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Measurements of altitude and geographic latitude in Latin astronomy, 1100–1300

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Abstract

This article surveys measurements of celestial (chiefly solar) altitudes documented from twelfth- and thirteenth-century Latin Europe. It consists of four main parts providing (i) an overview of the instruments available for altitude measurements and described in contemporary sources, viz. astrolabes, quadrants, shadow sticks, and the torquetum; (ii) a survey of the role played by altitude measurements in the determination of geographic latitude, which takes into account more than 70 preserved estimates; (iii) case studies of four sets of measured solar altitudes in twelfth-century Latin sources; (iv) an in-depth discussion of the evidence relating to altitude measurements performed in Paris in the period 1281–1290. The findings from the last part indicate that by the end of the thirteenth century Parisian astronomer had developed rigorous standards of observational practice in which altitudes were typically measured to a precision of minutes of arc and with a level of accuracy higher than $\pm\, 0;5°$, and sometimes exceeding $\pm\, 0;1°$.

1 Introduction

Much of the surviving evidence for the practical aspects of astronomy in the Latin Middle Ages resides on the pages of manuscripts that have rarely been studied, let alone made available in printed form. In the absence of any systematic examination of this corpus of material, it may be too early for any categorical remark about the "astonishing, almost complete absence of recorded observations before 1450,"[1] or other broad

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[1] Gingerich (1975: 88–89).

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generalizations about medieval astronomy's supposed lack of a strong empirical component.[2] The present article seeks to contribute towards a higher resolution picture of the empirical side of medieval astronomy by focusing on one specific type of observation: measurements of celestial altitude, which appear to have been the single most common type of angular measurement carried out by astronomers during the medieval period. Descriptions of altitude measurements were not only a routine feature of the literature on the uses of certain instruments, but the concrete surviving evidence of their practical application is probably richer than for any other type of observation.

Altitude measurements could serve a broad range of different purposes in the context of pre-telescopic astronomy. In addition to being an essential component of a variety of different time-telling methods, they were implicated in the construction and calibration of instruments and the use of astronomical tables, many of which were sensitive to geographic latitude. Medieval manuscripts accordingly preserve for us a wealth of latitude estimates for European cities, which in many instances would have resulted from an altitude measurement of the Sun or some fixed star. Beyond these practical uses, measured altitudes could play a crucial part in efforts to determine certain astronomical parameters such as the obliquity of the ecliptic, the length of the solar year, or the ecliptic coordinates of planets.

The following survey of medieval altitude measurements will concentrate on what might be called the nascent phase of Latin mathematical astronomy, from the beginning of the twelfth to the end of the thirteenth century. Its starting point is determined by the well-known fact that mathematical astronomy in what might be called the Hellenistic or Graeco-Arabic tradition fully began to take roots in Latin Europe only in the first half of the twelfth century, once translations from (mostly) Arabic provided access to the relevant sources.[3] The end point for this survey has been chosen pragmatically with a view to keeping the source material within manageable bounds. After 1300, the amount of preserved astronomical material increases very rapidly compared to the previous centuries. From about the 1320s onwards, it also undergoes certain technical or formal changes owing to the influence of the Alfonsine Tables.[4]

The article is divided into four main parts, each of which addresses a different aspect of the topic at hand. Part 2 offers a concise overview of the four main types of instruments that were available for altitude measurements during the period under review: astrolabes, quadrants, shadows (i.e., shadow sticks), and the torquetum. Part 3 considers the use of altitude measurements in medieval attempts to determine geographic latitude as well as the important role knowledge of geographic latitude played in some aspects of astronomical practice. To this end, I have surveyed on a comprehensive scale the sources that preserve latitudes for localities in Latin Europe, insofar as these latitudes were in all likelihood established during the twelfth and thirteenth

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[2] Statements of this nature abound in the literature. See, e.g., Martínez Usó and Marco Castillo (2019: 192–193), who opine that "there were no 'scientific' systematic observations of the celestial sphere until well into the fifteenth century". The even more sweeping verdict that "the idea of making observations or performing experiments was largely unknown to medieval thinkers" can be found in Poskett (2022: 16).

[3] See, e.g., Hugonnard-Roche (1996). This is not to deny that some level of astronomical knowledge and practice existed in the Latin West already before 1100. See, e.g., McCluskey (1998), Eastwood (2007), Borrelli (2008) and Nothaft (2017b).

[4] On Alfonsine astronomy in the late Middle Ages, see Kremer et al. (2022).

centuries. Part 3 concludes with a table assembling 74 such estimates for 40 different cities, accompanied by a brief analysis of their precision and accuracy. Parts 4 and 5 turn towards specific instances of altitude measurement that have been recorded in sources from the same period. In the case of Part 4, the focus is on four sources from the twelfth century that document measurements of solar altitudes, only some of which are clearly dated. Part 5 discusses a cluster of sources from late-thirteenth century Paris, which offer unusually detailed glimpses into the solar observations carried out by a group astronomers including Peter of Limoges and William of Saint-Cloud. They show how altitudes were used deliberately to investigate other astronomical parameters such as the obliquity of the ecliptic, the time of the vernal equinox, and the rate of precession. It is also evident from these sources that the astronomers who worked in this Parisian milieu operated with methods and instruments that enabled them to measure altitudes with a relatively high level of precision (to the minute of arc) as well as with an impressive degree of accuracy that could exceed $\pm$ 0;1°.

2 Instruments

While the main focus of this study will be on altitude measurements that are explicitly attested in Latin sources of the twelfth and thirteenth centuries, it should be underlined that these sources can only provide an extremely fragmentary picture of day-to-day observational practice in the relevant time period. It is possible to supplement this picture, however, by considering contemporary texts that explain the use of certain astronomical instruments. In addition to answering the basic question of which instruments and techniques were available for measuring altitudes, these sources also give us some idea of the purposes for which such measurements were typically intended.

2.1 Astrolabes

Astrolabes were by some margin the most frequently written about astronomical instrument of the Latin Middle Ages. The twelfth and thirteenth centuries are no exception in this regard.[5] Texts on astrolabe use from this period tend to devote at least one of their chapters to a description of how to measure the altitude of the Sun or a star with the alidade (sighting tool) fastened to the instrument's backplate, which also carried a circular degree scale.[6] Some of these accounts are fairly rich in technical detail, as when they mention the pinnules or sighting vanes that were attached at right angles

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[5] For information on the Latin astrolabe literature produced between 1100 and 1300, see Poulle (1954: 84–94, 1964c, 1972: 33–36; 1987, 1998: 80–83), Kunitzsch (1981, 1982: 483–507; 1994) and Borrelli (2008: 216–218, 240–241).

[6] See, for example, Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. John of Seville, c. 4 (ed. Millás Vallicrosa 1942: 265); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. Plato of Tivoli, c. 5 (ed. Lorch et al. 1994: 144); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. anon., c. 1 (ed. Nothaft 2023a: 395); pseudo-Māshā'allāh, *Practica astrolabii*, c. 2 (ed. Thomson 2022: iv. 50–57); Rudolf of Bruges, *Cum celestium sperarum diversam positionem...*, c. 13 ii (ed. Lorch 1999: 74); Abraham Ibn Ezra, *Genera astrolabiorum duo sunt...* (ed. Millás Vallicrosa 1940: 11); Adelard of Bath, *Libellus de opere astrolapsus* (ed. Dickey 1982: 175–176); Robert of Chester, *Liber de officio astrolabii*, c. 2 (MS Oxford, Bodleian Library, Canon. misc. 61, fol. 12v); pseudo-Hermes, *De sex rerum principiis* 24 (ed. Lucentini and Delp 2006: 206); *Liber*

to the ends of the alidade. According to Abraham Ibn Ezra and an obscure twelfth-century Latin writer named Arialdus, each vane ideally contained two perforations of differing size, the smaller for pinpointing stars during the night, the larger for capturing the rays of the Sun even on cloudy days.[7]

The purposes for which such measurements could be harnessed were numerous and wide-ranging, from finding the azimuthal directions of cities to determining the ecliptic positions of planets and stars. Above all, they played a crucial role in providing input data for the instrument's various horological functions. Most records of time measurements made with the help of an astrolabe or horary quadrant (on which see below) can, accordingly, be regarded as indirect evidence of altitude measurements, even if the sources are rarely explicit on this point. The earliest detailed record of a time determined with the help of an astrolabe in Latin Europe comes from Walcher of Malvern, who measured the altitude of the Moon (15°) during the approximate mid-point of the total lunar eclipse of 18 October 1092 and used it to derive the current quarter-hour.[8]

Multiple altitude measurements in the context of a single eclipse observation are attested in MS Laon, Bibliothèque municipale Suzanne Martinet, 425, fol. 89r (s. XIII$^{1/2}$), which records the phases of a total lunar eclipse visible near Melun (Dept. Seine-et-Marne) during the night of 21/22 October 1222. The time of mid-eclipse is here given as 02:14 h, or 14 equinoctial hours and 14 min from previous noon. In addition, however, the note in question mentions the altitudes of Aldebaran (α Tau) and Regulus (α Leo) at the beginning and end of the main phases of the eclipse. For instance, Regulus was observed 13° above the eastern horizon at the beginning of the phase of totality and had reached an altitude of 30° by the time the Moon exited totality, when Aldebaran was 51° above the eastern horizon.[9] According to modern calculation, the approximate local mean time at the cited altitudes of Regulus was 01:06 h and 02:50 h on 22 October,[10] which agrees fairly well with the modern calculated times for the beginning and end of total eclipse (01:05 h and 02:46 h).[11]

The use of stellar altitudes as time markers is also attested in a few other manuscripts from this period. They include MS London, British Library, Harley 4350, fol. 2r (s. XIII$^{2/2}$), where added notes mention the altitudes of Aldebaran and the Pleiades as observed at births that took place in 1258 and 1264; and MS Metz, Bibliothèque municipale, 1223, fol. 116v (s. XIII^{ex}), which records the altitude of Vega at the end

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Footnote 6 continued

de wazalkora, pt. 3, c. 7 (MS London, British Library, Arundel 339, fol. 91v); Peter of Maricourt, *Nova compositio astrolabii particularis*, c. 24 (ed. Thomson in Sturlese and Thomson 1995: 180).

[7] Abraham Ibn Ezra, *Genera astrolabiorum duo sunt…* (ed. Millás Vallicrosa 1940: 9, 12); Arialdus, *Cum inter omnia instrumentorum genera…* (MS Paris, Bibliothèque nationale de France, lat. 16652, fol. 31r).

[8] Walcher of Malvern, *De lunationibus* 4.1 (ed. Nothaft 2017b: 114–117).

[9] MS Laon, Bibliothèque municipale, 425, fol. 89r: "Et ad inicium more fuit super orizontem in parte orientali Cor Leonis 13 gradus. Et ad horam exitus Lune de tenebris fuit Cor Leonis super orizontem orientaliem 30 gradus. Et Aldevaran super orizontem occidentalem 51."

[10] Unless otherwise stated, all modern computations of astronomical phenomena referred to in this article were obtained with the open source software *Stellarium* 23.1 (https://stellarium.org/) and are given with a precision to the nearest minute (of time or arc).

[11] These times were obtained using NASA's *Java Lunar Eclipse Explorer*: https://eclipse.gsfc.nasa.gov/JLEX/JLEX-index.html, using GMT and a time correction of + 00:10 h for the longitude of Melun.

of an unidentified eclipse.[12] Explicit testimonies to the use of an astrolabe in the context of such altitude measurements are nevertheless quite rare. The *Liber iudiciorum* by Raymond of Marseilles, which dates from 1141 or shortly thereafter, contains an extended account of the techniques that helped him find the ecliptic longitudes of the planets from their observed altitudes.[13] He avers that he used 'a most exact astrolabe' (*astrolabium certissimum*) for these observations,[14] but does not specify when they took place and offers only limited data regarding the results achieved. Other observers from the twelfth century who are known to us by name include Rudolf of Bruges, whose measurement of the solar noon altitude on 24 April 1144 (discussed in part 4) was made for the purpose of constructing an astrolabe's horizon plate. From approximately the late 1170 s, we have the testimony of Roger of Hereford, who used an astrolabe to determine the Sun's noon altitudes at its sidereal sign entries. As we shall see below (Part 4), this determination could have consisted in a computational use of the astrolabe as much as in an empirical measurement made with an alidade.

Some contemporary writers on the astrolabe demonstrate a sharp awareness that the accuracy of any altitude measurement—and of the operations based on it—will crucially depend on the precision of the instrument used for it, which is in part a function of size. Relevant comments appear in the twelfth-century astrolabe treatises by John of Seville (on construction), Raymond of Marseilles (on construction and use), and the aforementioned Arialdus (on construction and use).[15] In one astronomical treatise of the final quarter of the twelfth century, an astrolabe large enough to display every minute of a degree on its scales is explicitly recommended for certain observations. The author states that this level of precision requires a size of 'at least two or three cubits' (*bicubitum ad minus vel tricubitum*), presumably meaning the length of the instrument's diameter.[16]

An even larger instrument is conjured up in a treatise beginning *Est quedam celestis machine spera*…, which is extant in three manuscripts written between the late twelfth and late thirteenth centuries. The work in question provides an introductory account of the celestial sphere, the Ptolemaic planetary models, and the construction and use of astronomical tables, reflecting in particular the influence of the Toledan Tables.[17] In the longer recension preserved in two thirteenth-century copies, it includes a passage

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[12] On the former source, see Juste (2018: 72) (n. 208). One may also mention the note on an observation of Venus and the Moon on 4 February 1292 in MS Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 4572, fol. 6v (ed. Pedersen 2014: 61), which states the altitude of Alchimech/Spica (α Vir).

[13] Raymond of Marseilles, *Liber iudiciorum* (MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 17ra–va). Compare the announcement of this discussion in Raymond of Marseilles, *Tractatus astrolabii* 16c (ed. Alverny et al. 2009: 94): "[…] loca planetarum sive stellarum fixarum cum in medio celi fuerint certius cognoscantur quemadmodum in judiciorum libro dicturi sumus."

[14] Raymond of Marseilles, *Liber iudiciorum* (MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 17ra).

