



Galileo Galilei and the centers of gravity of solids: a reconstruction based on a newly discovered version of the conical frustum contained in manuscript UCLA 170/624

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Abstract

The manuscript UCLA 170/624 (ff. 75–76) contains Galileo’s proof of the center of gravity of the frustum of a cone, which was ultimately published as *Theoremata circa centrum gravitatis solidorum* in *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Leiden 1638). The UCLA copy opens the possibility of giving a fuller account of *Theoremata* dating and development, and it can shed light on the origins of this research by the young Galileo. A comparison of the UCLA manuscript with the other extant copies is carried out to propose a new dating for the composition of the *Theoremata*. This dating will then be reconsidered in light of the mathematical content. The paper ends with a complete description of the content of the UCLA manuscript and the edition of Galileo’s text there contained.

1 Introduction

1.1 Galileo’s *Theoremata*

The *Theoremata circa centrum gravitatis solidorum*—an appendix to *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Leiden 1638)—gather together Galileo’s studies on the centers of gravity of solids. Although these studies date to the years of his formation (he wrote those theorems beginning about 1587), they were printed only in 1638 after several plans to publish them fell through.

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Galileo's aim was to establish himself as a mathematician and to obtain the chair of mathematics at the University of Bologna. To this end he circulated the theorems among some noted mathematicians of his time, including Christopher Clavius, Guidobaldo dal Monte, Giuseppe Moleti, and Abraham Ortelius.¹

In these theorems Galileo determined the center of gravity of the paraboloid, the cone, and their frusta. In the Leiden version the *Theoremata*² begin with a postulate and a lemma necessary for the proofs to follow. Then for each whole solid two lemmas and two theorems are proved, and for each frustum one lemma and one theorem. The argumentative structure for all the proofs is the same: lemmas required for the proofs are proved ad hoc; in the first theorem, the positions of the centers of gravity of the inscribed and circumscribed solids are determined; in the second theorem, it is shown that the center of gravity of the solid divides the axis in a certain ratio. For the frusta, there is one theorem for each case where the ratio is found directly.

Besides the printed edition, there was previously thought to be only one other incomplete copy on these theorems. This is in the Milan, Biblioteca Ambrosiana, MS A 71 Inf., which contains on folios 95–96 only the lemma and theorem on the frustum of a cone.

Recently another incomplete witness of the *Theoremata* had been discovered. This is in the manuscript now in the Charles E. Young Research Library, University of California-Los Angeles with the shelfmark 170/624.

1.2 The manuscript UCLA 170/624

The manuscript was described by Neville (1986) in “The Printer’s Copy of Commandino’s Translation of Archimedes, 1558.” It is a composite codex, formerly in the Albani Library (MS 670), made of three different codicological units:

- 1 the printer’s copy of Federico Commandino’s translation of Archimedes for his *Archimedis Opera nonnulla* (Venice, 1558) (folios 1–74);
- 2 miscellaneous materials linked to Guidobaldo dal Monte,³ partly written by himself (folios 75–91); and
- 3 a text related to François Viète’s algebra (folios 92–214).⁴

In recent years, several studies have examined Guidobaldo’s role not only as scientific heir to Commandino but also for the part he played in guiding the first steps of Galileo’s career and for his network of correspondents, which included Christopher Clavius in Rome and Giacomo Contarini in Venice.⁵ This justifies a deeper inquiry into this

¹ On the contents of the correspondence between these mathematicians see Sect. 4.

² See Sect. 3 for a complete description of the contents (Table 1) and the witnesses of the text.

³ Guidobaldo dal Monte (1545–1607), one of the most distinguished students of Commandino, was a key figure at the end of the sixteenth century especially renowned for his contributions to mechanics and perspective. On his mechanics see Frank (2011) and Laird (2013).

⁴ See also Enrico Narducci’s catalogues of the Boncompagni collection (Narducci 1862, 1898).

⁵ On the personal and scientific relationship between Galileo and Guidobaldo, see Frank and Napolitani (2015). The background against which Guidobaldo worked in the Duchy of Urbino is described in Gamba and Montebelli (1988), and more recently in Frank (2013b).

second part of the manuscript, which is also related to other texts by Guidobaldo, for example the *Meditatiunculae de rebus mathematicis*.⁶

1.3 A notebook by Guidobaldo dal Monte: the *Colibeto*

Muzio Oddi (1569–1639), engineer and mathematician, studied in Urbino with Guidobaldo.⁷ Oddi referred on various occasions to Guidobaldo's writings which he proved to know very well.⁸ For example, in a letter dated 21 February 1635 Oddi described a text written by Guidobaldo:

A paraphrase of that passage in Hyginus on the meridian line, which I printed and I know that in his [Guidobaldo's] book that he called *Colibeto* there are various treatments of three, four, and more propositions together on various matters.⁹

In this passage Oddi affirmed that Guidobaldo wrote a *Colibeto*, a booklet or folder, with problems and mathematical notes in no definite order. According to Oddi, the *Colibeto* contained “propositions together on diverse matters,” and in particular a discussion on the meridian line by Hyginus. Oddi also published a longer description of Guidobaldo's account on Hyginus:

To know how to draw the meridian line on a plane to the horizon ...I wanted on this occasion to refer here to one [solution] written by Hyginus, the ancient and famous astronomer, in the book *Gromaticus* ...this passage being so badly marred by the multitude of years, one can only make the construction poorly who has not seen the exposition that, at the instance of Signor Giovanni Vincenzo Pinelli of Padua, Signor Guidobaldo Marchese del Monte made for you, from whose kindness I confess to have learned all of what little knowledge I have of mathematics.¹⁰

The text to which Oddi refers, citing Guidobaldo and his corrections to Hyginus, is found in the UCLA manuscript at folios 77–78 and 83–3^o, which can therefore be identified as the *Colibeto* (or at least a part of it).¹¹

⁶ See Tassora (2001).

⁷ On Muzio Oddi's life and works see Montebelli (2001) and Marr (2011).

⁸ See Frank (2011, 64–65).

⁹ “Una Parafrasi sopra quel luogo d'Igeno della linea meridiana che io stampai e so che in un suo libro, che chiamava *Colibeto* ci havea diverse cose di tre quattro e più proposizioni insieme sopra diverse materie.” Pesaro, Biblioteca Oliveriana, MS 413, f. 259r-v, available on line at <https://echo.mpiwg-berlin.mpg.de/content/mpiwglib/pesaro>

¹⁰ “Sapere disegnare in piano all'orizzonte la linea meridiana ...ho voluto, con questa occasione, refferirme qui uno, scritto da Higeno antico e famoso astronomo nei libri *Gromatici* ...per essere quel luogo tanto mal ridotto dalla moltitudine degl'anni, che malamente ne può trar costruito chi non ha veduto l'esposizione che, ad istanza del Sig. Gio. Vincenzo Pinelli da Padova, vi fece l'illustrissimo Sig. Guidobaldo de' Marchesi del Monte, dalla benigna humanità del quale confesso di riconoscere tutto quel poco di cognitione che ho delle matematiche.”(Oddi 1614, 18–19).

¹¹ See the description of the content below.

1.4 The purpose

In this article, our attention will focus on UCLA folios 75–76, containing Galileo’s proof of the center of gravity of the frustum of a cone, which was ultimately published among the results of *Theoremata*.¹²

The discovery of the UCLA copy opened the possibility of giving a fuller account of *Theoremata* dating and development, and it can shed light on the origins of this research by the young Galileo. Moreover, it helps to explain the connections between Galileo and Guidobaldo, starting from 1587–88, in the context of Galileo’s early work. But before turning to this manuscript, it is necessary first to broaden the view by presenting the genesis of Galileo’s studies in this field and following his mathematical and academic career.

So in Sect. 2 we shall give an overview of sixteenth-century studies on centers of gravity. Then, after comparing the UCLA manuscript with other extant copies, we shall propose a dating for the composition of the *Theoremata circa centrum gravitatis solidorum* based on Galileo’s correspondence with the Jesuit Christopher Clavius (1538–1612) and with Guidobaldo. This dating will then be reconsidered in light of the mathematical content, which will be described in Sect. 5. We shall end with an appendix (Sect. 7) containing a description of the manuscript and an edition of the text of the lemma and theorem.

2 Centers of gravity of solids at the end of the sixteenth century

The study of centers of gravity during the sixteenth century developed as a direct consequence of the restoration of Archimedes’ corpus, and in particular of the work *On the Equilibrium of Planes*. This Archimedean work is in two books: the first deals with the law of the lever and the center of gravity of the triangle and the trapezium; in the second, Archimedes determined the center of gravity of a segment of a parabola and of the mixtilinear trapezium.¹³ In *On the Equilibrium of Planes* no mention is made of the centers of gravity of solid figures, although references to this problem appeared in other Archimedean texts available to sixteenth-century mathematicians. A passage from the second book of *On Floating Bodies* in Commandino’s version reads:

¹² See the “Lemma Galileus” on f. 75r. On f. 85r, a page containing three constructions of conics, there is a similar reference: “Del Galileo.” This text on conic sections is however more problematic from the codicological point of view: to the page is attached a slip of paper, partially overlapping the underlying text. At the bottom of the page Guidobaldo noted “Del Galileo.” It is difficult to evaluate the meaning of this remark by Guidobaldo, because just before it (on 85r) some words are written concerning gnomonics (or the planisphere): “verticali perché tutti passano per il vertice che è il Zenit.” The attribution to Galileo is, therefore, at least uncertain and deserves further investigation. Juvenile studies of conics by Galileo are little known; it would be interesting if this reference to Galileo could be confirmed in some way. Renn et al. (2000) suggested a connection between Galileo and Guidobaldo in the study of conic sections. Around 1592, they suggested, the two mathematicians could have carried out together experiments to trace parabolas; later both explained the same methods in their works: see Guidobaldo, *Meditatiunculae* in Tassora (2001, 545–546); and Galilei (1638, 145–146); ed. Favaro, *Opere di Galileo Galilei, Edizione nazionale (Galilei 1891-1909, hereafter cited as EN)*, VIII, 185–186; trans. Drake (1989, 142–143).

¹³ This figure is obtained by cutting a segment of a parabola with two lines perpendicular to the axis.

The center of gravity of a portion of a right conoid is on the axis, which it so divides that the part that ends at the vertex is double the remaining part at the base.¹⁴

This passage conveys the critical information that Archimedes had determined the precise location of the center of the paraboloid of revolution. The demonstration of this result was not present in any writings of Archimedes available at that time.¹⁵

Unfortunately, the fate of *On Floating Bodies* was not a happy one. Although it was translated into Latin by William of Moerbeke in 1269, his translation did not become part of any of the editions of Archimedes' works printed in the first half of the sixteenth century. The two books *On Floating Bodies* are absent from Luca Gaurico's *Tetragonismus id est quadratura circuli* (Gaurico 1503), which contains only *The Measurement of a Circle* and *The Quadrature of the Parabola*. Tartaglia's more complete edition (Archimedes 1543) contained only the first book of *On Floating Bodies*.¹⁶

Interpretation by Commandino of the second book in his *De iis quae vehuntur in aqua* (Archimedes 1565a) cleared up many of the obscurities in Archimedes' text and Moerbeke's translation.¹⁷

Besides the difficulties of Archimedes' texts on centers of gravity, there was also a lack of sources. For instance, a definition of center of gravity was absent from all the works at disposal of the scholars of that period.¹⁸ In his *Liber de centro gravitatis solidorum* of 1565, Commandino declared that the *quaestio de centro gravitatis corporum solidorum* was truly *perdifficilis et perobscura*.

2.1 Maurolico's results

Francesco Maurolico (1494–1575) did not have access to Moerbeke's translation when he devoted himself to restoring Archimedes' theory of equilibrium and so adopted a different approach. His *De momentis aequalibus libri quattuor* represents a complete remaking of Archimedes' work on the subject.¹⁹ In the fourth book he tackled the question of finding the center of gravity of solids and in 1548 he managed to devise a

¹⁴ "Portionis enim conoidis rectanguli centrum gravitatis est in axe quem ita dividit ut pars eius quae ad verticem terminatur reliquae partis quae ad basim sit dupla." Archimedes (1565a, f. 11r); see also the critical edition of J. L. Heiberg (Archimedes 1910-1915, II, 351): "Nam in libro De aequilibrata demonstratum est, cuiusvis segmenti rectanguli conoidis centrum gravitatis in axe positum esse ita diviso, ut pars axis ad verticem posita duplo maior sit reliqua" (proposition II.2).

¹⁵ A demonstration of the center of gravity of the paraboloid was given in the *Method*, a previously lost work of Archimedes contained in Codex C, a palimpsest found only in 1906.

¹⁶ *On Floating Bodies* is absent from the *editio princeps* of Basel (Archimedes 1544), which depends on a textual tradition different from that of Moerbeke's text.

¹⁷ Tartaglia's version, completed with the addition of the second book, was printed by Curtius Troianus in Venice as *De insidentibus aquae* (Archimedes 1565b), but this edition cannot stand in comparison with Commandino's.

¹⁸ Something similar to a definition of center of gravity (non-static) is found in the Maurolico's (*Cosmographia*). See Maurolico 1543, 18v (lines 17-20), 19r (lines 34-37), and 19v (lines 1-8).

¹⁹ Maurolico's Archimedean sources can be traced back to Giorgio Valla's *De expetendis et fugiendis rebus* (1501). For an analysis of the genesis of this work see the note to the text in the *Edizione nazionale dell'opera matematica di Francesco Maurolico*, vol. VII.B.1 (forthcoming); see also Giusti (2001) and Tucci (2004).

brilliant demonstration for the pyramid and the regular solids, as he points out in the preface:

It now remains to treat of finding the center of gravity in solids; for this was the subject of his work that I marvel not a little was omitted by Archimedes. ...Nevertheless, the center of the pyramid could be sought out with no less labour—not to say with more—than the center of the plane triangle. So since we shall have treated in the first book the general theory of weights, in the second the centers of planes, and in the third of conic sections called parabolas, in order to understand more clearly what Archimedes wrote. Now in this fourth book we shall undertake the task.²⁰

Napolitani and Sutto (2001) determined that Maurolico resumed his study of the centers of gravity of solids only in 1565, extending it to the paraboloid of revolution. In that year Maurolico made a capital discovery: that the center of gravity of the paraboloid could be resolved back to that of the triangle, already known from the works of Archimedes and extensively treated by Maurolico in the second book of *De momentis aequalibus*.

