Social Choice and Welfare
https://doi.org/10.1007/s00355-024-01514-5

ORIGINAL PAPER

Berge equilibrium, altruism and social welfare

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Received: 30 April 2022 / Accepted: 15 February 2024

Abstract
Welfare and other properties of Berge equilibria are investigated. In particular, we address the questions to what extent Berge equilibrium can select from multiple Nash equilibria; can serve as a substitute for Nash equilibria; can Pareto improve upon Nash equilibrium. Furthermore, some of the recent results on the relation between Berge equilibria and Kantian equilibria are summarized.

1 Introduction

The first aim of this paper is to explore the implications of the altruism inherent in Berge equilibrium play, to reexamine and assess the main assumptions and conclusions.[1] The second aim is to embark on a short tour d'horizon of the literature related to Berge equilibrium, to unify some of the results and fill in some of the gaps. In a relatively sparse but growing literature, a Nash equilibrium of the dual of a two-person game is called a Berge equilibrium in the sense of Zhukovskiy. In the dual game, each player maximizes the objective payoff of the other player. More generally, the solution concept of Berge equilibrium in the sense of Zhukovskiy as defined by Courtois et al. (2017), of **Berge equilibrium** for short, has been defined for any strategic game with a finite set of players. In a Berge equilibrium, each coalition of all players but one maximizes the payoff of the non-member, given the non-member's strategic choice.

The standard solution concept for strategic games is Nash equilibrium. However, coordinating on a Nash equilibrium (in pure strategies) can prove problematic. It can be difficult when the game has multiple Nash equilibria. It can be impossible if the

[1] Originally, the introductory sentence of the paper said "normative implications" instead of "implications". While "normative implications" would be correct, it falls short of the full scope of the analysis. For like most game-theoretic investigations, our analysis tells us how the game will or might be played as well as how it should be played. So there is a positive and a normative side to it.

I would like to thank two referees for valuable suggestions and Andrew Colman and Michael Moehler for insightful comments.

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Published online: 28 March 2024

game has no Nash equilibrium in pure strategies. It can be undesirable if the Nash equilibrium is Pareto dominated by another outcome like in the Prisoner's Dilemma. Several reasons for studying Berge equilibria have been forwarded in the literature. They are related to the said potential problems of playing Nash equilibria:

(a) Multiple Nash equilibria, implicitly suggesting that Berge equilibrium can serve as selection from the set of Nash equilibria—or at least as an alternative solution concept with a unique outcome.
(b) Non-existence of Nash equilibria, implicitly suggesting that Berge equilibria might exist in that case and could serve as substitutes for Nash equilibria.
(c) Fostering of cooperation which in turn might yield outcomes that Pareto dominate the Nash equilibria of the game.

Reasons (a) and (b) date back to Zhukovskiy (1985) and are usually mentioned in passing. Reason (c) is the main concern of Colman et al. (2011), Courtois et al. (2015) and Salukvadze and Zhukovskiy (2020).[2] We are going to examine all three reasons, with an emphasis on (c).

Regarding (c), the proverb *"The road to hell is paved with good intentions"* does not rule out that good intentions can have good consequences. But it points out that sometimes good intentions produce bad outcomes. In practice, the bad outcomes are often side-effects or unintended or unexpected consequences of well intended actions. Rent control can lead to less construction and poor maintenance of affordable housing. The use of pesticides may not only kill harmful animals and plants, but also useful species. The excessive use of antibiotics can foster the prevalence of drug-resistent strands of bacteria.

Altruistic or other-regarding preferences are commonly considered desirable character traits that improve social welfare. Indeed, an altruistic person's actions tend to benefit others—possibly to that person's detriment.[3] And everybody becoming an altruist may improve social welfare. However, it can also be the case that everybody's net benefits are negative when everybody becomes an altruist. The first possibility is exhibited by the Prisoner's Dilemma (PD) game. In the original game, deviation is a strictly dominant strategy and the Nash equilibrium is strictly Pareto dominated by the cooperative outcome. This phenomenon is well understood: When choosing the strictly dominant strategy, a player prohibits the two best payoffs for the other player. In the dual game, where every player cares only about the other player's payoff, cooperating is a strictly dominant strategy. Hence the Berge equilibrium of the PD yields a better outcome. Yet this prime example of positive consequences of altruism in a strategic game also illustrates the second possibility, since the dual of the dual game is the original PD game. To develop some intuition for playing Berge can lead to a worse outcome, notice that in the dual of the PD, altruistic behavior requires a sacrifice in own payoff that outweighs the benefit for the other player so that, indeed, everybody's net benefits are negative when everybody becomes an altruist.

We find that moving from Nash equilibrium to Berge equilibrium can yield better outcomes, like in the Prisoner's Dilemma. But moving from Nash equilibrium to

[2] Further references to the pertinent literature are given in subsequent sections.

[3] According to a frequent narrow definition, helping others is only altruistic if it involves a sacrifice by the altruist.

Berge equilibrium can also yield worse outcomes as in the dual game of the Prisoner's Dilemma. It can also be the case that moving from Nash equilibrium to Berge equilibrium adds further equilibria that are worse than the worst Nash equilibrium. It is possible, too, that either all Nash and Berge equilibria yield the same payoffs or cannot be Pareto ranked. In any case, reason (c) is valid in some cases and not in others. When there are multiple Nash equilibria, Berge equilibrium may or may not select one of those. One possibility is that Nash and Berge equilibria coincide. Another possibility is that no Berge equilibrium exists. Thus reason (a) proves valid in some cases and not in others. There are games without a Nash equilibrium in pure strategies, but with a Berge equilibrium. In some games neither a Nash equilibrium nor a Berge equilibrium exists so that Berge equilibria cannot serve as substitute for lacking Nash equilibria. Hence reason (b) may or may not be valid.

The study of Berge equilibria is of potential interest to several scientific disciplines. First of all, it provides new perspectives in game theory and mathematics. The comparison of Berge equilibrium and Nash equilibrium accentuates the pros and cons of the latter concept. Novel techniques may be warranted to analyze Berge equilibria. Second, experimental findings in psychology, behavioral economics and game theory have motivated researchers to look for conceptual alternatives to Nash equilibrium, with Berge equilibrium among those. However, I am unaware of any experiments devoted to Berge equilibria per se—which may indicate fertile ground for future research. Third, the normative aspects of Berge equilibrium touch upon issues in moral philosophy and welfare economics. Some of these issues and ideas, especially the golden rule and Kant's categorical imperative, will be discussed briefly in Sect. 6. Inspired by Kantian reasoning, Roemer (2010), Roemer (2019) introduces the concept of Kantian equilibrium, which constitutes an alternative to both Nash equilibrium and Berge equilibrium. Section 7 reports on a comparison of additive Kantian equilibrium and Berge equilibrium by Crettez and Nessah (2020) on the one hand and a comparison of multiplicative Kantian equilibrium with Berge equilibrium by Ünveren et al. (2023) on the other hand.