[15] John of Seville, *Dixit Iohannes: Cum volueris facere astrolabium…* (ed. Millás Vallicrosa 1942: 324); Raymond of Marseilles, *Tractatus astrolabii* 2i (ed. Alverny et al. 2009: 58); Arialdus, *Cum inter omnia instrumentorum genera…* (MS Cambridge, Jesus College, Q.G.29, fol. 188v): "Sunt etiam quidam qui instrumentum tante magnitudinis faciunt ut minuta per tricenarium vel alium numerum continentem exterius depingant, quod nos per omnia approbamus."

[16] *Investigantibus astronomiam primo sciendum*, Jn222 (ed. Pedersen 1990: 267).

[17] MSS Cambridge, Fitzwilliam Museum, McClean 165, fols. 49v–50v (s. XII[4/4]); Florence, Biblioteca Medicea Laurenziana, San Marco 194, fols. 62ra–65rb (s. XIII[2/2]); Oxford, Bodleian Library, Digby 47, fols.

that seeks to explain how the ecliptic longitudes of the Sun's apogee and perigee might found by measuring solar noon altitudes in order to gauge the lengths of the season. The author of this passage emphasizes the necessity of using an astrolabe with a degree scale large enough to show tiny sexagesimal fractions such as minutes and seconds. More than that, he even mentions 'fractions up to tenths' (*fractiones etiam usque ad decima*). Realizing that the instrument would have to be scaled up considerably for this purpose, he elaborates by claiming that a certain individual was said to be in possession of an astrolabe with a truly gigantic diameter of 17 cubits ($\approx$ 25.5 ft).[18]

If such enormous contraptions were ever built, which one ought to consider sceptically, it seems unlikely that they would have closely resembled an ordinary planispheric astrolabe in incorporating a horizon plate and rete. Rather, one ought to consider the possibility that some of the larger instruments in use during this period reduced the astrolabe to the measuring device on its backplate.[19] A case in point is a passage in the *Astrologia* composed in 1220 by William the Englishman, citizen of Marseilles, which describes an instrument constructed for the specific purpose of determining the length of the tropical year on the basis of measured solar altitudes. It involves a graduated circle engraved onto the perfectly flat surface of a large stone square, which is erected in the meridian plane. Altitudes are measured with an alidade pivoted at the centre of this circle.[20] It seems probable that this simple design evolved out of the meridional armillary described in Ptolemy's *Almagest*, which consisted of two concentric rings, one of them equipped with sights and turning inside the other.[21] The Latin *Almagesti minor*, which was among the sources William used for his *Astrologia*,[22] modified Ptolemy's account by imagining the inner ring as fitted inside the circular cavity of a square-shaped object.[23] It would appear that William simplified this design proposal by leaving the square intact and replacing the inner ring with an alidade. Another such *instrumentum altitudinis* features briefly in the little-known *Canones* by

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Footnote 17 continued

2r–8r (s. XIII^{ex}). In the Oxford MS, the text carries the title *Liber omnium spherarum celi et compositionis tabularum translatus a magistro G. Cremonense de Arabico in Latinum*, but the same claim of origin is made about extracts from the canons to the Toledan Tabls on fols. 61r–67v, which are headed *Liber coequationis planetarum translatus a magistro G. Cremonense de Arabico in Latinum*. See Pedersen (2002: ii. 727–728, 734–736) (CcF).

[18] MS Florence, Biblioteca Medicea Laurenziana, San Marco 194, fol. 62vb: "Nota autem istarum quartarum altitudinem non per astrolabium ubi solum insigni essent gradus posse accipi, sed etiam ubi et minuta et secunda et tertia et fractiones etiam usque ad decima. Unde quidam astrolapsum 17 cubitorum diametrum habentem dicitur habuisse." See also MS Oxford, Bodleian Library, Digby 47, fols. 3v–4r.

[19] According to a thesis frequently advanced by Emmanuel Poulle, virtually all references in medieval sources to altitude measurements with an 'astrolabe' are in fact to an instrument of this type. This interpretation goes too far in my opinion. See Poulle (1969: 17; 1972: 40; 1980: 47–48; 1981: 8–10; 1991: 255–257; 2011: 73–76). Overly dismissive comments on the astrolabe's observational utility also appear in Poulle (1964b: 148; 1969: 8–9; 1983: 23–24; 1987: 130; 1998: 77).

[20] William of Marseilles, *Astrologia* (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 357, fol. 3r).

[21] Ptolemy, *Almagest*, trans. Gerard of Cremona, I.12. See fol. 9r in the 1515 edition transcribed by Stefan Georges at https://ptolemaeus.badw.de/print/1/70/text/1. For an English translation of the Greek text of the relevant chapter, see Toomer (1998: 61–63). For a discussion of the instrument, see Price (1957: 588–589).

[22] See Zepeda (2018: 7–11).

[23] *Almagesti minor* I.15 (ed. and trans. Zepeda 2018: 158–161).

Roger of Cotum, a thirteenth-century English friar, according to whom the device was supposed to assist in measurements of local latitude.[24]

2.2 Quadrants

An alternative to the rotating alidade that is very frequently described in medieval astronomical manuscripts is the handheld quadrant equipped with a degree scale, perforated pinnules, and plumbline. The most widely attested variant of this instrument in the period under review is the so-called *quadrans vetus*, which served the dual purpose of finding time during the day (via the solar altitude) and measuring heights, distances, and depths (by means of a shadow square).[25] Owing to an invention made by Profatius Judaeus (Jacob ben Makhir ibn Tibbon) and first described in a Hebrew treatise of 1288, a *quadrans novus* entered the Latin astronomer's toolkit towards the end of the thirteenth century. Profatius's design combined the time-finding functions of the *quadrans vetus* with the computational uses of the planispheric astrolabe and accordingly exploited altitude measurements in much the same way as the latter instrument.[26] Both the *vetus* and *novus* variants of the quadrant were portable and thus relatively small. They must not be confused with another type of quadrant, where the device was erected in a fixed position in the meridian plane. To this class belongs Ptolemy's 'plinth', which was equipped with cylindrical pegs, of which one served as a gnomon casting a shadow against an engraved degree scale of 90°.[27] Aside from the *Almagest* and its Latin offshoot, the *Almagesti minor*,[28] astronomers could read about this instrument in two Arabic works translated during the first half of the twelfth century: al-Battānī's *Ṣābi' Zīj* (trans. Plato of Tivoli) and a commentary on the tables of al-Khwārizmī by Ibn al-Muthannā (trans. Hugo Sanctelliensis). Both authors emphasize the importance of building the quadrant at a large scale so as to improve one's chances of making an accurate measurement.[29]

In Bernard of Verdun's *Tractatus super totam astrologiam*, written in the second half (and probably towards the end) of the thirteenth century, a quadrant's graduated arc of 90° is imagined as engraved onto the surface of a square-shaped copper plate that is fitted inside a wooden frame. The frame has a diameter of four cubits, which is supposed to be large enough to allow for the arc to be divided into minutes and even some multiples of seconds. Altitudes are here measured with an alidade whose pivot is located on the corner that serves as the centre for the graduated arc (see Fig. 1).[30] The resulting instrument may be considered a 90°-variant of that depicted

[24] Roger of Cotum, *Canones* I.22 (MS Oxford, Bodleian Library, Laud. Misc. 644, fol. 215v).

[25] Poulle (1972: 36–39; 1983: 9–13; 1998: 83–84), Hahn (1982), Archinard (1990), Knorr (1997a, 1997b) and Nothaft (2020).

[26] Boffito and Melzi d'Eril (1922), Poulle (1964b, 1983: 24–28), Dekker (1995, 2008) and King (2005: 80, 183, 225–229).

[27] Ptolemy, *Almagest*, trans. Gerard of Cremona, I.12 (see n. 21 above); Price (1957: 587–588) and Evans (1999: 272–274).

[28] *Almagesti minor* I.15 (ed. and trans. Zepeda 2018: 158–163).

[29] al-Battānī, *De motu stellarum*, trans. Plato of Tivoli, c. 57 (ed. 1537, fols. 88r–89r); Ibn al-Muthannā, *Commentary on the Tables of al-Khwārizmī*, trans. Hugo Sanctelliensis (ed. Millás Vendrell 1963: 131–132).

[30] Bernard of Verdun, *Tractatus super totam astrologiam* II.2.2 (ed. Hartmann 1961: 40).
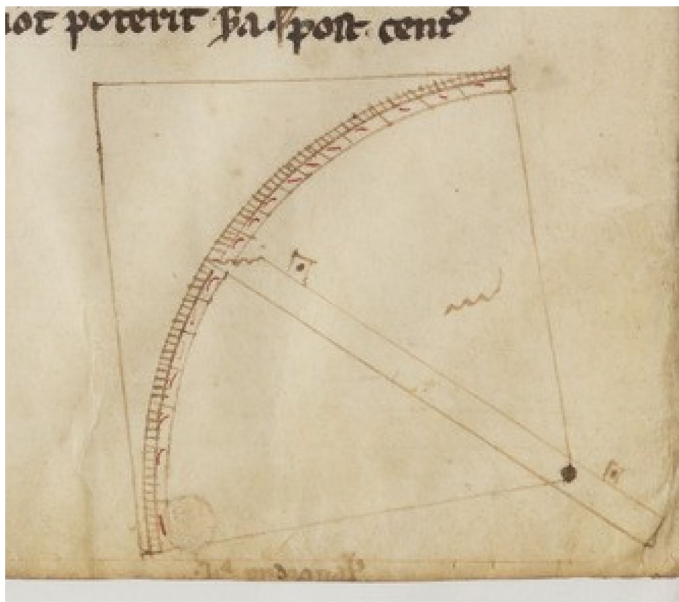
Fig. 1 Quadrant with alidade in MS Paris, Bibliothèque nationale de France, lat. 7333, fol. 6vb (Bernard of Verdun, *Tractatus super totam astrologiam*). *Source*: BnF

in William the Englishman's *Astrologia* in 1220 (see above), where the alidade pivots around the centre of a 360°-scale. As described in Bernard's *Tractatus*, the device in question was intended primarily for obtaining the obliquity of the ecliptic, which involved measurements of the solar noon altitude for 3 or 4 consecutive days around the solstices.[31] He also recommended it for observations around the autumn equinox with the goal of deriving the length of the tropical year.[32]

2.3 Shadows

In the specific case of solar altitudes, there was the further possibility of deriving this result indirectly from the length of a shadow. Perhaps the easiest way to accomplish this was to apply the known ratio between the length of the shadow and the height of the object casting it to the shadow square that was engraved on the surfaces of many astrolabes and portable quadrants. By following a set of simple arithmetical rules, one could use the square to convert the ratio in a quasi-mechanical fashion into a degree value for the solar altitude.[33] Another option was to use tables for the mutual conversion of altitudes and shadow lengths, as were commonly found in larger sets

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[31] Ibid.

[32] Ibid. III.1.2 (ed. Hartmann 1961: 61–62).

[33] For examples of relevant accounts in instrument treatises, see Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. John of Seville, c. 29 (ed. Millás Vallicrosa 1942: 277–278); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. Plato of Tivoli, c. 31 (ed. Lorch et al. 1994: 163–164); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. anon., c. 21 (ed. Nothaft 2023a: 404); pseudo-Māshā'allāh, *Practica astrolabii*, c. 43 (ed.

of astronomical tables.[34] In all these techniques, the shadow is commonly measured according to digits or segments that represent 1/12th of the height of the shadow-casting object. The angle of solar altitude (h) as a function of the length of the shadow (s) will then be: $h = \arctan(\frac{12}{s})$. Medieval texts instead tend to solve for h using the equivalent formula: $\sin(90 - h) = \frac{60s}{\sqrt{s^2 + 144}}$.[35] An anonymous work on practical geometry (*Artis cuiuslibet consummatio…*) composed in Paris in 1193 proposes that one apply the same principle to the altitude of a star at night, by employing horizontal and vertical sticks in the manner of a cross-staff.[36] The final chapter of this text describes a gnomon surrounded by a scale made up of concentric circles, which could be used to measure the digits in the length of the gnomon's shadow.[37] This type of device makes another appearance in the aforementioned *Canones* by Roger of Cotum, who suggests that it can be used to find the Sun's zenith distance.[38]

It remains unclear to what extent practising astronomers followed such advice in the sense that they inferred altitudes from shadow lengths, rather than measuring them directly. The only known hint that this may have been attempted in the period under discussion comes from the Parisian scholar Peter of Limoges, who recorded the length of the noon shadow on 11 March 1290 as 14;3 (= 14.05) digits, adding that the corresponding solar altitude was 40;30°.[39] This correspondence happens to be exact, both according to the available shadow tables and the formulae given above, which raises suspicions about Peter's mode of procedure. As I shall argue below in Part 5, Peter's 14;3 digits are less likely to have resulted from a direct measurement than from a computation that began with the known value of the solar noon altitude.

Footnote 33 continued

and trans. Thomson 2022: iv. 424–429); Robert of Chester, *Liber de officio astrolabii*, c. 28 (MS Oxford, Bodleian Library, Canon. misc. 61, fol. 20r–v); *Tractatus magistri Habrahe de tabulis planetarum* (MS London, British Library, Arundel 377, fols. 60vb–61ra); *Quelibet ars suum habet artificem…* (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 352, fol. 112vb); Peter of Maricourt, *Nova compositio astrolabii particularis*, c. 27 (ed. Thomson in Sturlese and Thomson 1995: 183). On the wider topic of shadows in medieval practical geometry, see Lička (2022: 198–216).

[34] Chabás and Goldstein (2012: 24–25).

[35] See, e.g., *Ezic Elkaurezmi*, trans. Adelard of Bath, c. 28a (ed. Suter 1914: 21–22); al-Battānī, *De motu stellarum*, trans. Plato of Tivoli, c. 10 (ed. 1537, fols. 14r); Ibn al-Muthannā, *Commentary on the Tables of al-Khwārizmī*, trans. Hugo Sanctelliensis (ed. Millás Vendrell 1963: 153–154); Toledan Tables canons Ca65, Cb123–124, Cc118–119 (ed. Pedersen 2002: i. 240; ii. 428, 634); *Artis cuiuslibet consummatio…* II.1, 4 (ed. and trans. Victor 1979: 220–223, 228–229); Bernard of Verdun, *Tractatus super totam astrologiam* II.4.8 (ed. Hartmann 1961: 52).