This discovery was presented, with a not completely rigorous proof, in a manuscript (Paris, Bibliothèque Nationale, MS lat. 7466, folios 8–14) addressed to an unknown recipient, probably a Jesuit with whom Maurolico was in contact. Unfortunately, this text remained unpublished until the nineteenth century. However, the determination of the center of gravity of the paraboloid was later formalized by Maurolico and added to the fourth book of *De momentis*. There the proof in the manuscript was replaced by one using double contradiction, usually known as the method of exhaustion, which was closer to Archimedean rigor. Maurolico's *De momentis aequalibus libri quattuor* was not printed until 1685,²¹ but Maurolico's results circulated among the mathematicians and influenced research on centers of gravity in the second half of the sixteenth century.²²

Commandino and Clavius were two of those who received Maurolico's legacy and took up the study of centers of gravity.

2.2 Commandino's Challenge

Federico Commandino started an in-depth study of Archimedes following a request made by the cardinal of the Vatican library Marcello Cervini who was looking for

²⁰ "Superest nunc agere de centri gravitatis inventione in solidis; hic enim erat eius speculationis locus quem ab Archimede omissum non parum admiror. ... Tamen centrum pyramidis non minori industria quam centrum plani trianguli, ne dicam maiori exquiri poterat. Itaque cum in primo libello doctrinam gravium universalem tradiderimus, in secundo centra planorum, in tertio conicae sectionis quae parabola dicitur, ad ea distinctius intelligenda, quae scripsit Archimedes. Nunc in hoc quarto solidorum negotium exequemur." Maurolico, *De momentis aequalibus, Praefatio, Edizione nazionale dell'opera matematica*, vol. VII.B.1.4 (forthcoming).

²¹ Archimedes (1685, 86–180).

²² Maurolico described his works in the *Index lucubrationum*, a list produced by Maurolico himself of *propria* and *aliena* texts. This list was printed in the dedicatory letter to the *Cosmographia* (Maurolico 1543) and as an appendix to the *Theodosii sphaericorum elementorum libri III ex traditione Maurolyci* (Maurolico 1558).

explanations concerning two works—*De insidentibus aquae*, William of Moerbeke’s Latin translation of Archimedes’ *On floating bodies*, and Ptolemy’s *De analemmate*—contained in a copy of Rome, Biblioteca Apostolica Vaticana, MS. Ott. lat. 1850. Cervini also asked for a corrected Latin edition thereof. Commandino began the examination of Archimedes’ texts and in 1558 he published in Venice the *Archimedis opera nonnulla*, a new translation of many Archimedean works, but not *On Floating Bodies* (Archimedes 1558).

In 1565 he completed the task entrusted to him fifteen years earlier by Cervini by printing in Bologna his edition of Moerbeke’s translation of *On Floating Bodies* and an original work, the *Liber de centro gravitatis solidorum* (Commandino 1565). In writing this text Commandino, always philologically faithful to his model, was inspired by the method of Archimedes’ *On the Equilibrium of Planes*.

We have evidence of a correspondence between Maurolico and Commandino on some theorems of *On Conoids and Spheroids* although nothing that demonstrates a direct exchange of ideas regarding centers of gravity of solids.²³

In the preface, Commandino wrote that he delayed publication after coming into possession of Maurolico’s book on centers of gravity:

But when I was in the process of writing this, the book of Francesco Maurolico of Messina was sent to me, in which that most learned and most accomplished man in these disciplines affirmed that he had written on centers of gravity of solid bodies. When I learned this, I held myself back for a time and waited silently while the work of this most renowned man, whom I always mention with honor, was brought to light.²⁴

The *Liber de centro gravitatis solidorum* starts with the definition of center of gravity and provides some results for plane figures already demonstrated by Archimedes. Commandino determined the center of gravity of simpler solids such as the prism and the cylinder; then he moved on to the more complex ones: the pyramid, cone, paraboloid and frustra (propositions XXII, XXIX and XXVI, XXXI). Commandino had thrown down the gauntlet to other mathematicians. But the challenge was not over. The theorems for frustra were not fully satisfactory: the enunciations were long and convoluted and the proofs were difficult to understand.²⁵ In particular, the proof for the frustum of a paraboloid was so unclear that Guidobaldo himself wrote in a letter to Galileo that it was not good, because it was not general (*universale*).²⁶ The mathematicians after Commandino took up the challenge and thus began to study the centers of gravity of solids.

²³ For the dating of the correspondence between the two, see Sisana (2021).

²⁴ “Cum autem ad hoc scribendum aggressus essem, allatus est ad me liber Francisci Maurolici Messanensis in quo vir ille doctissimus et in iis disciplinis exercitatissimus affirmabat se de centro gravitatis corporum solidorum conscripsisse. Cum hoc intellexissem, sustinui me paulisper tacitusque expectavi dum opus clarissimi viri, quem semper honoris causa nominino, in lucem proferretur.” Commandino (1565, f. 112r-v).

²⁵ Commandino (1565, f. 34v) (for the frustum of a cone) and f. 46r (for the frustum of a paraboloid).

²⁶ Guidobaldo to Galileo, 16 January 1588, EN, X, 25.

2.3 From Clavius to Galileo, towards the Epilogue

Clavius should be mentioned for his indirect contribution to the development of the study of centers of gravity in the second half of the sixteenth century. He got to know Maurolico both in correspondence (from 1569) and in person during a stay in Messina in 1574.²⁷ On this occasion he consulted some working copies of Maurolico's mathematical texts and brought them back to Rome, once he had returned from his travels in Sicily.²⁸

At the Collegio Romano Clavius established a course in higher mathematics that included centers of gravity of solids. Although he never published anything on the subject, various testimonies suggest that from the 1580s he was considered an expert.²⁹ The most significant comes from a letter from Guidobaldo to Galileo dated 16 January 1588, containing a reference to the theorem on the center of gravity of the frustum of a paraboloid:

Among some letters that on many days passed between Father Clavius and me, I wrote him that the last [proposition] of Commandino's *De centro gravitatis solidorum* was not sound because it was not general; he sent me then his own proof, very different from yours.³⁰

Traces of Clavius's proof remain today among Guidobaldo's papers in the *Meditatiunculae de rebus mathematicis*.³¹ Clavius's demonstration, transcribed by Guidobaldo, follows Commandino's deductive pattern, but with even greater attention to detail. Galileo would be the first to propose a proof for the frustum of a paraboloid in a fully Archimedean style, although with a substantial change in the demonstrative structure. In 1587, still at the beginning of his career, Galileo met Clavius in Rome and gave him a copy of his demonstration for the frustum of a paraboloid. In Galileo's correspondence of the following year, there are some letters regarding a lemma necessary for the demonstration of the center of gravity of the whole paraboloid. The nature of this correspondence and the relationship between the two will be studied in the next section. Galileo's results on centers of gravity, which were published as an appendix to his *Discorsi e dimostrazioni matematiche* in 1638, concern the paraboloid and its frustum and also the pyramid, the cone, and their frusta.

The key book on centers of gravity, which according to Galileo made further research in the field unnecessary, was the *De centro gravitatis solidorum libri tres*

²⁷ See d'Alessandro and Napolitani (2001).

²⁸ For the optical works see Bellé (2017); for the connection between Maurolico and the Jesuits, see Moscheo (1998).

²⁹ Valerio, for example, mentioned Clavius as among "the most notables" in matters of equilibrium (Valerio 1582, 21).

³⁰ "Fra alcune lettere, che molti giorni sono occorsero fra il Padre Clavio et me, io le scrissi che l'ultima del Commandino, *De centro gravitatis solidorum* non era buona per non esser universale; il qual Padre mi mandò poi la sua dimostrazione, assai diversa da questa di V. S." Guidobaldo to Galileo, 16 January 1588, EN, X, 25–26.

³¹ Paris, Bibliothèque Nationale, MS lat. 10246, pp. 123–125; ed. Tassora (2001, 396–399); see also Tassora (2001, 32).

(1604) by Luca Valerio (1553–1618).³² In it Valerio brought to full fruition the ideas already present in Maurolico’s manuscript cited above.³³

Not surprisingly, Valerio’s book precluded any other attempt to determine the centers of gravity of solids. Clavius was certainly deterred by it and never published his book on centers of gravity.³⁴ Valerio’s work also caused Galileo’s delay in releasing his theorems on the subject, which were ready for publication by the end of the sixteenth century.

3 Another witness of the *Theoremata*

The witnesses known today of the *Theoremata circa centrum gravitatis solidorum* are

- Galileo Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze ... con una Appendice del centro di gravità d’alcuni solidi* (Leiden 1638, 289–314) (but numbered 306) (siglum **L**);
- Los Angeles, Charles E. Young Research Library, University of California-Los Angeles, MS 170/624, folios 75–76 (siglum **U**);
- Milan, Biblioteca Ambrosiana, MS A 71 Inf., folios 95–96 (siglum **A**).³⁵

As can be seen in Table 1, the two manuscripts include only the last lemma and theorem on the conical frustum. In his edition, Favaro gave the Ambrosiana text (**A**) in parallel with Leiden edition (**L**).³⁶ We publish in Sect. 7 for the first time, the text contained in the UCLA manuscript (**U**).

For the sake of exposition, we can divide the content of the UCLA manuscript into five parts:

- the *Lemma* on folio 75r, which corresponds to EN, I, 204–205;

³² At the end of *Discorsi*, by way of introducing the appendix containing the theorems on centers of gravity, Galileo has Salviati say, “Ma incontratosi, dopo alcuno tempo, nel libro del Sig. Luca Valerio, massimo geometra, e veduto come egli risolve tutta questa materia senza niente lasciar in dietro, non seguitò più avanti, ben che le aggressioni sue siano per strade molto diverse da quelle del Sig. Valerio,” Galilei (1638, 288; EN, VIII, 313); trans. Drake (1989, 259–260).

³³ See in Sect. 2.1. It is not known whether Valerio’s ideas developed independently from Maurolico’s work or if there was some influence through the Jesuits.

³⁴ Although today this work is lost, its existence is known from a letter from the Jesuit Giovanni Giacomo Staserio to Clavius dated 9 July 1604: “Et lei [Clavius] attenda a governarsi, et insieme prepari qualche altra bella cosa De centro gravitatis, quale V. R. mi promese, et poi si ritenne per rivederlo con l’occasione del libro uscito ultimamente della medesima materia.” See Clavius (1992), V, 113, letter no. 225. The following letter (no. 226) also contains a reference to this work: “De centro gravitatis, *tanto del suo* quanto di quel nuovo stampato aspetto risposta” and “almeno le cose fatte, come è de centro gravitatis et altre che con poca fatica potria accomodare, vorrei si forzasse a darle fuori” (our italics).

³⁵ There is also a copy made from the Ambrosiana manuscript by G. B. Venturi (in Florence, Biblioteca Nazionale, MS Gal. 72, ff. 3–5), which is listed, at the beginning of the manuscript, as: “Indice delle cose meccaniche del Galileo contenute nel presente Tomo. Determinazione del centro di gravità di una piramide tronca, lemma di Galileo, scritto nel 1587, copia moderna a noi pervenuta dal P. Venturi, che la estrasse dalla Biblioteca Ambrosiana” (MS Gal. 72, f. 2r). Moreover, in MS Gal. 79 there is a copy of the Leiden edition text with additions by Vincenzo Viviani, which has been transcribed by Giusti (1990, 297–319).

³⁶ See EN, I, 204–208.

Table 1 Content and witnesses

	L	A	U
Postulate	✓	X	X
<i>Paraboloid</i>			
Lemma 1 and Lemma 2	✓	X	X
Theorem 1 and Theorem 2	✓	X	X
<i>Paraboloid frustum</i>			
Lemma 1	✓	X	X
Theorem 1	✓	X	X
<i>Cone or pyramid</i>			
Lemma 1 and Lemma 2	✓	X	X
Theorem 1 and Theorem 2	✓	X	X
<i>Conical or pyramidal frustum</i>			
Lemma 1	✓	✓	✓
Theorem 1	✓	✓	✓

- a brief passage on the center of the cone: “that the center of gravity of any cone or pyramid so divides the axis that the part towards the vertex is triple the remainder was already proved by Commandino and *by me in another way*” [our italics];³⁷
- the enunciation and beginning of the proof of the center of gravity of the frustum of a cone (75v–76r; EN, I, 205–206);³⁸
- two lines of text, probably an addition or a comment by Guidobaldo, since they are underlined, as was usual when he wanted to indicate his own words;³⁹ and
- the actual proof, which corresponds to the Leiden text (EN, I, 206–208).

At first sight, this manuscript does not seem to add anything significant to our knowledge of Galileo’s theory of centers of gravity: although it differs in some passages from the Leiden text, the differences are not very significant. It can however shed light on some features of the young Galileo’s research.

4 When were the *Theoremata* written?

The *Theoremata* were published only in 1638. At the end of the fourth day, Salviati presents the theorems:

These are some propositions pertaining to the center of gravity of solids which our Academician [that is, Galileo] discovered in his youth, when it appeared to him that there were still some defects in what had been left written on the

³⁷ An analogous passage in the Leiden edition is found in the statement of theorem on the whole cone: “Cuiuslibet coni vel pyramidis centrum gravitatis axem dividit ut pars ad verticem reliquae ad basin sit tripla” (Galilei 1638, 309, numbered 301).

³⁸ Whereas the enunciation is similar in the Leiden version, there are significant differences in the setting out (*ekthesis*), as we shall see later, on in Sect. 5.2.