In the next section, the basic concepts are defined. In Sect. 3 two-person games are investigated. Section 4 deals with several games with more than two players. Section 5 on the one hand identifies situations where it is beneficial to be an altruist and on the other hand presents instances where it is beneficial to behave like an altruist. Section 6 offers brief comments on several guiding principles (and their relationship) that may but need not induce altruistic behavior: the golden rule, Kant's categorical imperative, the veil of ignorance, and pragmatic equilibrium selection. Section 7 is devoted to Roemer's concept of Kantian equilibrium, an alternative to both Berge equilibrium and Nash equilibrium. Section 8 reports on the concept of unilateral support equilibrium that encompasses both Berge equilibrium and Nash equilibrium. In Sect. 9 in a sense the opposite of Berge equilibrium is considered. In Sect. 10 existence and computational complexity are addressed. Section 11 concludes.

2 Finite games and equilibria

A finite game in strategic or normal form is a tuple $G = (I, (S_i, u_i)_{i \in I})$ where I is a nonempty finite set, the set of players; for each player i, S_i is a nonempty finite set, the players's strategy set (or strategy space, action set, action space); for each player i, $u_i : S = \prod_{j \in I} S_j \to \mathbb{R}$ is player i's payoff function. Although in the interpretation of some prominent games the payoffs are just utilities and not monetary or material, we shall use the terminology *objective payoffs* for the given game G. An element in $S = \prod_{i \in I} S_i$ is called a joint strategy or strategy profile. For $i \in I$, the set $S_{-i} = \prod_{j \neq i} S_j$ consists of the joint strategies of all players but i. Whenever convenient and appropriate, we shall treat $s \in S$ as an element of $S_i \times S_{-i}$ and write $s = (s_i, s_{-i})$. Finally, let $I_{-i} = I \setminus \{i\}$ for $i \in I$.

For our purposes, only nontrivial games and players are of interest. Therefore, throughout the paper we make the

Assumption 1 $|I| > 1$ and $|S_i| > 1$ for $i \in I$.

With the exception of Sect. 10and a few explicit references to the literature, only equilibria in pure strategies will be considered.

Definition 1 Let $(I, (S_i, u_i)_{i \in I})$ be a finite strategic game and $s^* = (s_i^*)_{i \in I} \in S$.

(i) s^* is a **Nash equilibrium** of G if
$u_i(s^*) \geq u_i(s_i, s_{-i}^*)$ for all $i \in I,\ s_i \in S_i$.

(ii) s^* is a **Berge equilibrium** of G if
$u_i(s^*) \geq u_i(s_i^*, s_{-i})$ for all $i \in I,\ s_{-i} \in S_{-i}$.

(iii) s^* is a **Berge-Nash equilibrium** of G if
it is both a Berge and a Nash equilibrium of G.

(iv) s^* is a **strong Berge equilibrium** of G if
$u_j(s^*) \geq u_j(s_i^*, s_{-i})$ for all $i \in I,\ j \in I_{-i},\ s_{-i} \in S_{-i}$.

In a Nash equilibrium s^*, no player has an incentive to deviate from the chosen strategy given the other players' equilibrium strategies. In a strong Berge equilibrium, no coalition of $|I| - 1$ players can improve the payoff of one of its members by deviating from their chosen joint strategy, given the equilibrium strategy of the non-member. This constitutes a refinement of Nash equilibrium. In a Berge equilibrium, the objective of any $(|I| - 1)$-player coalition tends to be the opposite from strong Berge equilibrium play: They aim to maximize the payoff of the non-member, given that player's equilibrium strategy. Most importantly, strong Berge equilibrium is not a refinement of Berge equilibrium, contrary to what the terminology might suggest.

Let $NE(G)$ denote the set of Nash equilibria and $BE(G)$ the set of Berge equilibria of a game G.

Musy et al. (2012) point out parallel reformulations of Nash equilibrium and Berge equilibrium. Namely, let again $G = (I, (S_i, u_i)_{i \in I})$ be a finite strategic game. For $i \in I$, let $BR_i : S_{_i} \twoheadrightarrow S_i$ be i's best response or best reply relation.[4] That is,

[4] Some scholars use the term best response correspondence whereas most economists and game theorists reserve that terminology to non-empty valued relations.

$BR_i(s_{-i}) = \arg max_{s_i \in S_i} u_i(s_i, s_{-i})$ for $s_{-i} \in S_{-i}$.

Let $\overline{BR_i}$ be the graph of BR_i. Then
$NE(G) = \bigcap_{i \in I} \overline{BR_i}$.

Finally, let $BR : S \twoheadrightarrow S$ be the joint best response relation given by $BR(s) = \prod_{i \in I} BR_i(s_{-i})$ for $s = (s_i)_{i \in I} \in S$. Then
$s^* \in NE(G)$ if and only if s^* is a fixed point of BR.

In a similar vein, Musy et al. (2012) define player i's best support relation $BS_i : S_i \twoheadrightarrow S_{-i}$:
$BS_i(s_i) = \arg max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ for $s_i \in S_i$.

Let $\overline{BS_i}$ be the graph of BS_i. Then
$BE(G) = \bigcap_{i \in I} \overline{BS_i}$.
The parallel would be perfect, if one could define a joint best support relation
$BS : S \twoheadrightarrow S$, which is impossible. Instead, let us consider the relations $\widehat{BS}_i : S \twoheadrightarrow S$ given by $\widehat{BS}_i(s_i, s_{-i}) = \{s_i\} \times BS_i(s_i)$ for $s = (s_i, s_{-i}) \in S$, $i \in I$, and the relation $BS : S^I \twoheadrightarrow S^I$ given by $BS((s^i)_{i \in I}) = \prod_{i \in I} \widehat{BS}_i(s^i)$ for $(s^i)_{i \in I} \in S^I$. Then
$s^* \in BE(G)$ if and only if $(s^*, \ldots, s^*)$ is a fixed point of BS.

3 2-Player games

Many prominent "named" games are two-player games that prove useful to our investigation. To each two-player game $G = (\{i, j\}, S_i, S_j, u_i, u_j)$, one can associate its dual game $G^o = (\{i, j\}, S_i, S_j, u_i^o, u_j^o)$ where $u_i^o = u_j$ and $u_j^o = u_i$. Two obvious, but important facts obtain:

Fact 1 $BE(G) = NE(G^o)$.

