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ORIGINAL PAPER

Dorm augmented college assignments

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Abstract
In college assignments, a common practice is that students receive their dorm allocation after the realization of college placements. This causes wasted resources and unfair allocation. To fix this, we consider a college assignment problem where students simultaneously receive their college and dorm assignments. We first introduce the so-called "*Dorm Augmented Deferred Acceptance*" (DDA) and show that it is stable and efficient. However, it is not student-optimal stable. We then introduce our next mechanism, "*Student-Improving Dorm Augmented Deferred Acceptance*" ($SDDA$). It is mainly built on DDA, but with some extra steps to neutralize the student-harming rejection cycles. We show that $SDDA$ is student-optimal stable, efficient, and unanimously preferred to DDA by students. Stability and strategy-proofness are incompatible, implying that neither of these mechanisms is strategy-proof. None of these mechanisms is more manipulable than the other; hence $SDDA$ improves the students' welfare without an extra strategic cost.

1 Introduction

Dorms are essential for many college students to pursue their education. In today's practice, students are placed at colleges first, and then apply for a dorm, leaving uncertainties regarding dorm assignments by the realization of college placements. This sequentiality in the assignment yields severe problems, including wasted resources and unfairness. For instance, consider a scenario where a student is placed at a college. She then applies for a dorm, but cannot get it. If the student cannot offer college life without a dorm, because of high off-campus housing rental rates, she cannot go to the college, causing a wasted quota. Moreover, such a student might envy someone else even though the former has a better ranking, causing unfair assignments.

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This problematic phenomenon is not only hypothetical but rather happening in practice. For instance, in Turkey, many college students did not enroll or had to freeze their enrollments because of not receiving a dorm assignment.[1] Even college officials might suggest students not complete their registration unless they secure a stable accommodation. For instance, Glasgow University does not guarantee a dorm to newcomers, and they give that advice.[2]

A remedy for the problem is to design a centralized matching market where students report their preferences over college-accommodation pairs so that the mechanism designer is aware of how students evaluate each contingency before concluding an assignment. This, coupled with a well-working assignment mechanism allocating colleges and dorms simultaneously, would avoid all the problems above. The current work pursues this research direction and offers a dorm-augmented college assignment problem. As extensively discussed in Sect. 2, our paper admits some unique features, which have not been considered before.

Essentially, two goods are allocated in the problem: College seats and dorms. This calls for fairness restrictions for both goods assignments. Yet another feature making the problem even more convoluted is that students might be treated differently in the assignment of these goods. That is, colleges' preferences for students might very well be different from dorm priorities.[3] This, in turn, raises tradeoffs between fairness notions, manifested in our solution. Nevertheless, we manage to introduce two plausible and compatible fairness concepts: C-fairness and D-fairness (fairness with respect to the college and dorm assignments, respectively). C-fairness rules out any pair of students being placed at different colleges, and either of them envies the other while the former is better ranked by the latter's college. D-fairness, on the other hand, only considers students in the same college. It ensures that if a dorm is given to a student at the expense of another with a higher dorm priority, either only the former has a strong dorm demand or she has a better ranking at the college.[4]

Stability requires the standard individual rationality and non-wastefulness in addition to C-fairness and D-fairness.[5] When it comes to the mechanism design, we first introduce the so-called "*Dorm-Augmented Deferred Acceptance*" (DDA). It is mainly built on the Gale and Shapley (1962)'s deferred acceptance (DA) mechanism. Students apply to college-accommodation pairs in order of their preferences. Each college first tentatively accepts the best-ranked students up to its quota without rejecting the rest. Those applying for a dorm among the tentatively-accepted group are assigned a dorm according to an ordering based on the preferences and dorm priorities until the dorm capacities are exhausted. Each student in this group is rejected upon not receiving a dorm, and someone else who is not tentatively accepted previously can replace her in the next step.

[1] https://www.voaturkce.com/a/barinamiyoruz-diyen-ogrenciler-parklarda-sabahliyor/6240312.html.

[2] https://www.bbc.com/news/uk-scotland-glasgow-west-62982938.

[3] For instance, academic standing does matter for colleges when it comes to admissions. On the other hand, students' addresses and their parents' income levels might be determining factors in dorm allocations.

[4] A student has a strong dorm demand at a college if conditional on being placed at the college, she would rather receive some other college-accommodation alternative unless a dorm is assigned to her.

[5] Individual rationality ensures that no student would rather be unassigned. Non-wastefulness, on the other hand, eliminates wasted resources.

We show that DDA is stable and efficient. However, it is not student-optimal stable.[6] We then introduce our second mechanism—*Student-Improving Dorm-Augmented Deferred Acceptance* ($SDDA$). $SDDA$ neutralizes the student-harming requests in DDA, manifested by futile applications to college-dorm pairs where the applicants ultimately receive these colleges without a dorm. We show that $SDDA$ is student-optimal stable and efficient. It is also unanimously preferred to DDA by students.

In terms of strategic properties, we find a general tension between stability and strategy-proofness, implying that none of these mechanisms is strategy-proof.[7] We also compare their degree of vulnerability à la Pathak and Sönmez (2013) and find that none of them is more vulnerable than the other. Hence, $SDDA$ improves the students' welfare over DDA without an additional strategic cost.

2 Literature review

This paper extends the classical college admission problem of Gale and Shapley (1962) by incorporating dorm assignments. It allows for separate rankings for both college and dorm assignments and addresses fairness with respect to both assignments. While our setting is similar to Hatfield and Milgrom (2005)'s matching with contract formulation, there are critical differences. Dorms cannot be interpreted as a contract term in the latter's formulation since they are real resources, which entails a fairness restriction. The double fairness imposition, empowered by separate rankings, constitutes a critical departure from the matching with contract setting and significantly affects the results, including mechanism designs and matching properties. For instance, stability and strategy-proofness are incompatible in our formulation, while the opposite is true in Hatfield and Milgrom (2005).

There is an extensive literature on generalizing college admission problems in various dimensions. Dur et al. (2019) introduce a school choice problem where certain priorities can be violated. They weaken the usual stability and propose a stable and constrained efficient mechanism. Afacan (2019) incorporates vouchers into the school choice problem, where poor students need to have a voucher to receive a private school. He introduces a stability notion and proposes a constrained efficient and stable mechanism. Abizada (2016) studies a college admission problem where colleges distribute seats and budget as stipends for the admitted students. Students have preferences over college-stipend pairs. He achieves stability and strategy-proofness despite the absence of the substitutes condition. Ehlers and Morrill (2019) define legal assignments in school choice and examine their properties.