³⁹ For example, see on ff. 87v–88r the studies of propositions 22 and 23 of Luca Valerio’s *Subtilium indagatum liber* (1582), where Guidobaldo distinguished Valerio’s text from his own comments.

subject by Federico Commandino. He thought that these propositions which you see written here might supply that which Commandino's book left to be desired, and he applied himself to this study at the instance of the illustrious Marquis Guidobaldo del Monte, a very great mathematician of his time as shown by his various published works. Our Author gave a copy of these to that gentleman, intending to pursue the subject for other solids not touched on by Commandino. But some time later, he ran across the book of Luca Valerio, a prince of geometers, and saw that this resolved the entire subject without omitting anything; hence he went no further, though his own advances were made along quite a different road from that taken by Valerio.⁴⁰

This passage is interesting for at least three reasons. First, because it explains that Galileo decided to deal with the centers of gravity of solids to make up for what was missing in Commandino's book. Secondly, because it explicitly states that Galileo sent a copy of these studies to Guidobaldo. The UCLA manuscript represents what remains today of the copy delivered to him. Galileo, as young man, had accepted Commandino's challenge and began to study centers of gravity to mend some flaws present in the theorem on the frustum of a paraboloid. Then, encouraged by Guidobaldo, he continued his research. But in 1604 Luca Valerio's *De centro gravitatis solidorum libri tres* was published—and this is the third reason—which put an end to Galileo's studies on the subject. In the *Discorsi* there are two mentions of Luca Valerio: the one just quoted before the appendix containing the *Theoremata*, the other in the Second Day, after the quadrature of the parabola:

A beautiful and ingenious demonstration This proves something that Archimedes demonstrated by two different trains of many propositions, both of them admirable, and which was also demonstrated more recently by Luca Valerio, a second Archimedes according to our age, whose demonstration is given in the book he wrote on the center of gravity in solids. A book truly not to be placed below anything written by the most famous geometers of the present or all past centuries. When it was seen by our Academician [Galileo], it caused him to desist from pursuing the discoveries that he had been writing about the same subject, since he saw the whole thing so happily revealed and demonstrated by Signor Valerio.⁴¹

⁴⁰ “Queste sono alcune proposizioni attenenti al centro di gravità de i solidi, le quali in sua gioventù andò ritrovando il nostro Accademico, parendogli che quello che in tal maniera aveva scritto Federigo Comandino non mancasse di qualche imperfezzione. Credette dunque con queste proposizioni, che qui vedete scritte, poter supplire a quello che si desiderava nel libro del Comandino; ed applicossi a questa contemplazione ad istanza dell'Illustrissimo Sig. Marchese Guid'Ubaldo Dal Monte, grandissimo matematico de' suoi tempi, come le diverse sue opere pubblicate ne mostrano, ed a quel Signore ne dette copia, con pensiero di andar seguitando cotal materia anco ne gli altri solidi non tocchi dal Comandino.” Galilei, *Discorsi*, EN, VIII, 313; trans. Drake (1989, 259–260) (our italics).

⁴¹ “Bella e ingegnosa dimostrazione . . . provando quello che Archimede con due tra di loro diversissimi, ma amendue ammirabili, progressi di molte proposizioni dimostrò; como anco fu dimostrata ultimamente da Luca Valerio, altro Archimede secondo dell'età nostra, la qual dimostrazione è registrata nel libro che

Footnote 40 continued
egli scrisse del centro di gravità de i solidi. Libro veramente da non esser posposto a qual si sia scritto da i più famosi geometri del presente e di tutti i secoli passati; il quale quando fu veduto dall'Accademico nostro, lo fece desistere dal proseguire i suoi trovati, che egli andava continuando di scrivere sopra 'l medesimo

But when were the *Theoremata* written and in what order were they proved?

Favaro dated them to 1585 on the basis of a well-known letter written by Galileo to Elia Diodati (6 December 1636).⁴² According to this letter, Galileo obtained the results published in 1638 when he was “twenty-one years of age and with two years of geometrical study.”⁴³ So Favaro placed them in chronological order between the *Iuvenilia* and *La bilancetta*. This letter from 1636, however, is the only document on which this dating is founded. Other more direct sources suggest that Galileo had elaborated the theorems at different times and continued to work on them until the middle of 1588.

4.1 Assessment and testimonial

The first document that provides us with evidence for determining a date of composition is the Ambrosiana copy, which contains an attestation of Galileo’s authorship signed by four Florentine gentlemen and dated 12 December 1587.⁴⁴ Immediately below with the date 29 December 1587 there is also the assessment and signature of Giuseppe Moleti (1531–1588), then professor of mathematics at Padua:

On 29 December 1587, I, Giuseppe Moleti, professor of mathematics at the University of Padua, state that I have read the present lemma and theorem, which seem sound to me, and I judge their author to be a well experienced geometer. The same Giuseppe has written this in his own hand.⁴⁵

Moreover, in the MS Gal. 72 there is an anonymous and undated letter that contains an appraisal of the lemma and the theorem on the conical frustum given orally to the sender by an unnamed friend. The sender reports that his friend “has infinite praise for the discoverer of this theorem and together with Signor Moleti judges him to be well versed in mathematics.”⁴⁶ The letter then goes on to relate his judgment of the lemma and the theorem.⁴⁷ Favaro could not offer a reliable conjecture on the identity of the

soggetto, già che vedde il tutto tanto felicemente ritrovato e dimostrato dal detto Sig. Valerio (*Discorsi*, EN, VIII, 184; trans. Drake (1989, 141–142); see ed. Giusti (1990, 159), with variant readings from Florence, Biblioteca Nazionale, MS *Banco Rari* 31, formerly A. 5, p. 2, n. 13). The work by Valerio on the quadrature of the parabola is *Quadratura parabolae per simplex falsum* and not “the book on the center of gravity of the solids” (as wrongly cited here by Galileo).

⁴² See EN, I, 182.

⁴³ “essendo di età di 21 anno e di 2 di studio di geometria,” EN, XVII, 524.

⁴⁴ “Fassi fede per me Giovanni Bardi de’ Conti di Vernio, come le presenti conclusioni e dimostrazioni sono state ritrovate da M. Galileo Galilei; e in fede ò fatto la presente questo di dodici di Dicembre 1587, manu propria. Io Gio. Batta Strozzi affermo il medesimo; e in fede mi sono sottoscritto di mia mano. Io Luigi di Piero Alamanni affermo il medesimo; et in fede ho sottoscritto di mia propria mano questo di 12 Dicembre 1587. Io Gio. Batta da Ricasoli Baroni confermando il medesimo mi sottoscrivo di man propria il dì 12 detto 1587” (A, f. 96r).

⁴⁵ “A di 29 di Dicembre del 1587. Io Gioseppe Moletto, Lettor publico delle Mathematiche nello studio di Padova, dico haver letti i presenti Lem[m]a et Theorema i quali mi son parsi buoni, e stimo l’autor d’essi esser buono et esercitato Geometra. Il medesimo Gioseppe ha scritto di man propria” (A, f. 96r).

⁴⁶ “Loda infinitamente lo inventore di questa speculatione et insieme col sign.r Moletto lo giudica molto versato nelle matematiche.” Florence, Biblioteca nazionale MS Gal. 72, f. 6; EN, X, 21–22.

⁴⁷ This manuscript also contains a copy of the Galilean material in the Ambrosiana manuscript (see n. 34).

sender, but suggested that the friend and assessor was perhaps Pier Antonio Cataldi (1552–1626), who held the chair of mathematics at Bologna from 1582–1626.⁴⁸

Although it is only a hypothesis, we certainly agree that the author of this judgment was very familiar with the mathematical debate concerning the centers of gravity. He cited three of the mathematicians of the time who had studied the Archimedean works and had treated the subject themselves. The first reference was to Niccolò Tartaglia's *Secundus Archimedis Tractatus*.⁴⁹ The friend explained the analogy between proposition IX of second book *On the Equilibrium of Planes* and the lemma on frustum of a cone in meticulous and technical detail.⁵⁰ The author then cast doubt on the theorem, for the point designated as the center of gravity did not conform to the definition of center of gravity posited by Pappus and used by Guidobaldo in *Le mechaniche* (dal Monte (1581)), could be valid in this theorem. He finally decided to give credit to proposition XXVI of Commandino's *De centro gravitatis solidorum* Commandino (1565)—the third mathematician cited—as conforming better to this definition.⁵¹ The doubt arose probably from a misunderstanding of Pappus's definition of center of gravity, as reported in the Italian translation of Guidobaldo's *Mechaniche*, with a reference to Commandino's version of this definition:⁵²

The center of gravity of any solid shape is that point within it around which are disposed on all sides parts of equal moments, so that if a plane be passed through this point cutting the said shape, it will always be divided into parts that *weight equally* (*peseranno ugualmente*) [our italics].

⁴⁸ EN, I, 183–185. The index of MS Gal. 72, on f. 2 lists a “Lettera autografa del Marsili, nella quale si dà ragguaglio del giudizio fatto in Bologna sopra questa proposizione di Galileo.” As pointed out by Favaro this must be a mistake, for the only Marsili of Bologna connected to Galileo was born after this letter was written.

⁴⁹ This is *Liber Archimedi de centrum gravitatis, vel duplationis aequerepentibus*, in Archimedes (1543, 11–19). Proposition IX, also called Theorema VIII, is on pp. 15–17.

⁵⁰ “Et benchè questo lemma non sia il medesimo con la nona d’Archimede, nel 2° trattato del Tartaglia, par non di meno nato di là, et sotto la forma di quella proposizione constretto, et simile ad una proposizione che egli già molti anni fece, nella quale, sì come Archimede toglie i due quinti della massima et l’amico di V. S. un quarto, egli toglieva un ottavo, seguendo, ne l’altre, con simili proportionalità, nel lor genere. Et dice non esser molta fatica, seguendo la forma d’Archimede, formarsene assaissime.” EN, X, 21–22.

⁵¹ “Quanto al teorema, egli dubita se il centro del pezzo della piramide sia il punto o: per ciò che, stando la definizione del centro delle gravità de’ corpi posta da Pappo et adoprata dal Marchese Del Monte nelle *Mecaniche*, non segue che se per lo centro o supposto passerà un piano, quel pezzo si divida in due parti ugualmente pesanti, come dovria quando fosse veramente il centro. Et il Comandino, che la medesima materia tratta nel libro *De centro gravium* alla XXVI proposizione, molto più s’acosta a trovar il centro, che non par che faccia questa demonstratione, quantunque da quella del Comandino non sia molto differente.” EN, X, 21–22.

⁵² See dal Monte (1581, 1): “Il centro della gravezza di ciascun corpo è un certo punto posto dentro, dal quale se con la imaginatione s’intende esservi appeso il grave, mentre è portato sta fermo, et mantine quel sito, che egli havea da principio, ne in quel portamento si va rivolgendo. Questa diffinitione del centro della gravezza insegnò Pappo Alessandrino nell’ottavo libro delle raccolte methematiche. Ma Federico Commandino nel libro del centro della gravezza de’ corpi solidi dichiarò l’istesso centro in questa maniera descrivendolo. Il centro della gravezza di ciascuna figura solida è quel punto posto dentro, d’intorno al quale le parti di momenti eguali da ogni parte si fermano. Peroche se per tale centro sarà condotto un piano, che seghi in qual si voglia modo la figura, sempre la dividerà in parti, che peseranno ugualmente.” See also the Latin version, *Mechanicorum Liber*, (dal Monte 1577, 1). The English translation is taken from Drabkin and Drake (1969, 259). A thorough explanation of the two definitions is found in Frank (2011, 196–197).

Table 2 The correspondence between Galileo and Clavius

Date	From	To
8/01/1588	Galileo	Clavius
16/01/1588	Clavius	Galileo
25/02/1588	Galileo	Clavius
5/03/1588	Clavius	Galileo

The assessor's interpretation of *peseranno ugualmente* was wrong; with this expression Commandino did not mean that the two parts of the solid were *equal in weight*, but rather that they *weighed equally*, that is, that they were in balance with each other.⁵³

In any case, what matters most to us is the date of the work and its contents. The reference to Moleti's judgment of 29 December 1587 strongly suggests that this letter was written later than the end of 1587 but, according to Favaro, before the end of 1588.⁵⁴ Furthermore, the assessor evaluated only the theorems on the conical frustum, which—according to him—were similar to Archimedes' demonstrative style, and not the other lemmas and theorems that are found in the Leiden edition (Table 1).

A first certain conclusion is that by 12 December 1587 Galileo had found the proof for the conical frustum and its lemma, which he then presented to his mathematical colleagues. No document proves that he already had all the proofs of the other theorems. So it is possible that the order of presentation in the Leiden edition is different from that of the discovery of the theorems.

Consistent with this is more evidence that helps to determine more precisely the entire process of composition. Along these lines, much useful information emerges from Galileo's correspondence with Clavius (Table 2) and with Guidobaldo (Table 3).

4.2 The correspondence with Clavius

In the second half of 1587 Galileo went to Rome, where he met Clavius. At that time Galileo aspired to become professor of mathematics at the University of Bologna and was seeking approval from the most famous mathematicians of his time (Favaro 1922). To get a recommendation from Clavius, on the occasion of his visit he gave him a copy of the theorem on the frustum of a paraboloid, as emerges from a letter of 8 January 1588, in which Galileo apparently wanted to resume interrupted contacts.⁵⁵ In the letter, he sent a correction of an oversight in the theorem on the frustum of a

⁵³ The debate on the definition of the center of gravity and its consequence for the concept of equilibrium are discussed in Renn and Peter (2012, 102–107).

⁵⁴ EN, I, 184, Moleti's assessment is also mentioned in a letter from Antonio Riccoboni to Galileo on 11 March 1588: "quella sua compositione che da tanti valent'huomini è stata approvata e sottoscritta" where it is emphasized that "S.or Moletto l'ama medesimamente da buon senno" (EN, X, 30).

⁵⁵ "Io credo che nella dimostrazione di quel teorema del centro della gravezza del frusto del conoidale rettangolo, che lasciai a V. S. M. R., vi sia una scorrezione, poi che è ancora nell'originale d'onde la copiai; Footnote 54 continued

et dove credo che dica: *Quam autem rationem habet composita ex ns et tripla sx ad compositam ex ns et dupla sx, si deve leggere: Quam autem rationem habet composita ex ns et dupla sx ad compositam ex tripla utriusque simul ns, sx.* Questa scorrezione è di poca importanza; ma se ci fossero errori di momento, desidero che la mi favorisca avvertirmene," (EN, X, 22). In the Leiden edition this passage is slightly

paraboloid and asked for an opinion on a lemma necessary for the proof of the center of gravity of the whole paraboloid. The lemma in question concerns the center of gravity of a particular disposition of weights in a balance.⁵⁶ Galileo wrote to Clavius that some Florentine friends “said they were not entirely satisfied with it” (“dicono non ci haver l’intera soddisfazione”).⁵⁷ On this lemma, they exchanged other two letters: Galileo’s reply of 25 February 1588, and the last letter from Clavius of 5 March 1588. At the end of this exchange, Galileo and Clavius had apparently not changed their minds. Clavius remained convinced that there was a *petitio principii* whereas Galileo thought that the proof was sound. This question will be discussed in detail below (see Sect. 4.4).

From these letters (Table 2) it seems that Clavius’s attitude towards Galileo’s research was ambiguous. On the one hand, he appears in his answers to be not well-disposed towards Galileo;⁵⁸ on the other he showed interest in the same research topics. In fact, a letter from Guidobaldo to Galileo of the same period (16 January 1588) states that Clavius had dealt with centers of gravity of solids and that he had proved the paraboloid frustum theorem: “Father [Clavius] then send me his proof, very different from yours [Galileo’s].”⁵⁹ Moreover, Clavius’s attitude was due not only to scientific reasons. He knew the mathematician Giovanni Antonio Magini (1555–1617), who was then in Rome. Magini was Galileo’s direct competitor for the chair of mathematics in Bologna. Clavius, when consulted by the ambassadors sent to Rome by the Senate of Bologna, had shown preference for Magini, who, in the end, was chosen for the chair.⁶⁰

different: “Quam autem proportionem composita ex ns et dupla sx ad compositam ex tripla ns et tripla sx ” (EN, I, 197).