Fact 2 $G^{oo} = G$.

Fact 1 is shown in Colman et al. (2011) as well as a generalization to games with more than two players. Courtois et al. (2015) argue that computation of a Berge equilibrium of G can be reduced to the computation of a Nash equilibrium of G^o, referring to Fact 1. Validity of Fact 1 and their argument is not confined to finite strategy sets. We shall explore each of the two facts, in particular the combination of the two.

Pottier and Nessah (2014) elaborate on Colman et al. (2011) and consider utility transformations $F : \mathbb{R}^2 \to \mathbb{R}^2$ of the form $F(u_1, u_2) = (F_1(u_2), F_2(u_1))$ with strictly increasing functions F_1 and F_2.[5] They show that these are precisely the utility transformations where $BE(G) = NE(G')$ holds for each 2-player game $G = (\{1, 2\}, S_1, S_2, u_1, u_2)$ and the transformed game $G' = (\{1, 2\}, S_1, S_2, F_1(u_2), F_2(u_1))$. They also show that a similar result does not hold

[5] Berge-Vaisman equilibrium as defined by Pottier and Nessah (2014) is a synonym for Berge equilibrium whereas Berge-Vaisman equilibrium in the sense of Colman et al. (2011) is a special case of Berge-Nash equilibrium.

for games with more than two players.

Symmetric games. Colman et al. (2011) consider the symmetric 2×2 games in which both players have strict preferences among the four outcomes. There are 12 equivalence classes of such games (or ordinally distinct such games). There exist Nash and Berge equilibria in all twelve cases. Specifically:

(A) In four of the cases, there are two equilibria which are both Berge-Nash equilibria.
(B) In two cases, there exists a single equilibrium which is Berge-Nash.
(C) In one case, there are two Nash equilibria, one of which is Berge.
(D) In one case, there are two Nash equilibria and a unique Berge equilibrium that is not Nash.
(E) In one case, there are two Berge equilibria, one of which is Nash.
(F) In one case, there are two Berge equilibria and a Nash equilibrium that is not Berge.
(G) In two cases, there exists a unique Nash equilibrium and a unique but distinct Berge equilibrium.

In the instance of (A), Berge equilibrium does not constitute a selection from the set of Nash equilibria. In (C), Berge equilibrium selects a Nash equilibrium. In (D), Berge equilibrium does not select a Nash equilibrium, but is unique. Hence the implicit claim of reason (a) given in the Introduction, that Berge equilibrium selects from multiple Nash equilibria, holds in some games and not in others. To check the implicit claim of reason (b) given in the Introduction, that games without Nash equilibria do have a Berge equilibrium, one needs to consider games without Nash equilibria. Examples of such games will be analyzed when we deal with zero-sum games and games with more than two players.

That Berge equilibrium play can improve upon Nash equilibrium play is exemplified by the Prisoner's Dilemma, one of the two classes of games under (G). Because of Facts 1 and 2, the dual of the Prisoner's Dilemma, which constitutes the second class under (G), is an example where Nash equilibrium play is better than Berge equilibrium play. Hence the claim inherent in reason (c) given in the Introduction, that Berge equilibrium play may Pareto dominate Nash equilibrium play, is confirmed in some cases and starkly refuted in others.

Let us consider a narrative which demonstrates that the dual of a Prisoner's Dilemma game can be more than a mere mathematical artifact. Consider a couple that has to decide how to spend Saturday evening. The couple own a single ticket for one person for the sold out Garth Brooks concert that Saturday. Each person decides whether to stay home (IN) or to go out (OUT). If both choose IN, they share a nice evening at home. If both choose OUT, they will have dinner in a fancy restaurant and go to a disco thereafter. If one chooses IN and the other chooses OUT, the former stays home alone and the latter attends the Garth Brooks concert.

"Selfish" individuals have preferences given by the following utilities (payoffs):

- In case both choose IN and share the evening at home, the individual has utility 3.
- In case both choose OUT and go to the restaurant and disco, the individual has utility 4.

- If the individual chooses OUT and the partner chooses IN, then the individual goes to the Garth Brooks concert and obtains utility 8.
- If the individual chooses IN and the partner chooses OUT, then the individual stays home alone and obtains utility 2.

Let us look at three different couples.

(I) *Two selfish individuals*, Adam and Eve. They play a game with the following payoff matrix:

Eve

		OUT	IN
Adam	OUT	(4, 4)	(8, 2)
	IN	(2, 8)	(3, 3)

For instance, (8,2) means that Adam gets payoff 8 and Eve gets payoff 2 if Adam chooses OUT and Eve chooses IN.

(II) *Two perfect altruists*, Mike and Liz. Each cares only about the payoff the other would have as a selfish player. They play a game with the following payoff matrix:

Liz

		OUT	IN
Mike	OUT	(4, 4)	(2, 8)
	IN	(8, 2)	(3, 3)

(III) *A couple with heterogeneous preferences*, a selfish player (Jake) and a perfect altruist (Jane). They play a game with the following payoff matrix:

		Jane	
		OUT	IN
Jake	OUT	(4, 4)	(8, 8)
	IN	(2, 2)	(3, 3)

In each of the three games, there is an equilibrium in strictly dominant strategies. In game (I), both achieve payoff 4. In game (II), both achieve payoff 3. Notice that (II) is a specification of the Prisoner's Dilemma and (I) is its dual. In (III), both achieve the highest payoff 8, provided that Jane is truly altruistic. The distinction between altruistic preferences and altruistic behavior by egoists will be discussed in Sect. 5

Zero-Sum Games. A two-player game $G = (\{i, j\}, S_i, S_j, u_i, u_j)$ is a zero-sum game if $u_i(s) + u_j(s) = 0$ for all $s \in S$ or, equivalently, $u_j = -u_i$. If 0 is replaced by another constant, one has a constant-sum game—which is strategically equivalent to a zero-sum game, as is every strictly competitive two-player game; see Adler et al. (2009).

In a zero-sum game, there do not exist any two joint strategies s and t such that s Pareto dominates t. Hence switching from Nash equilibrium play to Berge equilibrium play can never cause a Pareto improvement and reason (c) from the Introduction becomes obsolete in zero-sum games.

There are two-person zero-sum games that have neither a Nash equilibrium in pure strategies nor a Berge equilibrium. An example is the well known Matching Pennies game, also known as "hide and seek" or "land and sea". Another example is the Rock, Paper, Scissors game. In these games, the rationale behind reason (b) of the Introduction proves obsolete.

