesetzen der Bewegung des Wassers und jeder Flüssigkeit überhaupt. [...] Hydrodynamik. Die Lehre von den Kräften und Bewegungen des Wassers, aber auch jedes anderen flüssigen Körpers im Allgemeinen. Die Hydrodynamik ist ein Zweig der Hydraulik" (Funke 1805).

28 "Wie wichtig aber beyde Wissenschaften [Hydrostatik und Hydraulik] für das bürgerliche Leben sind, zeigt sich dadurch, dass sie die nothwendige Grundlage der ganzen Hydrotechnik oder Wasserbaukunst ausmachen, und auch der Maschinenlehre unentbehrlich sind..." (Funke and Lippold 1825)

vein, *Gehler's Physikalisches Wörterbuch* associated hydraulics with technical applications: "Under this label one has often summarized all doctrines which concern the motion of fluid, incompressible bodies". However, in view of the greek origin of the word hydraulis (meaning water organ), the editor of this encyclopedia found it "more appropriate to associate the name hydraulics only with the technical applications that arise from the movement of water; I have therefore presented the theoretical part of the doctrines about the motion of water under the title: hydrodynamics."[29] Subsequent encyclopediae followed this semantics. The *Enzyklopädie der Experimental-Physik* defined hydrodynamics as "the doctrine of the motion of fluid bodies" and hydraulics as "either the same as what one regards under hydrodynamics, or preferably only the technical application of the same."[30]

Encyclopaediae in other countries displayed subtle differences with respect to the meaning of hydraulics in relation to hydrodynamics, but the overall tendency was the same. The *Encyclopedia Britannica*, for example, defined in its fifth edition in 1815 hydraulics as "the science of the motion of fluids and the construction of all kinds of instruments and machines relating thereto", and hydrodynamics as "divided into two branches, hydrostatics and hydraulics." In 1824, in a supplement to the fourth, fifth, and sixth editions, it extended the earlier presentation of "the general principles of Hydraulics, as they have been detailed in the article Hydrodynamics". Now, it was considered "necessary to give some account of the later attempts that have been made to improve the theory of this department of science"—pointing to publications by Prony and others which accounted for engineering applications. In the course of further editions, the growing impact of hydraulic applications gave rise to the new entry in the ninth edition in 1881 on "hydromechanics" as the science of "the mechanics of water and fluids in general. The science is divided into three branches: Hydrostatics, which deals with the equilibrium of fluids; Hydrodynamics, which deals with the mathematical theory of the motion of fluids, neglecting the viscosity; and Hydraulics, in which the motion of water in pipes and canals is considered, and hydrodynamical questions of practical application are investigated."[31]

6 The appropriation of the efflux problem by engineers

The semantics provide information on how the meaning of hydraulics changed in the course of the nineteenth century. A more detailed view of this change will be obtained from examples which illustrate the rise of engineering in this period. The efflux problem was appropriated by engineers and entered textbooks that were explicitly addressed to engineers. In this process, hydraulics became firmly associated

[29] "da aber der Name von hydraulis, die Wasser-Orgel, herkommt, so ist es wohl angemessener, den Namen Hydraulik nur auf die technischen Anwendungen zu beziehen, die man von der Bewegung des Wassers macht; ich habe deshalb den theoretischen Theil der Lehren von der Bewegung des Wassers unter dem Titel: Hydrodynamik vorgetragen." (Gehler 1829).

[30] "Unter Hydraulik (v. d. griech. hydraulis Wasserorgel), versteht man entweder dasselbe was unter Hydrodynamik, oder vorzugsweise nur die technische Anwendung derselben" (Marbach 1836).

[31] The various editions of the Encyclopedia Britannica are available online: https://digital.nls.uk/encyclopaedia-britannica/archive/188936619.

with engineering—distinct from hydrodynamics as a science with little prospect for practical applications.