⁵⁶ “If any number of magnitudes equally exceed one another, the excesses being equal to the least of them, and they are so arranged on a balance as to hang at equal distances, the center of gravity of all these divides the balance so that the part on the side of the smaller [magnitudes] is double the other part” EN, I, 187–8; trans. Drake (1989, 262). See the fuller discussion Sect. 4.4.

⁵⁷ “Il parer suo circa alcune mie difficultà; delle quali una è questa, che con la presente gli mando, intorno alla dimostrazione dell’infra scritto lemma, la quale desidero saper da lei se interamente gli quietà l’intelletto, atteso che alcuni, a i quali qui in Firenze l’ho mostrata, dicono non ci haver l’intera soddisfazione, non tollerando volentieri quel doppio modo di considerare le medesime grandezze in diverse bilancie, come benissimo V. S. M. R. nella dimostrazione scorgerà. Io ho cercato molti giorni con diligenza qualche altra dimostrazione, ma non trovo cosa alcuna, salvo che a dimostrarla per induzione, il qual modo di dimostrare a me non satisfà molto” (Galileo to Clavius, 8 January 1588, EN, X, 22–23). The lemma is partly cited by Clavius in his reply of 16 January 1588. See Sect. 4.4.

⁵⁸ See, for example, these passages from Clavius’s letters: “adesso sto molto rimoto di queste speculationi de aequponderantibus, le quali, come V. S. sa bene, ricercano grande attuazione ... Io non ho ancora havuto tempo di vedere detta dimostratione. Spetto occasione che possi un poco rinfrescare la memoria di questo studio, et gli scriverò sinceramente quello che io sentirò” (Clavius to Galileo, 16 February 1588, EN, X, 18); and “Ho ricevuto la risposta alla mia scrittali, et mi dispiace di non potere, per le continue mie occupationi, attendere con più studio alla materia del centro gravitatis, per satisfare a V. S. nel suo quesito” (Clavius to Galileo 5 March 1588, EN, X, 22).

⁵⁹ “il qual Padre mi mandò poi la sua dimostratione, assai diversa da questa di V. S.” (EN, X, 26); see also Sect. 2.3. This proof is preserved in Guidobaldo’s *Meditatiunculae* with the title “Ultima propositio Federici Commandini de centro gravitatis solidorum, ut notavimus in ipso libro, falsa existit; ac ratione restitui poterit. Et haec demonstratio est Cristophori Clavii e Societate Jesu” (*Meditatiunculae*, 123–125; ed. Tassora 2001, 396–399; see also Tassora 2001, 32–33).

⁶⁰ Galileo nevertheless obtained two recommendations, one from of Giovanni dall’Armi and the other from Enrico Caetani. The first is testified by a document in the State Archives in Bologna dated 1587 (Favaro

Table 3 The correspondence between Galileo and Guidobaldo

Date	From	To
16/01/1588	Guidobaldo	Galileo
24/03/1588	Guidobaldo	Galileo
31/03/1588	Coignet	Galileo
28/05/1588	Guidobaldo	Galileo
17/06/1588	Guidobaldo	Galileo
16/07/1588	Galileo	Guidobaldo
22/07/1588	Guidobaldo	Galileo
16/09/1588	Guidobaldo	Galileo
07/10/1588	Guidobaldo	Galileo
30/12/1588	Guidobaldo	Galileo

In all events it is likely that the proof for the paraboloid frustum was already in a suitable form at the time of the visit to Rome, so that Galileo could leave a copy of it with Clavius. The proposition on the whole paraboloid, in contrast, was not yet available in a suitable version by the beginning January 1588 (and probably not before 5 March 1588), as is suggested by the discussion of the proof of the lemma with Clavius.

4.3 The correspondence with Guidobaldo

This is where Guidobaldo comes in. Guidobaldo's correspondence with Galileo is wide-ranging and covers an extended period of time (Table 3). The first interesting evidence for the dating of the *Theoremata* appears in a letter sent by Guidobaldo to Galileo on 16 January 1588.⁶¹ In that letter, Guidobaldo thanked Galileo for having sent him the theorem in which he had imitated Archimedes in the last propositions of *On the Equilibrium of Planes*. Guidobaldo was referring to the paraboloid frustum. Guidobaldo also reported that Clavius had sent him his own proof, very different from Galileo's. Since Commandino's *De centro gravitatis solidorum* ends with the demonstration for the center of gravity of the paraboloid frustum, it is conceivable that the theorem to which Guidobaldo referred is precisely that on the paraboloid frustum. The letter concludes with a request that Galileo send him more on centers of gravity ("le altre cose sui centri di gravezza"). Although not explicitly stated, it is possible that this reference is to the proof of the conical frustum, authenticated in 1587.

Another letter from Guidobaldo to Galileo, dated 24 March 1588, confirms that Galileo fulfilled Guidobaldo's request and sent to him a proof.⁶² In the following letter

1922, 152), the second by a letter from Caetani dated 10 February 1588 addressed to the Senate of Bologna (EN, X, 20).

⁶¹ EN, X, 25–26.

⁶² "La sua dimostrazione ultima, che mi ha mandato, mi ha piaciut' assai. E le bascio le mani" (Guidobaldo to Galileo, 24 March 1588, EN, X, 31). Guidobaldo was probably referring to the proof on the frustum of a cone, already read by the four Florentine men and by Moleti at the end of December 1587. In fact, the

(28 May 1588) Guidobaldo raised doubts about the soundness of the same lemma that Galileo had discussed with Clavius:⁶³

Please do not omit attending to these matters on the center of gravity that you have begun, since they are most beautiful and subtle. I have seen your lemma, and to speak freely my opinion, I doubt that it begs the question [petat principium], because in the proof where it says “the center of all [the weights] is truly x , therefore x divides the lines ba and ad into the same ratio,” it seems that one could deny this conclusion.⁶⁴

These doubts were resolved only a few days later by Guidobaldo himself who, in a letter to Galileo, dated 17 June 1588, confirmed the validity of the lemma, with a proviso:

When I wrote to you about that proof of yours, it occurred to me a few days later where I had gone wrong. . . . So it seems to me that the proof is very sound, founded on that supposition, which could perhaps be proved with a little trouble.⁶⁵

Galileo, heartened by this judgment,⁶⁶ on 16 July 1588 replied to Guidobaldo and sent him the proof in which this lemma is applied, the center of gravity of the whole paraboloid. Finally, he promised him another proof, on the hyperboloid:

the center of gravity of the obtuse-angle conoid [hyperboloid] divides the axis such that the part at the vertex to the rest has the same ratio as the sum of the axis plus double the addition to the axis has to the sum of the addition and a third part of the axis.⁶⁷

theorem on the paraboloid frustum had already been sent to Guidobaldo with the previous letter (16 January 1588). It can be conjectured that Galileo’s notes transcribed by Guidobaldo in the UCLA manuscript are those received with this letter.

⁶³ Clearly Galileo had asked for Guidobaldo’s opinion, not having been satisfied with Clavius’s answer.

⁶⁴ “La prego a non mancar di attendere a queste cose del centro della gravità, che ha cominciato, essendo cose bellissime et sottilissime. Ho veduto il suo lemma, e per dirgli liberamente il parer mio, dubbito che petat principium, perchè nella dimostrazione dove dice: *Verum centrum omnium est x , quare x eadem ratione dividet ba et ad lineas*, pare che si possa negare questa conseguenza” (Guidobaldo to Galileo, 28 May 1588, EN, X, 34).

⁶⁵ “Quand’io scrissi a V. S. intorno a quella sua dimostrazione, di lì a due giorni io mi accorsi dove havevo pigliato errore. . . . sì che a me pare che la dimostrazione stia benissimo, fondata in quella suppositione, la quale si potrebbe forse dimostrare con poca cosa” (Guidobaldo to Galileo, 17 June 1588, EN, X, 34–35).

⁶⁶ “Ho havuto contento che la dimostrazione del lemma gli sia parsa buona, però che il giudizio di due uomini illustri, qual è V. S. Ill.ma et un altro [Clavius] che pur due volte mi ha replicato che petit principium, mi facevano assai dubitare di essere abbagliato; e l’haver ancora con gran fatica cercatane altra dimostrazione, e non l’haver trovata mi sbigottiva” (Galileo to Guidobaldo, 16 July 1588, EN, X, 35).

⁶⁷ “in conoide obtusiangulo centrum gravitatis axem ita dividit, ut pars ad verticem ad reliquam eandem habeat rationem, quam composita ex axe et dupla ad axem adiectae habet ad compositam ex adiecta et tertia parte axis” (Galileo to Guidobaldo, 16 July 1588, EN, X, 36).

This enunciation is quite different from what Luca Valerio wrote a few years later in his *De centro gravitatis solidorum libri tres* (1604).⁶⁸ Today there is no other trace of this theorem neither in Galileo's correspondence nor in his printed works.

4.4 The Doubtful lemma

The lemma discussed by Galileo with Clavius and Guidobaldo concerns the center of gravity of any number of magnitudes that equally exceed one another, with the excesses equal to the least of them, hung on a balance beam at equal distances in order of weight. Galileo proved that, in this case, the point of equilibrium divides the beam in the ratio 2:1. He considered the magnitudes in two different ways, by imagining them divided and regrouped. In the first way there are five *vertical* magnitudes hanged at points *a, c, d, e, b*, from the greatest (five blocks in the figure) to the smallest (only one block). In the second we should consider the five magnitudes as disposed *horizontally*, hanged at points *d, i, c, m, a*, from the greatest (five blocks horizontally disposed) to the smallest, only one block hung at point *a* (see figure 1).⁶⁹ The size and position of the magnitudes along the beam remained unchanged, so that their common center of gravity also remained at the same point.⁷⁰

As we saw above in Galileo's correspondence, "some Florentine friends," Clavius, and even Guidobaldo were not persuaded of the soundness of the proof.⁷¹ The Florentine friends were not satisfied with considering the magnitudes in two different ways on the same balance; according to his first letter, of 16 January 1588, however, Clavius claimed to have no problem with the reconsideration of weights, but only with the assertion that the centers of gravity of the weights considered in the two ways coincide:

this double way of considering the same magnitudes on different balances gave me no trouble, since Archimedes does almost the same thing in Proposition 6 of Book 1 of *On Plane Equilibrium*; but when, on balance *ad*, the greatest [magnitude] is hanging at *d* and the least at *a*, you now suppose that the same point *x* would be the point of equilibrium of the whole, as though the same

⁶⁸ "Omnis conoidis hyperbolici centrum gravitatis est punctum illud, in quo duodecima pars axis ordine quarta ab ea, quae basim attingit, sic dividitur, ut pars basi propinquior sit ad reliquam, ut sesquialtera transversi lateris hyperboles, quae conoides describit ad axim conoidis." (Valerio 1604, II.43, 78). We also point out that in Archimedes' *Method*, rediscovered only at the beginning of the twentieth century, in proposition 11, without proof, it is stated: "centrum gravitatis conoidis obtusianguli in axe positum esse ita secto, ut pars ad verticem posita ad reliquam eam rationem habeat, quam habeat triplum axis octuplumque rectae ad axem adiectae ad axem ipsius conoidis quadruplumque ipsius rectae ad eum adiectae" (Archimedes 1910-1915, 484-485). Galileo's statement is wrong but can be corrected by replacing "*adiecta ad axem*" with "*4/3 of adiecta ad axem*." For a reconstruction of Archimedes' proof see Hayashi (1994).

⁶⁹ Note that this is not what Archimedes did in *On the Equilibrium of Planes* I.6-7: there he actually did divide and redistribute the weights. Galileo has not done this; he has merely considered the same weights in the same places in two different ways, and this is what Clavius could not see, perhaps because he thought that Galileo was doing what Archimedes had done.

⁷⁰ EN, I, 187-188; for a brief discussion see Drake (1978, 13-14).

⁷¹ It is evident that what we read now is a text published some 40 years later, and so it is not known what Clavius and the others would have read, although some passages of the proof are cited in the letters exchanged on this theme.

point x is placed the point of equilibrium when the greatest [magnitude] hangs at a and the least at b on balance ab . This seems to be what was sought to be proved, in other words it seems to me that it begs the question. If you posit that point x is the point of equilibrium of the arm ad , as it is of the arm ab , it seems to me, according to my small judgement (being now so remote from these speculations), that your proof proceeds well.⁷²

Galileo tried to persuade Clavius that x , the center of gravity of the balance ab , is also the center of the balance ad :

If we stipulate that the equilibrium [point] of the composite of all the magnitudes is x when the component parts are f, g, h, k , and n , the point of equilibrium of the same composite will still be the same point x , even though I consider it to be composed of parts n, o, r, s , and t , since the point of equilibrium is one and the same of the composite and its component parts; they do not change place or size by considering them in different ways. But perhaps the diagram that I have attached will better explain my meaning, in which (and so serves my purpose) I have shown the magnitudes conjoined.⁷³

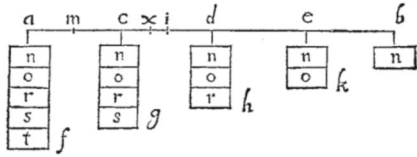
Galileo redrew the diagram with the “grandezze congiunte,” i.e., so that each new magnitude is now made up by small blocks joined together and not by separated columns of blocks as in the first diagram (see Fig. 1). It was an attempt to persuade Clavius of the soundness of his proof, removing the doubt caused by reconsidering the magnitudes. In the second diagram there is nothing to rearrange, the magnitudes remain the same and in the same places, only the points of suspension have changed.⁷⁴ But Galileo, despite the long explanation and the new diagram, was not successful in persuading Clavius, who replied soon after (on 5 March) reasserting his doubts. In Clavius’s last letter there are two crucial passages (EN, X, 29):

- “mi pare che egeat demonstratione che ’l punto x resti il punto del equilibrio nella libra ad ” (it seems to me that it needs be proved that the point x remains the point of equilibrium in the balance ad);

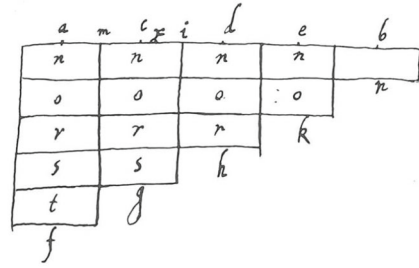
⁷² “Non mi dà fastidio quel doppio modo di considerare le medesime grandezze in diverse bilancie, perché Archimede fa quasi il medesimo nella prop. 6 del lib. 1 *De aequiponderantibus*; ma quando, nella libra ad , nel d pende la massima et nel a la minima, suppone V. S. che al hora il medesimo punto x sia il punto dell’equilibrio di tutte, sì come il medesimo x si pone il punto dell’equilibrio quando la massima pende nel a et la minima nel b , nella libra ab ; il che pare che ricerca d’essere dimostrato, altrimenti mi pare quod petitur principium. Se costasse che ’l punto x fosse il punto dell’equilibrio della libra ad , sì come gl’è nella libra ab , mi pare secondo il mio poco giuditio (stando adesso così remoto di queste speculationi), che la sua dimostrazione proceda bene.” (Clavius to Galileo, 16 January 1588, EN, X, 24).