A surging literature is on the frontier of incompatible properties in school choice. Abdulkadiroğlu et al. (2020) obtain that the Top-Trading Cycles mechanism admits minimal justified envy in the class of efficient and strategy-proof mechanisms whenever schools have unit capacity. Doğan and Ehlers (2022) extends this result to a larger set of justified envy comparison measures. Doğan and Ehlers (2021) characterize the

[6] A matching is student-optimal stable if it is stable, and no other stable matching is unanimously preferred to the former by students.

[7] A mechanism is strategy-proof if no student ever benefits by misreporting her preferences.

priority profiles ensuring the existence of an improvement over the DA outcome that is also minimally unstable among efficient assignments.

3 Model

A problem consists of $(S, C, P, \succ, \rhd, q)$ where the elements are as follows

- S and C are the finite sets of *students* and *colleges*, respectively.
- $P = (P_i)_{i \in S}$ is the *preference* profile of the students, where each P_i is a complete, asymmetric, and transitive binary relation over $(C \times \{D, N\}) \cup \{\emptyset\}$—the set of college-accommodation pairs along with being unassigned, denoted by $\emptyset$. The terms of D and N stand for the "*with dorm*" and "*without dorm*" options, respectively. We write R_i for the "*at-least-as-good-as*" relation, defined as follows: For each college-accommodation pair (c, t) and (c', t'), $(c, t)\ R_i\ (c', t')$ if and only if $(c, t)\ P_i\ (c', t')$ or $(c, t) = (c', t')$. We say that (c, t) is *acceptable* to student i if $(c, t)\ P_i\ \emptyset$; and otherwise, *unacceptable*. We write $P_{-i} = (P_j)_{j \in S \setminus \{i\}}$ for the preference profile of all students but student i.
- Each college c has a strict preference $\succ_c$ over the subsets of students, which is a complete, asymmetric, and transitive binary relation over 2^S. We assume that it is responsive (Roth 1985) to the rankings over the individual students.[8] We also make the assumption that the colleges are *acceptant*, meaning that for each student i and college c, $i \succ_c \emptyset$, indicating that no college prefers to keep its slot empty. This assumption, as discussed in Section 5, simplifies the analysis, and the entire work can still be carried out with minor modifications. Let $\succeq_c$ be the weak version of $\succ_c$, where for any pair of student groups S', S'', $S' \succeq_c S''$ if and only if $S' = S''$ or $S' \succ_c S''$. Let $\succ = (\succ)_{c \in C}$ be the profile of the college preferences.
- Each college c has a separate quota for enrollment and dorms. The enrollment (dorm) quota limits the maximum number of students that can be placed (assigned a dorm) at college c. Let $q_c = (q_c^e, q_c^d)$ be the quota vector of college c where q_c^e and q_c^d are the enrollment and dorm quotas, respectively. Let $q = (q_c)_{c \in C}$ be the quota profile of the colleges. Unless otherwise stated, we refer to the enrollment quota as the quota.
- Each college c has a strict priority ordering over S for its dorm assignments. It is denoted by $\rhd_c$, and that might very well differ from the students' rankings under $\succ_c$. Let $\rhd = (\rhd_c)_{c \in C}$.

For ease of writing, let $A = \{D, N\}$—the set of accommodation types. In the rest of the paper, we fix all the primitives except the students' preferences and denote the problem by P.

A *matching* μ is an assignment of students to the college-accommodation pairs such that no student receives more than one such pair, and no college receives more students than its quota and assigns more dorms than its dorm quota. We write μ_i for the assignment of student i under matching μ. We write μ_i^C and μ_i^A for the college and the accommodation components of μ_i, respectively. For instance, if $\mu_i = (c, D)$

[8] $\succ_c$ is responsive if, for each $S' \subset S$, and pair of students $i, j \in S \setminus S'$, (i) $S \cup \{i\} \succ_c S \cup \{j\}$ if and only if $i \succ_c j$, and (ii) $S \cup \{i\} \succ_c S$ if and only if $i \succ_c \emptyset$.

[$\mu_i = (c, N)$], it means that student i is placed at college c with a dorm [without a dorm], and we write $\mu_i^C = c$ and $\mu_i^A = D$ [$\mu_i^A = N$]. Let $\mu_c = \{i \in S : \mu_i^C = c\}$ and $\mu_c^d = \{i \in \mu_c : \mu_i^A = D\}$, the sets of students at college c and that with a dorm, respectively.

For any college c, we define $N^c(P) = \{i \in S : (c, D)\ P_i\ (c, N)$, and there is no $k \in (C \times A) \cup \{\emptyset\}$ such that $(c, D)\ P_i\ k\ P_i\ (c, N)\}$. It is the set of students, each ranking college c with a dorm and without a dorm back to back. We say that these students do not have a strong demand for a dorm at college c in that conditional on being placed at college c, they do not prefer to drop the college or to switch to another even when they do not receive a dorm. Let $D^c(P) = \{i \in S : (c, D)\ P_i\ k\ P_i\ (c, N)$ for some $k \in (C \times A) \cup \{\emptyset\}\}$. This is the set of students having a strong demand for a dorm at college c as conditional on being placed at college c, they would rather change their assignment unless they receive a dorm.

We are now ready to introduce our properties. A matching μ is *individually-rational* if $\mu_i\ R_i\ \emptyset$ for each student i. This is a standard requirement ensuring that no student would rather be unassigned. Non-wastefulness below is also canonical, eliminating wasted resources.

Definition 1 Matching μ is *non-wasteful* if there is no triplet $(i, c, t) \in S \times C \times A$ such that
(i) $(c, t)\ P_i\ \mu_i$,
(ii) $|\mu_c| < q_c$, and
(iii) $t = N$ or $t = D$ and $|\mu_c^d| < q_c^d$.

Definition 2 Matching μ is *fair with respect to the college-assignments* if there is no pair of students i, j such that $\mu_i^C \neq \mu_j^C = c$ and $i \succ_c j$, and any of the followings holds:
(i) $\mu_j^A = D$ and $(c, t)\ P_i\ \mu_i$ for some $t \in \{D, N\}$,
(ii) $\mu_j^A = N$ and $(c, N)\ P_i\ \mu_i$, or
(iii) $\mu_j^A = N$, $(c, D)\ P_i\ \mu_i$, and $|\mu_c^d| < q_c^d$.

The fairness notion above only considers student pairs placed at separate colleges, making it specific to college assignments. It eliminates envy among student tuples where the envying student is also preferred by the college, and she can be given a dorm if desired. Its only difference from the standard fairness notion is that envies by better-ranked students are tolerated as long as they demand a dorm, which is already exhausted.

The next condition takes care of fairness in dorm assignments within the colleges.