Next let us examine the relationship between Nash equilibria and Berge equilibria in zero-sum games in more detail. Let $s^* \in S$ be a Nash equilibrium in the zero-sum game $G = (\{i, j\}, S_i, S_j, u_i, u_j)$. There are two possibilities:

1.) $u_i(s_i^*, s_j^*) = u_i(s_i, s_j^*)$ for all $s_i \in S_i$ and
$u_j(s_i^*, s_j^*) = u_j(s_i^*, s_j)$ for all $s_j \in S_j$. Then
$u_j(s_i^*, s_j^*) = u_j(s_i, s_j^*)$ for all $s_i \in S_i$ and
$u_i(s_i^*, s_j^*) = u_i(s_i^*, s_j)$ for all $s_j \in S_j$.
Therefore, s^* is a Berge equilibrium of G.

2.) $u_i(s_i^*, s_j^*) > u_i(s_i, s_j^*)$ for some $s_i \in S_i$ or
$u_j(s_i^*, s_j^*) > u_j(s_i^*, s_j)$ for some $s_j \in S_j$. Then
$u_j(s_i^*, s_j^*) < u_j(s_i, s_j^*)$ for some $s_i \in S_i$ or
$u_i(s_i^*, s_j^*) < u_i(s_i^*, s_j)$ for some $s_j \in S_j$.
Consequently, s^* is not a Berge equilibrium of G.

Thus "very weak" Nash equilibria are Berge-Nash whereas others are not Berge. Numerical examples are the following.

i \ j	ℓ	m	r
T	$(-1, 1)$	$(0, 0)$	$(1, -1)$
M	$(0, 0)$	$(0, 0)$	$(0, 0)$
B	$(1, -1)$	$(0, 0)$	$(-1, 1)$

i \ j	ℓ	m	r
T	$(-1, 1)$	$(0, 0)$	$(-1, 1)$
M	$(1, -1)$	$(1, -1)$	$(1, -1)$
B	$(-1, 1)$	$(0, 0)$	$(-1, 1)$

In the left-hand example, (M, m) is a very weak Nash equilibrium which is also Berge. In the right-hand example, (M, m) is a Nash equilibrium that is not Berge. There also exist two Berge equilibria, (T, m) and (B, m) which are not Nash. Incidentally, in its dual, the Berge equilibrium is not a selection from the set of Nash equilibria.

4 More than two players

Here we present several games to complete the inquiry into the three reasons in favor of Berge equilibrium forwarded in the Introduction.

First of all, Figure 1(b) in Colman et al. (2011) depicts a $2 \times 2 \times 2$ game that has four Nash equilibria and no Berge equilibrium. Hence Berge equilibrium fails to select from the set of Nash equilibria because of lack of Berge equilibria. In case (A) of Sect. 3 Berge equilibrium fails to select from the set of Nash equilibria because of multiplicity of Berge-Nash equilibria. Finally, in case (D) of Sect. 3 Berge equilibrium fails to select one of the two Nash equilibria because the unique Berge equilibrium is not Nash. This is also the case in some zero-sum games as mentioned at the end of the previous section.

But would Berge equilibrium select the best Nash equilibrium if it did select? Case (C) in Sect. 3is the so-called Stag Hunt game. In that case, the Berge-Nash equilibrium Pareto dominates the other Nash equilibrium. But it is possible as well that the Berge-Nash equilibrium is worse than all other Nash equilibria. To develop such an example, let us first consider a $2 \times 2 \times 2$ game that is a special case of an example in Nessah et al. (2007). Let $I = \{1, 2, 3\}$, $S_i = \{0, 1\}$ for $i \in I$ and

$u_1(s_1, s_2, s_3) = s_1 + s_2 + s_3,$
$u_2(s_1, s_2, s_3) = s_1 + s_2 - s_3,$
$u_3(s_1, s_2, s_3) = s_1 - s_2 + s_3$

for $(s_1, s_2, s_3) \in S$. This game has the unique Nash equilibrium $s^* = (1, 1, 1)$ and no Berge equilibrium. Let us next consider the $3 \times 3 \times 3$ game with player set I, strategy sets $X_i = \{0, 1, 2\}$ for $i \in I$ and payoff functions v_i given by $v_i(x) = u_i(x)$ for $x \in S$,

$v_i(2, 2, 2) = 1/2$, and $v_i(x) = -2$ otherwise. This game has two Nash equilibria, s^* and $x^* = (2, 2, 2)$ and the unique Berge equilibrium x^*. Hence Berge equilibrium selects x^* from the set of Nash equilibria. Yet $u_i(x^*) = 1/2 < 1 \leq u_i(s^*)$, which shows that Berge equilibrium may select the worst Nash equilibrium.

It is also interesting to see what happens when the set of equilibria expands as one moves from Nash to Berge equilibria. If one takes the dual of the Stag-Hunt game as the primal game, then a Pareto inferior equilibrium gets added when one moves from Nash to Berge equilibria. This phenomenon can also be observed in the following $2 \times 2 \times 2$ game: $I = \{1, 2, 3\}$, $S_i = \{0, 1\}$ for $i \in I$ and

$u_1(s) = s_1,$
$u_2(s) = \min\{s_1, s_2\},$
$u_3(s) = \min\{s_1, s_3\}$

for $s = (s_1, s_2, s_3) \in S$. This game has the unique Nash equilibrium $s^* = (1, 1, 1)$ which is also Berge. $s^{**} = (0, 0, 0)$ is a further Berge equilibrium with $u_i(s^{**}) < u_i(s^*)$ for all $i \in I$.

It is possible as well that a better equilibrium gets added. To demonstrate this possibility, let us begin with the following 3×3 game: $I = \{1, 2\}$, $S_i = \{a_i, b_i, c_i\}$ for $i = 1, 2$ and the payoff matrix

2

1

	a_2	b_2	c_2
a_1	$(-2, 6)$	$(2, 4)$	$(3, 2)$
b_1	$(0, 2)$	$(0, 4)$	$(0, 2)$
c_1	$(2, 2)$	$(-2, 4)$	$(-4, 6)$

Let u_1 and u_2 denote the corresponding payoff functions. The game has no Nash equilibrium and the unique Berge equilibrium $s^* = (b_1, b_2)$. Therefore, it supports reason (b) given in the Introduction, that the Berge equilibrium may serve as a substitute for the non-existing Nash equilibria. We use the game to specify a 4×4 game with player set I, strategy set $X_i = \{a_i, b_i, c_i, d_i\}$ for $i = 1, 2$ and payoff functions v_i given by $v_i(x) = u_i(x)$ for $x \in S$, $v_i(d_1, d_2) = -2$, and $v_i(x) = -6$ otherwise. This game has two equilibria, the Berge equilibrium s^* which is not Nash and the Berge-Nash equilibrium $s^{**} = (d_1, d_2)$ with $v_i(s^*) > v_i(s^{**})$ for $i = 1, 2$. Thus moving from Nash to Berge equilibria adds a better equilibrium.