[36] *Artis cuiuslibet consummatio…* II.2 (ed. and trans. Victor 1979: 224–225).

[37] Ibid. II.38 (ed. and trans. Victor 1979: 322–325). This contraption was also described in the canons to the Tables of Jaén by Ibn Muʿādh, which were translated in the twelfth century by Gerard of Cremona. The relevant part of the text is edited in Samsó and Mielgo (1994: 23–24).

[38] Roger of Cotum, *Canones* I.21 (MS Oxford, Bodleian Library, Laud. Misc. 644, fol. 215va). See also *Quelibet ars suum habet artificem…* (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 352, fol. 112vb).

[39] See below, n. 116.

2.4 The torquetum

Worth at least a brief mention is the so-called torquetum, a relatively complex, multi-functional instrument, whose origins are shrouded in some degree of obscurity.[40] What the available evidence allows us to conclude is that the torquetum became known among astronomers working in Paris at some point before July 1284. It appears to have been designed from the outset with the intention of combining the observational functions of Ptolemy's armillary instrument, which could measure celestial positions directly in the ecliptic coordinate system, with some of the computational and horological uses of a standard astrolabe.[41] For measurements of ecliptic latitudes, the torquetum came equipped with the so-called *crista* (a word that could also be used to describe a chicken comb), which was a graduated circular disc equipped with an alidade. A further component attached to this alidade was a semi-circle with plumbline, which could be used for altitude measurements in the horizontal coordinate system. The semi-circle, or *semis*, also carried an hour-line diagram for unequal hours, which allowed it operate in much the same way as a quadrant. Depending on how the torquetum was constructed, it was possible to 'fold' the apparatus containing the *crista* into the horizontal plane, in which case altitudes could be measured directly with its alidade.[42] It should be borne in mind, however, that an astronomer intent on finding celestial altitudes could rely on much simpler and cheaper instruments for this specific purpose, which makes it less than surprising that no known evidence links the torquetum to any altitude measurements made before 1300.

3 Geographic latitudes

Knowledge of geographic latitude was a prerequisite for a host of commonly documented astronomical activities. These included the construction of astrolabes and sundials as well as the correct handling of 'universal' instruments such as horary quadrants, not to mention the task of setting up a torquetum or armillary instrument for observational use in a particular location. In a similar vein, many common types of astronomical tables were geographically specific and therefore had to be recast or newly constructed before they could be applied to a location of different latitude.[43] Prior to the 'translation movement' of the twelfth century, the concept of latitude had already been a feature of the earliest Latin texts on the astrolabe, which first took shape in the late tenth century. Where such texts address the actual measurement of

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[40] The two most important recent publications on the torquetum are Kremer (2019) (on the historiography, sources, and question of origin) and Włodarczyk (2022) (on its observational utility). See also the earlier discussions in Poulle (1964a, 1983: 31–35) and Lejbowicz (1997: 202–205). One thirteenth-century mention of the instrument not already recorded in these studies is Roger Bacon, *Communia mathematica* 1.3.3 (ed. Steele 1940: 45, l. 4), the precise date of which is not known.

[41] The claim that the torquetum was a computational rather than observational instrument has been made by scholars such as Poulle (1969: 7–8; 1983: 33; 1991: 264) and Samsó (2020: 495), but it is at odds with the sources. See on this point Dekker and Lippincott (1999: 118–121).

[42] Włodarczyk (2022: 275–276).

[43] For a survey and editions of latitude-dependent tables in the corpus of the Toledan Tables, see Pedersen 2002: iii. 996–1130 (BD–BK).

this parameter, they usually do so on the basis of the equinoctial solar noon altitude (i.e., the first of the three methods listed below).[44] More detailed accounts of how to establish one's local latitude began to circulate in the first half of the twelfth century as a consequence of Arabic-to-Latin translations of instrument treatises and canons to astronomical tables.[45] The three methods one typically encounters in such sources are as follows:

1. measure the solar noon altitude on the day of the vernal or autumn equinox to obtain the local co-latitude; subtract the result from 90°.
2. measure the solar noon altitude on any day and add/subtract the declination corresponding to the Sun's current ecliptic longitude; subtract the result from 90°.
3. measure the upper and lower culmination altitudes of a circumpolar star and halve their difference to find the elevation of the north celestial pole.

There is no shortage of Latin works of the twelfth and thirteenth centuries in which these same three methods are presented at some level of detail. Relevant examples include the following:

- Works on the astrolabe: Raymond of Marseilles, *Tractatus astrolabii* (*c.*1141)[46]; Adelard of Bath, *Libellus de opere astrolapsus* (1149/1150)[47]; Abraham Ibn Ezra, *Genera astrolabiorum duo sunt…*(s. XII^med)[48]; Robert of Chester, *Liber de officio astrolabii* (s. XII^med)[49]; *Liber de wazalkora* (s. XII^2/2)[50]; pseudo-Māshā'allāh, *Practica astrolabii* (s. XII/XIII)[51]; Arialdus, *Cum inter omnia instrumentorum genera…* (s. XII^2/2)[52]; *Quelibet ars suum habet artificem…* (s. XIII^1/2)[53]; Peter of Maricourt, *Nova compositio astrolabii particularis* (1263/64)[54]

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[44] See the texts edited in Millás Vallicrosa 1931: 287, 307–308, 321, as well as *De utilitatibus astrolabii* 18.2 (ed. Bubnov 1899: 140–141); *In qua parte climatis cuiuslibet quisque sit…* (MS Avranches, Bibliothèque municipale, 235, fol. 31v; s. XII^med). See also Poulle (1969: 3–5), Wiesenbach (1991: 134–135) and Gautier Dalché (2000: 407–408).

[45] Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. John of Seville, c. 23 (ed. Millás Vallicrosa 1942: 274); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. Plato of Tivoli, c. 25 (ed. Lorch et al. 1994: 157–158); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. anon., c. 18 (ed. Nothaft 2023a: 402–403); *Ezich Elkaurezmi*, trans. Adelard of Bath, c. 24 (ed. Suter 1914: 18); Toledan Tables Ca47–49, Cb67, Cc049–52 (ed. and trans. Pedersen 2002: i. 234–235; ii. 408–409, 608–611); al-Battānī, *De motu stellarum*, trans. Plato of Tivoli, c. 14 (ed. 1537, fol. 19r–v); Ibn al-Muthannā, *Commentary on the Tables of al-Khwārizmī*, trans. Hugo Sanctelliensis (ed. Millás Vendrell 1963: 133–136); Ibn Moforngh, *Canones tabularum universalium*, sermo 38 (MS Oxford, Bodleian Library, Ashmole 361, fol. 79rb–va); Ibn al-Zarqālluh, *Liber operacionis tabule que nominatur saphea*, trans. Profatius Judaeus and John of Brescia, c. 16, 39–40 (ed. Millás Vallicrosa 1933: 121–122, 135).

[46] Ed. and trans. Alverny et al. (2009: 96–103) (§22).

[47] Ed. Dickey (1982: 191–193).

[48] Ed. Millás Vallicrosa (1940: 16, 20).

[49] MS Oxford, Bodleian Library, Canon. misc. 61, fol. 17r–v (c. 22).

[50] MS London, British Library, Arundel 339, fols. 92r, 93r–94r (pt. 3, c. 14, 42, 52).

[51] Ed. Thomson 2022: iv. 216–231 (c. 21–22).

[52] MS Paris, Bibliothèque nationale de France, lat. 16652, fol. 35r–v.

[53] MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 352, fol. 112rb (s. XIII^1/2). This is the text recorded in Kunitzsch (1982: 506–507), who knew it only from MS Oxford, Bodleian Library, Ashmole 361, fols. 35ra–41vb (s. XIV^med). The copy in the Erfurt MS has a different beginning (*Cuius artis instrumentum est astrolabium…*) and abridges the text somewhat.

[54] Ed. Thomson in Sturlese and Thomson (1995: 180) (c. 24).

- Works on the horary quadrant: John of Sacrobosco, *Tractatus de quadrante* (1239/56)[55]; *Ad quadrantem faciendum fiat lamina…* (1244/72)[56]; *Geometrie due sunt partes…* (1260/71)[57]; *Cum quadrantem componere volueris…* (s. XIII²/²)[58]
- Works on astronomical tables/computational astronomy: Abraham Ibn Ezra, *Liber de rationibus tabularum* (1154)[59]; *Tractatus magistri Habrahe de tabulis planetarum* (*c.*1150)[60]; commentary *Sicut dicit Hermes* on Toledan Tables canons Cb (*c.*1290)[61]; John of Sicily, *Scriptum super canones Azarchelis* (1291/93)[62]; Roger of Cotum, *Canones* (s. XIII)[63]
- Other astronomical works: *Artis cuiuslibet consummatio…* (1193)[64]; Gerard of Feltre, *Summa de astris* (1264)[65]; Robert the Englishman, *Commentary on John of Sacrobosco's* Tractatus de spera (1271)[66]; Friar John, *Summa astrologie* (1276)[67]; Bernard of Verdun, *Tractatus super totam astrologiam* (s. XIII²/²).[68]

It is usually implied in this literature that measurements of solar or stellar altitude will be taken with the alidade of an astrolabe or with some type of quadrant. Only occasionally are we reminded of the additional option of inferring the noon altitude of the Sun from the length of a shadow.[69] While the solar noon altitude could, in principle, be measured on any day of the year, the lower and upper culmination of the same circumpolar star occurs twelve equinoctial hours apart, meaning that method (3) was feasible only during long winter nights. It was possible, however, to modify this approach by modelling it after method (2): measure a star's altitude at upper culmination and subtract its known declination to find the elevation of the celestial equator (to be subtracted from 90° to yield the local latitude). This option is raised briefly in a treatise on astrolabe use by Ibn al-Ṣaffār (d. 1035), which was translated into Latin at least three times before the middle of the twelfth century.[70] From one of these translations (by John of Seville), it passed into the *Practica astrolabii* of

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[55] MS Paris, Bibliothèque nationale de France, lat. 7196, fols. 26vb–27ra; cf. Knorr (1997a).

[56] MS Paris, Bibliothèque nationale de France, lat. 7475, fol. 63r; cf. Thorndike and Kibre (1963: 58) and Knorr (1997a: 212).

[57] Ed. Hahn 1982: 54–57 (§§41–43).

[58] MS Oxford, Bodleian Library, Digby 98, fol. 163rb.

[59] Ed. Millás Vallicrosa (1947: 155–156).

[60] MS London, British Library, Arundel 377, fol. 60ra.

[61] Ed. Pedersen 2001–2002, pt. 1: 207–210 (Ap109–115).

[62] Ed. Pedersen (1986: 74–77) (J146–149).

[63] MS Oxford, Bodleian Library, Laud. Misc. 644, fol. 215vb (I.22).

[64] Ed. and trans. Victor (1979: 230–231) (II.5), 234–237 (II.7–8).

[65] MS London, Wellcome Library, 308, fol. 29r–v (I.8).

[66] Ed. Thorndike (1949: 181) (lectio XI).

[67] MS Paris, Bibliothèque nationale de France, lat. 7293A, fol. 63v (III.3).

[68] Ed. Hartmann 1961: 51–52 (II.4.6).

[69] al-Battānī, *De motu stellarum*, trans. Plato of Tivoli, c. 14 (ed. 1537, fol. 19v); *Liber de wazalkora*, pt. 3, c. 59 (MS London, British Library, Arundel 339, fol. 94r–v); Toledan Tables Cc051 (ed. Pedersen 2002: ii. 608).

[70] Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. John of Seville, c. 23 (ed. Millás Vallicrosa 1942: 274); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. Plato of Tivoli, c. 25 (ed. Lorch et al. 1994: 158); Ibn al-Ṣaffār, *On the Uses of the Astrolabe*, trans. anon., c. 18 (ed. Nothaft 2023a: 402).

pseudo-Māshāʾallāh, which was the most frequently copied medieval Latin treatise on astrolabe use (with *c.*200 copies still extant).[71] The method receives slightly more attention in the treatise by Arialdus, who gives the example for a star with a declination of + 10° that is found at the zenith, thereby revealing a geographic latitude of 80°.[72]

Method (1) will be inexact insofar as the swift change of solar declination around the equinoxes usually means that the altitude measured on the noon closest to the vernal or autumn equinox deviates by several minutes from the altitude of the celestial equator, which is the local co-latitude. An attempt to find the co-latitude with greater exactitude had to use measurements of the annual maximum and minimum noon altitudes the Sun reaches around the solstices. Their difference is halved to obtain the obliquity of the ecliptic, which is then added to the minimum noon altitude, or subtracted from the maximum noon latitude, to yield the co-latitude. Latin astronomers were no doubt familiar with these operations, as seen from a number of descriptions and allusions in the available literature.[73] The clearest evidence we have of their practical application comes from 1280s Paris, as will be discussed in more detail below (in Part 5).

Instances where a source from this period informs us directly about the method that was used to find a particular latitude are exceedingly rare. All we have in the vast majority of cases is the latitude value itself, leaving us uncertain about the means by which it may have been established. An important type of source in this regard are the twelfth- and thirteenth-century copies of the tabular list of geographic coordinates commonly included among the Toledan Tables.[74] In its original form, this 'Toledan Coordinate Table' (hereafter abbreviated as TCT) was mostly restricted to locations in the Islamic world (Spain, North Africa, Levant). Over time, however, those copying it augmented it with data for locations in Latin Europe. For the present survey, I have examined the following 24 manuscripts containing such additions[75]:

Ba = Basel, Universitätsbibliothek, O.II.7, fol. 162r (s. XIII^ex); *Bf* = Berlin, Staats-bibliothek, lat. fol. 610, fol. 234r (s. XIII^ex); *Ca* = Cambridge, University Library, Hh.6.8, fol. 184r (s. XIII^2/2); *Ed* = Edinburgh, Royal Observatory, Crawford Collec-tion, Cr. 2.5, fol. 73v (s. XIII^med); *Eq* = Erfurt, Universitätsbibliothek, Dep. Erf. CA 8° 82, fol. 88r (s. XIII^ex); *Fj* = Florence, Biblioteca Nazionale Centrale, Conv. soppr. J.V.5, fol. 66r (s. XIII^2/2); *Fl* = Florence, Biblioteca Medicea Laurenziana, San Marco

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[71] pseudo-Māshāʾallāh, *Practica astrolabii*, c. 22 (ed. Thomson 2022: iv. 226–229). Attention to this passage was drawn by Poulle (1969: 5–6). On the origin of this treatise, see Kunitzsch (1981: 48–56; 1982: 501–502).