Already by the end of the eighteenth century, authors of treatises on hydrodynamics or hydraulics which aimed at practical applications were often associated with institutions close to engineering, even if engineering was not yet a professional activity in the modern sense. Langsdorf had worked as an officer of salt works ("Salineninspektor") before he was called to Erlangen as professor of machine technology ("Maschinenkunde"). Bossut, a jesuit ("Abbé Bossut"), was professor of mathematics at the École du Génie in Mézières and later examiner at the École Polytechnique in Paris. Venturi, another jesuit, was professor of physics at the military academy in Modena. By the early nineteenth century, the affiliation with engineering became more explicit. Eytelwein was a co-founder of the Berlin Bauakademie, from which Prussia's first engineering school, the Technische Universität Berlin, emerged. Prony was closely affiliated with engineering as professor at the École Polytechnique and director of the École Nationale des Ponts et Chaussées, role model institutions for educating engineers.

The list could be extended with names of textbook authors from the nineteenth century like Jean François d'Aubuisson de Voisins, chief engineer at the Royal Corps des Mines, and Julius Weisbach, professor at the mining academy in Freiberg. Their textbooks presented the efflux problem as a gateway to hydraulics as an engineering discipline of its own right. D'Aubuisson's *Traité d'Hydraulique à l'Usage des Ingénieurs* declared its subject as "much more in the realm of the observational sciences, the physical sciences, than the mathematical sciences; it's a treatise on experimental and applied hydraulics, not rational hydraulics." It started with a section of hundred pages "On the efflux of water contained in a reservoir" that led from Torricelli's theorem to recent measurements of efflux coefficients.[32] Weisbach's *Lehrbuch der Ingenieur-und Maschinen-Mechanik* also adopted the perspective of engineers. "Always keeping in mind the practical application," the author explained his motives, "I have always endeavoured, in compiling this work, to explain the teachings presented as much as possible with appropriate examples from life."[33] The section on the "dynamics of fluid bodies" contained nine chapters, six of which were dedicated to the efflux problem. The specifics of each chapter were clarified through examples presented as student exercises along with the solutions. Weisbach has laid the foundation for such a comprehensive treatment of efflux problems already a few years before with a book-length handbook article in which he also corrected erroneous views like Baader's wrong outflow-velocity formula (3) which had arisen from inappropriate criticism of classical hydrodynamics. "Therefore, several newer theories have emerged,

[32] "Ainsi mon ouvrage est par sa nature bien plus du domaine des sciences d' observation, des sciences physiques, que des sciences mathématiques; c'est un traité d'hydraulique expérimentale et appliquée, et non d'hydraulique rationnelle." (d'Aubuisson de Voisins 1834, IX). It was translated into German with the title *Handbuch der Hydraulik. Mit besonderer Rücksicht auf ihre Anwendung bei den Ausführungen des Ingenieurs* (d'Aubuisson de Voisins 1835).

[33] "Immer die Anwendung im Praktischen vor Augen habend, bin ich beim Aufsetzen dieses Werkes stets bemüht gewesen, die vorgetragenen Lehren durch passende Beispiele aus dem Leben soviel wie möglich zu erläutern." (Weisbach 1845, VII)

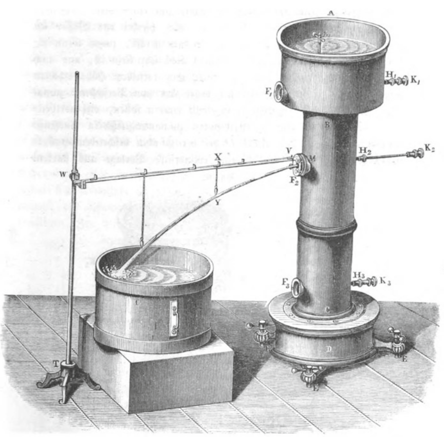

Fig. 7 The "Ausfluss-Versuchsapparat" used in Weisbach's courses on hydraulics (Weisbach 1855, 7)

the incorrectness of which we must briefly demonstrate here in order to warn practitioners against using them, which is all the more necessary since they have mostly come from men who are more practitioners than theorists."[34]

The textbook presentation of the efflux problem as an issue of primary engineering interest was based on decades of practical experience. D'Aubuisson and Weisbach had directed themselves outflow measurements at the water tower in Toulouse and at the mining academy Freiberg, respectively, which entered their tables of empirical outflow coefficients. Furthermore, they included measurements performed at the request of the French War Ministry by Jean-Victor Poncelet and Joseph Aimé Lesbros at the fortress ditch in Metz (Poncelet and Lesbros 1832); or measurements by Giorgio Bidone made at the water tower in Turin (Bidone 1830), to cite only two recent references to experimental sources of which they made use in their textbooks.