5 Beneficial altruistic preferences or altruistic behavior

We found that Berge equilibrium may or may not serve as a substitute if Nash equilibria do not exist. Berge equilibrium may or may not select one of several Nash equilibria.

Social welfare in Berge equilibrium compared to Nash equilibrium turns out to be better or worse as well. First, moving from Nash equilibrium to Berge equilibrium may improve or worsen social welfare. Second, when Berge equilibrium selects from several Nash equilibria, it may select the best or the worst. Third, when Berge equilibrium expands the set of equilibria, better or worse equilibria may be added.

5.1 When it is good to be an altruist

We have seen that in some games, both players fare better if they are both altruists rather than egoists. This is obvious in the Prisoner's Dilemma. According to interdependence theory, players may enter a game with a social value orientation.[6] In a two-person game with players i and j and objective payoff functions u_i and u_j, various social value orientations amount to transformed payoff functions v_i and v_j. Player i's social value orientation is

individualistic or egoistic if $v_i = u_i$;
altruistic if $v_i = u_j$;
cooperative if $v_i = u_i + u_j$;
competitive or antagonistic if $v_i = u_i - u_j$;
adversarial if $v_i = -u_j$;
equality-seeking if $v_i = \min\{u_i - u_j, u_j - u_i\}$;
Rawlsian if $v_i = \min\{u_i, u_j\}$.

The players play the game with the same game form $\{i, j, S_i, S_j\}$, but possibly transformed payoffs. The dual G^o of a two-player game G has the same game form as G while the payoff functions stem from altruistic social value orientation for both players. In certain games, this social value orientation bodes well for the players. In some games, however, altruistic social value orientation per se as embodied in the concept of Berge equilibrium is not enough to yield superior outcomes. Reconsider first the story told by Colman et al. (2011) of a jazz-loving man married to a classical music lover. Each wishes to choose a musical recording as a wedding anniversary gift for the other, knowing that they will spend many hours listening to the music together. They might each choose the other's favorite music, ending up with a jazz and a classical recording. That conforms with Berge equilibrium. But if they followed their egoistic preferences, they would have ended up with a jazz and a classical recording as well. In order to enhance the couple's welfare, the basic altruistic social value orientation must be supplemented by an additional warm glow caused by the act of mutual giving and receiving, that is, mutual giving and receiving has intrinsic value to the agents.

Following Coleman et al. (2011), let us consider next two particular symmetric 2×2 games:

[6] See Kelley (1991), Rusbult and Van Lange (2003), Colman et al. (2011).

		j	
		X	Y
i	X	(9, 9)	(0, 8)
	Y	(8, 0)	(7, 7)

		j	
		H	L
i	H	(9, 9)	(0, 0)
	L	(0, 0)	(7, 7)

The left-hand game is a specification of the Stag-Hunt game. The right-hand game is a pure coordination game. Each game has two symmetric Nash equilibria in pure strategies which are also the Berge equilibria and the equilibria under cooperative social value orientations. So neither altruistic nor cooperative social value orientations lead the players to select the payoff dominant equilibrium. Harsanyi and Selten (1994, p. 359) argue that payoff dominance must be explicitly incorporated into the concept of rationality. Otherwise, the payoff dominant equilibrium will not be the unique solution. In a similar vein, Sugden (1993) argues that it ought to be common knowledge that members are rational members of a team in order to obtain the payoff dominant outcome as the unique equilibrium. He further argues that this kind of common knowledge is as plausible as the standard assumption of common knowledge of rationality. Yes, but 2 years later Aumann and Brandenburger (1995) have shown that in the two-player games that populate Sugden's paper and most of the literature, "epistemic conditions not involving common knowledge in any way already imply Nash equilibrium."

Symmetric Cournot duopoly shares the key feature of the Prisoner's Dilemma in that joint profit maximization (collusion) Pareto dominates the Cournot-Nash equilibrium. But one would not expect that a duopolist foregoes profit opportunities to boost the rival's profits. Thus altruistic social value orientation is unlikely in some economically important situations even if it would be beneficial. Collusion may be sustained in repeated interactions, both in theory and in practice.

In some scenarios, egoists would contribute nothing to a public good while pure altruists would contribute their entire monetary endowment. Of these two polar cases, the second one is much less likely to be observed even when it is Pareto optimal that everybody donates everything.

In game (III) of Sect. 3 an egoist and an altruist benefit from being matched. If in the earlier zero-sum game with a "very weak" Nash equilibrium, player i and only player i becomes an altruist, then the modified game has four additional Nash equilibria in which both players have higher payoffs than in the original Berge-Nash equilibrium. Thus being matched with an egoist can be beneficial for a true altruist and for the egoist. For instance, if in a board game, the grandparent wants the grandchild to win, both might be very happy about the result.

5.2 When it is good to play like an altruist

In some dynamic games, a player can benefit from playing like another type of player. In mechanism design, agents may pretend to be of another type. Incentive compatibility means that it is in every agent's interest not to mimic another type: All participants can achieve their best outcomes by acting according to their true preferences.

Courtois et al. (2015) forward the idea that players may behave like altruists in the pursuit of their egoistic interests. They acknowledge that moving from Nash to Berge equilibrium may lead to better or worse outcomes. Therefore, they propose a pragmatic approach to two-person games where in some games the players behave according to their objective preferences whereas in others they behave like altruists, but always have only their own objective payoffs in mind. The choice of behavior rule, Berge or Nash, depends on which equilibrium yields the better outcome. But in the Prisoner's Dilemma it is still optimal for me to deviate no matter what the other player does or believes. Consequently, the other player has no reason to trust me, even if I announce to follow the Berge rule. By the same token, I cannot trust the other player's announcement. Courtois et al. (2015) go on to propose that the two players should hold the believe that the other is a conditional cooperator, that is the other player cooperates if I cooperate and deviates otherwise. This induces me to cooperate, provided that the other can observe my choice. In a one-shot game, however, players move simultaneously. My co-player does not observe my choice and has no reason to follow through with the rule of conditional cooperation. Hence mutual knowledge of conditional cooperation need not result in cooperation. The outcome is different, if each player believes to be a first mover and further that the other player can observe his first move and is a conditional cooperator. With such believes, cooperation obtains. It remains to explain, though, how the players succeed in "coordinating" their beliefs in this way.