[72] Arialdus, *Cum inter omnia instrumentorum genera…* (MS Paris, Bibliothèque nationale de France, lat. 16652, fol. 35v).

[73] Ptolemy, *Almagest*, trans. Gerard of Cremona, I.12 (see n. 21 above); al-Battānī, *De motu stellarum*, trans. Plato of Tivoli, c. 4 (ed. 1537, fol. 8r–v); *Almagesti minor* I.15 (ed. and trans. Zepeda 2018: 160–161); *Liber de wazalkora*, pt. 3, c. 51, 57 (MS London, British Library, Arundel 339, fols. 93v–94r); John of Sicily, *Scriptum super canones Azarchelis*, J545 (ed. Pedersen 1986: 257–258).

[74] See the critical edition of this list with discussion of its offshoots and variants in Pedersen 2002: iv. 1509–1518 (MA11–13). It makes its earliest datable appearance in Latin in Raymond of Marseilles, *Liber cursuum planetarum* (ed. and trans. Alverny et al. 2009: 198–199), which was begun in 1141. See also Wright 1923: 85–93; Toomer 1968: 134–139; Laguarda Trías 1990: 64–73 (with Tables 1 and 2); Gautier Dalché 2009: 101–103; Huth 2013: 221–228, 473–475 (unreliable); Schaldach 2020: 41–44.

[75] In what follows, I shall consider only additions to such lists if they pre-date the year 1300 with a high degree of likelihood. Several further copies of the list were examined for this project, but ultimately excluded on the same chronological grounds.

185, fols. 51r, 83r (s. XIII^{ex}); Lh = London, British Library, Harley 3734, fol. 40r (s. XIII$^{2/2}$); Ma = Madrid, Biblioteca nacional de España, 10,016, fol. 70v (s. XIII); Me = Metz, Bibliothèque municipale, 1223, fols. 16v, 110r (XIII^{ex}); Ny = New York, Columbia University, Rare Book & Manuscript Library, Smith Western Add. 06, fol. 116v (s. XIII^{ex}); Ol = Oxford, Bodleian Library, Laud. Misc. 644, fol. 91v (s. XIII$^{2/2}$); Ot = Oxford, Bodleian Library, Savile 22, fol. 67r (s. XIII^{med}); Pa = Paris, Bibliothèque de l'Arsenal, 877, fol. 75r (s. XIII$^{2/2}$); Pb = Paris, Bibliothèque de l'Arsenal, 1128, fol. 28v (s. XIII$^{2/2}$); Pc = Paris, Bibliothèque nationale de France, lat. 7198, fol. 90r (s. XIII^{ex}); Pe = Paris, Bibliothèque nationale de France, lat. 7409, fol. 37v (s. XIII$^{4/4}$); Pg = Paris, Bibliothèque nationale de France, lat. 7421, fol. 203v (s. XIII^{ex}); Ph = Paris, Bibliothèque nationale de France, lat. 11249, fol. 79r–v (s. XIII$^{1/2}$); Pl = Paris, Bibliothèque nationale de France, lat. 16211, fol. 93v (s. XIII^{ex}); Pn = Paris, Bibliothèque nationale de France, n.a.l. 3091, fol. 74v (s. XIII^{ex}); Pr = Princeton (NJ), University Library, Garrett 99, fol. 78v (s. XIII^{ex}); Vp = Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1414, fol. 138v (XIII$^{2/2}$); Vi = Vienna, Österreichische Nationalbibliothek, 2385, fol. 36v (s. XIII).

To these, one might add a unique list of the latitudes and co-latitudes (*elevatio capitis Arietis*) of 12 cities found in Cs = Cambridge, St John's College, F.25, fol. 49r, which seems to date from the end of the thirteenth century. Another relevant source is the *Summa de astris* composed in 1264 by Gerard of Feltre, a Dominican friar. The first of its three books includes a table of geographic coordinates that derives most of its entries from the TCT and Ptolemy's *Handy Tables*, with some more unusual additions. For the analysis below, I have selected Gerard's latitudes for Cremona, Feltre, London, Piacenza, and Paris, which appear to be independent of Ptolemy or the TCT.[76] I have chosen to disregard the two tables of coordinates in MS Cambridge, University Library, Hh.6.8, fols. 38v–39r (s. XIII^{in}), of which one contains the coordinates of Toulouse (latitude 43°), while the other is a version of the TCT with additions for Pisa and Bourges (both 45°). They belong to a set of astronomical tables for Toulouse stating in 1110, which were composed or otherwise influenced by the Iberian Jewish scholar Abraham bar Ḥiyya.[77] Their entries for the mentioned cities are hence likely to have originated in a non-Latin source, as is probably also true of the coordinate list in the *Liber canonum* composed around 1150 by Robert of Chester, who was active in Spain as a translator of Arabic texts.[78]

Further evidence of medieval European latitude estimates is provided by astronomical tables that were calculated for specific cities. An example that is attested in numerous manuscripts since the second half of the twelfth century is a table for oblique

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[76] Gerard of Feltre, *Summa de astris* I.8 (MS Milan, Biblioteca Ambrosiana, C 245 inf., fol. 7r). See Nothaft (2022: 11–12).

[77] Mercier (1987: 103–104; 2014: 185–203).

[78] Robert of Chester, *Liber canonum*, pt. 2, Lc5 (ed. and trans. Nothaft 2023b: 246–249).

ascensions at Cremona, whose latitude is given as 45°.[79] Other known cases from the period 1100–1300 are the following:

- Oblique ascensions for Palermo with latitude given as 37;57°: MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 2° 38, fol. 23r–v (s. XIII[med])[80]
- Oblique ascensions and houses for Toulouse with latitude given as 42;45°: MS Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1414, fols. 96r–100v (s. XIII[2/2])[81]
- Oblique ascensions for Cahors with latitude given as 46°: MS Paris, Bibliothèque nationale de France, lat. 7422, fols. 79v–81r (s. XIII[1/2])[82]
- Oblique ascensions for Le Mans with undefined latitude in the probable range of 47;30°–47;56°: MS Oxford, Bodleian Library, Savile 21, fols. 98v–99r (s. XIII[1/2]).[83]
- Oblique ascensions for Magdeburg with latitude given as 52°: part of Friar John, *Compotus philosophicus* (1273)[84]
- Houses for Paris with latitude given as 48°. Extant copies include MSS Metz, Bibliothèque municipale, 1223, fols. 61r–62v (s. XIII[ex]); Paris, Bibliothèque nationale de France, lat. 16211, fols. 50r–51r (s. XIII[ex]); Paris, Bibliothèque nationale de France, n.a.l. 3091, fols. 30r–31v (s. XIII[ex]).[85]

According to a seventeenth-century extract from the *Liber daticus* formerly in the possession of Roskilde Cathedral, this lost book contained a calendar as well as an associated astronomical table, which both showed the lengths of day and night at the latitude of Roskilde (and apparently also the corresponding solar longitudes and noon altitudes). With respect to the astronomical table, the text notes that it was drawn up in 1274 with the help of an astrolabe and trigonometric tables (*per astrolabium et Gardagas sinuum*).[86] A second extract from the same source notes that Roskilde is 23;12,16° further north than Jerusalem,[87] which in the TCT is normally placed at a latitude of 32°. The best explanation for the assumed latitude of 55;12,16°, which is confirmed by another manuscript from Roskilde,[88] is that it was generated with the help of the solar declinations provided by the Toledan Tables. According to their standard declination table, which was computed using an obliquity of 23;33,30°, the

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[79] Pedersen (2002: iii. 1004–1010) (BD20). Copies of this table that are not already recorded by Pedersen include MSS Naples, Biblioteca nazionale 'Vittorio Emanuele III', VIII.C.49, fols. 58v–59r (s. XIII[ex]; omits columns for hour-lengths); Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 5714, fols. 36r–37v (s. XIII[2/2]); Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1410, fols. 48v–50r (s. XIII[2/2]). There was also a table of houses for the same location and latitude: Pedersen (2002: iii. 1109–1111) (BH13).

[80] Pedersen (2002: iii. 1024) (BE15).

[81] See Pedersen (2002: iii. 1026) (BE30), 1119 (BH30). This latitude is also mentioned in headings to mean-motion tables for Toulouse, which go back to the first half of the thirteenth century. See Poulle (1994: 63–64); Pedersen (2002: iii. 1197–1205) (CC).

[82] Pedersen (2002: iii. 1027) (BE40).

[83] Pedersen (2002: iii. 1027–1028) (BE45). See also Toomer (1968: 153).

[84] MS Lüneburg, Ratsbücherei, Miscell. D 4° 46, fols. 13v–14r. See Nothaft (2014: 576).

[85] Pedersen (2002: iii. 1112–1116) (BH14).

[86] The text is printed in Pedersen (1983–1984: i. 56–57).

[87] Pedersen (1983–1984: i. 55).

[88] MS Stockholm, Kungliga Biblioteket, X 767, p. 131 (s. XIV). The note specifying Roskilde's latitude was added by a later hand (s. XIV or XV). Roskilde is given a latitude of 55° in a list of coordinates added (by a later hand) to MS Fermo, Biblioteca Civica Romolo Spezioli, 85, fol. 77v (s. XIII[2/2]).

declination is ± 22;12,16° at solar longitudes of 71°, 109°, 251°, and 289°.[89] This suggests that the latitude of Roskilde may have been found by subtracting 22;12,16° from an observed solar noon altitude of 57° on a day when the Sun was assumed to be at 71° or 109°, resulting in a co-latitude of 34,47,44°.[90] Modern calculation indicates that the Sun at Roskilde reached a noon altitude of 56;39° on 26 May 1274, which according to the Toledan Tables saw the Sun occupy the 71st ecliptic degree (c.70;59°).

The same method of adding or subtracting a declination value probably explains the unusually precise latitudes for Palermo (38;15,50°) and Ely in Cambridgeshire (52;35,6°) that are recorded in thirteenth-century glosses in MS Oxford, Bodleian Library, Digby 20, fol. 44r and MS London, British Library, Arundel 377, fol. 56r. One case where the use of a declination value in determining local latitude is directly attested is the anonymous note in MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 5r (s. XII^(2/2)), which reports a latitude measurement for Troyes made in 1165 (see Part 4).

The three examples just provided belong to a list of 14 manuscripts that contain particularly informative notes and glosses concerning the latitudes of places in Latin Europe. I shall refer to them by the following sigla for the remainder of this section:

Ck = Cambridge, University Library, Kk.1.1 (s. XIII)
Fl = Florence, Biblioteca Medicea Laurenziana, San Marco 185 (s. XIII^(ex))
La = Laon, Bibliothèque municipale Suzanne Martinet, 425 (s. XIII^(1/2))
Lb = London, British Library, Arundel 377 (s. XIII)
Li = London, British Library, Harley 3735 (s. XIII^(ex))
Lj = London, British Library, Harley 4350 (s. XIII^(2/2))
Od = Oxford, Bodleian Library, Digby 20 (s. XIII)
Os = Oxford, Bodleian Library, Savile 21 (s. XIII)
Ot = Oxford, Bodleian Library, Savile 22 (s. XIII^(2/2))
Pf = Paris, Bibliothèque nationale de France, lat. 7411(B) (s. XIII^(ex))
Pj = Paris, Bibliothèque nationale de France, lat. 16208 (s. XII^(2/2))
Pm = Paris, Bibliothèque nationale de France, lat. 16658 (s. XIII^(med))
Po = Pommersfelden, Gräflich Schönbornsche Bibliothek, 66 (2640) (s. XIII)
Vp = Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1414 (s. XIII^(2/2)).

Table 1 represents an effort to collect from these and other available sources a corpus of latitudes of Western European cities that were—with a high degree of likelihood—established between 1100 and 1300. In selecting data from the manuscripts mentioned above, I have generally erred on the side of parsimony in the sense of not including information from hands that are likely to be later than 1300. By the same token, I have excluded latitudes that are attested in manuscripts from this period, but were already established before 1100 or otherwise derived from non-Latin sources. I have, accordingly, disregarded any latitudes that trace their origin to Ptolemy or to Islamic coordinate lists.[91]

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[89] See Pedersen (2002: iii. 961–966) (BA21).

[90] The suggestion was already made by Pedersen (1983–1984: i. 56).

[91] For the Islamic lists, see Kennedy and Kennedy (1987). On the presence of geographic coordinates from Ptolemy's *Handy Tables* in thirteenth-century Italy, see Nothaft (2022).