In 1855, Weisbach further stressed the importance of the efflux problem for hydraulic engineering with a textbook titled *Die Experimental-Hydraulik*. The laws of "rational mechanics" are insufficient to account for the motion of water, he argued in the preface, "therefore there is nothing left to do but to carry out experiments under the most varied circumstances and conditions and, on the basis of the general laws of mechanics, to seek out special rules of hydraulics and to determine empirical coefficients for them." He designed an apparatus for performing efflux experiments in the classroom (Fig. 7). "These experiments allow the efflux, contraction and velocity coefficients for different orifices and mouthpieces to be determined, and from these the corresponding resistance coefficients [...] to be calculated."[35]

[34] "Es sind deshalb mehrere neuere Theorien entstanden, deren Unrichtigkeiten wir hier kurz nachweisen müssen, um Practiker vor Anwendung derselben zu warnen, was um so nöthiger ist, da sie grösstentheils von Männern ausgegangen sind, die mehr den Practikern als den Theoretikern beizuzählen sind" (Weisbach 1841, 587).

[35] "Es bleibt deshalb nichts weiter übrig, als unter den verschiedenartigsten Umständen und Verhältnissen Versuche anzustellen und aus denselben, mit Zugrundelegung der allgemeinen Gesetze der Mechanik, spezielle Regeln der Hydraulik aufzusuchen, und Erfahrungscoeffizienten für dieselbe zu bestimmen. [...] Durch diese Versuche lassen sich die Ausfluss-, Contractions- und Geschwindigkeits

The “empirical coefficients” became a distinguishing feature between hydraulics and hydrodynamics. No textbook on hydraulics from the second half of the nineteenth century lacked definitions of velocity, contraction and efflux coefficients and methods for their experimental determination. Moritz Rühlmann, for example, professor at the Polytechnic Institute (Polytechnische Schule) in Hanover, introduced the velocity coefficient (“Geschwindigkeitscoefficient”) ψ as a correctional factor with which the ideal efflux velocity (2) had to be multiplied to obtain the actual velocity for openings in thin walls. According to Weisbach’s experiments, he specified ψ between 0.958 and 0.988. Together with the contraction coefficient α (“Contractionscoefficient”), defined as the ratio of cross sections of the outflowing jet at the most contracted site to that at the orifice (a), the discharge rate was given by

$$Q = \psi \alpha a V = \mu a V$$

with $\mu = \psi\alpha$ as efflux coefficient (“Ausflusscoefficient”) (Rühlmann 1857, 149–162). Experimental measurements were the only means to determine these coefficients.

In 1880, Rühlmann published the second edition of his textbook under the new title *Hydromechanik oder die technische Mechanik flüssiger Körper*. Besides references to recent efflux studies, he added historical notes which showed to what extent outflow measurements had become an important research subject of hydraulic engineering. Because theory was unable to account for the contraction of water at the efflux from a vessel, Rühlmann conluded that a new route had to be chosen to advance our knowledge about the outflow of water from vessels:

> This way consists in not representing the complication of the elements that occur in the phenomena by calculation, but rather in basing the whole theory on the phenomena themselves and finally correcting the final results with the help of suitable experiments. And indeed, this path, where the calculation goes hand in hand with experience, as it were, may be called a very happy one, since it has led to many beautiful discoveries and to truly practical formulas that are usually in good agreement with experience.[36]

An example may illustrate this approach. Rühlmann referred to the following experiment originally conducted by Weisbach (Rühlmann 1880, 233). Given a prismatic container of cross section A filled with water to a height h that is emptied through an orifice in the bottom with cross section a. Find the efflux coefficient μ from measuring the time t within which the water level in the container sinks from h to h_1. The calculation proceeds as follows:

Footnote 35 continued
coefficienten für verschiedene Mündungen und Mundstücke bestimmen, und hieraus wieder die entsprechenden Widerstandscoefficienten [...] berechnen” (Weisbach 1855, V, IX–X).