⁷³ “Se noi diamo che del composto di tutte le grandezze l’equilibrio sia x quando le parti componenti sono f, g, h, k, n , del medesimo composto sarà ancora il punto dell’equilibrio il medesimo x , con tutto che io lo consideri esser composto dalle parti n, o, r, s, t , atteso che *del medesimo composto uno è il punto dell’equilibrio* [our italics] et le sue parti componenti per il diverso modo di considerarle non variano sito o grandezza. Ma forse meglio dichiarerò l’intenzione mia la figura che con questa gli mando, nella quale (e tanto serve al mio bisogno) pongo le grandezze congiunte.” (Galileo to Clavius, 25 February 1588, EN, X, 27–28).

⁷⁴ Although not essential to our argument, the diagrams for the following lemmas on the weights in balances (used in the proof for the center of gravity of the whole cone) have the weights disposed like the new diagram provided in the letter by Galileo. See Fig. 2.

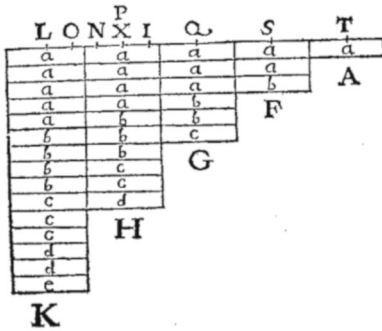


(a) Leiden, p. 291

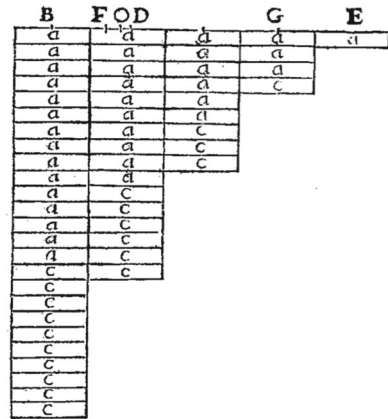


(b) Letter 25/02/1588

Fig. 1 The diagrams of the first “balance lemma”



(a) Leiden, p. 303



(b) Leiden, p. 305 (wrongly 297)

Fig. 2 The diagrams of the other “balance lemmas”

– “ma dirà uno che ’l detto punto nella libra *ad* sarà un altro diverso dal *x*” (but one could say that the point in the balance *ad* would be another different from *x*).

Clavius remained convinced that it must prove explicitly that *x* remains the center of gravity. But, as Galileo tried to explain, the center of gravity remains the same because the weights had not changed their size or position: the composite body of magnitudes remained exactly the same and so therefore does the point of equilibrium.

Guidobaldo had the same difficulty, as expressed in his letter of 28 May 1588: “Perhaps one could say that the balance *ad* will be divided not at *x* but at another point.”⁷⁵ But in the following letter (17 June), Guidobaldo said that he now understood correctly Galileo’s idea: the center of gravity is *x* for both considerations and so it divides the two beams in the same ratio.

it occurred to me a few days later where I had gone wrong. Since the first proof was very concise, it seemed to me that *bx* to *xa* should have had the same ratio

⁷⁵ “Si potrebbe forse dire che la libra *ad* sarà divisa non in *x*, ma in un altro punto” (Guidobaldo to Galileo, 20 May 1588, EN, X, 34).

as ax to xd , it followed that x was the center of gravity ...But it is quite the opposite, since x is the center of gravity, it follows that as bx is to xa so ax is to xd . And so it seems to me that the proof is perfectly sound, based on that assumption, which could perhaps be easily proved.⁷⁶

He persuaded himself of the soundness of the proof (“Io mi accorsi dove avevo pigliato errore”) considering two alternative lines of argument:

1. the beams are divided in the same ratio, therefore the centers of gravity are the same point (“avendo havere la medesima proportione bx a xa come ax a xd , che di qui ne seguitasse poi che x fusse il centro della gravità”);
2. the centers of gravity are at the same point, therefore the beams are divided in the same ratio (“essendo x il centro delle gravità, ne séguita che bx a xa sia come ax a xd ”).

Apparently he had first thought that Galileo had argued the first, but then he realized (“ma è al contrario”) that Galileo had meant the second. Assuming that the centers of gravity are the same point, he said that the argument was sound (“sì che a me pare che la dimostrazione stia benissimo, fondata in quella suppositione”), where “quella suppositione” refers to the coincidence of the centers of gravity of the two balances. This supposition he then suggested could be easily proved (“la quale si potrebbe forse provare con poca cosa”).⁷⁷

4.5 Dating the composition of *Theoremata*

In light of this reconstruction, Galileo first found the proof for the paraboloid frustum,⁷⁸ then that for the conical frustum and only afterwards did he pass on to the whole paraboloid.

⁷⁶ “di li a due giorni io mi accorsi dove havevo pigliato errore. Perché nella prima dimostrazione per esser assai succinta, mi parve che havendo havere la medesima proportione bx a xa come ax a xd , che di qui ne seguitasse poi che x fusse il centro della gravità ...ma è al contrario, che essendo x il centro delle gravità, ne séguita che bx a xa sia come ax a xd , sì che a me pare che la dimostrazione stia benissimo, fondata in quella suppositione, la quale si potrebbe forse dimostrare con poca cosa.” (Guidobaldo to Galileo, 17 June 1588, EN, X, 35).

⁷⁷ Note that later, in the *Theoremata*, Galileo introduced another lemma on the balance—with a different disposition of weights—necessary for the proof of the center of gravity of the cone. In the proof of this lemma, there is a clear explanation of the reason for the coincidence of the centers of gravity: “But there is [only] one center of the composites of the said magnitudes.” (Unum est autem centrum compositae ex dictis magnitudinibus) (EN, I, 199), trans. Drake (1989, 273). It is not known whether Clavius or Guidobaldo had seen this part of the *Theoremata* yet, or whether Galileo added this explanation after the controversy over the first lemma.

⁷⁸ In a letter dated 31 March 1588, Michel Coignet thanked Galileo for the proof he had sent to Abraham Ortelius, who then passed it on to him, regarding the paraboloid frustum. Here Galileo was also seeking the approval of eminent mathematicians. Galileo’s proof in the Leiden version presents a synthetic procedure. Coignet retraced Galileo’s demonstration backwards, through a process of analysis. In the letter Coignet emphasized that the strength of the Galilean proof was the simplicity with which it is possible to find the center of gravity of the frustum: “Sed centrum hoc multo facilius nova tua inventione investigare doces”. In his proof Coignet recalled the Galilean lemma, highlighting its key role in the demonstrative process: “et dicis, adminiculo tui lemmatis, quod centrum gravitatis dati frusti erit inter l et o .” This confirms that Coignet had at hand the lemma as well as the paraboloid frustum theorem (Michel Coignet to Galileo, 31 March 1588, EN, X, 31–33).

Thanks to the correspondence with Clavius, we can assume that before March 1588 Galileo did not have a valid proof for the whole paraboloid, because of the doubtful lemma. Evidence coming from the correspondence with Guidobaldo push this date further. Only in July 1588 did Galileo send the proof for the paraboloid to his mentor, leaving us to suppose that before then and until the required lemma had been approved, he considered the theorem unsatisfactory.

A final element that concurs with the other evidence for the order of composition is the complexity of the mathematical content of these theorems. The proof for the paraboloid frustum is a simple application of the Archimedean theorem on the trapezium from *On the Equilibrium of Planes*, whereas the proof for the conical frustum is more complex. Galileo passed from an application in a simple case to a new proof that drew on the previous one for its inspiration.

The same holds for the proofs of the paraboloid and of the cone. Galileo devised the lemma relating to the paraboloid because, like Maurolico years earlier, he had in mind the Archimedean theorem of the center of gravity of the triangle and found a possible extension of it to the case of the paraboloid. For the cone the proof arose from noticing a similarity with the Archimedean demonstration of the center of gravity of the parabola.

From the documents discussed, and taking into account the increasing mathematical complexity of the theorems, the order of discovery would have been as follows

- paraboloid frustum (second half of 1587, ready when Galileo went to Rome);
- conical frustum (December 1587, from the certificate of authenticity in the Ambrosiana copy);
- paraboloid (July 1588, from Galileo's letter to Guidobaldo);
- cone (post July 1588);
- hyperboloid (post July 1588).

The study of the centers of gravity of solids and the composition of the *Theoremata* in a form he considered acceptable can therefore be traced back to the years 1587 and 1588. In fact, it was only at the end of 1588 that Galileo showed any intention to publish these works, an intention encouraged by Guidobaldo:

I heard also to my greatest satisfaction that you wish to publish your work on centers of gravity, which truly will bring you much honor.⁷⁹

Even when Luca Valerio's *De centro gravitatis solidorum libri tres* was published in 1604, Galileo did not abandon his intention to print his early theorems. In 1607 Castelli wrote to Galileo: "I shall be awaiting the publication of your treatise On the Centers of Gravity of Solids."⁸⁰ Some years later Galileo attempted to publish the *Theoremata* in the Lincean Academy with the intervention of Prince Federico Cesi. In a letter to Cesi (5 January 1613), Galileo himself, through Cesi, asked Valerio to approve the printing of his works. This request was at least curious: Galileo was aware of a conflict with the

⁷⁹ "Ho anche con grandissima mia satisfatione sentito ch'ella vogli mandar fuori le sue cose del centro della gravezza, che in verità V. S. ne acquistarà molto honore" (Galileo to Guidobaldo, 30 December 1588, EN, X, 39).

⁸⁰ "Starò aspettando in luce il trattato suo *De centro gravitatis solidorum*" (Benedetto Castelli to Galileo, 1 April 1607, EN, X, 170).

publication of Valerio but still expressed the desire to publish these works: “it would not seem good for me to throw away the not little effort that I have already made.”⁸¹ The project however was not pursued. Even if Cesi and Valerio seemed inclined to accept Galileo’s request, nevertheless the adverb *risolutamente* (resolutely) hinted at Valerio’s intention to stall.⁸² Not until fifty years after their elaboration in the last phase of Galileo’s scientific career, would these theorems finally see the light in the form we read today.

The correspondence with Guidobaldo provides evidence for dating not only the composition of the *Theoremata* but also the UCLA manuscript. There are two letters (16 November and 7 October 1588) in which Guidobaldo said he had encountered the problem of the three circles.⁸³ It was a problem proposed by Pappus in his *Mathematical Collection* and that had re-emerged thanks to Commandino’s Latin translation (Pappus (1588)).⁸⁴ The solution was extensively treated in Guidobaldo’s *Meditatiunculae* with the title *Problema a Comandinum propositum ad Pappum pertinens* dating back to 1588.⁸⁵ In the UCLA manuscript this question is mentioned twice: the first time, in the index of *Meditatiunculae* on folio 86r, the second time, on folio 90r, where a solution is proposed. Considering the dating of the *Theoremata* and of the three circles problem, the UCLA manuscript can be dated to the second half of the 1580s, at least for these texts.

5 The evolution of Galileo’s studies

The reconstruction we made confirms that between 1587 and 1588 Galileo was looking for approval for his new role as a mathematician. It is no coincidence that he elaborated and wrote the *Theoremata* to show his competence. In fact, although the results were not new, the proofs exhibit a full understanding of the Archimedean deductive system

⁸¹ “Quando habbia parlato al Signor Luca di quel particolare, sentirò volentieri la sua resolutione, perchè in effetto non par bene che io butti via una fatica non piccola già fatta: et il Signor Salviati, che ultimamente l’ha veduta, non vuol per niente che la resti morta. Ma spero che il Signor Luca non doverà ricusar ciò, perchè, a mio potere, tenderà più alla sua gloria che alla mia; nè io mi asterrò di celebrarlo, e di conceder la preminenza alle sue veramente divine inventioni; le quali sicome mi concitorono a bramar la sua amicitia, così mi faranno vivergli sempre servitore, et ammiratore del suo felicissimo ingegno” (Galileo to Federico Cesi, 5 January 1613, EN, XI, 460). On scientific and personal relationships between Galileo and Luca Valerio, see Napolitani (1987).

⁸² “Col S.r Luca parlai già di quel particolare, e se ne mostrò sodisfattissimo: glie lo dirò di nuovo risolutamente.” (Federico Cesi to Galileo, 11 January 1613, EN, XI, 463). In a later letter Valerio seemed to want to delay again, saying that the Lincean Academy would like to print *new things*. “Ciò dico, perciò che il S^r. Velsero ha scritto al S^r. Principe pregandolo a far che si stampino qualch’altre cose nuove de’ Lincei; et per ciò penso di dar in luce li detti tre trattati, dovendo poi dare appresso, se Dio vorà, il libro De centro gravitatis solidorum, migliorato et accresciuto in guisa, che forse V. S. n’havrà diletto. All’opera De pyramide spesso ritorno. V. S. mi facci gratia d’avisarmi s’ell’ha mai ritrovata la dimostrazione del centro della gravità del conoide hiperbolico per la via d’Archimede; cosa nel vero anch’essa difficile per la potenza dell’applicate, composta di sì tra di loro diverse altre potenze” (Valerio to Galileo, 31 August 1613, EN, XI, 559).

⁸³ “problema delli tre circoli” (Guidobaldo to Galileo, 16 November and 7 October 1588, EN, X, 37–38).

⁸⁴ This edition was published posthumously in Pesaro 1588 and seen through the press by Guidobaldo himself.

⁸⁵ *Meditatiunculae*, 37–38bis; ed. Tassora (2001, 275–279); see Tassora (2001, 32–35 and 53–72).

in general, and specifically of the contents of *On the Equilibrium of Planes*. In these theorems Galileo applied Archimedean techniques to more complex cases, obtaining new proofs based on the use of geometric properties. If we exclude Maurolico's results, knowledge of which was confined to Jesuit circles, the only work that circulated freely was Commandino's, which however had some demonstrative flaws. Galileo's demonstrations thus allowed him to find interlocutors to establish a scientific exchange. Clavius replied with ambiguity and little enthusiasm; Guidobaldo in contrast showed a great interest in Galileo's skills and established a lasting collaboration with him.

