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CONSTRUCTIVE APPROXIMATION

CORRECTION

Correction to: Lattice Algorithms for Multivariate L_∞ Approximation in the Worst-Case Setting

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The correct expression for the worst-case error

The expression for the worst-case L_∞ approximation error derived in the original article, Lemma 1, was incorrect because some nonnegative terms were erroneously left out. Here we present the correct error expression and explain that the main theorem of the paper holds with enlarged constants.

We follow the notation and argument in the original article, page 481, to arrive at (add missing conjugates in some exponential basis functions along the way)

$$e^{\mathrm{wor}}(A;L_\infty) = \sup_{\boldsymbol{x}\in[0,1]^d} \left| \sum_{\boldsymbol{h}\in\mathbb{Z}^d} \sum_{\boldsymbol{p}\in\mathbb{Z}^d} \langle \tau_{\boldsymbol{h}}, \tau_{\boldsymbol{p}} \rangle_d \, e^{2\pi\mathrm{i}(\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{x}} \right|^{1/2}, \qquad (1.1)$$

The original article can be found online at https://doi.org/10.1007/s00365-009-9075-x.

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where

$$\langle \tau_{\boldsymbol{h}}, \tau_{\boldsymbol{p}} \rangle_d = \begin{cases} \displaystyle\sum_{\substack{\boldsymbol{\ell}\in\mathbb{Z}^d\setminus\{\boldsymbol{0},\boldsymbol{p}-\boldsymbol{h}\} \\ \boldsymbol{\ell}\cdot\boldsymbol{z}\equiv 0 \pmod n}} \frac{1}{r_d(\boldsymbol{h}+\boldsymbol{\ell})} & \text{if } \boldsymbol{h},\boldsymbol{p}\in\mathcal{A}_d \text{ and } (\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{z}\equiv 0 \pmod n, \\ \frac{1}{r_d(\boldsymbol{p})} & \text{if } \boldsymbol{h}\in\mathcal{A}_d,\ \boldsymbol{p}\notin\mathcal{A}_d, \text{ and } (\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{z}\equiv 0 \pmod n, \\ \frac{1}{r_d(\boldsymbol{h})} & \text{if } \boldsymbol{h}\notin\mathcal{A}_d,\ \boldsymbol{p}\in\mathcal{A}_d, \text{ and } (\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{z}\equiv 0 \pmod n, \\ \frac{1}{r_d(\boldsymbol{h})} & \text{if } \boldsymbol{h}=\boldsymbol{p}\notin\mathcal{A}_d, \\ 0 & \text{otherwise}. \end{cases} \tag{1.2}$$

The second and third cases in (1.2) were erroneously left out in the original article, page 482.

Since all values of (1.2) are real and nonnegative, the supremum over $\boldsymbol{x}\in[0,1]^d$ in (1.1) is attained by $\boldsymbol{x}=\boldsymbol{0}$. Hence,

$$\begin{aligned} [e^{\text{wor}}(A;L_\infty)]^2 &= \sum_{\boldsymbol{h}\in\mathbb{Z}^d}\sum_{\boldsymbol{p}\in\mathbb{Z}^d} \langle \tau_{\boldsymbol{h}}, \tau_{\boldsymbol{p}} \rangle_d \\ &= \sum_{\boldsymbol{h}\notin\mathcal{A}_d} \frac{1}{r_d(\boldsymbol{h})} + 2\sum_{\boldsymbol{h}\in\mathcal{A}_d} \sum_{\substack{\boldsymbol{p}\notin\mathcal{A}_d \\ (\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{z}\equiv 0 \pmod n}} \frac{1}{r_d(\boldsymbol{p})} + \text{sum}(T), \end{aligned} \tag{1.3}$$

where $\text{sum}(T)$ is the sum of all elements of the matrix $T := [\langle \tau_{\boldsymbol{h}}, \tau_{\boldsymbol{p}} \rangle_d]_{\boldsymbol{h},\boldsymbol{p}\in\mathcal{A}_d}$. The middle term in (1.3) was erroneously left out in the original article, Lemma 1.

The strategy to address the extra term

The middle term in (1.3) can be estimated as:

$$\begin{aligned} \sum_{\boldsymbol{h}\in\mathcal{A}_d} \sum_{\substack{\boldsymbol{p}\notin\mathcal{A}_d \\ (\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{z}\equiv 0 \pmod n}} \frac{1}{r_d(\boldsymbol{p})} &\le \sum_{\boldsymbol{h}\in\mathcal{A}_d} \sum_{\substack{\boldsymbol{p}\in\mathbb{Z}^d\setminus\{\boldsymbol{h}\} \\ (\boldsymbol{p}-\boldsymbol{h})\cdot\boldsymbol{z}\equiv 0 \pmod n}} \frac{1}{r_d(\boldsymbol{p})} \\ &= \sum_{\boldsymbol{h}\in\mathcal{A}_d} \sum_{\substack{\boldsymbol{\ell}\in\mathbb{Z}^d\setminus\{\boldsymbol{0}\} \\ \boldsymbol{\ell}\cdot\boldsymbol{z}\equiv 0 \pmod n}} \frac{1}{r_d(\boldsymbol{h}+\boldsymbol{\ell})} = \text{trace}(T) \le \text{sum}(T), \end{aligned}$$

where $\text{trace}(T)$ denotes the trace of the matrix T, i.e., the sum of the diagonal elements. The first inequality is due to the simple enlargement of the sum from $\boldsymbol{p}\notin\mathcal{A}_d$ to $\boldsymbol{p}\in\mathbb{Z}^d\setminus\{\boldsymbol{h}\}$. Hence, we obtain

$$[e^{\mathrm{wor}}(A;L_\infty)]^2 \le \sum_{\boldsymbol{h}\notin\mathcal{A}_d} \frac{1}{r_d(\boldsymbol{h})} + 3\,\mathrm{sum}(T).$$

Consequently in the original article, Lemma 3 can be repaired by multiplying each of the constants $c_{1,d,q,\lambda,\delta}$, $c_{2,d,q,\lambda,\delta}$, $c_{3,d,q,\lambda,\delta}$ by 3. The main result in the original article, Theorem 4 stands.

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