Remarks.

1. Incidentally, the belief that oneself is the first mover and the other is a conditional cooperator, is akin to the beliefs in a particular conjectural variations equilibrium, a quasi-dynamic concept dating back to Bowley (1924).

2. Neither Berge equilibrium nor any of the amendments by Sugden (1993), Harsanyi and Selten (1994), and Courtois et al. (2015) resolve the indeterminacy of equilibria in the Battle of the Sexes or anti-coordination games. Gauthier's 2013 concept of "agreed Pareto-optimizers" does not resolve the indeterminacy either.

3. In ultimatum games, altruistic social value orientations induce the responder to accept any division, with indifference between accepting and rejecting the proposal to grant all the money or the entire cake to the responder. With cooperative value orientations and any rule that mandates efficient outcomes, acceptance becomes a dominant strategy. Then any proposed division is a subgame-perfect equilibrium outcome. When playing against a responder with an altruistic or cooperative social value orientation, a proposer with egoistic social value orientation fares best, ending up with the entire cake or stake of money. None of the concepts discussed in this paper explains the fact that in experiments certain proposals occur frequently and often thresholds for

rejection are observed. Different determinants like fairness or inequity aversion à la Fehr and Schmidt (1999) have to be invoked to explain the evidence.

4. Even the best of all equilibria, Nash or Berge, may not be the best possible outcome. Namely, consider the following 4×4 game with row player i and column player j:

j

i		b_1	b_2	b_3	b_4
	a_1	(8, 6)	(0, 0)	(0, 0)	(6, 8)
	a_2	(0, 0)	(4, 4)	(2, 5)	(0, 0)
	a_3	(0, 0)	(5, 2)	(3, 3)	(0, 0)
	a_4	(6, 8)	(0, 0)	(0, 0)	(8, 6)

The game restricted to strategy sets $S_i' = \{a_2, a_3\}$ and $S_j' = \{b_2, b_3\}$ is a PD game. The game restricted to strategy sets $S_i'' = \{a_1, a_4\}$ and $S_j'' = \{b_1, b_4\}$ is a constant-sum game equivalent to Matching Pennies. The 4×4 game has the Nash equilibrium (a_3, b_3) with payoff pair $(3, 3)$ and the Berge equilibrium (a_2, b_2) with payoff pair $(4, 4)$. However, the Berge equilibrium is Pareto dominated by (a_1, b_1).

6 The golden rule and the categorical imperative

Here I briefly comment on several guiding principles (and their relationship) that may but need not induce altruistic behavior.

6.1 Golden rule

Salukvadze and Zhukovskiy (2020) elaborate on the golden rule, which is widely known and has a long history, as documented by the authors. There are various nuances of the rule, for instance Luke 6:31:"And as ye would that men should do to you, do ye also to them likewise."[7] There are also negative forms, requiring not to do to others what one does not want to be done to oneself. In any case, one need not be an altruist to follow the rule. For example, Exodus 22:18 says: "Thou shalt not suffer a witch to live." In that case, the golden rule merely means that if I approve that witches be

[7] This and the next quote are taken from the King James Bible.

killed, then I should agree that I ought to be killed when proven to be a witch. While many agree with the golden rule in principle, few follow it to the letter.

6.2 Categorical imperative

Popular interpretations of Kant's categorical imperative resemble the golden rule. But Kant's idea of a universal law means it should be applicable to everybody including oneself, without invoking any reciprocity or other-regarding preferences. For instance, for truth telling to be universal, one should not tell lies to anybody, not even white lies when facing a murderer whereas not answering a question at all might be alright. Though the golden rule might lead to the same conclusion in that case: Since I would prefer not to be lied to if I was a murderer, I should not lie to the murderer. In other instances, the conclusions might differ: By the categorical imperative, either one should never lie or one should always lie, regardless of the other person's identity or circumstances. According to the golden rule, if I would prefer to be lied to about my health under certain conditions, then I would not tell the truth to someone else under the same conditions. The difference would disappear in this case if the categorical imperative had a domain restriction: Never lie to a person unless the person does not want to learn the truth.

6.3 Veil of ignorance

The concept of veil of ignorance suggests that decision makers like the drafters of a constitution or social contract should pretend not to know anything about their particular abilities, preferences, social status and positions they are going to assume in society. This principle is consistent with the golden rule, but does not imply the golden rule mainly because a constitution or social contract does not prescribe behavior in all types of interaction. Moreover, social contract theory may allow for "deep moral diversity" that includes liberal moral agents, nonliberal moral agents and nonmoral agents. See Moehler (2018), Moehler (2020a), Moehler (2020b).

6.4 Pragmatic choices

Salukvadze and Zhukovskiy (2020) submit that the golden rule should be applied in the well known cases of symmetric 2×2 games where the best Berge equilibrium Pareto dominates the best Nash equilibrium. They indicate that Nash equilibrium should be played in zero-sum games. They mention that in some symmetric 2×2 games, the best Nash equilibrium Pareto dominates Berge equilibrium, implying, but not explicitly saying that then the golden rule should not be applied. In any case, they favor equilibria that are Pareto optimal among all equilibria. This leaves the problem how to select one of several Berge-Pareto or Nash-Pareto equilibria. In particular, there exist symmetric two-player games where the best Berge equilibria and the best Nash equilibria differ, but yield identical payoffs.

There are highly asymmetric games in which it is impossible to say what a player can do for or against another player so that strictu sensu the golden rule proves obsolete. Nonetheless, such a game may have a unique Berge equilibrium and no Nash equilibrium. For example, take the 3×3 game depicted in Sect. 4

7 Berge versus Kant

Roemer's (2019) concept of "Kantian optimization" and "Kantian equilibrium" shares with Sugden (1993) the notion of group rationality. It is also reminiscent of conjectural variations equilibrium. There are several versions which differ in the "deviation in the same direction" by others that a deviating player assumes. A comparison of Berge equilibrium and Kantian equilibria is warranted and has been performed in two recent publications.[8]

Crettez and Musy (2021) present an interesting application and comparison of Kantian equilibrium and Berge equilibrium. The problem at hand is to implement legal unification, that is the substitution of new and unique legislation for multiple national or local rules. They analyze the problem by means of a legal standardization game with $N \geq 2$ players, each player representing a country (or a local jurisdiction such as a federal state). Player i's strategy is a real number ℓ_i that constitutes i's legal system. Disregarding other countries' choices, the country has single-peaked preferences with respect to its own legal system, with bliss point θ_i, where the θ_i differ across countries. However, the country faces a tradeoff: On the one hand, it wants ℓ_i to be as close as possible to θ_i. On the other hand, it wants to deviate as little a possible from each of the other countries choices ℓ_j, $j \neq i$. The aim is to identify solution concepts that result in legal unification, i.e., where there exists $\ell \in \mathbb{R}$ such that the resulting strategy profile satisfies $\ell_i = \ell$ for all i.