Definition 3 Matching μ is *fair with respect to the dorm-assignments* if, for each pair of students i, j such that $\mu_i^C = \mu_j^C = c$ for some college c, $\mu_i^A = D$, $\mu_j^A = N$, $(c, D)\ P_j\ \mu_j$, and $j \rhd_c i$, we have either [$i \in D^c(P)$ and $j \notin D^c(P)$], or [$i, j \in D^c(P)$ and $i \succ_c j$].

Definition 3 imposes that if dorm priorities are not respected within the colleges, it must be due to the fact that either the envying student does not have a strong demand for a dorm or both have a strong demand, but the envying student is less preferred by the college.[9] From now on, we refer to fairness with respect to the college-assignments and dorm-assignments as *C-fairness* and *D-fairness*, respectively.

Remark 1 In Appendix A, we consider a more stringent fairness notion for dorm assignments, ruling out envies by the students with a higher dorm priority. However, it turns out to be incompatible with C-fairness, as formally shown in the Appendix. Thus, we consider the above weakening. It's our modeling choice to weaken the fairness requirement for dorm assignments instead of for colleges. This is because college assignments are central to the problem; dorms complement them.

A matching is *stable* if it is individually rational, non-wasteful, C-fair, and D-fair. Matching μ *Pareto dominates* μ' where $\mu \neq \mu'$ if, for each student i and college c, $\mu_i \; R_i \; \mu'_i$ and $\mu_c \succeq_c \mu'_c$. Matching is *efficient* if it is not Pareto dominated. Matching μ is *student-optimal stable* if it is stable, and there is no other stable matching μ' such that for each student i, $\mu'_i \; R_i \; \mu_i$, where it holds strictly for some student.

Remark 2 Whenever no student ever prefers receiving a dorm, no dorm is assigned at an efficient matching. Thus, D-fairness becomes vague, and our problem reduces to the standard student placement model without dorms. Likewise, our stability comes to be equivalent to the Gale and Shapley (1962)'s stability.

Mechanism ψ is a function producing a matching for each problem P. We write $\psi(P)$ to denote its outcome at P. Mechanism ψ is $<$ *stable, efficient, student-optimal stable* $>$ if $\psi(P)$ is $<$ stable, efficient, student-optimal stable$>$ at each problem P. Mechanism ψ is *strategy-proof* if there are no P, student i, and P'_i such that $\psi_i(P) \; P_i \; \psi_i(P'_i, P_{-i})$. Mechanism ψ *Pareto dominates* ϕ if, for each problem, either $\psi(P) = \phi(P)$ or $\psi(P)$ Pareto dominates $\phi(P)$, where the latter holds for some problem.

4 The mechanism designs and the results

Before delving into the mechanisms, let us present a result that highlights some notable distinctions in our framework compared to the well-known facts concerning conventional stability in a standard college assignment setting without dorms. In the absence of dorms, stability implies efficiency, and stable matchings form a lattice. However, these established results do not hold in our framework, as demonstrated below.

Proposition 1
(i) *Stability does not imply efficiency.*
(ii) *There might be multiple student-optimal stable matchings, implying that stable matchings do not form a lattice.*

In what follows, we pursue stable and efficient mechanism designs. For this purpose, we first introduce an artificial ordering over the students, which is used in the

[9] In a sense, students promote their dorm priorities through their preference reporting by revealing their strong demand for dorms. A similar approach exists in Sönmez and Switzer (2013) where cadets improve their assignment odds by preferring extended service duration.

mechanisms' formulations later. For each college c, let $\succ'_c$ denote this ordering over S, which is defined as follows: For each $i, j \in S$, $i \succ'_c j$ if and only if any of the followings holds:

$$(I)\ i \in D^c(P) \text{ and } j \notin D^c(P), \text{ or } (II)\ i, j \in D^c(P) \text{ and } i \succ_c j, \text{ or}$$
$$(III)\ i, j \in N^c(P) \text{ and } i \rhd_c j, \text{ or } (IV) \text{ None of them holds and } i \succ_c j.$$

4.1 The dorm augmented deferred acceptance

Let us first describe the mechanism with words. Students apply to college-accommodation pairs in order of their preferences. Each college collects its applicants and tentatively accepts the best of them up to its quota. A critical difference from Gale and Shapley (1962)'s DA is that those who are not tentatively accepted are not rejected right away.[10] Instead, their applications are kept for possible late acceptance. This is because some of the tentatively accepted students might not be able to receive a dorm, causing them to withdraw their application. Hence, some quotas might come to be available, which can be filled by those who are not tentatively accepted at first.

After the tentative acceptances, colleges tentatively assign dorms to those applying for a dorm by following the artificial priorities constructed above. If a dorm-demanding student does not receive one, she is rejected for her application. She may apply to the same college again but without a dorm application.

Here is the formal definition of our first mechanism: Given a problem P,

Step 1. Each student applies to her best acceptable college-accommodation pair. Let S_c^1 be those applying to college c for any accommodation type (that is, they are applying to (c, D) or (c, N)).

College c tentatively accepts its most preferred students in S_c^1 up to its quota, and let T_c^1 be the set of these students. A twist here is that college c does not reject the rest now, as some of the tentatively accepted students may be rejected later on, because of not receiving a dorm, yielding a room for tentatively accepting the others.

Let $T_c^1(D) \subseteq T_c^1$ be the set of tentatively accepted students applying to (c, D). We order the students in $T_c^1(D)$ following $\succ'_c$. That is, for any $i, j \in T_c^1(D)$, i comes before j if and only if $i \succ'_c j$.

The first $\min\{q_c^d, |T_c^1(D)|\}$ students in $T_c^1(D)$ tentatively receive a dorm. If no student in $T_c^1(D)$ is left without a dorm, then college c rejects all the students in $S_c^1 \setminus T_c^1$. Otherwise, only the students in $T_c^1(D)$ without a dorm assignment are rejected for their (c, D) application. Let $K_c^1 \subseteq S_c^1$ be the set of students who are not rejected by college c, called "*the set of tentatively kept students*." We then move to the next step.

[10] In DA, if a student is rejected by a college, the rejection is permanent. However, in our mechanism, tentatively accepted students might withdraw their applications if they do not receive a dorm. As a result of these withdrawals, the newly available quotas can be used to assign previously rejected students.

In general,

Step m. Each rejected student applies to her next best acceptable college-accommodation pair. College c tentatively accepts its most preferred students from $S_c^m \cup K_c^{m-1}$ up to its quota without rejecting the rest. Let T_c^m and $T_c^m(D)$ denote the sets of tentatively accepted students, and those with a dorm demand, respectively. That is, $T_c^m(D) = \{ i \in T_c^m : (c, D)\ P_i\ (c, N)\}$.