Table 1 Synopsis of latitudes of cities in Latin Europe established between 1100 and 1300

i	ii	iii	iv
City	LAT	Δ	Sources
Amiens	49;36°	− 0;18°	Richard de Fournival, *Nativitas* (before 22 Oct 1239) (MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1261, fol. 60r); Richard de Fournival, *Biblionomia* (*c.*1250) (ed. Delisle 1874: 520); *Po*, fol. 65v (s. XIII)
Arezzo	42;15°	− 1;13°	Restoro d'Arezzo, *La composizione del mondo*, bk. 1, ch. 15 (*c.*1282) (ed. Morino 1997: 37)
Arras	50;30°	+ 0;12°	*Po*, fol. 65v (s. XIII)
Bath	52°	+ 0;37°	Adelard of Bath, *Libellus de opere astrolapsus* (1149/50) (ed. Dickey 1982: 166)
Bordeaux	45°	+ 0;10°	*Pf*, fol. 93v (s. XIII[ex])
Cahors	46° [!]	+ 1;33°	Ascension tables for Cahors (s. XIII[1/2]) (See above, n. 82)
Canterbury	51;36°	+ 0;19°	Table in *Cs*, fol. 49r (s. XIII[ex])
Cremona	45°	− 0;8°	Ascension tables for Cremona (s. XII[2/2]) (See above, n. 79); *Liber de wazalkora* (s. XII[2/2]) (MS London, British Library, Arundel 339, fol. 94v); appendix to *Liber erarum* (1191) (ed. Nothaft 2014: 124); Gerard of Feltre, *Summa de astris* (1264) (above, n. 76); John of Sicily, *Scriptum super canones Azarchelis* (1291/93) (ed. Pedersen (1986: 100) (J193)); *Od*, fol. 44r (s. XIII); table in *Cs*, fol. 49r (s. XIII[ex]); TCT: *Ba*; *Bf*; *Ca*; *Eq*; *Fl*, fols. 51r, 83r; *Fj*; *Lh*; *Me*, fols. 16v, 110r; *Ny*; *Ol*; *Pa*, *Pb*; *Pc*; *Pe*; *Ph*; *Pl* (s. XIII[1/2])
Ely	52;35,6°	+ 0;11,13°	*Lb*, fol. 56r (s. XIII)
Feltre	45;30°	− 0;31°	Gerard of Feltre, *Summa de astris* (1264) (See above, n. 76)
Florence	42;30° [!]	− 1;16°	TCT: *Pl* (s. XIII[ex])
–	43;30°	− 0;16°	*Fl*, fols. 51r, 83r (s. XIII[ex]); TCT: *Fl*, fol. 83r (s. XIII[ex])
Genoa	41;30°	− 2;55°	TCT: *Eq* (s. XIII[ex])
Hereford	51;30°	− 0;34°	Ascension tables in *Ma*, fols. 85r (s. XIII)
–	52°	− 0;4°	Roger of Hereford (*c.*1178) (see Part 4); TCT: *Ma* (s. XIII)
–	52;40°	+ 0;36°	TCT: *Ca* (s. XIII[2/2])
Lincoln	53°	− 0;14°	Calendar of 'Roger Bacon' (*c.*1292) (MS Oxford, Bodleian Library, Bodley 464, fols. 58v–59r)
London	51;40°	+ 0;10°	*Od*, fol. 33v (s. XIII); table in *Cs*, fol. 49r (s. XIII[ex]); TCT: *Ca* (s. XIII[2/2])
–	52°	+ 0;30°	*Post chilindri compositionem*… (MS Verdun, Bibliothèque municipale, 25, fol. 121r; cf. Thorndike and Kibre 1963: 1062); table of solar altitudes in *Investigantibus nature chilindri compositionem*…(s. XIII[2/2]) (MS Oxford, Bodleian Library, Laud. Misc. 644, fol. 220r)

Table 1 (continued)

i	ii	iii	iv
–	53°	+ 1;30°	*Po*, fol. 65v (s. XIII)
Lyon	45;50°	+ 0;4°	TCT: *Ed* (s. XIII^med); Possibly Ptolemaic
–	45;55°	+ 0;9°	Table in *Cs*, fol. 49r (s. XIII^ex)
Magdeburg	52°	– 0;8°	Ascension tables for Magdeburg in the *Compotus philosophicus* (1273) (above, n. 84)
Marseilles	44°	+ 0;42°	Table in *Cs*, fol. 49r (s. XIII^ex); TCT: *Ca*; *Ed*; *Eq*; *Ny*; *Ol*; *Pc* (s. XIII^med)
–	45°	+ 1;42°	Raymond of Marseilles, *Tractatus astrolabii* (*c.*1141) (ed. Alverny et al. 2009: 64 (4b)); TCT: *Bf*; *Me*, fol. 110r (s. XIII^ex)
Mechelen	51;12°	+ 0;11°	Henry Bate, *Magistralis compositio astrolabii* (1274) (ed. Gunther 1932: i. 368b); Henry Bate, *Tables of Mechelen* (*c.*1280) (MS Paris, Bibliothèque nationale de France, n.a.l. 3091, fol. 79v; Nothaft 2018a: 284)
Melfi	21;30° [!]	– 20°	*Od*, fol. 44r (s. XIII)
Melun	48;19°	– 0;13°	*La*, fol. 89r (s. XIII^1/2)
Milan	45°	– 0;28°	Text on co-ascending degrees of fixed stars in 1240 (MS Naples, Biblioteca nazionale 'Vittorio Emanuele III', VIII.C.49, fol. 110r)
Montpellier	43;40°	+ 0;3°	TCT: *Ca* (s. XIII^2/2)
–	44°	+ 0;23°	*Geometrie due sunt partes…* (1260/71) (ed. Hahn 1982: 59 (§44)); anonymous commentary on John of Sacrobosco's *Tractatus de spera* (1267) (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 188, fol. 39ra); John of Sicily, *Scriptum super canones Azarchelis* (1291/93) (ed. Pedersen 1986, pt. 2: 76 (J148k))
–	45°	+ 1;23°	*Po*, fol. 65v (s. XIII)
Naples	30° [!]	– 10;50°	TCT: *Ny* (s. XIII^ex)
–	40°	– 0;50°	TCT: *Me*, fol. 16v (s. XIII^ex)
–	42;10°	+ 1;20°	TCT: *Ol*; *Pc* (s. XIII^2/2)
–	42;20°	+ 1;30°	TCT: *Eq* (s. XIII^ex)
Northampton	52;50°	+ 0;36°	TCT: *Ot* (s. XIII^med)
Novara	45°	– 0;27°	Campanus of Novara, *Tables of Novara* (before 1261) (MS Paris, Bibliothèque nationale de France, lat. 7411(B), fol. 1r); TCT: *Ba*; *Me*, fol. 110r; *Pg*; *Pl*; *Pn*; *Pr;* Vi (s. XIII^ex)
–	45;30°	+ 0;3°	TCT: *Bf* (s. XIII^ex)
Oxford	51;50°	+ 0;5°	*Ot*, fol. 3r (s. XIII^2/2)
–	52°	+ 0;15°	*Os*, fol. 29r (s. XIII); *Post chilindri compositionem…* (s. XIII^2/2) (MS Verdun, Bibliothèque municipale, 25, fol. 121r); Oxford calendar of 1292 (MS Oxford, Bodleian Library, Digby 149, fol. 124v); table in *Cs*, fol. 49r (s. XIII^ex)

Table 1 (continued)

i	ii	iii	iv
–	52;20°	+ 0;35°	TCT: *Ca* (s. XIII[2/2])
Palermo	37;57°	– 0;10°	Ascension tables for Palermo (s. XIII[med]) (above, n. 80)
–	38;15,50°	+ 0;9,10°	*Od*, fol. 44r (s. XIII)
Paris	48°	– 0;51°	*Artis cuiuslibet consummatio…* (1193) (ed. Victor 1979: 230 (II.5), 234 (II.7), 240 (II.10)); *Pj*, fol. 5r (s. XII/XIII); John of Pavia, *Canones super tabulas Humeniz* (1239) (ed. Millás Vallicrosa 1943–1950: 389); John of Sacrobosco, *Tractatus de quadrante* (1239/56) (see Knorr 1997a: 194, 199–200); astrolabe treatise *Quelibet ars suum habet artificem…* (s. XIII[1/2]) (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 352, fol. 111ra) *Geometrie due sunt partes…* (1260/71) (ed. Hahn 1982: 59 (§44)); Gerard of Feltre, *Summa de astris* (1264) (above, n. 76); anonymous commentary on John of Sacrobosco's *Tractatus de spera* (1267) (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 188, fol. 39ra); Robert the Englishman, *Commentary on John of Sacrobosco's* Tractatus de spera, lectio IV (1271) (ed. Thorndike 1949: 158); tables of houses for Paris (s. XIII[ex]) (above, n. 85); TCT: *Ed*; *Me*, fol. 16v; *Pb* (s. XIII[med])
–	48;13°	– 0;38°	Commentary *Sicut dicit Hermes* on Toledan Tables canons Cb (*c.*1290) (ed. Pedersen 2001–2002, pt. 1: 219, 223, 236–237 (Ap139, 148, 192); the value of 48;38° cited in Ap 192 is an evident error for 48;13°, seeing as the co-latitude in this text is reported as 47;47° (Ap 148, 192))
–	48;30°	– 0;21°	Star table of 1223 (see Kunitzsch 1966: 67–71 (type XI); Pedersen (2002: iv. 1506–1507) (LA21); Thomson 2022: iii. 42 (Tabula 3-A)); *La*, fol. 89r (s. XIII[1/2]); *Pm*, fol. 115r (s. XIII[2/2]); TCT: *Ny* (s. XIII[ex])
–	48;33° [!]	– 0;18°	TCT: *Ca* (s. XIII[2/2])
–	48;35°	– 0;16°	*Vp*, fol. 140r (s. XIII[2/2])
–	48;40°	– 0;11°	Table in *Cs*, fol. 49r (s. XIII[ex]) (This same value is assigned to the middle of the seventh climate in John of Sacrobosco, *Tractatus de spera*, c. 3 (ed. Thorndike 1949: 112))
–	48;48°	– 0;3°	Roger Bacon, *Opus maius* (1266/68) (ed. Bridges 1897–1900: ii. 178 (pt. 6, c. 4), 183 (pt. 6, c. 6)); John of Sicily, *Scriptum super canones Azarchelis* (1291/93) (ed. Pedersen (1986), pt. 2: 257 (J544e) cf. ibid., p. 77 (J152b–c): "48 graduum et 50 m'orum fere."); *Li*, fol. 73r (s. XIII[ex]); *Lj*, fol. 119v (s. XIII[2/2]); TCT: *Eq* (s. XIII[ex])

Table 1 (continued)

i	ii	iii	iv
–	48;50°	– 0;1°	Observations by Peter of Limoges and Franco, 1281–1290 (below, n. 116); William of Saint-Cloud, *Almanach planetarum* (*c.*1290) (ed. Pedersen 2014): 9); William of Saint-Cloud, *Kalendarium regine* (1296) (ed. Harper 1966: 159); TCT: *Ba*; *Bf*; *Me*, fol. 110r; *Pg; Pl*; *Pn*; *Pr* (s. XIII^[ex])
–	48;52°	+ 0;1°	Giles of Lessines, *Summa de temporibus* (*c.*1264) (ed. Steele 1926: 160, ll. 3–4 (III.3.4))
–	49°	+ 0;9°	*Po*, fol. 65v (s. XIII)
–	49;6°	+ 0;15°	*Speculum astronomiae* (1260/70?) (ed. Caroti, Pereira, Zamponi, and Zambelli in Zambelli 1992: 218, ll. 64–66 (c. 2))
Piacenza	44;50°	– 0;13°	Gerard of Feltre, *Summa de astris* (1264) (above, n. 76)
Pisa	42;30°	– 1;13°	Gerard of Feltre, *Summa de astris* (1264) (above, n. 76)
–	43;30°	– 0;13°	TCT: *Pb* (s. XIII^[2/2])
Rome	42°	+ 0;6°	*Po*, fol. 65v (s. XIII)
Roskilde	55;12,16°	– 0;26,44°	*Liber daticus Roskildensis* (*c.*1274) (Pedersen 1983–1984: i. 55)
Sarum	51;35°	+ 0;29°	TCT: *Ca* (s. XIII^[2/2])
Sens	48°	– 0;12°	*La*, fol. 89r (s. XIII^[1/2])
Stafford	53°	+ 0;11°	TCT: *Ca* (s. XIII^[2/2])
Strasbourg	48;20°	– 0;15°	*Liber theoreumacie* (1214) (Nothaft 2020: 56–57) possibly Ptolemaic
Toulouse	42°	– 1;36°	TCT: *Me*, fol. 16v (s. XIII^[ex])
–	42;45°	– 0;51°	Tables of Toulouse (s. XIII^[1/2]) (above, n. 81); TCT: *Ed*; *Ny* (s. XIII^[med])
–	43°	– 0;36°	TCT: *Ca*, fols. 38v (s. XIII^[in]), 184r (s. XIII^[2/2])
–	46° [!]	+ 2;24°	TCT: *Pl* (s. XIII^[ex])
–	48;50° [!]	+ 5;14°	TCT: *Bf* (s. XIII^[ex])
–	49;6° [!]	+ 5;30°	TCT: *Ba*; *Me*, fol. 110r; *Pg*; *Pn*; *Pr*; *Ps*, *Vp* (s. XIII^[2/2])
Troyes	47;36°	– 0;42°	*Pj*, fol. 5r (s. XII/XIII)
Winchester	50°	– 1;4°	*Ck*, fol. 128v (s. XIII); *Od*, fol. 44r (s. XIII); addition to eclipse treatise *Ars inveniendi eclipsim lune…* (*Ck*, fol. 138v; s. XIII)
–	51;37°(?)	+ 0;33°	TCT: *Ca* (s. XIII^[2/2])
York	53;30°	– 0;27°	Table of solar altitudes in *Investigantibus nature chilindri compositionem* (s. XIII^[2/2]) (MS Oxford, Bodleian Library, Laud. Misc. 644, fol. 220r)

The table above lists the cities for which latitudes have been transmitted from the period in question in alphabetical order. Where multiple values are attested for one and the same locality, they are ordered according to increasing latitude. The four columns in Table 1 show (i) the name of the city; (ii) the attested latitude (LAT); (iii) the discrepancy (Δ) of the attested latitude with respect to modern geographic data; (iv) the source(s) in which the attested latitude is found. Sources in column (iv) are placed in an approximate chronological order, with dates shown in parentheses. Notes and glosses are cited according to the manuscript sigla indicated above. If a particular longitude appears in one or more copies of the standard list of coordinates in the Toledan Tables, this is indicated by adding 'TCT', followed by the relevant manuscript sigla (as indicated above), followed by the approximate date or time window of its earliest appearance (shown in parentheses).

Table 1 lists a total of 74 latitudes for 40 different cities, numbers which clearly refute the idea that efforts to determine latitude did not become common in Latin Europe before the fifteenth century.[92] Cities for which more than one latitude value are transmitted from the twelfth and thirteenth centuries are: Florence (2), Hereford (3), London (3), Lyon (2), Marseilles (2), Montpellier (3), Naples (4), Novara (2), Oxford (3), Palermo (2), Paris (11), Pisa (2), Toulouse (6), and Winchester (2). The number of reported estimates is particularly high in the case of Paris, for which we have eleven different values ranging from 48° to 49;6°. Since this level of variety is best explained as the result of multiple measurements carried out in the same approximate location, it goes to confirm Paris's role as a major centre of astronomical activity in Latin Europe during the thirteenth century.[93] The highest of the Parisian latitude values is 49;6°, which appears in the *Speculum astronomie* in the context of a passage mentioning the Tables of Toulouse and the supposed fact that Toulouse is found on the same meridian as Paris.[94] It is probably owing to a misinterpretation of this passage that a relatively large number of late-thirteenth century copies of the TCT (in *Ba*, *Me*, *Pg*, *Pn*, *Pr*, *Ps*, *Vp*) assign to Toulouse an identical value of 49;6°, which would be 5;30° too high.