[36] “Dieser Weg besteht darin, daß man die Complication der Elemente, welche in den Erscheinungen vorkommen, nicht durch die Rechnung darstellt, sondern vielmehr die ganze Theorie auf die Erscheinung selbst gründet und schließlich die Endresultate mit Hülfe geeigneter Versuche corrigirt. Und in der That darf dieser Weg, wo die Rechnung gleichsam mit der Erfahrung Hand in Hand geht, ein sehr glücklicher genannt werden, da er zu mannigfachen schönen Entdeckungen und zu wahrhaft practischen, mit der Erfahrung meist gut stimmenden Formeln geführt hat.” (Rühlmann 1880, 196)

In time dt, the water level at height x sinks by dx. The change of volume in the container is balanced by the efflux volume

$$-Adx = \mu a V(x)dt = \mu a\sqrt{2gx}dt,$$

which yields

$$t = -\frac{A}{\mu a\sqrt{2g}}\int_h^{h_1}\frac{dx}{\sqrt{x}} = \frac{2A}{\mu a\sqrt{2g}}(\sqrt{h} - \sqrt{h_1}).$$

Thus

$$\mu = \frac{2A(\sqrt{h} - \sqrt{h_1})}{at\sqrt{2g}}.$$

With the values from Weisbach's experiment, this yielded $\mu = 0.605$. If the orifice was replaced by a short cylindrical tube, the result was $\mu = 0.822$. For a conical convergent orifice, it was $\mu = 0.970$ (Rühlmann 1880, 282, 288). The agreement with experience was still not perfect, as Rühlmann admitted, because the true outflow caused a vortical motion around the axis of the orifice which was neglected in the preceeding calculation. Nevertheless, this way of dealing with the efflux problem in terms of experimentally determined coefficients proved useful for practical tasks such as the design of weirs and locks. The discharge over a sharp crested weir, for example, was given by

$$Q = \frac{2}{3}\mu bH\sqrt{2gH},$$

where b is the width of the weir and H is the pressure height (i.e., the height difference some distance before and after the weir).[37] However, the efflux coefficient μ in this formula was dependent on the geometry of the weir and other circumstances and became subject of more detailed experimental investigations. Outstanding efforts in this regard were undertaken by Henri Bazin, inspector general of the Corps des ponts et chaussées, the leading engineering society in France. Bazin had made a name as an engineer/scientist with experimental studies on water waves and channel flow (Darrigol 2005, Chapter 2.4 and 6.2); his weir overflow experiments published in a sequence of articles in the *Annales des Ponts et Chaussées* under the title "Expériences nouvelles sur l'écoulement en déversoir" between 1890 and 1898 were equally pathbreaking and regarded as pillars of hydraulics as an experimental engineering science.[38]

Overall, Rühlmann's textbook as well as the experiments of Bazin and other hydraulic engineers show how the efflux problem had become subject of sophisticated elaboration throughout the nineteenth century. Physicists and mathematicians

[37] Furthermore, it is assumed that the velocity with which the water approaches the weir is much smaller than that of the overflow at the weir; see Rühlmann (1880, 295–299).

[38] On 12 December 1922, a monument to Henri Bazin was inaugurated in Dijon, where he had performed these experiments. Bazin was counted among the founders of hydraulics in France. For a review of his work, see the hommage "Hydraulique: la Mémoire d'Henri Bazin," *La Houille Blanche*, 1923, 20:2, 52–56. https://www.tandfonline.com/doi/abs/10.1051/lhb/1923008.

had little to compare with these investigations in their lectures and seminars on hydrodynamics. In 1900, the author of a textbook on hydrodynamics lamented about the gap between theory and practice as follows:

> Here the actual processes are so inconsistent with the theoretical conclusions that engineering developed a special way of treating hydrodynamic tasks, which is usually called hydraulics.[39]