This collaboration is illustrated in the theorem on the conical frustum, where Guidobaldo's transcription in **U** shows some interesting differences and novelties compared to the version preserved in **A**.

5.1 The lemma for the conical frustum

The lemma on proportional means that accompanies the theorem on the conical frustum presents only negligible differences between the three versions; but in the UCLA manuscript after the end of the proof, Galileo recalled the result for the center of gravity of the whole cone:

That the center of gravity of any cone and of any pyramid divides the axis so that the part at the vertex is triple the remainder was proven by Commandino and in a different way by me.⁸⁶

This passage is significant for two reasons: first because it is absent from **L** and **A**, and secondly because it provides information on the nature of the note transcribed by Guidobaldo. The proof of the center of gravity of the conical frustum is based on two results: the lemma proved specifically for this purpose and the theorem on the whole cone. This last result is recalled explicitly as necessary for the main theorem. The reason for the absence of this passage in **L** is obvious, for the proof of the whole cone is found just before the lemma. Its absence from **A** is strange but it can be explained: first, the result of the whole cone was already known from Commandino's works, and secondly the Ambrosiana copy was intended only to display Galileo's mathematical competence, and only a complex theorem could do this.⁸⁷

The passage cited above, then, seems to be of a confidential nature, as though coming from private correspondence, both for its position and for the difference in tone from what comes before and after. Its brevity, well expressed by the adverb *aliter*, refers to something different from what is presented here or from what is known and is in contrast to the rigorous arguments of the lemma and the theorem. The quick reference to Commandino and the expression *a me* recall something already known or leave it pending, with the desire to investigate the matter later.

⁸⁶ "Cuiuslibet cone et cuiuslibet pyramidis centrum gravitatis axem dividere ut pars ad verticem reliquae sit tripla, a Comandino et aliter a me demonstratum est" (Los Angeles, Charles E. Young Research Library, University of California-Los Angeles, MS 170/624, 75r).

⁸⁷ See the attestations of authenticity by Florentine gentlemen and the assessment by Moleti in the Ambrosiana copy discussed above, Sect. 4.1.

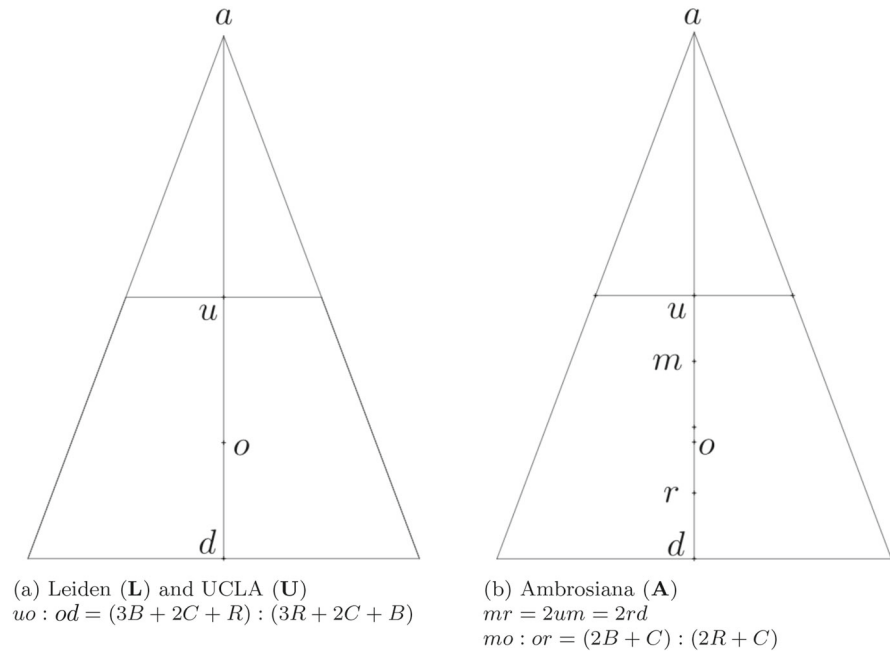


Fig. 3 The center of gravity of the conical frustum

According to Galileo and his colleagues, Commandino's *De centro gravitatis solidorum* had some flaws, in particular in the proofs for the frusta. Therefore the theorem concerning the center of gravity of the whole cone, which was considered acceptable, could be neglected for the moment and taken up later.

5.2 The theorem on the conical frustum

In the enunciation of the theorem there are some differences that put **U** closer to **L** than to **A** (Fig. 3).

In **U**, the position of the center of gravity is expressed by the ratio between the part of the axis towards the smaller base and the remainder. In **U** and **L** the ratio is referred to the whole axis, whereas in **A** the ratio is given for the axis divided into four portions.

U: Cuiuslibet frusti pyramidis seu conii, plano basi aequidistante abscissi, centrum gravitatis in axe consistit, eumque ita dividit ut pars versus minorem basem ad reliquam sit ut triplum maioris basis, cum duplo spaciae medii proportionalis inter basem maiorem et basem minorem, et basi minori ad triplum basis minoris cum duplo eiusdem spaciae medii et basi maiori.

A: Cuiuscunque frusti pyramidis seu conii, plano basi aequidistante abscissi, centrum gravitatis in axe consistit, ita ut prius ab eo utrinque quarta sui parte dempta, centrum gravitatis in reliqua consistit; eaque sic dividit, ut pars versus minorem basem ad reliquam eandem habeat rationem, quam spaciae quod

basium sit medium proportionale cum duplo maioris basis habet ad idem spacium inter bases proportionale cum duplo minoris basis [our italics].

If we designate (following Galileo's notation in **U**) the smaller base with R , the greater with B and the proportional mean between the two bases with C , then in **L** and **U** the center of gravity o of \mathcal{F} frustum will divide the ud axis in a such way that

$$uo : od = (3B + 2C + R) : (3R + 2C + B)$$

On the other hand, in **A**, the axis of the frustum ud is divided into four parts, and the center of gravity o will be on the two middle quarters mr according to the ratio:

$$mo : or = (2B + C) : (2R + C)$$

Consistent with the enunciation, the settings out and the first part of the proofs also show these differences.⁸⁸ Before going into some details about the proofs, we should note that **U** has two passages not present in the other witnesses. The first is found in the setting out:

Frusti autem maior basis aequalis sit spacio B , minor vero R . Spaciorum autem $B R$ medium esto proportionale C .

The letters $B R C$ are introduced both into the text and into the diagram to designate areas equal to the bases (see Fig. 4). In **L** and **A** periphrases are used to designate these areas.⁸⁹

The second passage unique to **U** is another linking sentence, right at the end of the determination and before the beginning of the actual proof:

But it is obvious that the center of gravity of the frustum lies on the axis, since the centers of gravity of the whole cone or pyramid, and of the abscissa of the cone or pyramid, lie on the same axis.⁹⁰

This comment, which is underlined in the manuscript, was probably an addition by Guidobaldo himself and does not make a substantial contribution to the argument of the theorem.

Once the setting out is finished, the proofs in the three witnesses proceed in the same manner. Four auxiliary lines in continuous proportion are introduced so that a specific lemma can be applied.⁹¹ It is then possible to prove that point n is the center

⁸⁸ For details, see the critical apparatus of the edition below.

⁸⁹ For example, in **L** the ratio is expressed as “tripla maximae basis cum dupla mediae et minima ad triplam minimae cum dupla mediae et maxima” whereas in **U** as “tres B cum duobus C et unum R ad tres R cum duobus C et unum B ”.

⁹⁰ “Quod autem centrum gravitatis frusti in axe consistat manifestum est cum totius conii vel pyramidis, et conii vel pyramidis abscissae centra gravitatum in eodem axe consistent”.

⁹¹ The auxiliary lines hx kx xl and xs (where hx is equal to the axis of the cone C , kx is equal to the axis of the frustum \mathcal{F} , and xl and xs are third and fourth proportional lines) are introduced in order to express the key ratio $(3B + 2C + R) : (3R + 2C + B) = (3hx + 2kx + xl) : (3xl + 2kx + hx) = uo : od$, from which it follows that $od : du = (hx + 2xk + 3xl) : 4(hx + xk + xl)$ in **L** and **U**; and $(2B + C) : (2R + C) = (2hx + xk) : (2xl + xk) = mo : or$ in **A**.

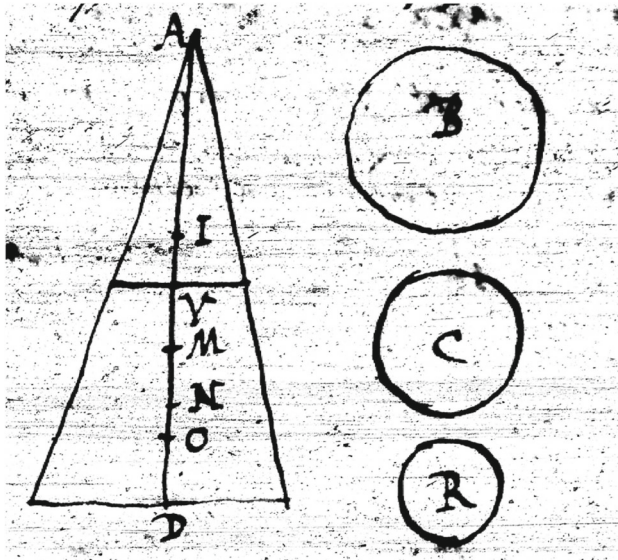


Fig. 4 The diagram in the UCLA manuscript U

of gravity of the whole cone C .⁹² Next point i is found as the center of gravity of the smaller cone \mathcal{K} , the difference between C and \mathcal{F} (see Fig. 5).

The proof is thus reduced to proving that $in : no = \mathcal{F} : \mathcal{K}$.⁹³

This proportion is obtained by the application of the law of the lever (*On the Equilibrium of Planes*, I.8) to the cones C and \mathcal{K} .⁹⁴ In fact, given the whole cone C with center of gravity n , and the cone part of it \mathcal{K} with center of gravity i , the center of gravity of their difference (frustum \mathcal{F}) will be a point o on in such that $no : in = \mathcal{K} : \mathcal{F}$.

L and U thus offer the same mathematical point of view, with two differences: in the manuscript

1. the whole discourse seems to be more amalgamated as can be seen from the connecting sentences between the parts;⁹⁵ and

⁹² The point n is placed such that $hs : sx = md : no$.

⁹³ By following some implicit steps, the proportion $hs : xs = \mathcal{F} : \mathcal{K}$ is obtained; by construction $hs : sx = md : no$ and $md = \frac{3}{4} du = in$ from which the theorem is proved.

⁹⁴ This proposition was used by Archimedes to find the center of gravity of the trapezium (I.15) and of the mixtilinear trapezium (the portion of a parabola, II.10). In the first case the trapezium is seen as the difference between two triangles; in the second case the portion of a parabola as the difference between two parabolas. In the *Theoremata*, Galileo imitated Archimedes: the frustum of a paraboloid is seen as the difference between two paraboloids and the frustum of a cone as the difference between two cones; having found the centers of gravity of the whole solids, he could apply the Archimedean proposition I.8 to find the center of gravity of the frustra.

⁹⁵ For instance: “Cuiuslibet coni ... a Comandino et aliter a me demonstratum est.”; “Quod autem centrum gravitatis frusti in axe consistat, manifestum est”; “sunt enim $hx \ xk \ xl$ continue proportionales”.

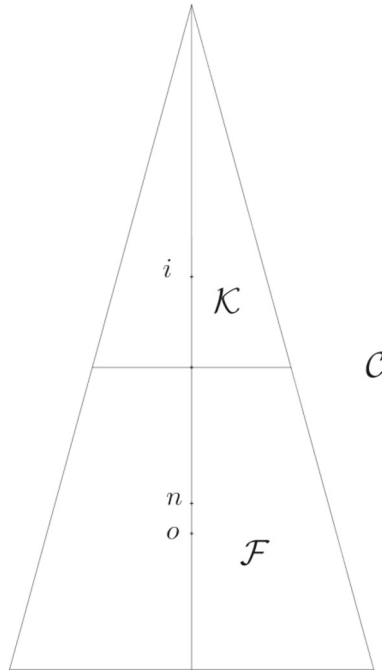


Fig. 5 The proof of the center of gravity of the frustum.

2. there is an attempt to simplify some periphrases.⁹⁶

A in our opinion is the oldest version and presents a more Archimedean determination. Archimedes (in *On Equilibrium of Planes* II.10) divided the axis of the mixtilinear trapezium into five equal parts, and showed that the center of gravity lies in the middle part and divides it in a certain ratio.⁹⁷ Similarly, Galileo divided the axis of the conical frustum into four equal parts and proved that the center of gravity lies in the two middle parts and divides them in a certain ratio.

5.3 Reasons for a change?

Why this difference of style between the various versions of the conical frustum?

Actually, a similar change occurs in the theorem on the center of gravity of the frustum of a paraboloid in **L**.

In this case, Galileo also expounded the proposition by taking as his model the Archimedean formulation of the mixtilinear trapezium (*On the Equilibrium of Planes*, II.10): he divided the axis into three equal parts and proved that the center of grav-

⁹⁶ For instance: “Frusti autem maior basis aequalis sit spacio B , minor vero R ”; “Frusti autem *du* axis dividatur in o , ita ut pars ... eandem habeat rationem quam tres B cum duobus C et unum R ad tres R cum duobus C et unum B .”; “per lemma praecedens”.

⁹⁷ “Centrum gravitatis est in linea recta, quae frusti existit diametros, qua in quinque partes aequas divisa, centrum in quinta eius media existit, atque in eo eius puncto quo ipsa quinta sic dividitur, ut portio eius propinquier minori basi frusti ad reliquam eius portionem eam habet proportionem quam habet solidum” (Archimedes 1544, 140–141).

ity lies in the middle part and divides it according to a certain ratio.⁹⁸ The proof, however, closely follows Archimedes' proof of the center of gravity of the trapezium, which presents an enunciation similar to that of the Leiden version and of the UCLA manuscript described above.⁹⁹ Galileo, aware of the demonstrative analogies between the frustum of a paraboloid and the trapezium, realized this stylistic difference and at the end of the demonstration added the following note:

Therefore it is clear that the center of gravity of frustum *ULMC* is point *I*, and the axis is so divided [by it] that the part toward the smaller base is to the part toward the larger base as double the larger base plus the smaller is to double the smaller plus the larger. *Which is the proposition, but more elegantly expressed [out italics].*¹⁰⁰

He considered an enunciation nearer to the Archimedean style “more elegantly expressed” (*elegantius*). The same choice was repeated for the conical frustum in **U**: the first “less elegant” statement survived only in **A**, the oldest version.