We order the students in $T_c^m(D)$ following $\succ'_c$. The first $\min\{q_c^d, |T_c^m(D)|\}$ students in $T_c^m(D)$ tentatively receive a dorm. If each student in $T_c^m(D)$ receives a dorm, then college c rejects all in $(S_c^m \cup K_c^{m-1}) \setminus T_c^m$. Otherwise, those in $T_c^m(D)$ without a dorm are rejected for their (c, D) application. Let $K_c^m \subseteq S_c^m \cup K_c^{m-1}$ be the set of students not rejected in this step.

The algorithm terminates whenever each student is tentatively accepted or has gotten a rejection from each of her acceptable alternatives. The assignments by the end of the terminal step constitute the algorithm's outcome. We call the algorithm "*Dorm augmented deferred acceptance*" (DDA).

Before formally showing the properties of DDA, we run it on an example below.

Example 1 Let us consider six students, $i_1, .., i_6$, and three colleges, $c_1, .., c_3$. Each college has a capacity of 2 and a dorm capacity of 1. The preferences and priorities are as follows:

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\rhd_{c_1}$
(c_1, D)	(c_1, D)	(c_2, D)	(c_3, D)	(c_3, D)	(c_3, N)	i_3	i_2	i_4	i_2
(c_2, D)	(c_2, D)	(c_1, D)	$\emptyset$	$\emptyset$	$\emptyset$	i_1	i_3	i_5	i_1
(c_1, N)	$\emptyset$	$\emptyset$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\emptyset$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Note that $D^{c_1}(P) = D^{c_2}(P) = \{i_1, i_2, i_3\}$, and $D^{c_3}(P) = \{i_4, i_5\}$. Therefore, the relevant artificial priorities directly come from the colleges' preferences. That is, $i_3 \succ'_{c_1} i_1 \succ'_{c_1} i_2$; $i_2 \succ'_{c_2} i_3 \succ'_{c_2} i_1$; and $i_4 \succ'_{c_3} i_5 \succ'_{c_3} i_6$.

Step 1. Students i_1 and i_2 apply to (c_1, D). College c_1 tentatively accepts both students. As $i_1 \succ'_{c_1} i_2$, student i_1 tentatively receives a dorm, and i_2 is rejected for her (c_1, D) application.

Student i_3 applies to (c_2, D), and she is tentatively accepted with a dorm. Students i_4 and i_5 apply to (c_3, D) while i_6 applies to (c_3, N). i_4 and i_5 are tentatively accepted. However, as $i_4 \succ'_{c_3} i_5$, student i_5 does not receive a dorm, hence she is rejected. Note that i_6 is tentatively kept.

Step 2. Students i_2 applies to (c_2, D). As $i_2 \succ'_{c_2} i_3$, student i_2 tentatively receives a dorm, and student i_3 is rejected. On the other hand, student i_6 is tentatively accepted by college c_3 in this step. As she is not applying for a dorm, student i_4's dorm assignment is not affected.

Step 3. Student i_3 applies to (c_1, D). As $i_3 \succ'_{c_1} i_1$, student i_3 is tentatively accepted with a dorm; hence student i_1 stops receiving dorm and is rejected.

Step 4. Student i_1 applies to (c_2, D). As $i_2 \succ'_{c_2} i_1$, she cannot receive a dorm; hence she is rejected.

Step 5. Student i_1 applies to (c_1, N). She is tentatively accepted without a dorm.

The algorithm terminates by the end of Step 5. If we write μ for the DDA outcome, we have $\mu_{i_1} = (c_1, N)$, $\mu_{i_2} = (c_2, D)$, $\mu_{i_3} = (c_1, D)$, $\mu_{i_4} = (c_3, D)$, $\mu_{i_5} = \emptyset$, and $\mu_{i_6} = (c_3, N)$.

Theorem 1 *DDA terminates in a finite number of steps and produces a matching. It is stable and efficient. However, DDA is not student-optimal stable.*

The reason why DDA fails to achieve student-optimal stability is the fact that stability constraints are not fixed but rather change depending on the students' assignments. More specifically, the constraints a student imposes on college c assignments differ when she receives the college c without a dorm and when she is placed at a college different from c. However, DDA remains rigid in respecting the priorities, causing more rejections than necessary for stability. This harms students beyond stability, explaining why it fails to achieve student-optimal stability. For example, in the above scenario, i_2 is rejected for her (c_1, D) application. However, conditional on student i_1 receiving (c_1, N), student i_2 could obtain (c_1, D) under stability as $i_2 \rhd_{c_1} i_1$.

Our next mechanism below improves students' welfare over DDA while preserving stability.

4.2 The student-improving dorm-augmented deferred acceptance (SDDA)

This mechanism—$SDDA$—is built on Kesten (2010)'s efficiency-adjusted deferred acceptance mechanism. Informally, $SDDA$ identifies students rejected for their dorm demand at their colleges under the DDA matching. It then removes the alternatives consisting of these colleges with a dorm from those students' preferences and runs DDA with a twist that any tentatively accepted student should have a higher artificial or dorm priority than those students. Below is its formal description: Given a problem P,

Step 0. We apply DDA to problem P and write μ^0 for the outcome. For each college c, let $I_c = \emptyset$.

Step 1.

Substep 1.1. Let $N^1 = \{(i, c) \in S \times C : (c, D) \; P_i \; (c, N) \text{ and } \mu_i^0 = (c, N)\}$. Note that a student cannot be paired with more than one college in N^1.

If $N^1 = \emptyset$, then the algorithm ends with the final outcome of μ^0. Otherwise, for each $(i, c) \in N^1$, we identify the step in which student i's (c, D) application is rejected in the previous DDA application. We next consider the pairs in N^1 having this rejection step latest. For each such pair (i, c), we define artificial preferences for student i by demoting (c, D) to the least preferred unacceptable alternative while keeping the rest of the rankings the same. We keep the other students' preferences the same. We also add i to I_c. Let P^1 denote these artificial preferences. We then move to the next substep.

Substep 1.2. We apply DDA to P^1 with a twist that a tentatively accepted student j at a college c is rejected for her dorm demand whenever $i \succ'_c j$ and $i \rhd_c j$ for some $i \in I_c$. We write μ^1 for the outcome and move to the next step.

In general,

Step k.