A few years later, the schism between hydrodynamics and hydraulics was expressed most clearly in the *Enzyklopädie der mathematischen Wissenschaften* by Philipp Forchheimer from the Technical University in Graz. "Practical hydraulics shows a significantly different character than theoretical hydrodynamics", he introduced a book-length article on "Hydraulik":

> Thus, practical hydraulics today is still primarily an area of power of the coefficients and its working method is often only an interpolation of empirical data. It should be noted that the technical tasks are not freely chosen by the researcher like the physical ones, but are forced upon him by practical need. A theoretically unsatisfactory solution, if it only proves useful within the limits within which the technique uses it, is still better than none at all.[40]

Forchheimer's *Hydraulik* was preceeded by a two-part encyclopedia article on *Hydrodynamik* authored by Augustus Edward Hough Love, a professor of natural philosophy from Oxford University (Love 1901a, b). The overlap between Forchheimer's and Love's articles was minimal. Whereas Forchheimer listed dozens of references for the experimental determination of efflux coefficients (Forchheimer 1905, 396–419), the only theoretical calculation of an efflux coefficient presented in Love's article was due to Gustav Kirchhoff's theory of jets which resorted to potential theory and made use of conformal mapping. This method was based on Hermann von Helmholtz's concept of complex potentials for two-dimensional flows confined partly by rigid boundaries and partly by the surface of the fluid (Darrigol 2005, Chapter 4.3.2). The form of the latter, however, was not known in advance and had to be determined in the course of the calculation, which limited its application to a few plane outflow configurations such as the efflux through a slit in the bottom of a rectangular extremely wide container. In this case, the theory yields the contraction coefficient (Love 1901b, 98–99)

$$\frac{\pi}{2+\pi} = 0.611.$$

[39] "Hier stimmen die thatsächlichen Vorgänge mit den theoretischen Folgerungen vielfach so ungenügend überein, dass die Technik sich für ihre Zwecke eine besondere Behandlungsweise hydrodynamischer Aufgaben, die meistens den Namen Hydraulik führt, zurechtgemacht hat." (Wien 1900, III)

[40] "Die praktische Hydraulik zeigt ein wesentlich anderes Gepräge als die theoretische Hydrodynamik. [...]. So ist die praktische Hydraulik heute noch vornehmlich ein Machtgebiet der Koeffizienten und ihre Arbeitsmethode vielfach nur eine Interpolation empirischer Daten. Dabei ist festzuhalten, dass die technischen Aufgaben nicht gleich den physikalischen vom Forscher frei gewählt, sondern ihm durch das praktische Bedürfnis aufgezwungen werden. Eine theoretisch unbefriedigende Lösung, wenn sie nur in den Grenzen, innerhalb welcher die Technik sie verwendet, sich noch brauchbar zeigt, ist dann immer noch besser als gar keine." (Forchheimer 1905, 327)

The large number of different discharge configurations (orifice shape, flow over weirs, etc.) eluded the grasp of theoretical hydrodynamics. Felix Klein, the founder of the *Enzyklopädie der mathematischen Wissenschaften* and editor of the volumes on mechanics to which he assigned both hydrodynamics and hydraulics, expressed his discomfort about the gap between theory and practice in this field. Although he categorized efflux among the "well defined problems" of hydrodynamics, in contrast to turbulence ("somewhat poorly defined") or river flow ("very poorly defined"), he regarded Love's encyclopedia article as "deficient" even with respect to such well-defined problems like efflux. Klein arrived at this evaluation in the course of a seminar on "Selected Chapters of Hydrodynamics" in the wintersemester 1903/04 which he introduced as follows:

> Since mechanics has developed in different directions after the beginning of the nineteenth century, namely the mathematical side on the one hand and the side of applications on the other hand, the connection between both directions has become quite loose and it might be time to strengthen it again. From this point of view the coming seminar is to be considered. [...] What was said above for mechanics in general applies to hydrodynamics in particular. On the one hand, we have the theoretical hydrodynamics, where the hydrodynamic differential equations (which may include friction elements) are set up and integrals are searched for, and on the other hand, we have the practical hydrodynamics, the so-called hydraulics, which is the science of the water movements that actually take place. [...] The (ideal) program of the coming seminar is to understand hydraulics from the point of view of theoretical hydrodynamics.[41]