Latitude values that probably stem from scribal accidents or from misapprehensions like the one just noted are marked in Table 1 with an '[!]'. Once these are put aside, we are left with 66 reported estimates. Of these, only 15 or roughly 23% exhibit an inaccuracy greater than 1°. For smaller discrepancies up to 1°, the percentages among the extant estimates are as follows:

$\leq$ 1°: 77%
$\leq$ 0;30°: 62%
$\leq$ 0;15°: 42%
$\leq$ 0;10°: 24%
$\leq$ 0;5°: 12%
$\leq$ 0;1°: 3%.

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92 Zinner 1931: 350 ("[s]ehr selten").

93 On the importance of Paris in this period, see Juste (2018: 68–80) and Nothaft (2021a: 10–11).

94 *Speculum astronomie*, c. 2 (ed. Caroti, Pereira, Zamponi, and Zambelli in Zambelli 1992: 218, ll. 61–67): "Et multi multos libros canonum ad civitates suas super annos domini conscripserunt, ut est ille qui est [...] ad mediam diem Tolosae, quae est sub eodem meridiano cum civitate Parisius, cuius longitudo est ab occidente quadraginta graduum et quadraginta septem minutorum fere, latitudo vero quadraginta novem graduum et decima unius.".

4 Measurements of solar altitudes in the twelfth century

While the latitude estimates discussed above can in many cases be considered indirect evidence of altitude measurements, a close analysis of the methods used and accuracies achieved would require more explicit accounts of specific measurements, which are relatively rare. Perhaps the first datable example of a solar altitude being measured anywhere in Latin Europe appears in an astrolabe treatise composed by Rudolf of Bruges, who was a student of the well-known translator Hermann of Carinthia. Rudolf tells us that he was in Béziers in 1144 when he sought to construct a horizon plate for the latitude of this region. He did so on the basis of the solar noon altitude on 24 April, which he gives as 62°, while also assuming that the Sun occupied the fourth degree of Taurus (33°–34°) on the date in question.[95]

There is a striking discrepancy between Rudolf's placement of the Sun's degree in this particular instance and the data he provides elsewhere in his treatise for constructing the calendar scale on the astrolabe's backplate. According to his own text, the first degree of Aries corresponds to 16 March, which would imply the ninth degree of Taurus for 24 April.[96] In astronomical fact, the Sun's ecliptic longitude at noon on 24 April 1144 would have been within the eleventh degree. Rudolf's incorrect claim that the Sun was in the fourth degree might be explained by his use of the tables in Ptolemy's *Almagest*, which no longer gave good results in his own time.[97] Had Rudolf used this incorrect solar longitude to obtain the noon altitude on 24 April through calculation, he could have hardly found an accurate result. For instance, an astrolabe constructed on the basis of a round obliquity value of 24° would have given the declination of the fourth degree of Taurus (here taken as 34°) as $\sin^{-1}$ (sin 24° × sin 34°) ≈ 13;9°. Adding this to the co-latitude of Béziers (46;39°) would have produced a solar noon altitude of 59;48°. Yet, the Sun's apparent noon altitude at Béziers on 24 April 1144 was in fact 61;43°, which agrees to the nearest degree with Rudolf's stated result of 62°. There is hence every reason to conclude that this result was actually observed.

It is more difficult to draw such firm conclusions from altitudes whose values and calendar dates are recorded, but whose year of observation is unknown. A source of this kind is MS Avranches, Bibliothèque municipale, 235, fol. 47r (Normandy, s. XII[med]), where an isolated paragraph specifies the solar noon altitudes for *Constantiis* on six different dates: 21 March in a bissextile year; 21 March in a year before the bissextile intercalation; 20 June and 1 July, both presumably in the same year before the bissextile intercalation; 21 September and 22 December in the third year after the bissextile intercalation.[98] The author of this note relied on an established computistical tradition of placing the equinoxes and solstices on the 12th day before the respective

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[95] Rudolf of Bruges, *Cum celestium sperarum diversam positionem*, c. 10 (ed. Lorch 1999: 70).

[96] Rudolf of Bruges, *Cum celestium sperarum diversam positionem*, c. 3 (ed. Lorch 1999: 62–63).

[97] Rudolf's use of the *Almagest* was suggested by Mercier (1987: 114–115). See also Lorch (1999: 90).

[98] MS Avranches, Bibliothèque municipale, 235, fol. 47r: "Equinoctio vernali, quod est XII kl. Aprilis, hora VI est altitudo solis Constantiis in anno bissextili, intercalato scilicet iam bissexto, XLV gradibus et in anno preterito, id est precedente bissextum, XLIII gradibus terminatur. In estivo solstitio, quod est XII kl. Iulii, eadem hora terminatur LXV gradibus, id est integris XIII[cim] partibus. Duodecimo ad hoc die in medio Geminorum, scilicet kalendis Iulii, terminatur in eadem hora LXIIII[or] gradibus et semisse, id est dimidio gradu. Tercio anno a bissexto in XI kl. Octobris XL gradibus terminatur. Primo quoque post hiemale

Table 2 Comparison between dates and solar altitudes (α) reported in MS Avranches, Bibliothèque munici-pale, 235, fol. 47r and the local apparent solar altitudes at the latitudes of Coutances (49;3°) and Constance (47;47°) in 1051–1055 and 1151–1155

Date	α	Coutances (49;3°)		Constance (47;40°)	
		1052	1152	1052	1152
21 March	45°	43;39°	43;58°	45;3°	45;22°
		1051	1151	1051	1151
21 March	43°	43;22°	43;40°	44;46°	45;4°
20 June	65°	64;28°	64;26°	65;52°	65;50°
1 July	64 ½°	63;47°	63;41°	65;10°	65;4°
		1055	1155	1055	1155
21 September	40°	39;34°	39;16	40;57°	40;38°
22 December	18°	17;33°	17;36°	18;59°	19;2°

Kalends (of April, July, October, and January). This convention puts the vernal equinox on 21 March, whereas its actual date in mid-twelfth century would have been 13/14 March. He also makes the mistake of placing the Sun on 1 July *in medio Geminorum* rather than in the sign of Cancer.

In light of the probable origin of the manuscript, which lies in Normandy (perhaps Mont-Saint-Michel),[99] an interpretation of *Constantiis* as the town of Coutances (Dept. Manche) suggests itself, although the possibility that we are dealing with a reference to Constance cannot be excluded a priori.[100] Table 2 takes into account both of these possibilities and applies them to two different mid-century year-ranges, 1051–1055 and 1151–1155, such that 1052 and 1152 are taken as the bissextile year mentioned in the text. Overall, a comparison of the apparent solar noon altitudes in these years with those recorded in the text favours the interpretation of *Constantiis* as Coutances. It turns out that most of the recorded altitudes are accurate within ± 1° for this local latitude, regardless of the whether the observations were made in the mid-eleventh or mid-twelfth century. The most serious discrepancy presents itself for 21 March in the bissextile year (i.e., 1052 or 1152), which is 45° in the text, but would have been 43;39° in 1052 and 43;58° in 1152. It is probably no more than a coincidence that the data for Constance agree much more closely in this case, with apparent solar noon altitudes of 45;3° in 1052 and 45;22° in 1152.

A more clear-cut analysis is possible when considering a note in the top margin of MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 5r (s. XII/XIII), which reports a latitude measurement for Troyes.[101] The author of this note argues that the

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Footnote 98 continued

solstitium die, id est XI kl. Ianuarii, terminatur XVIII gradibus." For a description of the manuscript, see Holtz (2000).

[99] Holtz (2000: 35–39).

[100] This ambiguity was already noted by Jacquemard (2000: 58 n. 57), whose article cautiously suggests a connection between this source and Richard the Bishop, archdeacon of Coutances between 1158 and 1170.

[101] See already Gautier Dalché (2000: 420–421).

latitude of Troyes is 47;36° on the basis of his (or somebody else's) observation that the solar noon altitude on 23 March was 46°. He infers the corresponding value of the declination from the assumption that the Sun was then in the 9th degree of Aries as well as from a rule of thumb that the Sun's declination around the equinoxes changes by 0;24° per degree of longitude. The same note adds a value for the equation or *motus* of the eighth sphere 'in this year 1165' (*in hoc anno MᵒCᵒLXᵒVᵒ*), which is given as 11;39°.[102] According to the Toledan Tables the equation at the end of 1165 was 8;11,23°.[103]

Troyes has a northern latitude of 48;18°, so the stated result of 47;36° is off by *c*.0;42°. Most of this relatively large error was probably due to the crudeness of the observed noon altitude. That 1165 was the year of this observation is not stated explicitly in the text, though it seems to follow from the phrase 'in this year' ahead of the cited value of the equation of the eighth sphere. On 23 March 1165, the apparent solar noon altitude at Troyes was 45;27° rather than 46°, accounting for a difference of 0;33°. The author subtracted from the found altitude a declination value of 9 × 0;24° = 3;36° (not 3;24° as mistakenly given in the manuscript) to arrive at a co-latitude of 46° – 3;36° = 42;24° and a corresponding latitude of 90° – 42;24° = 47;36°. The Toledan Tables would have broadly supported this calculation, as they predict a declination value of 3;35,5° for a solar longitude of 9°,[104] which is close to what the author found from his rule of thumb. In actual fact, however, the Sun's apparent declination at the putative moment of the altitude measurement (noon of 23 March 1165) was closer to 3;45°, which accounts for the remaining 0;9° in the discrepancy between Troyes's actual latitude (48;18°) and the result found by our astronomer (47;36°).

The most extensive record of solar noon altitudes to survive from the twelfth century appears in the left margin of MS London, British Library, Arundel 377, fol. 86va, in close proximity to a text (fol. 86vb) that accompanies astronomical mean-motion tables cast by Roger of Hereford in 1178 (fol. 87r). All altitudes are expressed in round degrees and given for the beginning of each sign in a sidereal, rather than tropical, zodiac.[105] We are informed that the values apply to noon at Hereford and that they are given 'according to the astrolabe' (*Hic secundum meridiem Hereford et astrolabium dicta sunt*), which could refer to computation as much as to observation. A prefatory remark in the same note places Hereford at a latitude of 52° and asserts that the noon Sun will reach altitudes of 14° and 62° at the solstices and 38° at the equinoxes, which is consistent with a round value of 24° for the obliquity of the ecliptic. The latitude value of 52° for Hereford is also recorded twice in MS Madrid, Biblioteca nacional de España, 10016, fols. 70v–71r (s. XIII), once again in close vicinity to

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[102] MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 5r: "Latitudo civitatis Trecarum ab equinoctiali linea est 47 graduum et 36 puncta. Nam 23 die Marcii altitudo solis in meridie fuit 46 gradus et eadem die erat sol in 9 gradu Arietis. Nonus Arietis gradus: est declinatio a linea equinoctiali 3 graduum et 24 puncta. Nam 24 puncta fuit declinatio cuiuslibet gradus Arietis ab equinoctiali angulo. In hoc anno in hoc anno MᵒCᵒLXᵒVᵒ nat < ivitatis > Christi circulus signorum, id est, motus VIIIᵉ spere sit XI gradus et 39 puncta."

[103] Here and elsewhere in this article, information about the predictions made by medieval sets of astronomical tables was drawn from the software *Deviations* developed by Raymond Mercier (http://www.raymondm.co.uk/).

[104] See n. 89.

[105] For the relevant details, see Nothaft (2021b: 55–56).

Table 3 Comparison between the solar noon altitudes reported in MS London, British Library, Arundel 377, fol. 86va, with the modern data for the apparent solar noon altitudes on the dates of the sidereal sign entries in 1177–1178 ('apparent') and with the hypothetical values shown by an astrolabe constructed for a latitude of 52° and an obliquity of 24° ('astrolabe')

Sign entries	α	Apparent	Astrolabe
Capricorn (23 Dec)	14°	14;46°	14;15°
Aquarius (21 Jan)	20°	19;45°	19;18°
Pisces (20 Feb)	30°	29;35°	29;14°
Aries (22 Mar)	42°	41;15°	41;15°
Taurus (22 Apr)	52°	52;19°	52;30°
Gemini (24 May)	60°	59;52°	60;9°
Cancer (24 Jun)	62°	61;13°	61;45°
Leo (24 Jul)	56°	56;28°	56;42°
Virgo (25 Aug)	46°	46;28°	46;46°
Libra (24 Sep)	34°	34;53°	34;45°
Scorpio (25 Oct)	24°	23;31°	23;30°
Sagittarius (23 Nov)	16°	16;15°	15;51°

Roger's astronomical tables. In one of these instances, the value is mentioned as part of an explanation of a table of solar zenith distances (fols. 71v–72v), which is here explicitly attributed to Roger of Hereford. Another table in the same manuscript, which is for oblique ascensions at Hereford, instead assumes 51;30° (fol. 85r). The note in the London manuscript offers no context that would explain why the author found it necessary to record the Sun's noon altitudes at its sidereal sign entries. One possibility is that they were used to cast the aforementioned table of zenith distances, which likewise uses sidereal signs, in a sequence from Cancer to Gemini. The zenith distances it displays for noon when the Sun is the first degree of a given sign are consistent with the noon altitudes in the London manuscript in the majority of instances, whereas the values for Leo, Libra, Sagittarius, and Gemini differ by 1°.

As can be seen from Table 3, the solar altitudes reported in the London manuscript are all within ± 1° of the solar noon altitudes at Hereford that could have been observed on those dates that a user of the Toledan Tables would have identified as the dates of the sidereal sign entries in 1177–1178. Yet, this is not enough to eliminate the alternative hypothesis according to which the author derived his noon altitudes from the rete of an astrolabe with a horizon plate constructed for 52° latitude. If one assumes that the beginning of sidereal Aries coincides with 8° tropical Aries, as Roger of Hereford is known to have done,[106] the resulting noon altitudes (rounded to minutes of arc) will be those shown in the rightmost column of Table 3. The values in this column were computed according to the relation $\alpha = 90° - \varphi + \delta$, where α is the solar noon altitude, φ is the local latitude (52°), and δ is the declination. The value of the declination was obtained from the relation $\sin \delta = \sin \lambda \times \sin \varepsilon$, where λ is the tropical solar longitude (i.e., the sidereal longitude – 8°) and ε the obliquity of the ecliptic (here: 24°). Once again, the reported altitudes are within ± 1° of the computed ones.