By putting together what emerges from the dating and from the mathematical analysis, it is possible to conclude that Galileo proved the theorems at different times and in a different order from the one he used when he belatedly published the work. The oldest text, unrevised stylistically, is preserved in **A**, whereas **U** takes an intermediate position between **A** and **L**.

The link between the young Galileo and Guidobaldo was not hierarchical but one of equality: Guidobaldo deeply esteemed the Galileo's results on centers of gravity, deeming them worthy of publication. This emerges both from the UCLA manuscript, which preserves Guidobaldo's copy of the notes sent to him by Galileo, and from their correspondence. In his letters, Guidobaldo constantly encouraged Galileo to continue his mathematical studies, always waiting for new findings.¹⁰¹

Galileo's studies of the centers of gravity were part of a research field in which the most important mathematicians of the time had already expressed themselves, achieving considerable results. Even if Galileo's proofs fit into this tradition, however, they clearly revealed his affinity with the Archimedean way of thinking. Galileo not only showed he had read and understood *On the Equilibrium of Planes* but he also adapted its Archimedean proofs to more complex cases. Galileo was faithful to his model in the individual arguments, in the presentation of the results obtained and even in style.

⁹⁸ “centrum gravitatis est in linea recta quae frusti est axis; qua in tres partes divisa, centrum gravitatis in media existit, eamque sic dividit, ut pars versus minorem basim ad partem versus maiorem basim, eandem habeat rationem quam maior basis ad basim minorem” (*Theoremata*, EN, I, 196, lines 26–30).

⁹⁹ “Sic divisa sit, ut pars eius terminata ad minus laterum aequedistantium in duo aequa sectorum ad reliquam partem eam habeat proportionem quam habet utraque simul duplum maioris aequidistantium cum minore ad duplum minoris cum maiore” (Archimedes 1544, 132).

¹⁰⁰ “Constat igitur, frusti *ulmc* gravitatis centrum esse punctum *i* et axem ita dividere ut pars versus minorem basin ad partem versus maiorem sit ut dupla maioris basis una cum minor ad duplam minoris una cum maiori. Quod est propositum elegantius explicatum” (EN, I, 198, lines 2–6; trans. Drake 1989, 272).

¹⁰¹ On the link between Galileo and Guidobaldo, see Frank and Napolitani (2015).

6 Some conclusions

As we have seen, Galileo began to deal with centers of gravity of solids to show his mathematical competence. He came into a research field in which the results achieved by his contemporary colleagues were not fully satisfactory.

The *Theoremata* does not seem to have been born in the same order as the Leiden printing, and their composition responded to different needs: Galileo first wanted to enter into the field of mathematical research, accepting the challenge thrown by Commandino; besides this, he aspired to become a mathematician, showing that he knew how to extend Archimedean ideas to more complex cases. Thus, at the beginning, not fully satisfied with the treatment by his contemporaries, he solved the theorems on the center of gravity of the frusta, and only afterwards did he go on to the centers of the whole solids, proposing a new approach. The idea of publishing these theorems dates to the end of 1588, when he believed he had produced an acceptable work.

The comparison between the three versions affords a glimpse into some of Galileo's traits as a young mathematician. He was a careful and deep expert in Archimedes' mathematics. He was aware that the ideas and principles of Archimedes' work could be successfully applied to solids, and that it was only necessary to change the lemmas accordingly. Galileo was following in the footsteps of Archimedes who did the same when he built the proof for the portion of a parabola on that for the trapezium.

Finally, the Ambrosiana copy, the oldest of the three extant copies, shows stylistic differences from both the UCLA manuscript and the Leiden printing. It seems from this that Galileo wanted to follow Archimedes not only in mathematics but also in style, using in his own proofs the same elegant stylistic features that he had found in Archimedes.

7 Appendix: The text of the UCLA manuscript

7.1 Description of the codex

Manuscript U (Los Angeles, Charles E. Young Research Library, University of California-Los Angeles, MS 170/624), is divided into three parts. For our purpose only the second part, mainly written by Guidobaldo, is relevant.¹⁰²

This section of the manuscript is made up of sheets of different kinds of paper with different dimensions, written in different hands and inks, and on various topics. It was assembled from miscellaneous materials written at various times.

It consists of 21 folios, numbered in the upper right corner 75–91: four folios are numbered 83–1^o, 83–2^o, 83–3^o, 83–4^o, the last folio (92) is not numbered.¹⁰³ The

¹⁰² See Sect. 1.2 for a brief description of the contents of the other parts.

¹⁰³ There are three folios (83–1^o, 89 and 91) of smaller size and three leaves attached: 83–2^o, 83–4^o and one leaf attached to folio 85r. It is impossible to be more precise since we could not examine the manuscript itself but only photographic reproductions. For the same reason it was impossible to determine the gatherings, the watermarks and the ink changes. Folios 76v, 79v and 90v are blank, whereas folios 80, 81, 87 and 88 are bound upside down. Probably because of the upside-down binding, folios 80 and 81 are numbered on the lower left corner.

text is written mostly in Latin, although folios 79–82, 85 and 91 are in Italian.¹⁰⁴ There are at least four hands: Guidobaldo wrote most of the folios, except folios 79, 83–3^o, 89 and 91v. The manuscript is not dated, but folio 83 contains astronomical data referring to October and November 1580.¹⁰⁵

On folio 84r there is a reference to a diagram (*ut in secunda figura*) on folio 82r. For this reason, it is probable that folios 82 and 84 form a bifolio.¹⁰⁶

On the basis of content, paper dimensions, ink and internal references it is possible to guess that folios 75–76, 77–78, 80–81, 82–84 and 87–88 also form bifolios.

7.2 Content

- | | |
|--------------------------------------|--|
| ff. 75–76 | Galileo's proof for the conical frustum with two geometrical diagrams (76v blank); |
| ff. 77–78, 83–3 ^o | On 77r an extract from <i>De limitibus constituendis</i> by Hyginus: “Verba Hygeni de limitibus constit. Pag. 117.” Quotation (and diagram) copied from pages 117–118 of Hyginus (1554). On 83–3 ^o the same text as 77r is repeated in a different hand. Folio 83–3 ^o v, however, has a passage not present on 77r: “Eisdem ...convenit,” with another geometrical diagram from the same edition (page 118, lines 13–16); 77v and 78 contain notes and diagrams by Guidobaldo on Hyginus: “Haec Hygeni verba una cum figura hoc modo corrigi posse videntur” (77v); “Huius constructionis demonstratio” (78r) and “Praxis” (78v). ¹⁰⁷ |
| f. 79, 80–81 | Two problems “Proposti dal S. Marchese di Carrara”; solutions to the problems are proposed on 80–81. ¹⁰⁸ |
| f. 82r | “Sia il cono scaleno <i>abc</i> ...” two geometrical diagrams |
| f. 83 1 ^o –2 ^o | “Multi spernunt ...,” a theoretical text on mathematics. |
| f. 83–4 ^o | Astronomical data for a comet dated 1580 at the top of the page. The observations are on October 10th, October 16th and November 20th. |
| f. 84r | “Propositio 8. Problema. Duabus datis rectis lineis in eodem plano non existentibus lineam ambobi perpendicularem ducere.” This folio has no mathematical diagram but instead notes: “ut in secunda figura,” in reference to the second figure on 82r. |

¹⁰⁴ Folios 86 and 89 are exceptions since they contain two indexes of materials related to the *Meditatiunculae in rebus mathematicis*, they are therefore partly in Italian and partly in Latin according to the language of the items listed. The last leaf (92) contains only four diagrams and no text.

¹⁰⁵ Some pages of this manuscript have already been published: folio 79r was published in Frank (2011), 121–122; folios 86r, 89 and 90r in Tassora (2001), 221–224.

¹⁰⁶ In addition, the pages numbered 83–1^o to 83–4^o are different in dimensions, paper, and writing and were inserted later within the bifolio 82–84. This explains the numbering 83–1^o to 83–4^o.

¹⁰⁷ These folios probably contain “l’ espositione [on Hyginus’s meridian line] che, ad istanza del Sig. Gio. Vincenzo Pinelli da Padova, vi fece l’ illustrissimo Sig. Guidobaldo de’ Marchesi del Monte. See Sect. 1.3.

¹⁰⁸ Frank, who transcribed this text (Frank 2011, 121–122), dates this page to 1570, when Alderano Cybo (the “Marchese di Carrara”) and Guidobaldo attended Commandino’s lectures. The folios 80–81 are written in two columns and bound upside down; the columns are numbered at the top and the bottom. The ink is very dark with stains, and many passages are unreadable. See Frank and Napolitani (2015, 186).

- f. 85r Constructions of the three conic sections (with four diagrams) written in Italian. A leaf is attached to the folio, partially covering the text. At the end of the folio (apparently in different ink) there are two notes: “verticali perché tutti passano per il vertice che è il zenit” and “Del Galileo.”
- f. 86r A list in Guidobaldo’s hand of some topics treated in the *Meditatiunculae* (86v blank).¹⁰⁹
- ff. 87–88 Bound upside down, notes by Guidobaldo on some propositions from Valerio (1582). The text starts on 88v with proposition XXII and continues on 88r with proposition XXIII, and 87v with proposition XXIV (XXVIII, in the manuscript); and ends on 87r with proposition 8 (=VIII).
- f. 89 Another list of topics in the *Meditatiunculae* (not autograph).¹¹⁰
- f. 90r Pappus’s three circles problem “Tribus datis circulis inaequalis qui se non contingant ...circulum describere qui omnes contingat.”¹¹¹
- f. 91r “Sia A l’occhio, B il punto della distanza ...” No geometrical diagram, although two diagrams are cited in the text: “avvertendo che nella prima figura”; “sia nella seconda figura.”
- f. 91v At the bottom of the page, upside down, there is a short letter (six lines of text, in Italian) quite faded and almost unreadable; in this letter, the name Guidobaldo appears twice.
- f. 92* On the recto and the verso of this unnumbered leaf, two geometrical diagrams of an ellipse are found.

7.2.1 A possible dating for the manuscript

It is possible to conjecture that this part of the manuscript was written mainly around 1580–1590, in particular because

- folio 83 contains astronomical data of a comet recorded in October and November 1580;¹¹²
- folios 87–88 report some theorems from Valerio (1582);
- the folios on the center of gravity of the conical frustum are datable to 1587–1588, as we have seen; and

¹⁰⁹ The list is transcribed in Tassora (2001, 221–222); see also Tassora 2001, 27–28).

¹¹⁰ Frank (2011, 658), n. 4 attributes this page to Pier Matteo Giordani, a close friend and scientific collaborator of Guidobaldo’s.

¹¹¹ Guidobaldo also dealt with this problem also in the *Meditatiunculae* (Tassora 2001, 224; see also Tassora 2001, 53–71). The topic was also treated in two letters to Galileo (16 September 1588 and 7 October 1588, EN, X, 37), on which see Sect. 4.3.

¹¹² Tycho Brahe also reported this comet; see Dreyer (1890, 160): “On the 10th October 1580 Tycho found a comet in the constellation Pisces. It was observed at Hveen till the 25th November, and again after the perihelium passage on the morning of the 13th December.” See also Brahe (1925–1926, 13, 305–333). The observations of this comet in the period are discussed in detail by Green (2004), 197–200.

- the part of the *Meditatiunculae* that shows connections with this manuscript can be dated between 1586–87 and 1593.¹¹³

7.3 Editorial criteria

In the edition we have written out abbreviations and abridgements; for example *prima* for “p.a.,” *quarta* for “4.a.,” *dupla* for “2pla.,” and so on. Punctuation, capitalization and use of the letters “u” “v” and “i” “j” have been adjusted to modern usage. Concerning the mathematical lettering, we have adopted the style of Favaro’s *Edizione nazionale* and used small letters in italics, both in the text and in the diagrams.¹¹⁴ Reference to the pagination of the witnesses is in the margin of the page. The text is subdivided by paragraph numbers in smaller, boldface type.

We have not recorded in the apparatus the graphical variants and misprints that we report here: spacium **U** **A** spatium **L**; basem **U** **A** basin **L**. The copist of **A** sometimes writes either *aequidixtante* or *aequidistancte*, and *frustrum* (in one occurrence *frux-trum*). There are two misprints in mathematical lettering: *hxx* for *hx xk*; *abo* for “ab o,” and some others less important.

The UCLA manuscript has some erasures and imperfections; we have noted the most important ones, leaving out those that involve only one letter. For example, at the top of folio 76r there is, at least in our photographic reproduction, a patch that covers the space of a few words.

The diagrams are similar in the three witnesses and we have reproduced the diagrams in **U**. As noted above (see Sect. 5.2) **U** has three circles representing the areas of the greater base, the smaller base and their mean proportional.

7.4 Sources and internal references

In the UCLA manuscript there are three mathematical references:

1. *per conversam 24^{ae} quinti*;¹¹⁵
2. *A Comandino et aliter a me demonstratum est*;
3. *per Lemma praecedens*.

The first reference is to Euclid, *Elements*, book 5, proposition 24. The second is to Commandino’s theorem on the center of gravity of the cone, proposition 18 of his *De centro gravitatis solidorum* (Commandino 1565), folios 27v–30r. The third is an internal reference to the *Lemma* at the beginning of the text.

¹¹³ The dating of the *Meditatiunculae* is discussed in depth in Tassora (2001, 19–51). Recently Frank (2013b), proposed a different period of composition for the first part of the *Meditatiunculae*, dating it to the 1570s. Frank confirms, however, that the part of the UCLA manuscript connected with *Meditatiunculae* dates to the 1580s (p. 287) : “it seems plausible that, at the time when he [Guidobaldo] wrote about it to Galileo in September 1588, he did not refer to his earliest studies on the problem, as constituted by pp. 37–38–38bis of the *Meditatiunculae*, but to studies that had reached a certain degree of completion and mathematical depth: i.e., possibly those nowadays contained in UCLA ms 170/624 or even further elaborations” (287).

¹¹⁴ The UCLA manuscript has small letters in the text but capital letters in the diagrams.

¹¹⁵ See Euclid *Elements*, V.24, ed. Heiberg (Euclides 1883–1888, II (1884), 66–67).

In the Ambrosiana manuscript there are two other references in the margin:

1. “11 et 12.6”; and
2. “20.6 vel 2.12 vel 7 Conoid. Archim.”