First of all, the authors observe that legal unification never maximizes aggregate social welfare—though it may maximize very specific utilitarian social welfare functions.[9] Second, Crettez and Musy show that there is no Nash equilibrium in which legal unification prevails when other-regarding preferences are introduced in a legal standardization game. Next they demonstrate that legal unification is not achieved at any additive Kantian equilibrium à la Roemer (2019). Finally, they find that Berge equilibria exist and each of them yields legal unification in the legal standardization game, which is good news. Yet there are also some drawbacks: There is a continuum of Berge equilibria. Moreover, if payoff functions are concave, then the Pareto optimal Berge equilibria are those where all countries choose a select country's bliss point θ_i so that a utilitarian social welfare function is maximized in which only country i has positive weight.

Ünveren et al. (2023) embark in a systematic comparison of Kant and Berge equilibria, reflecting self-regarding and other-regarding preferences, respectively. They resort to the notion of multiplicative Kantian equilibrium that dates back to Roemer

[8] Borissov et al. (2023) study a Ramsey model where decisions are made by temporal selves of the representative agent. In their intrapersonal context, the authors distinguish between Berge policy and Kantian policy in analogy to Berge equilibrium and Kantian equilibrium, respectively, in our interpersonal setting.

[9] This impossibility result is meant by the terminology "paradox of legal unification".

(2010) and differs from additive Kantian equilibrium adopted by Crettez and Nessah (2020). Their investigation is confined to two classes of two-player games: (α) $n \times n$ games where equilibria in mixed strategies are considered and (β) continuous games with the unit interval as strategy set and a smooth and strictly concave payoff function for each player. The authors first obtain four key results for class (α):

1. Efficient Berge equilibria are Kantian for generic symmetric games.
2. Efficient Berge-Nash equilibria are Kantian in any game.
3. The set of interior (i.e., completely mixed) Berge equilibria coincides with the set of interior (i.e., completely mixed) Nash equilibria in any game.
4. The intersection of the set of interior (i.e., completely mixed) Berge equilibria and the set of interior (i.e., completely mixed) Kantian equilibria is empty for generic games.

Analog results are shown for class (β). The first two results say that under certain circumstances, efficient Berge equilibria are Kantian equilibria as well. Hence under these circumstances, existence of efficient Berge equilibria begets existence of efficient Kantian equilibria.

Ideally, a multiplicative Kantian equilibrium of an $n \times n$ game would be defined as a joint mixed strategy pair $q^* = (q_i^*)_{i \in I}$ such that

(v) $Eu_i(q_i^*, q_j^*) \geq Eu_i(rq_i^*, rq_j^*)$ for all $i, j \in I, i \neq j, r \in \mathbb{R}_+^n$,

where Eu_i stands for i's expected payoff and $rq_i = (r_1 q_{i1}, \ldots, r_n q_{in})$ for $r = (r_1, \ldots, r_n), q_i = (q_{i1}, \ldots, q_{in})$. However, for any n-dimensional probability vector q_i, the rescaled vector rq_i is not necessarily a probability vector. In particular, at most $n - 1$ components can be uniformly rescaled. In an admissible rescaling, the last component tends to be the exception. This treatment has several consequences. To begin with, there exists a Kantian equilibrium in pure strategies where each player chooses her n^{th} strategy.[10] Any joint strategy pair can become the special equilibrium after the strategies are relabeled. The special equilibrium can be inefficient. It can be efficient and differ from the also efficient unique Berge equilibrium in mixed strategies like in Matching Pennies.

It follows from the last two results above that for generic games, an interior equilibrium point is either Berge-Nash or Kantian. Thus, while Berge equilibria and Kantian equilibria are related under certain circumstances, they are not in general.

8 Unilateral support equilibrium

In a Berge equilibrium, for each player i, the members of coalition I_{-i} coordinate their strategies in such a way that i's payoff is maximized—given i's equilibrium strategy. One may find this kind of coordination too demanding when there are more than two players. In response, Schouten et al. (2018) have suggested an equilibrium concept in the spirit of Berge equilibrium, but without this kind of cooperation:

[10] For an $n \times n$ game G, the dual game G^o has a Nash equilibrium in mixed strategies. Hence G has a Berge equilibrium in mixed strategies. The restriction to 2-player games is crucial. See the counter-examples in 5.6 and Corley (2015).

Definition 2 Let $(I, (S_i, u_i)_{i \in I})$ be a finite strategic game and $s^* = (s_i^*)_{i \in I} \in S$.

(vi) s^* is a **unilateral support equilibrium** of G if
$u_i(s^*) \geq u_i(s_{-j}^*, s_j)$ for all $i \in I,\ j \in I_{-i}, s_j \in S_j$.

In a unilateral support equilibrium, each player unilaterally maximizes the payoff of everybody else. Crettez and Nessah (2020) have shown that with more than two players, every strong Berge equilibrium is a unilateral support equilibrium. Obviously, Berge equilibria are unilateral support equilibria irrespective of the number of players. In two-player games, unilateral support equilibria and Berge equilibria coincide. Schouten et al. (2018) give an example with a unilateral support equilibrium that is not Berge. While the set of unilateral support equilibria can be rather large, Schouten et al. (2018) further show that unilateral support equilibria need not exist.

9 The opposite of Berge equilibrium

In order to model Hobbes's state of nature, Crettez (2017) proposes a solution concept that in a sense is the opposite of Berge equilibrium and has been studied for 2×2 games before. Like throughout the paper, the following definition is formulated for finite games but can be generalized to games with arbitrary strategy spaces.

Definition 3 Let $G = (I, (S_i, u_i)_{i \in I})$ be a finite strategic game and $s^* = (s_i^*)_{i \in I} \in S$.

(vii) s^* is a **state of nature** of G if
$u_i(s^*) \leq u_i(s_i^*, s_{-i})$ for all $i \in I,\ s_{-i} \in S_{-i}$.

(viii) s^* is a **rational state of nature** of G if
it is both a state of nature of G and a Nash equilibrium of G.