Substep k.1. Let $N^k = \{(i, c) \in S \times C : (c, D) \; P_i^{k-1} \; (c, N)$ and $\mu_i^{k-1} = (c, N)\}$. If $N^k = \emptyset$, the algorithm ends with the final outcome of μ^{k-1}. Otherwise, for each $(i, c) \in N^k$, we identify the step in which student i is rejected for her (c, D) application in the previous step DDA application. We pick the student-college pairs admitting this rejection step latest. For each such pair (i, c), we change the ranking of (c, D) under P_i^{k-1} and put it at the very end of the preferences below the being unassigned alternative, while keeping the other alternatives' positions the same. We also add i to I_c. We keep the other students' preferences the same as under P^{k-1} and write P^k for the newly created preferences.

Substep k.2. We apply DDA to P^k with a twist that a tentatively accepted student j at a college c is rejected for her dorm demand whenever $i \succ'_c j$ and $i \rhd_c j$ for some $i \in I_c$. We write μ^k for the outcome and move to the next step.

As formally shown in Theorem 2, N^k becomes empty in some Step k, meaning that the algorithm terminates in a finite time. We call the algorithm "*Student-improving dorm-augmented deferred acceptance*" ($SDDA$).

Let us run $SDDA$ on the problem in Example 1 to see its difference from DDA.

Example 2 **Step 0.** This step consists of DDA to be applied to the problem P. It is already performed in Example 1, and the outcome μ^0 is such that $\mu_{i_1}^0 = (c_1, N)$, $\mu_{i_2}^0 = (c_2, D)$, $\mu_{i_3}^0 = (c_1, D)$, $\mu_{i_4}^1 = (c_3, D)$, $\mu_{i_5}^1 = \emptyset$, and $\mu_{i_6}^1 = (c_3, N)$.

Step 1.

Substep 1.1. In the DDA application in Example 1, we see that student i_1 receives (c_1, N) while $(c_1, D) \; P_{i_1} \; (c_1, N)$. Thus, $(i_1, c_1) \in N^1$. On the other hand, student i_6 receives (c_3, N), but since $(c_3, N) \; P_{i_6} \; (c_3, D)$, $(i_6, c_3) \notin N^1$. No other student is placed at a college without a dorm, hence $N^1 = \{(i_1, c_1)\}$.

We define $P_{i_1}^1 : (c_2, D), (c_1, N), \emptyset..$; and $P_{-i_1}^1 = P_{-i_1}$. We also add i_1 to I_{c_1}.

Substep 1.2. We next apply DDA to P^1 with a twist that any tentatively accepted student j at college c_1 is rejected for her dorm demand whenever $i \succ'_{c_1} j$ and $i \rhd_{c_1} j$.

If we run DDA to P^1, it is easy to verify that the outcome μ^1 is such that $\mu_{i_1}^1 = (c_1, N)$, $\mu_{i_2}^1 = (c_1, D)$, $\mu_{i_3}^1 = (c_2, D)$, $\mu_{i_4}^1 = (c_3, D)$, $\mu_{i_5}^1 = \emptyset$, and $\mu_{i_6}^1 = (c_3, N)$. Note that as $i_2 \rhd_{c_1} i_1$, she is not rejected for her (c_1, D) application.

Step 2.

Substep 2.1. $N^2 = \emptyset$, thus the algorithm terminates, with the final outcome of μ^1.

As we see from the outcomes, μ^1 is stable, efficient, student-optimal stable, and unanimously preferred to μ^0 by the students. Notice that the reason why $SDDA$ differs from DDA at the problem is the fact that $i_2 \rhd_c i_1$ so that i_2 is not rejected for her (c_1, D) application in $SDDA$. However, if it were $i_1 \rhd_c i_2$, then i_2 would have been rejected for (c_1, D) in Substep 1.2 above, thereby the $SDDA$ outcome would remain the same as μ^0.

Theorem 2 *$SDDA$ is stable, efficient, and student-optimal stable. Moreover, its outcome is always at least weakly better than DDA's for each student.*

By Proposition 1, there might be multiple student-optimal stable matchings. $SDDA$ selects a particular one among them. Its selection in a given problem depends on the dynamics of DDA in that problem, which in turn relies on the primitives.

Let us next investigate the strategic properties of the mechanisms. A mechanism ψ is *manipulable* by a student i at problem P if $\psi_i(P_i', P_{-i})\ P_i\ \psi_i(P)$ for some P_i'. Mechanism ψ is *strategy-proof* if there is no problem at which it is manipulable by some student.

Proposition 2 *There is no stable and strategy-proof mechanism.*

Corollary 1 *Neither DDA nor $SDDA$ is strategy-proof.*

The tension between stability and strategy-proofness emerges because of D-fairness. Without it, Gale and Shapley (1962)'s deferred acceptance where dorms are assigned to the demanders with the better ranking at the college preferences would satisfy all the properties except D-fairness. It is also strategy-proof. However, under D-fairness, students with a strong dorm demand (i.e., those in $D^c(P)$) have an advantage in dorm assignment. Thus, they might benefit from false reporting, which puts them in the strong dorm demanders.

We next compare the degree of strategic vulnerability of DDA and $SDDA$ á la Pathak and Sönmez (2013).[11]

Proposition 3 *There is some problem at which DDA is manipulable, but not $SDDA$; and the converse is also true.*

Thus, neither of these mechanisms is more manipulable than the other. Hence, $SDDA$ improves students' welfare over DDA without a strategic cost in the sense of Pathak and Sönmez (2013).

5 Discussion

We assume a single type of dorm at each college. However, the analysis can be extended to accommodate multiple dorm types. In this extended framework, students will have further preferences over the dorm types within each college. D-fairness becomes more stringent, dealing with the students' preferences over dorm types. We define strong dorm demand, similar to what we do in the main body, for each dorm type. We also construct artificial priorities for each dorm type similarly. When it comes to the mechanisms, each dorm type is assigned to its applicants based on the artificial priorities defined for that type, and so on.

Yet another easily generalizable aspect of the modeling is regarding the colleges' preferences. The base model assumes that no college finds any student unacceptable.

[11] Pathak and Sönmez (2013) say that a mechanism is more manipulable than another if the former is manipulable whenever the latter is, but the converse does not hold.

However, the whole analysis could be carried out for that more general class of college preferences through simple modifications to stability and mechanisms.

An interesting restricted domain is one where a group of students consistently prefers colleges with dormitories, and the opposite for the rest of the students. We can categorize the former group as “poor" and the latter as “rich" students. If each college is considered unacceptable without a dormitory for the poor group, then DDA becomes equivalent to the following: Tentatively allocate the dorms to the poor applicants based on their rankings in the college preferences up to the capacity and reject those without a dorm assignment. Subsequently, the tentative dorm holders and all the other rich applicants compete for the college seats, and they are tentatively accepted following the college preferences. Upon being rejected, the dorm holders lose their dorms as well, and the rejected students apply to their next best alternative, and so on. It is important to note that no rich student is allocated a dorm. Furthermore, as no poor student ever receives a college without a dorm, DDA is equivalent to $SDDA$. Therefore, DDA is student-optimal stable and efficient. It is also strategy-proof, as all the poor students already have a strong demand for dormitories. Thus, they cannot improve their ranking in the artificial priorities used for dormitory assignments through misreporting. This special case resembles Afacan (2019), where the poor group has access to private schools only when they obtain a voucher. A critical distinction in the modeling is that a voucher can be used at any private school, whereas dormitories are college-specific in the current problem. This leads to different mechanisms, causing researchers to diverge even in this special case.