Six presentations of this seminar dealt with the efflux of water from vessels and weir overflow which were in principle "a pure task of the potential theory", as Klein concluded in a summary report, "only too difficult for the conventional methods, since the edges, along which for the potential φ certain conditions exist, must be found partly themselves first. Therefore, one is dependent on consulting the experiment".[42] Thus,

[41] "Seitdem sich nach Beginn des 19. Jahrhunderts die Mechanik nach verschiedenen Richtungen hin entwickelt hat, nämlich nach der mathematischen Seite einerseits und der Seite der Anwendungen andererseits, ist der Zusammenhang zwischen beiden Richtungen nachgerade ein recht loser geworden und es dürfte an der Zeit sein, ihn wieder zu festigen. Von diesem Gesichtspunkte aus ist das kommende Seminar zu betrachten. [...] Was oben für die Mechanik im Allgemeinen gesagt wurde, gilt für die Hydrodynamik im Speziellen. Wir haben einerseits die theoretische Hydrodynamik, wo man die hydrodynamischen Differentialgleichungen (welche eventuell mit Reibungsgliedern ausgestattet sind) aufstellt und Integrale dazu sucht, und wir haben andererseits die praktische Hydrodynamik, die sog. Hydraulik, das ist die Lehre von den Wasserbewegungen, welche wirklich stattfinden [...] Das (ideale) Programm des kommenden Seminars ist, die Hydraulik vom Standpunkte der theoretischen Hydrodynamik aus zu verstehen." Protokollbuch Nr. 20, WS 1903/04: Ausgewählte Kapitel aus der Hydrodynamik. Mathematisches Institut der Universität Göttingen. Online https://www.uni-math.gwdg.de/aufzeichnungen/klein-scans/klein/V20-1903-1904/V20-1903-1904.html.

[42] "Sowohl den Ausfluß als den Überfall theoretisch zu behandeln, ist eine reine Aufgabe der Potentialtheorie, nur zu schwer für die herkömmlichen Methoden, da die Ränder, längs deren für das Potential φ gewisse Bedingungen bestehen, zum Teil selbst erst gefunden werden müssen. Deshalb ist man darauf angewiesen, das Experiment zu konsultieren" Ibid.

the gap between hydrodynamics and hydraulics was solidified rather than bridged. However, it deserves to be mentioned again that the difficulties in the efflux problem were of a different nature than in turbulence where the challenge to find theoretical solutions lay in the chaotic motion at high Reynolds numbers.

7 Outlook

In the twentieth century, hydraulics developed into a well-established engineering science—whereas hydrodynamics remained an academic specialty associated with theoretical physics. New concepts such as boundary layer theory sparked the rise of modern fluid mechanics and entailed a convergence of theory and practice in some areas. When Hunter Rouse, director of the Iowa Center for Hydraulic Research, published in 1946 a textbook on *Elementary Mechanics of Fluids*, he emphasized that the so-titled field was "evidently not merely traditional hydraulics under another name, or even a combination of hydraulics with certain aspects of aerodynamics, but rather as fundamental a treatment of fluid behavior as the mechanics of solids is a behavior of rigid and solid bodies" (Rouse 1978, iii). Yet, this textbook shows a preponderance of practical applications that could easily have been presented under the name of hydraulics.

The efflux problem, in particular, remained a subject that reflected the divergent routes of hydraulics and hydrodynamics for many more decades. Theoretically oriented textbooks on *Hydrodynamics* gave it rather cursory consideration, such as Lamb (1945, 96–99) or Landau and Lifschitz (1966, 32–34), whereas it entered engineering textbooks as an ever more important topic in demand of sophisticated treatment; see, e.g., Jaeger (1949, Chapters BII, BIII) or Kozeny (1953, Chapters M, N). "There is hardly an area of technical hydraulics in which outdated formulas can be found so abundantly in the literature (even in modern literature!) as the area of 'outflow and discharge control'. In fact, even today we cannot get by without coefficients or coefficients in this area." This is how the handling of outflow problems was described in a hydraulics textbook as late as 1987. The author called for "a new way of representing" the discharge coefficient as a function of parameters chosen under similarity considerations according to the particular flow configuration, such as the Reynolds number when friction has to be taken into account.[43] As an example, he presented a diagram for the discharge coefficient as a function of the Reynolds number for the flow in a pipe through a diaphragma (Fig. 8).