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[106] Nothaft (2021b: 57).

Table 4 Comparison between observations of the solar noon altitudes in 1281–1290 reported by Peter of Limoges (α) with the apparent solar noon altitude according to modern calculation for a latitude of 48;50° (Paris/Saint-Cloud), here given to the nearest second of arc ('apparent')

Date	α	Apparent
15 June 1281	64;43°	64;42,14°
14 December 1281	17;37°	17;40,55°
13 December 1283	17;35°	17;41,10°
11 March 1290	40;30°	40;29,48°

5 Parisian solar observations in the late thirteenth century

As seen from the previous sections, a large percentage of the altitude measurements carried out by medieval astronomers would have served day-to-day practical purposes to do with instrument construction, the adaptation of astronomical tables, or with time-keeping. There are, nevertheless, some attested measurements that may be deemed 'scientific' insofar as they were made to establish a particular astronomical parameter. Perhaps the easiest parameter to obtain in this way was the obliquity of the ecliptic, here understood as equivalent to half the distance between the maximum and minimum noon altitudes attained by the Sun in the course of a year (see Part 3). The first clearly documented attempt to derive this parameter took place in the years 1281–1283, as is known from a handwritten note by the Parisian scholar Peter of Limoges.[107] In 1281, a certain Franco, who must probably be identified with Franco of Poland,[108] measured the solar noon altitudes on 15 June and 14 December, finding 64;43° at the summer solstice and 17;37° at the winter solstice. Peter joined Franco in another such observation on 13 December 1283, when they recorded a noon altitude of 17;35°. He writes that this specific observation was carried out using a large wooden quadrant that was graduated with divisions for 0;5°, which is consistent with the fact that 17;35° is a multiple of 0;5°.[109]

As can be seen from Table 4, the accuracy Peter and Franco managed to attained with this instrument in the case at hand is *c.*–0;6°, which comes close to the width of one division on the quadrant. By contrast, the results obtained by Franco in 1281 suggest that he used an instrument whose graduation was capable of showing individual minutes of arc. Indeed, both of his measurements remained well within ± 0;5° of the

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[107] MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1191, fol. 139r. This note was printed in Delorme (1936: 559), who wrongly attributed it to William of Saint-Cloud. See Nothaft (2017a: 219–220); (2018a: 295–296); Juste (2018: 76).

[108] On Franco of Poland, see Weijers (1996: 100).

[109] MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1191, fol. 139r: "Quod autem altitudo Solis existentis in equinoctiale in meridie Parisiensi sit 41 gradus et 10 minuta, vel circieter, habeo ex hac observatione que sequitur. Anno Domini 1281, 15 die Iunii, littera dominicalis E, dominica eadem die in meridie altitudo Cancri 64 gradus et 43 minuta. Eodem anno, Decembris die 14ᵃ dominica altitudo Capricorni in meridie Parisius 17 gradus, 37 minuta. Hec observavit Franco. 1283 in die sancte Lucie ego cum ipso inveni 17 gradus, 35 minuta. Hec fecimus cum magno quadrante, cuius latera erant de ligno et limbus de latone [*sic*] divisus usque ad 5 minuta."

apparent solar noon altitude on the dates in question, with an error of $< 0;1°$ in the case of the summer solstice of 15 June 1281.

Peter's note continues by claiming that the obliquity implied by these observations is 23;34°,[110] which suggests that he compared the altitude on 15 June 1281 with that on 13 December 1283: $(64;43° - 17;35°) \div 2 = 23;34°$. The 2 values obtained in 1281 would instead have implied an obliquity of $(64;43° - 17;37°) \div 2 = 23;33°$, which was a commonly cited variant of the more precise value of 23;33,30° assumed in the Toledan Tables for computing the solar declination.[111] One of the first Latin authors to mention the latter value was Raymond of Marseilles, whose Tables of Marseilles were derived from the Toledan Tables.[112] In a passage in his astrolabe treatise, which he wrote shortly before his *Liber cursuum planetarum* of 1141, Raymond recommended using it to confirm or disconfirm the 'modern' view according to which the equinoxes occurred 7 days ahead of the Sun's reaching the signs of Aries and Libra (which was a consequence of using a sidereal reference frame). What he had in mind was to measure the Sun's maximum and minimum noon altitudes at the solstices and compare them with the noon altitude measured on a date seven days before the computed entries into Aries and Libra. If the difference corresponded to the obliquity, the 'moderns' were proven correct.[113]

The precise date and time of the equinoxes was a concern shared by the Parisian astronomers who measured the obliquity in the early 1280s. Peter of Limoges, in the note just mentioned, reports that the noon shadow of 11 March 1290 had a length of 14 *puncta* and 3 *minuta*, to which corresponds a solar altitude of 40;30°.[114] As is evident from the shadow table included in the Toledan Tables,[115] the *puncta* here refer to the digits of the length of a gnomon and the *minuta* to the sexagesimals of a digit (3 *minuta* = 0.05 digit). The precision of the reported measurement is thus remarkably high and would imply that the gnomon Peter used for this purpose was equipped with a perpendicular measuring scale, perhaps one drawn in parallel to a meridian line. What raises suspicions, however, is the exactness of the relation between the reported shadow length and the corresponding solar altitude of 40;30°, which follows without any rounding both from the Toledan shadow table and from the appropriate formula $h = \arctan(\frac{12}{s})$ for $s = 14.05$. Rather than reporting his precise measurement of the noon shadow, Peter may have retrofitted its length to match the assumed solar altitude

[110] Ibid.: "Per hoc patet maximam declinationem fore circiter 23. gradus 34. minuta."

[111] For the Toledan declination table, see Pedersen (2002: iii. 961–966) (BA21). For some examples of sources mentioning 23;33°, see Abraham Ibn Ezra, *Liber de rationibus tabularum* (ed. Millás Vallicrosa (1947): 77, ll. 17–18; 93, l. 4; 143, l. 11); Abraham Ibn Ezra, *De mundo vel seculo*, trans. Henry Bate, §14.9 (ed. Sela 2022: 954); John of Seville, *Dixit Iohannes: Cum volueris facere astrolabium...* (ed. Millás Vallicrosa 1942: 324); *Artis cuiuslibet consummatio...* II.6 (ed. Victor 1979: 232); pseudo-Māshā'allāh, *De compositione astrolabii*, c. 7 (ed. Thomson 2022: ii. 136); John of Sacrobosco, *Tractatus de spera*, c. 2 (ed. Thorndike 1949: 90).

[112] See the declination table in Raymond of Marseilles, *Liber cursuum planetarum*, tab. 21 (ed. Alverny et al. 2009: 336–338).

[113] See Raymond of Marseilles, *Tractatus astrolabii* 22d–j (ed. and trans. Alverny et al. 2009: 98–99), together with the emendations noted in Caiazzo and Burnett (2011: 347). For the context, see Nothaft (2021b: 49–54).

[114] See below, n. 116.

[115] Pedersen (2002: iii. 993) (BC21).

of 40;30°, which may in fact have been derived from another measurement carried out one day later, on 12 March (see below).

Whatever its precise origin story, the solar altitude of 40;30° was 0;40° removed from an equinoctial altitude of 41;10° that, according to Peter, followed from the previously obtained solstitial noon altitudes (e.g., 64;43° − 23;33° = 41;10°). An approximate time of the moment when the Sun would reach 0° tropical longitude could be inferred from the rule of thumb, already mentioned above, according to which the Sun near the equinoxes will undergo a daily chance in declination of 0;24°, or approx. 0;1° per hour. Given a difference of 0;40°, Peter concluded that the Sun's centre reached the vernal point 16 h after noon of 12 March.[116]

He could have learned this method from a number of different sources available in Latin at the time.[117] One likely candidate among them is the first version of Abraham Ibn Ezra's *Sefer ha-ʿOlam*, a Latin translation of which had been completed by Henry Bate of Mechelen in October of 1281 (*Liber de mundo vel seculo*).[118] In Ibn Ezra's hypothetical example the local co-latitude is 47;30° and the noon altitude measured on the day before the day of the vernal equinox is 47;20°, which leads to the conclusion that the Sun will enter Aries 10 h after noon.[119] There is some circumstantial evidence to suggest that Bate, who studied in Paris in the 1260s and early 1270s, was personally acquainted with Peter of Limoges, which could explain how the latter became familiar with Ibn Ezra's work.[120] In the context of the *Sefer ha-ʿOlam*, the technique of deriving the time of the vernal equinox from measured altitudes appears in the context of a general explanation of how past astronomers were able to derive the length of the tropical year from observations separated by long time intervals.[121] It is possible that this passage inspired similar efforts on the part of Henry Bate, seeing as his Tables of Mechelen rely on an unusual value for the length of the tropical year that he claimed to be a result of comparing his own observations with earlier ones.[122]

A different way of exploiting the results of an equinox observation was to test the astronomical tables already available, by considering their prediction for the same

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[116] MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1191, fol. 139r: "Annis Christi 1289 perfectis, XIª die Marcii, que fuit dies sabbati ante Letare Ierusalem, fuit umbra in meridie apud Parisius 14 puncta et 3 minuta, quibus correspondent de altitudine 40 gradus et 30 minuta. Et cum altitudo Solis in meridie existentis in equinoctiali Parisiensi sit 41 gradus et 10 minuta, qui excedunt predictam altitudinem in 40 minutis, et Sol quolibet die circa introitum suum in equinoctialem addat altitudini sue 24 minuta fere, et hoc est in hora una unum minutum, sequitur, si vera est hec experiencia, quod Sol [*sic*] per 16 horas post meridiem diei dominice sequentis diem sabbati predictam venit centrum corporis Solis ad equinoctialem. Subtrahendo enim pro una die 24 minuta de 40 minutis predictis remanent 16, quibus correspondent 16 hore. […] Qualiter autem possit umbra rei precise haberi, et per hoc altitudo Solis, alias declarabo."

[117] See especially *Almagesti minor* III.1 (ed. and trans. Zepeda 2018: 222–225); William (the Englishman) of Marseille, *Astrologia* (MS Erfurt, Universitätsbibliothek, Dep. Erf. CA 4° 357, fol. 3r); Bernard of Verdun, *Tractatus super totam astrologiam* III.1.2 (ed. Hartmann 1961: 61–62); cf. al-Battānī, *De motu stellarum*, trans. Plato of Tivoli, c. 27 (ed. 1537, fol. 26v).

[118] Sela (2022: 59–64).

[119] Abraham Ibn Ezra, *De mundo vel seculo*, trans. Henry Bate, §§14–15 (ed. and trans. Sela 2022: 952–955).

[120] Juste (2018: 68–74).

[121] Abraham Ibn Ezra, *De mundo vel seculo*, trans. Henry Bate, §§16–17 (ed. and trans. Sela 2022: 954–957).

[122] Nothaft (2018a: 290–297).

time. Peter of Limoges pointed in this direction when he noted that tables that plotted ecliptic longitudes against a sidereal frame of reference placed the Sun on 13 March 1292, 4 a.m. (i.e., his previously established time for the vernal equinox) at 349;47°.[123] The difference between a celestial body's sidereal and tropical positions was a function of precession and could therefore also be referred to as the 'motion' (*motus*) of the eighth sphere. Peter's data showed this 'motion' to be 360° – 349;47° = 10;13°, which clashed with the 9;22° predicted for the same parameter by the Toledan Tables.[124] Peter of Limoges appears to have been aware of this discrepancy, even though he concludes his note with the disclaimer that it is too early to draw any conclusions 'until I have made several other observations, not only at the equinoxes, but also at the solstices'.[125]

A more definite response to the results obtained in 1290 is preserved as part of the introduction to William of Saint-Cloud's *Almanach planetarum*, which dates from 1292.[126] William, who was one of the most prominent Parisian astronomers of the late thirteenth century, reports maximum and minimum solar noon altitudes that differ only slightly from those found by Peter and Franco in the early 1280s (see Table 4), although in his case no specific dates are mentioned. He merely notes that he measured altitudes when the Sun was at the beginning and Cancer (64;44°) and Capricorn (17;36°), obtaining from them an obliquity value of (64;44° – 17;36°) ÷ 2 = 23;34° and a local co-latitude of 41;10°.[127] As with Peter of Limoges, William used these data to infer the time of the vernal equinox in 1290, although in his version of events the relevant observation concerned not the length of the noon shadow on 11 March, but the solar noon altitude on 12 March, which he found to be 40;54°. Assuming a daily change of 0;24°, this result was completely congruent with Peter's 40;30° for the solar noon altitude on 11 March. It according left unaffected the resulting equinox time, which according to both accounts was 16 h after noon on 12 March = 13 March, 4 a.m., implying an equation of the eighth sphere of 10;13°.[128] William famously took this to mean that the Toledan access-and-recess model for the motion of the eighth sphere was flawed and had to be replaced by a steady or linear precession model, as previously espoused by Ptolemy, al-Battānī, and al-Ṣūfī (who had all suggested different rates). He also used the data gathered from the observation of 1290 to compute the solar

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[123] MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1191, fol. 139r: "Et secundum hoc motus 8^e spere tempori isto est 10 gradus, 13 minuta […]. Equando enim Solem ad predictam horam, sciliter per 16 horas post meridiem diei dominice, per tabulas factas ad 8^am speram, invenitur verus Solis 11 signa, 19 gradus, 47 minuta, que subtracta de 12 signis ostendunt dictum numerum. Nichil precise per hanc observationem possum asserere quousque plures alias fecerim observationes, non solum in equinoctiis, sed etiam in solsticiis." The stated true solar longitude of 349;47° can be recomputed to the minute of arc with either the Tables of Toulouse (cf. Poulle 1994) or the Toledan Tables adjusted for a meridian + 12° east of Toledo.

[124] See Nothaft (2017a), for further details and a discussion of the wider background.

[125] See above, n. 123.