On the other hand, the positive strategy-proof result mentioned above breaks down even if only one poor student finds a college without a dorm option acceptable. To illustrate this, consider the proof of Proposition 2. In that scenario, both students are poor, with one of them still finding the college without a dorm acceptable, and each stable mechanism is manipulable in that instance. Furthermore, $SDDA$ is no longer equivalent to DDA in this case, as a student can be assigned to a college without a dormitory even though she prefers a college with a dormitory.

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Appendices

A. A stronger fairness notion for dorm asssignments

Definition 4 A matching μ is strongly fair with respect to the dorm-assignments if there is no pair of students i, j such that $\mu_i^C = \mu_j^C = c$ for some college c, $\mu_i^A = D$, $\mu_j^A = N$, $(c, D)\ P_j\ (c, N)$, and $j \rhd_c i$.

Proposition 4 *There does not always exist a non-wasteful and C-fair matching that is strongly fair with respect to the dorm-assignments.*

Proof Consider a problem with two students i, j and one college c. Let $q_c^e = 2$ and $q_c^d = 1$. Let the preferences be such student i's only acceptable alternative is (c, D), whereas student j prefers (c, D) to (c, N) while finding both acceptable. Let $i \succ_c j$ and $j \rhd_c i$. There is only one non-wasteful and C-fair matching where student i receives (c, D) and student j receives (c, N). This matching, however, is not stronlgy D-fair. $\square$

B. The proofs of Proposition 1, Theorem 1, and Theorem 2

Proof of Proposition 1 (i). Let us consider a problem with two students i, j and one college c, with $q_c^e = 2$ and $q_c^d = 1$. Let the preferences be such that $P_i : (c, N), (c, D), \emptyset$; and $P_j : (c, D), (c, N), \emptyset$. Suppose $i \succ_c j$ and $i \rhd_c j$. Let μ be a matching such that $\mu_i = (c, D)$ and $\mu_j = (c, N)$. This matching is stable. However, it is Pareto dominated by μ' where $\mu'_i = (c, N)$ and $\mu'_j = (c, D)$, showing that μ is not efficient.

(ii). In the same market above, let us consider preferences $P_i : (c, D), (c, N), \emptyset$; and $P_j : (c, D), \emptyset$. There are two stable matchings, say μ and μ': $\mu_i = (c, D)$; $\mu_j = \emptyset$; and $\mu'_i = (c, N)$; $\mu'_j = (c, D)$. Note that as $j \in D^c(P)$ and $i \notin D^c(P)$, μ' is D-fair. Neither of them is unanimously preferred to the other by the students, implying that both of them are student-optimal stable. This also implies that the stable matchings do not form a lattice. $\square$

The following Lemma will be used in the rest of the proofs.

Lemma 1 *Let P be a problem and μ be a stable matching. Suppose μ is such that for each student i with $\mu_i = (c, D)$ for some college c, $(c, D)\ P_i\ (c, N)$. Then, μ is efficient.*

Proof Assume for a contradiction that there is another matching μ' Pareto dominating μ. We first claim that for some college c, $\mu'_c \neq \mu_c$. Assume for a contradiction that $\mu'_c = \mu_c$ for each college c. This means that for some student i with $\mu_i'^C = \mu_i^C = c$, either $\mu'_i = (c, D)$ and $\mu_i = (c, N)$ or $\mu'_i = (c, N)$ and $\mu_i = (c, D)$. Suppose $\mu'_i = (c, N)$ and $\mu_i = (c, D)$. This means that $(c, N)\ P_i\ (c, D)$, contradicting our supposition. Thus, consider $\mu'_i = (c, D)$ and $\mu_i = (c, N)$. We have $(c, D)\ P_i\ (c, N)$. By the stability (hence, non-wastefulness) of μ. we have $|\mu_c^d| = q_c^d$. This, as well as $\mu_c = \mu'_c$ for each college c, implies that for some student j, we have $\mu_j = (c, D)$

and $\mu'_j = (c, N)$. This, however, falls into the previous case, yielding a contradiction. Thus, we conclude that $\mu_c \neq \mu'_c$ for some college c.

Let $i \in \mu'_c \setminus \mu_c$. Note that as μ' Pareto dominates μ, it means that $\mu'_c \succ_c \mu_c$. Suppose $\mu'_i = (c, N)$. By the non-wastefulness of μ, $|\mu_c| = q_c$. On the other hand, by the responsiveness of $\succ_c$, for some student $j \in \mu'_c \setminus \mu_c$, we have $i \succ_c j$. Thus, we have $(c, N)\ P_i\ \mu_i$, where $\mu_i^C \neq c$, and $i \succ_c j$ for some $j \in \mu_c$. This, however, contradicts the stability of μ. Next, suppose that $\mu'_i = (c, D)$. As the same as above, for some student $j \in \mu'_c \setminus \mu_c$, we have $i \succ_c j$. If $\mu_j = (c, D)$, then it contradicts the stability of μ. Thus, $\mu_j = (c, N)$. If $|\mu_c^d| < q_c$, it constitutes a contradiction to the stability of μ. Suppose that $|\mu_c^d| = q_c$. This implies that there exists some student $h \in \mu_c^d$ such that $\mu'_h \neq \mu_h$. Moreover, from above, $\mu'_h \neq (c, N)$. This, along with the responsiveness, in turn, implies that for some such student $h \in \mu_c^d$ and $\mu_h'^C \neq c$, we have $i \succ_c h$. Thus, we have $(c, D)\ P_i\ \mu_i$, $i \succ_c h$, and $\mu_h = (c, D)$, contradicting the stability of μ. Therefore, μ is efficient, finishing the proof. □

Proof of Theorem 1 In each step, some rejected student applies to her next best acceptable college-accommodation pair. This, along with the fact that there are finitely many students and college-accommodation types, implies that DDA terminates in a finite step. By its construction, no student receives more than one college-accommodation assignment, and no college has more students than its capacity and gives more dorms than its capacity. All these show that DDA terminates in a finite step, producing a matching; hence it is a well-defined mechanism.