Note that playing a state of nature in a 2-player game amounts to playing a Nash equilibrium when the players enter the game with adversarial social value orientations. Further note that the states of nature of G are the Berge equilibria of the game $G^- = (I, (S_i, -u_i)_{i \in I})$. Finally note that the set of rational states of nature of G need not coincide with the set of Berge-Nash equilibria of G^-. Therefore, many but not all our findings for Berge equilibria translate into statements for states of nature and vice versa. For instance, the numerical example in the next section shows that a finite game may not have a Berge equilibrium in mixed strategies. Take such a game G. Since $G^{--} = G$, the game G^-does not have a state of nature even if mixed strategies are considered.

10 Existence and computational complexity

The role of convexity. Existence proofs typically require convexity of best response sets. This can be achieved by assuming quasi-concavity of payoff functions as in Courtois et al. (2017), Crettez and Nessah (2020), by assuming uniqueness of specific best responses like in Nessah et al. (2007) and Larbani and Nessah (2008), or by directly assuming convexity of best response sets (Corley (2015)). When one moves from pure to mixed strategies, the unilateral best responses of each player are convex and, therefore, every finite game has a Nash equilibrium in mixed strategies. Essentially the same argument yields existence of a Berge equilibrium in mixed strategies for two-player games. In games with more than two players, Berge equilibria in mixed strategies need not exist, as Corley (2015) has shown. One reason is that best response sets may not be convex. A simple example is the following $2\times2\times2$ game: $I = \{1, 2, 3\}$, $S_i = \{0, 1\}$ for $i \in I$ and payoffs

$u_1(s) = -(s_2 - s_3)^2$,
$u_2(s) = s_1 - s_3$,
$u_3(s) = s_1 + s_2$

for $s = (s_1, s_2, s_3) \in S$. The only best responses (in mixed strategies) against any $s_1 \in S_1$ are the pure strategy pairs $(s_2, s_3) = (0, 0)$ and $(s_2, s_3) = (1, 1)$ whereas a best response against any $s_2' \in S_2$ requires $(s_1, s_3) = (1, 0)$ and any best response against $s_3' \in S_3$ requires $(s_1, s_2) = (1, 1)$. But the three conditions $s_2 = s_3,\ s_2 = 1, s_3 = 0$ cannot be satisfied simultaneously. Hence a Berge equilibrium in mixed strategies does not exist. Indeed, the technical reason is a lack of convexity of best response sets: For $\lambda \in (0, 1)$, the probability measure μ that assigns probability λ to $(0, 0)$ and probability $1 - \lambda$ to $(1, 1)$ is not a product measure on $S_{-1} = S_2 \times S_3$ and, therefore, is not a joint mixed strategy for players 2 and 3.

Salukvadze and Zhukovskiy (2020) obtain existence of a Berge equilibrium without any assumption that yields convex best response sets, a surprising result. They assert existence of Berge equilibrium in mixed strategies for arbitrary finite games, contrary to the last example.

Computational complexity. Nahhas and Corley (2018) have shown that the problem of finding a Berge equilibrium in mixed strategies of a finite k-player game is NP-complete. In contrast, the worst case time of finding a Berge equilibrium in pure strategies of a finite game is polynomial in input size. For let $I = \{1, \ldots, k\}$ with $k \geq 2$ and $|S_i| = n_i \geq 2$ for $i \in I$. Then $|S| = n = n_1 \cdot \ldots \cdot n_k$. To proceed, an algorithm needs as input the values $u_i(s), s \in S$ for each player i, a total of $N = kn$ numbers, and also the number of players, k. Hence the input size is $K = k + N$.

To determine whether a given joint strategy s is a Berge equilibrium (in pure strategies), for each player i, the payoff $u_i(s)$ may have to be compared with the payoffs $u_i(s_i, s'_{-i}),\ s_{-i} \in S_{-i}, s'_{-i} \neq s_{-i}$, a total of at most $|S_{-i}| - 1 < |S| = n$ comparisons. Doing this for all players would require at most $kn = N$ comparisons. In the worst case scenario, all n joint strategies s have to be checked, which amounts to $nN < K^2$ comparisons. This shows that the problem can be solved by means of a quadratic time algorithm. Of course, a limited length of the binary representation of the entries $u_i(s)$ has to be assumed.

The real problem lies in what could be called input complexity. To see this, suppose that $|S_i| = 2$ for all i. Then $|S| = 2^k$, that is the growth of input size is exponential in the number of players.

11 Final remarks

Altruism has many facets—notably biological, economic, neurological, philosophical, psychological, political, sociological, theological—and has been examined through many lenses. See, e.g., Post et al. (2002). The present study is devoted to Berge equilibrium, a game-theoretical solution concept that formalizes a particular type of altruism. We have focused on the potential of Berge equilibrium to

(a) serve as an equilibrium selection criterion;
(b) serve as substitute of Nash equilibrium;
(c) to improve upon Nash equilibrium.

Our findings are mixed: At some times but not at all times, Berge equilibrium has some of the desired properties. It has been suggested to choose the best Nash equilibria (Harsanyi and Selten (1988)), the best Berge equilibria (Colman et al. (2011), Salukvadze and Zhukovskiy (2020)) or the best of all equilibria (Courtois et al. (2015)).[11] This requires, however, to modify the epistemic premises and to add some group rationality to individual rationality. Similarly, with three or more players, Berge equilibrium assumes that for each player, the others as a group maximize the player's payoff, hence act according to some group rationality. The same applies to strong Berge equilibrium and strong Nash equilibrium as well as states of nature in the sense of Crettez (2017). Therefore, determining the epistemic status of group rationality in strategic games could be an interesting subject of future research.

A further promising research area could be the comparison of Nash equilibria and Berge equilibria—and, perhaps, Kantian equilibria—in aggregative games, that is games with payoff functions of the form $u_i(s_i, \sum_{j \in I} s_j)$.[12] Highly stylized models of this form could capture the impact of CO_2 emissions where the players are countries and s_i stands for country i's emission. The actually observed equilibrium very likely is a hybrid one, since heterogeneity prevails in countries' attitude (or social value orientation broadly conceived). Some act in pure self-interest when reducing, not reducing or even increasing their emissions. A second group is concerned with the welfare of others such as Pacific island nations or African countries. A third group invokes Kantian equilibrium reasoning: If they go ahead and serve as role model, then others will follow suit.

Last but not least, experimental studies designed specifically to test Berge equilibrium are warranted. So far, the findings of studies designed for other purposes have motivated theoretical research on Berge equilibrium while experiments devoted to Berge equilibria per se seem lacking.

Data Availability N/A.

[11] Still, the best of all equilibria may not be the best possible outcome.

[12] See Corchón (2021) for a survey on the subject.

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