[126] William of Saint-Cloud, *Almanach planetarum* §§2–6 (ed. Pedersen (2014: 8–10). The relevant passage was also printed by Delorme (1936: 560–561). It has been discussed numerous times in the literature. See, e.g., Duhem (1913–1959): iv. 16–17); Zinner (1936: 322–323); Millás Vallicrosa (1943–1950): 392–394); Harper (1966): 41–45); Mercier (1976–1977, pt. 1: 201–204); Hugonnard-Roche (1992: 65; 1996: 296–297); Lejbowicz (1997: 209–210); McCluskey (1998: 205); Nothaft (2017a: 220).

[127] William of Saint-Cloud, *Almanach planetarum* §§3–4 (ed. Pedersen 2014: 8–9).

[128] William of Saint-Cloud, *Almanach planetarum* §§2, 5 (ed. Pedersen 2014: 8–10).

longitudes for his *Almanach planetarum* as well as the times of the equinoxes and solstices that accompanied his later *Kalendarium regine* (1296).[129]

It should be noted that William's 40;54° for the solar noon altitude on 12 March 1290 is a mere 0;0,34° in excess of the 40;53,26° yielded by modern computation for the apparent noon altitude at a latitude of 48;50°. This makes his value barely worse than Peter's 40;30° for 11 March 1290, which is just 0;0,12° in excess of the modern value of 40;29,48° (see Table 4). In both cases, the excellent starting value led to a vernal equinox time that was less than 2 h too early compared to the modern computed value for the time when the true Sun reached the vernal point.[130] The idea that Peter and William arrived at this result completely independently is belied both by the close verbal overlap that exists between their accounts and by the fact that their starting altitudes differ by 0;24°, which is the precise daily change in declination required by the method they both used to determine the equinox. It therefore seems reasonable to suspect that only one of the two altitudes was actually observed, the other inferred by adding or subtracting these 0;24°. The fact that Peter of Limoges links his solar noon altitude of 40;30° on 11 March to a suspiciously exact shadow length may speak in favour of William's 40;54° as the original value, although one can hardly be certain.

Other than in Peter's note and in William's *Almanach*, the Parisian solar observations of 1281, 1283, and 1290 are also attested in a gloss added to MS Vienna, Österreichische Nationalbibliothek, 2311, fol. 96v (s. XIII^ex). Most of this gloss is a verbatim rendition of Peter of Limoges's account, albeit one that omits certain details such as Peter's mention of the large quadrant he and Franco used for the altitude measurement at the winter solstice of 1283. The Vienna glossator also converts the text from a first-person to a third-person report and changes the place where the noon shadow of 11 March 1290 was measured from Paris to Saint-Cloud (*c*.9.9 km west of Notre Dame cathedral).[131] An added sentence at the end gives 41;8° as an alternative value for the co-latitude, which would imply a latitude of 48;52°, as was assigned to Paris by Giles of Lessines in the 1260s (Table 1). The glossator suggests that this value matches the 'experience' of a certain master (*concordat hoc cum experientia magistri*) and concludes with *Hec H*. This could be potentially be a reference to Henry (*Henricus*) Bate of Mechelen, considering that the gloss appears at the end of an appendix to a copy of Haly Abenrudian's commentary on Ptolemy's *Quadripartitum* (as translated by Aegidius de Tebaldis), in which Haly's nativity horoscope is recalculated on the basis of Bate's astronomical tables for Mechelen.[132] There is, as David Juste

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[129] William of Saint-Cloud, *Almanach planetarum* §§6 (ed. Pedersen 2014: 10); Nothaft (2018b: 171–174).

[130] According to *Stellarium* 23.1 (see above, n. 10), the Sun reached a tropical longitude of 0° on 13 March 1290 at *c*.06:00 h, which is 17:52 h after its meridian transit at 12:08 h.

[131] MS Vienna, Österreichische Nationalbibliothek, 2311, fol. 96v: "Anno domini 1289 perfecto, 11ᵃ die Martii, que fuit die [*sic*] sabbati ante Letare Iehrusalem, fuit umbra in meridie apud Sanctum Clodoaltum 54 [*sic*] puncta et 3. minuta, quibus correspondet de altitudine 40 gradus et 30 minuta. […]. Quod autem altitudo Solis existentis in equinoctiale in meridie Parisiensi sit 21 [*sic*] gradus, 10 minuta habeo ex hac observatione. […] Hec observavit Franco. 1283 in die sancte Lucie ipse cum Petro invenerunt 17 gradus, 35 minuta. Et secundum hoc motus 8 spere tempore isto est 10 gradus, 13 minuta, 0 secunda. Equando enim Solem ad predictam horam per tabulas factas ad octavam < speram > invenio 11.19.47.0, que subtracta de 12 signis ostendunt dictum numerum." Cf. the parallel text quoted above in nn. 105, 116, 123.

[132] See Juste (2018: 78–79), who identifies the author of both the appendix and the gloss with William of Saint-Cloud.

has shown, good reason to believe that as an astronomer-astrologer Henry Bate was in close contact with his Parisian colleagues, most notably Peter of Limoges and William of Saint-Cloud, which would explain his familiarity with their activities well as with Peter's specific note on the observations of 1281, 1283, and 1290.[133]

Direct evidence of Bate's knowledge of Parisian observation is furnished by the 22nd book of his *Speculum divinorum et quorundam naturalium*, most of which was composed between 1301 and 1305. It includes a discussion of the fallibility of instrument-based measurements, which has Bate bring up the example of two astronomers working in Paris in his own time. According to Bate, these unnamed individuals used two extremely large quadrants to measure the Sun's solstitial noon altitude. These quadrants were 'skilfully constructed and their reliability confirmed with ingenuity' (*artificiose factis et ingeniose verificatis*), which did not keep their users from arriving at diverging results of 64;42° vs. 64;45°.[134] We have already seen that Peter of Limoges, in his mention of the measurement he made in 1283 in tandem with Franco, testifies to their use of a 'large quadrant', which resonates to some degree with what Bate has to say about the two Parisian astronomers, even as the specific altitude values in his example are not attested in the sources discussed above.

What seems to be a further glimpse at the observational activities that took place in Paris in the late thirteenth century is offered by some notes contained in MS London, British Library, Harley 3735, fol. 73r (s. XIII^ex).[135] The manuscript page records predictions of the equinoxes and solstices as they were going to occur in 1293, with the Sun entering Aries on 12 March, 9 p.m. This result comes fairly close to the one Peter of Limoges had obtained for the equinox of 1290, which fell 16 h after noon and thus implied an equinox in 1293 at 16 h + 18 h – 24 h = 10 h after noon. The same page in the London MS has notes documenting someone's personal finding (*inveni*) concerning the solar noon altitude at the summer solstice in some unspecified year: 64;46°. The astronomer in question subtracted an obliquity value of 23;34°, the same as Peter's and William's, to arrive at Parisian values for the co-latitude (64;46° – 23;34° = 41;12°) and latitude (90° – 41;12° = 48;48°).

It should be noted that this author's value of 48;48° for the latitude of Paris was apparently known as early as the 1260 s, judging by the fact that Roger Bacon mentions it in his *Opus maius* (1266/68).[136] It is also attested in a note added to the December page in a calendar of the second half of the thirteenth century preserved in MS London, British Library, Harley 4350, fol. 119v. It mentions not only the Parisian latitude of 48;48° but also the corresponding co-latitude or *elevatio capitis Arietis* of 41;12°. To this information, the same hand added the solar noon altitude at the Sun's maximum negative declination, here assumed to occur on 15 December (*altitudo Solis in meridie in maxima Solis declinatione, ut 18 kl. Ianuarii*): 17;38,47°. Rather than using the actual maximum declination value, which the Toledan Tables gave as 23;33,30°, this result was clearly obtained via the Toledan value for a solar longitude of 269° or

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[133] Juste (2018: 68–80).

[134] Henry Bate, *Speculum divinorum et quorundam naturalium* 22.17 (ed. Steel and Guldentops 1996: 337, ll. 33–49). See also Bate's own addendum to 22.18 (ed. Steel and Guldentops 1996: 346, ll. 1–8), which was written after January 1310 and discusses a Parisian observation of the solar eclipse of 12 April 1298.

[135] See Thorndike (1959: 157–158); Nothaft (2017a: 222).

[136] See Table 1.

271°,[137] which is + 23;33,13°. Using this declination value and a co-latitude of 41;12°, one arrives at a solstitial minimum of 41;12° − 23;33,13° = 17;38;47°.

A co-latitude of 41;12° is also among the parameters assumed in John of Sicily's extensive Parisian commentary on the most common Latin canons to the Toledan Tables, which he wrote at some point in the early 1290 s (probably 1291/93). In one of the many excursuses contained in this work, John turns the solar observations carried out by his Parisian contemporaries into a general recipe for determining empirically the equation of the eighth sphere. Parts of his account remain relatively vague. He writes that the 'place' (*locus*) of the Sun when it is near the vernal point must be examined with an 'instrument appropriate for this task, such as an astrolabe or torquetum' (*per instrumentum ad hoc opus conveniens inquiratur, sicut per astrolabium vel torquetum*).[138] Whatever examination John had in mind here, he presents it as an alternative to the familiar method of comparing the measured noon altitude of the Sun with the local co-latitude in order to find the time of the equinox. In the example his commentary provides, the noon altitude on 12 March is found to be 40;54°, which is the exact value obtained by William of Saint-Cloud (who goes unmentioned). The local co-latitude is nevertheless given as 41;12°, despite the fact that William had found 41;10°.[139]

The presence of these diverging parameters in John's work may reflect the circulation of observational results in late-thirteenth century Paris, where different astronomers were able to share and compare the results of their work, apparently without reaching a consensus on the size of certain values. This lack of unanimity, which in part must have been a consequence of measurements being taken at slightly different latitudes, is well exemplified by the maximum solar noon altitude at the summer solstice. The sources at hand report it as 64;42° (Henry Bate), 64;43° (Peter of Limoges), 64;44° (William of Saint-Cloud), 64;45° (Henry Bate), or 64;46° (anonymous in London MS). For the minimum solar noon altitude, the gamut of reported values includes 17;35° (Peter and Franco), 17;36° (William), and 17;37° (Peter and Franco), while 17;38° follows implicitly from the data recorded in the London MS.

6 Conclusion

The purpose of this article was to provide an overview as well as some specific examples of the types of altitude measurements that were carried out in the context of Latin astronomy in the period from 1100 to 1300. It does not pretend to be an exhaustive study of the topic, which would be difficult owing to the very diverse body of existing source material and the great number of uncatalogued or as-yet-overlooked manuscripts. I nevertheless hope to have achieved some level of representativeness for the two main uses of solar altitude measurements discussed above: those related to geographic latitudes and those made for 'scientific' purposes.

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[137] See above, n. 89.

[138] John of Sicily, *Scriptum super canones Azarchelis*, J544b (ed. Pedersen (1986), pt. 2: 257).

[139] Ibid. (J544d–f).

The material discussed above allows for the general conclusion that measurements of solar altitude were far more common, at least in a concretely documented form, than those for other planets or fixed stars. This is explicable in part with the Sun's special utility in attempts to establish local latitude, which are well documented even from this early period (see Part 3). We have also seen how measured solar altitudes could be employed successfully to derive the obliquity of the ecliptic as well as to make estimates of the time of the equinox. While the results of such measurements are in many cases reported only to the nearest degree of arc, a precision to minutes was not uncommon and appears to have become more widespread over time. It already characterizes the latitude measurement made in Troyes in 1165 (see Part 4) and is maintained across the different solar observations documented from Paris in the period 1281–1290 (see Part 5). Of the 74 estimates of geographic latitudes in Table 1, which are mainly from the thirteenth century, about 59% (44 values) are reported with a precision to minutes of arc. Latitude values that are precise even to seconds of arc were found in three instances (see Part 3), yet this precision is in all likelihood an artefact of the computed declination value. Indeed, the standard of observational precision that was practically attainable given the physiological limitations of the human eye is 0;1°.[140]

When it comes to assessing the accuracy of the values reported in our sources, we may follow Price in accepting ± 0;5° as the standard for reasonably good angular measurements attainable by naked-eye methods.[141] Ignoring some corrupt values, this standard of accuracy was reached by 12% of the latitude estimates in Table 1. It also characterizes most of the solar observations made in Paris in the late thirteenth century, as seen from Table 4 and the discussion in Part 5. Most noteworthy among these are the solar noon altitudes on 11 and 12 March 1290 reported by Peter of Limoges and William of Saint-Cloud, which were in error by no more than 0;1°.

According to a study by Said and Stephenson, published in 1995, Islamic astronomers in the period AD 832–1019 were capable of attaining a mean error of not much more than ± 0;1° in observing solstitial noon altitudes,[142] whereas their results at other times of the year were more varied. The most accurate of these were attained by al-Bīrūnī in Jurāniya in 1016, with an approximate mean error of ± 0;1°.[143] Mozaffari's recent analysis of observations made in the 1260s and 1270s at the Maragha observatory by Muḥyī al-Dīn al-Maghribī indicates a mean error of ± 0;1° for his measurements at the solstices in 1264, but a repeated error of + 0;4° in his other noon altitudes.[144]

The high degree of accuracy with which Parisian astronomers measured the solar noon altitude in March of 1290 translated into a similarly impressive result for the time of the vernal equinox, which was within 2 h of the correct time according to modern observation. This approximates the accuracy at which equinox times were determined by the aforementioned Islamic astronomers, whose mean error according to Said and

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[140] Price (1957: 582).

[141] Price (1957: 582–583).

[142] Said and Stephenson (1995: 122–125, 129–130).

[143] Said and Stephenson (1995: 125–126).

[144] Mozaffari (2018: 598, 618, 634–635).

Stephenson was 1.2 h, corresponding to a change in solar altitude of 0;1°.[145] It seems worth emphasizing that no equinox or solstice observation recorded from antiquity comes even close to this level of accuracy.[146]

Overall, the findings in this article highlight the potential for a wholesale re-evaluation of the observational activities of Latin astronomers in the central Middle Ages, which appears to have involved a greater degree of rigour and accuracy than has often been presupposed. It is to be hoped that future research can trace the continuation of this observational tradition in the fourteenth century, both in Paris and in other centres of astronomical activity.

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[145] Said and Stephenson (1995: 125, 127–128, 130).

[146] See Jones (2022: 19–24).

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