Let us next show that DDA is stable. No student ever applies to her unacceptable alternatives, implying that it is individually rational. Students apply in order of their preferences. A student with a dorm demand at a college is never rejected whenever the college has an available enrollment and dorm quota. If she does not have a dorm demand, then the college never rejects her unless it already fills the quota. All these mean that DDA is non-wasteful.

Let us next show that DDA is C-fair. Let P be a problem and $DDA(P) = \mu$. Let us consider a pair of students i, j with $\mu_i^C = c$ and $\mu_j^C = c'$ with $c \neq c'$. Let $j \succ_c i$. Suppose that $\mu_i^A = D$ and $(c, t)\ P_j\ \mu_j$ for some $t \in \{D, N\}$. This means that student j applies to either (c, D) or (c, N) in DDA. This, along with $j \succ_c i$, means that $\mu_j\ P_j\ (c, N)$. Because otherwise, she would have received (c, N) at DDA at most at the expense of student i. Hence, we have $(c, D)\ P_j\ \mu_j\ P_j\ (c, N)$. This means that $j \in D^c(P)$. On the other hand, $j \succ_c i$; hence $j \succ'_c i$. This, in turn, means that student j has a priority against student i in both the seat allocation at college c and its dorm assignment. This, however, contradicts our supposition.

Suppose $\mu_i^A = N$ and $(c, N)\ P_j\ \mu_j$. In DDA, student j applies to (c, N), but is rejected. As $j \succ_c i$ and $\mu_i = (c, N)$, it cannot happen, yielding a contradiction.

Suppose $\mu_i^A = N$, $(c, D)\ P_j\ \mu_j$ and $|\mu_c^d| < q_c$. Student j applies to (c, D) in DDA. Moreover, $\mu_j\ P_j\ (c, N)$, because, otherwise, we reach a contradiction from above (recall that $\mu_j^C = c' \neq c$, hence $\mu_j \neq (c, N)$). Thus, we have $(c, D)\ P_j\ \mu_j\ P_j$ (c, N), hence $j \in D^c(P)$.

As $j \succ_c i$, student j is rejected by college c because of dorm shortage. This, as well as $j \in D^c(P)$, shows that all the tentatively dorm-receiving students at college c by the rejection of student j are more preferred to the latter by college c. Hence, in particular, they all have a better artificial ranking than student j, implying that they are all in $D^c(P)$. On the other hand, $|\mu_c^d| < q_c^d$ implies that some of these students are rejected by college c for the sake of another, say k, applying to college c without a dorm demand. Thus, student k does not compete for a dorm at college c. From above, all the tentatively dorm receivers after the rejection of student j have a better ranking than student j, implying that each is preferred to student i by college c. This, as well as $\mu_i = (c, N)$, shows that none of them is rejected for the sake of student k as they are better ranked than student i, and student k does not have a dorm demand. This implies that college c cannot have an excess dorm, contradicting $|\mu_c^d| < q_c^d$. Therefore, μ is C-fair.

Let us next show that μ is D-fair. Suppose for a pair of students i, j, we have $\mu_i^C = \mu_j^C = c$ for some college c, $\mu_i^A = D$, $\mu_j^A = N$, $(c, D)\ P_j\ \mu_j$, and $j \rhd_c i$. As $(c, D)\ P_j\ \mu_j = (c, N)$, student j also applies for (c, D) in DDA. Hence, both compete for a dorm. This, as well as the fact that student i obtains a dorm, shows that $i \succ'_c j$. Thus, either $i \in D^c(P)$ and $j \notin D^c(P)$ or $i \succ_c j$. This shows that μ is D-fair. Hence, μ is stable, so is DDA.

By the definition of DDA, no student receives a dorm at a college unless she demands it. That is, for each student i with $\mu_i = (c, D)$ for some college c, we have $(c, D)\ P_i\ (c, N)$. Thus, we can invoke Lemma 1 to conclude that μ is efficient, and so is DDA. Example 2 reveals that DDA is not student-optimal stable. □

The Lemma below will be used in the proof of Theorem 2.

Lemma 2 *At any problem P, let μ^k be the Step k outcome in $SDDA$. For each student i, $\mu_i^k\ R_i\ \mu_i^{k-1}$. Moreover, if $(i, c) \in N^k$, and an artificial preference list is defined for student i in Step k, then $\mu_i^t = (c, N)$ for each Step t outcome in $SDDA$ where $t \geq k$.*

Proof Let us consider Step k in $SDDA$. Let us suppose that $(i, c) \in N^k$ (note that $N^k \neq \emptyset$, because, otherwise, the algorithm terminates with the final outcome of μ^{k-1}), and student i is rejected for her (c, D) application not earlier than any such pair in N^k. This means that $\mu_i^{k-1} = (c, N)$, and she is rejected for her (c, D) application latest among all such pairs in the previous step's DDA application. As she is rejected latest, she receives the same rejections in Step k's DDA application in $SDDA$, showing that $\mu_i^k = (c, N)$. Moreover, in Step k, the rejection rule for (c, D) is relaxed. This is because in order for a student j to receive (c, D) in the previous step, she has to be ranked better than student i with respect to $\succ'_c$. But in Step k, she can have a better ranking than student i based on $\succ'_c$ or $j \rhd_c i$. This, in turn, implies that no student receives an extra rejection in Step k's DDA application compared to the previous step. Thus, for each student j, $\mu_j^k\ R_j\ \mu_j^{k-1}$.

Suppose $(i, k) \in N^k$, and student i is rejected for her (c, D) application latest among all such pairs in N^k. From above, we have $\mu_i^k = (c, N)$. Suppose for a contradiction that $\mu_i^t \neq (c, N)$ for some $t > k$. This means that for some $(j, c') \in N^t$, we define artificial preferences for student j, causing student i to receive a better assignment (from above, we know that students are never worse off after each step). This means that student j's (c', D) application causes a rejection cycle, placing student i at (c, N). Moreover, as it is a rejection cycle, once student i receives (c, N), someone else is rejected from (c, N), and so on. These rejections ultimately lead student j to be rejected from (c', D). This shows that in Step $k-1$, where student i's preferences are changed in $SDDA$, student j is rejected from (c', D) in a later step than that in which student i is rejected from (c, D). However, as j's preferences are not changed in Step k, student j does not receive (c', N) in Step $k-1$, but in some later step. But then, this means that student j starts receiving (c', N) after some student h's preferences are changed in $SDDA$ through removing (c'', D) for some college c''. By the same token as above, it means that student h is rejected for (c'', D) in a later step than student j's rejection from (c', D). If student h receives (c'', N) in Step t, then student j's preferences are not changed in this step, a contradiction. Thus, student h does not receive (c'', N) in Step t. Note that all the students i, j, h are different from each other, as the step in which each is rejected from her relevant college-dorm application is different. If we continue applying the same arguments above for student h, we need to find a different student in each step, which is not possible due to the finite number of students. Thus, it eventually leads to a contradiction, showing that $\mu_i^t = (c, N)$ for each $t \geq k$, finishing the proof. □