Manuals from the late twentieth century show that hydraulics did not cease to flourish as a science that relied on empirically determined coefficients (see, e.g., Brater et al. 1996, Sections 4 and 5). Even the rise of Computational Fluid Dynamics as a new tool to solve practical problems by numerical methods did not bridge the gap

[43] "Es gibt kaum einen Bereich der Technischen Hydraulik, in dem überholte Formeln so reichlich in der Literatur (selbst in der modernen!) zu finden sind, wie den Bereich 'Ausfluß- und Abflußsteuerung'. Tatsächlich kommen wir gerade in diesem Bereich auch heute noch nicht ohne Beiwerte oder Koeffizienten aus. [...] wir können durch Umstellung auf die neue Darstellungsweise nur gewinnen [...] und schließlich gehören Strömungsparameter wie Froude-Zahl Fr, Reynolds-Zahl Re, Weber-Zahl We, Kavitationszahl Ka dazu, je nachdem welche der mit diesen Parametern berücksichtigten Kräftearten bzw. Fluideigenschaften die Strömungsgeometrie beeinflussen" (Naudascher 1987, 74–76).

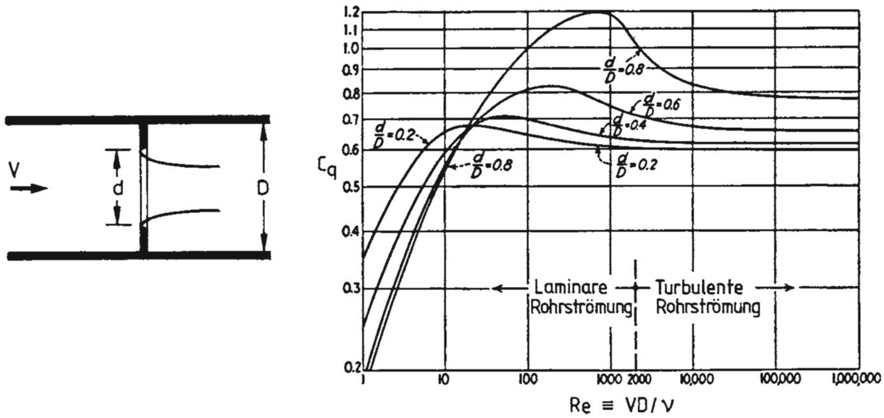

Fig. 8 Experimentally determined discharge coefficient C_q as a function of the Reynolds number VD/ν for the flow through a diaphragma (width d) in a pipe of inner diameter D (Naudascher 1987, 81)

between theory and practice. "There is not a great deal of difference with computational hydrodynamics or computational fluid dynamics, but these terms are too much restricted to the fluid as such." In this manner, a textbook on *Computational Hydraulics* juxtaposed these terms. Thus, the old schism emerged in a new guise with "computational hydraulics" as the tool that was better adapted to engineering purposes than "computational hydrodynamics" (Vreugdenhil 1989).

The history of hydraulics in the twentieth century deserves a historical inquiry in its own right. Furthermore, a comparison between different countries probably would reveal distinct national traditions and cultures. Yet, some lessons may be drawn already from this study on the efflux problem: The schism of hydrodynamics versus hydraulics is a phenomenon that emerged in the nineteenth century with the appropriation of hydraulic problems by engineers. From an epistemic and historiographic perspective, this study calls for inquiries that combine the history of science with that of engineering. The history of fluid mechanics may not be approached from one or the other side alone, and specific cases such as the efflux problem provide suitable probes in this quest.

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Declarations

Conflict of interest The author declares that he has no conflict of interest.

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