The first two refer to Euclid, *Elemets*, VI.11 and VI.12, where the existence of the third and the fourth proportional is proved.¹¹⁶ The second three refer to Euclid, *Elements*, VI.20 and XII.2, and Archimedes, *On Conoids and Sphaeroids*, proposition 6.¹¹⁷

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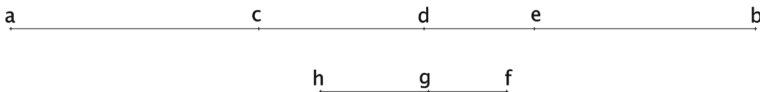
U	Los Angeles, Charles E. Young Research Library, University of California-Los Angeles, MS 170/624, folios 75–76
L	<i>Editio princeps</i> : Galileo Galilei, <i>Discorsi e dimostrazioni matematiche intorno a due nuove scienze ...con una Appendice del centro di gravità d'alcuni solidi</i> (Leiden, 1638), 289–314 (but numbered 306)
A	Milan, Biblioteca Ambrosiana, MS A 71 Inf, folios 95–96
Favaro del.	<i>Edizione Nazionale delle Opere di Galileo</i> delevit
om.	omisit
post corr.	post corectionem
add.	addidit
in marg.	in margine

¹¹⁶ Euclid, *Elements*, VI.11 and VI.12, ed. Heiberg (Euclides 1883-1888, II (1884), 106–110).

¹¹⁷ Euclid, *Elements*, VI.20, ed. Heiberg (Euclides 1883-1888, II (1884), 130–131); *Elements*, XII.2, ed. Heiberg (Euclides 1883-1888, IV (1885), 140–141); and *On Conoids and Sphaeroids* proposition 7, Archimedes (1544), 66; in Heiberg's edition this is proposition 6 (Archimedes 1910-1915, I, 282–285).

Lemma. Galileus¹¹⁸

Si fuerint quatuor lineae continue,¹¹⁹ proportionales et quam rationem habet minima earum ad excessum quo maxima minimam superat, eandem habuerit linea quaedam sumpta ad $\frac{3}{4}$ excessus quo maxima secundam superat, quam autem rationem habet linea his aequalis maximae, duplae¹²⁰ secundae et triplae tertiae ad lineam aequalem quadruplae maximae, quadruplae secundae et quadruplae tertiae, eandem¹²¹ habuerit alia quaedam sumpta ad excessum quo maxima secundam superat, erunt istae duae lineae simul sumptae quarta pars maximae¹²² proportionalium.



Sint enim quatuor lineae continue¹²³ proportionales $ab\ bc\ bd\ be$ et quam rationem habet be ad ea eandem habeat fg ad $\frac{3}{4}ac$ ¹²⁴, quam autem rationem habet linea aequalis ab et¹²⁵ duplae bc et triplae bd ad aequalem quadruplae ipsarum $ab\ bc\ bd$, hanc habeat hg ad ac . Ostendendum¹²⁶ est hf quartam esse partem ipsius ab .

Quia igitur $ab\ bc\ bd\ be$ sunt \mid continue¹²⁷ proportionales in eadem ratione erunt etiam $ac\ cd\ de$ et ut quadrupla ipsarum $ab\ bc\ bd$ ad ab cum dupla bc et tripla bd ita quadrupla ipsarum $ac\ cd\ de$, hoc est quadrupla ipsius ae , ad ac cum dupla cd et tripla de et sic est ac ad hg . Ergo ut tripla ipsius¹²⁸ ae ad ac cum dupla cd et tripla de ita $\frac{3}{4}$ ipsius ac ad hg . Est autem ut tripla ae ad triplam eb ita $\frac{3}{4}ac$ ad gf . Ergo, per conversam^{24^{ae}} quinti¹²⁹, ut tripla ae ad ac cum dupla cd et tripla db ita $\frac{3}{4}$ ipsius ac ad hf , et ut quadrupla ae ad ac cum dupla cd et tripla db , hoc est ad ab cum cb et bd , ita ac ad hf et permutando, ut quadrupla ae ad ac ita ab cum cb et bd ad hf , ut autem ac ad ae ita ab ad ab cum cb et bd . Ergo ex aequali proportione¹³⁰ perturbata ut quadrupla ae ad ae ita ab ad hf : quare constat hf quartam esse partem ipsius ab .

Cuiuslibet¹³¹ conii et cuiuslibet pyramidis centrum gravitatis axem dividere ut pars ad verticem reliquae sit tripla, a Comandino et aliter a me demonstratum est.

¹¹⁸ Lemma. Galileus *U* Lemma *A om. L*

¹¹⁹ continue *U L om. A*

¹²⁰ post duplae del. unum verbum *U*

¹²¹ eandem *LA* eadem *U*

¹²² maximae *U L* maxime *A*

¹²³ continue *U om. LA*

¹²⁴ ante *ac* add. ipsius *LA*

¹²⁵ et *U L om. A*

¹²⁶ Ostendendum *LA* Ostendendus *U*

¹²⁷ continue *U om. LA*

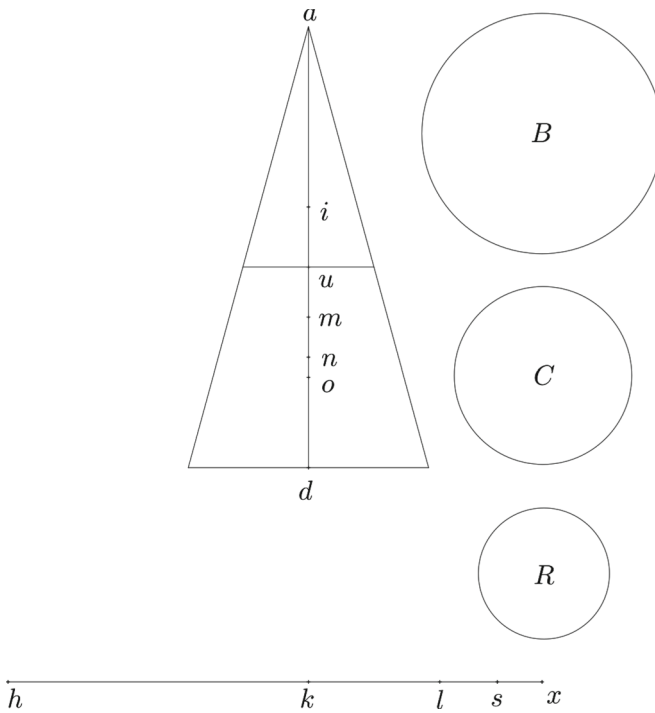
¹²⁸ ipsius \sim tripla *U L om. A*

¹²⁹ 24^{ae} quinti *U* 24^{ae} 5ⁱ *A* vigesimam quartam quinti *L*

¹³⁰ ante proportione add. in *LA*

¹³¹ Cuiuslibet \sim est *U om. LA*

Cuiuslibet¹³² frusti pyramidis seu conii, plano basi aequidistante abscissi¹³³, centrum gravitatis in axe consistit, eumque ita¹³⁴ dividit ut pars versus minorem basem ad reliquam sit¹³⁵ ut triplum¹³⁶ maioris basis, cum duplo spaciae medii proportionalis¹³⁷ inter basem maiorem et basem¹³⁸ minorem, et¹³⁹ basi minori ad triplum¹⁴⁰ basis minoris¹⁴¹ cum duplo eiusdem¹⁴² spaciae medii et¹⁴³ basi maiori.



¹³² Cuiuslibet *U* Cuiuscumque *L* Cuiuscunqae *A*

¹³³ abscissi *U A* secti *L*

¹³⁴ eumque ita *U L* ita ut prius ab eo utrinque quarta sui parte dempta centrum gravitatis in reliqua consistit eamque sic *A*

¹³⁵ sit \sim maiori *U L* eandem habeat rationem quam spaciae quod basium sit medium proportionale cum duplo maioris basis, habet ad idem spaciae inter bases proportionale cum duplo minoris basis *A*

¹³⁶ triplum *U* tripla *L*

¹³⁷ duplo spaciae medii proportionalis *U* spacio duplo medii *L*

¹³⁸ basem *U om. L*

¹³⁹ et *U* una cum *L*

¹⁴⁰ triplum *U* triplam *L*

¹⁴¹ basis minoris *U* minoris basis *L*

¹⁴² duplo eiusdem *U* eodem duplo *L*

¹⁴³ medii et *U* medii et cum *Favaro* medii etiam *L*

Sit¹⁴⁴ frustum cuius axis du sectum | a cono vel pyramide cuius axis da . Frusti¹⁴⁵ autem maior basis aequalis sit spacio B , minor vero R . Spaciorum autem $B R$ medium esto proportionale C . Frusti autem du axis dividatur in o , ita ut pars uo ad reliquam od eandem habeat rationem quam tres B cum duobus C et unum R ad tres R cum duobus C et unum B .

Dico¹⁴⁶ o centrum gravitatis frusti cuius axis ud ¹⁴⁷ existere. Quod¹⁴⁸ autem centrum gravitatis frusti in axe consistat, manifestum est, cum totius cono vel pyramidis, et cono vel pyramidis abscissae centra gravitatum in eodem axe consistant.

Sit um quarta pars ipsius ud ¹⁴⁹. Exponatur linea hx ipsi¹⁵⁰ ad aequalis sitque kx aequalis au , ipsarum vero hx xk tertia proportionalis sit xl , et quarta xs . Et quam rationem habet hs ad sx eandem¹⁵¹ habeat md ad lineam sumptam ab o versus a , quae sit on . Et quia maior basis ad eam quae inter maiorem et minorem est media proportionalis, est ut da ad¹⁵² au , hoc est ut hx ad xk , dicta autem media ad minorem est ut kx ¹⁵³ ad xl (sunt enim hx xk xl continue proportionales)¹⁵⁴, erunt maior, media et minor bases¹⁵⁵ in eadem ratione et lineae hx xk xl .

Quare tripla¹⁵⁶ maioris basis cum dupla mediae et minima ad triplam minimae cum dupla mediae et maiori¹⁵⁷, hoc est ut uo ad od , ita tripla hx cum dupla kx ¹⁵⁸ et xl ad triplam xl cum dupla xk et xh , et componendo et convertendo erit od ad du ut hx cum dupla xk et tripla¹⁵⁹ xl ad quadruplam ipsarum hx xk xl .

Sunt itaque¹⁶⁰ quatuor lineae continue¹⁶¹ proportionales hx xk xl xs ; et quam rationem habet xs ad sh hanc habet linea quaedam sumpta no ad $\frac{3}{4}$ ipsius du , nempe

¹⁴⁴ Sit $\sim da$ UA cono vel pyramide cuius axis ad secetur plano basi aequidistante frustum cuius axis ud . LA

¹⁴⁵ Frusti ~ 9 unum B U Et quam rationem habet tripla maximae basis cum dupla mediae et minima ad triplam minimae cum dupla mediae et maxima, hanc habeat uo ad od . L Ab ud autem utrinque quarta sui pars auferatur et reliqua intermedia sit mr quae in signo o dividatur ita ut mo ad or eandem habeat rationem quam dupla maioris basis cum ea quae inter maiorem et minorem basem est intermedia in ratione, habet ad eandem mediam una cum dupla minoris basis. A

¹⁴⁶ Dico U Ostendendum est LA

¹⁴⁷ cuius axis ud U om. LA

¹⁴⁸ Quod \sim consistant U om. LA

¹⁴⁹ Sit um quarta pars ipsius ud UL om. A

¹⁵⁰ ipsi LA ipsi U

¹⁵¹ eandem U hanc LA

¹⁵² ad UL om. A

¹⁵³ kx UL hx A

¹⁵⁴ (sunt enim hx xk xl continue proportionales) U om. LA

¹⁵⁵ bases UA basis L

¹⁵⁶ tripla $\sim hx$ xk xl UL ut dupla maioris basis cum media ad mediam cum dupla minoris, hoc est ut mo ad or ita erit dupla hx cum xk ad kx cum dupla xl A

¹⁵⁷ maiori U maxima L

¹⁵⁸ kx U xk L

¹⁵⁹ et tripla L ad 3plam U

¹⁶⁰ itaque UA igitur L

¹⁶¹ continue U om. LA

ad dm , hoc est ad $\frac{3}{4}$ ipsius hk . Quam autem U habet rationem¹⁶² hx cum dupla xk et tripla xl ad quadruplam ipsarum hx xk xl , eandem habet alia quaedam¹⁶³ sumpta od ad du , hoc est ad hk . Ergo per lemma praecedens¹⁶⁴ dn erit quarta pars ipsius l hx , hoc est¹⁶⁵ ad . Quare punctum n l erit gravitatis centrum coni, vel pyramidis, cuius axis ad ¹⁶⁶. Sit pyramidis vel coni cuius axis au centrum gravitatis i . Constat igitur centrum gravitatis frusti esse in linea in ¹⁶⁷ ad partes n extensa, in eoque eius puncto quod¹⁶⁸ cum puncto n lineam intercipiat ad quam in eam l habeat rationem, quam abscissum frustum habet ad pyramidem, vel conum, cuius axis au ¹⁶⁹.

Ostendendum itaque restat in ad no eandem habere rationem, quam frustum ad conum cuius axis au . Est autem ut conus cuius axis da ad conum cuius axis au , ita cubus da ad cubum au , hoc est cubus hx ad cubum xk . Haec autem eadem est ratio¹⁷⁰ quam habet hx ad xs , quare dividendo ut hs ad sx ita erit frustum cuius axis du ad conum vel pyramidem cuius axis ua .

Est autem ut¹⁷¹ hs ad sx ita md ¹⁷² ad on , quare frustum ad conum vel¹⁷³ pyramidem cuius axis au est ut md ad no . Et quia an est $\frac{3}{4}$ ipsius ad , ai autem¹⁷⁴ $\frac{3}{4}$ ipsius au , erit reliqua in $\frac{3}{4}$ reliquae ud . Quare in aequalis erit ipsi md . Et demonstratum est md ad no esse ut frustum ad conum vel pyramidem cuius axis¹⁷⁵ au . Constat ergo hanc eandem rationem habere etiam in ad no . Quare patet propositum.

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¹⁶² habet rationem U rationem habet LA

¹⁶³ quaedam LA quaedam U

¹⁶⁴ per lemma praecedens U per lemma superius *in marg. A* (per ea quae demonstrata sunt) L

¹⁶⁵ post est *add.* ipsius LA

¹⁶⁶ $ad \sim$ coni LA non legitur U

¹⁶⁷ post *in del.* linea A

¹⁶⁸ quod U qui LA

¹⁶⁹ post *au add.* Est autem dictum centrum oA

¹⁷⁰ ratio U proportio LA

¹⁷¹ ut LA *om.* U

¹⁷² ante *md add.* etiam L

¹⁷³ conum vel U *om.* LA

¹⁷⁴ post autem *add.* est LA

¹⁷⁵ vel pyramidem cuius axis U *om.* LA

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