Proof of Theorem 2 Let us consider N^k in some Step k in $SDDA$. If it is non-empty, we consider the pairs $(i, c) \in N^k$ who are rejected for her dorm application by college c not earlier than any such pair in N^k. We then define an artificial preference for student i where she reports (c, D) unacceptable. This implies that (i, c) cannot be included in N^{k+1}. As both students and colleges are finite, it shows that N^k comes to be empty after some step, hence $SDDA$ terminates in a finite time. As no student receives more than one assignment, and no college admits more students than its quota and gives more dorms than its dorm capacity in DDA (this clearly holds for the slight DDA modifications in the $SDDA$ steps), the $SDDA$ outcome in its terminal round is a matching. Hence, we conclude that $SDDA$ is a well-defined mechanism.

Let us next show that $SDDA$ is stable. Let P and μ be a problem and its outcome at P. In each $SDDA$ step, students only apply to their acceptable alternatives, meaning that they would never rather be unassigned. Hence, μ is individually rational. Let us next show that μ is non-wasteful. From Theorem 1, we know that μ^0 is non-wasteful. Let us consider the Step 1 outcome of $SDDA$, μ^1. We claim that μ^1 is non-wasteful. If $\mu^1 = \mu^0$, then there is nothing to prove. Otherwise, $N^1 \neq \emptyset$, and the preferences of some students from N^1 are changed, as defined in $SDDA$. Let us suppose that student i is one of them, and (c, D) is put at the very end of her preferences. Assume for a contradiction that μ^1 is wasteful. The rejection rules are the same as in DDA for each college accommodation pairs except those removed from the preferences. Thus, the only possibility is the existence of a waste at some college-accommodation pair that is removed from the preferences. However, if a student j is rejected by (c, D) because $i \succ_c' j$ and $i \rhd_c j$, then he cannot obtain it in DDA as well (because $i \succ_c' j$ and student i is rejected for (c, D)). In this case, the same rejections occur in Step 1's DDA as the original DDA, causing the same matching. This, along with the non-wastefulness of μ^0, implies that μ^1 is non-wasteful. If we apply the same arguments for μ^1 and μ^2, we conclude that μ^2 is non-wasteful, and so on. Thus, $SDDA$ is non-wasteful.

Let us next show that μ is C-fair. From Lemma 2, we know that if (c, D) is removed from the preferences of student i, then $\mu_i = (c, N)$. Moreover, student i continues to apply all the college-accommodation pairs except (c, D) in $SDDA$. Thus, she cannot violate C-fairness. On the other hand, all the other students whose preferences are not changed in $SDDA$ already apply to all alternatives in order of their preferences; thus, they do not violate C-fairness. All these show that μ is C-fair.

To show its D-fairness, let us consider a pair of students i, j such that $\mu_i^C = \mu_j^C = c$, $\mu_i^A = D$, $\mu_j^A = N$, and $(c, D)\ P_j\ (c, N)$. Then, by $SDDA$ definition, either $i \succ'_c j$ or $i \rhd_c j$ (the latter arises if (c, D) is demoted in student j's preferences in $SDDA$. Otherwise, the former is always the case). Thus, D-fairness is never violated, showing that $SDDA$ is D-fair. Hence, $SDDA$ is stable.

By the definition of $SDDA$, no student receives a dorm at a college unless she demands it. Thus, for each student i with $\mu_i = (c, D)$, we have $(c, D)\ P_i\ (c, N)$. Thus, by Lemma 1, μ is efficient, and so is $SDDA$.

From Lemma 2, $SDDA$ outcome is always at least weakly better than DDA's for each student. We lastly show that $SDDA$ is student-optimal stable. Assume for a contradiction that it is not. By following above, if we write μ for the $SDDA$ outcome at problem P, there exists another stable matching μ' such that for each student i, $\mu'_i\ R_i\ \mu_i$, where it holds strictly for some student. Let $W = \{i \in S : \mu'_i\ P_i\ \mu_i\}$. By our supposition, $W \neq \emptyset$. Let $i \in W$. By the individual rationality of μ, $\mu'_i \neq \emptyset$. Let us write $\mu'_i = (c, t) \in C \times A$. Because of the non-wastefulness of μ, there exists another student $j \in W$ such that $\mu_j = \mu'_i = (c, t)$. For the same reason, for some student $k \in W$, we have $\mu_k = \mu'_j$. That is, for each $i \in W$, there exists another student $j \in W$ such that $\mu_j = \mu'_i$. That is, the students in W are better off by trading their assignments under μ with each other. In other words, there are beneficial cyclic trades at μ among those in W. Moreover, once we implement these trades among the students in W, we obtain μ'. Note that as μ' is at least weakly better than μ for the student side, for each student $i \notin W$, we have $\mu_i = \mu'_i$.

In $SDDA$, students apply to the college-accommodation pairs in decreasing order of their preferences. This means that the students in W apply for their assignments under μ'. However, in $SDDA$, students in W receive their inferior options under μ, causing the beneficial cyclic trades discussed above. Moreover, these trades are stability preserving because of the stability of μ'. This shows that the rejection rule in $SDDA$ is more than what is needed for stability, implying that some student, say i, is rejected for (c, D) and ultimately receives (c, N). This is because, otherwise, D-fairness requires that no student j receives (c, D) whenever $i \succ'_c j$, implying that $SDDA$ exactly requires what is needed for stability. Thus, in $SDDA$, some student receives (c, N) after being rejected by (c, D) for some college c. This, however, cannot be possible as (c, D) is ranked as unacceptable in the artificial preferences of student i in $SDDA$. Hence, μ is student-optimal stable, and so is $SDDA$. □

C. Proofs of Proposition 2 and Proposition 3

Proof of Proposition 2 Let us consider a problem with two students i, j, and one college c. Let $q_c^e = 2$ and $q_c^d = 1$. Let P be a problem such that $P_i : (c, D), (c, N), \emptyset$; and $P_j : (c, D), \emptyset$. Let $i \succ_c j$ and $j \rhd_c i$.

Let ψ be a stable mechanism. There is only one stable matching μ where $\mu_i = (c, N)$, $\mu_j = (c, D)$. However, student i can benefit by reporting (c, D) as the only acceptable choice: Under the misreporting, there is a unique stable matching ν where $\nu_i = (c, D)$, $\nu_j = \emptyset$. This shows that student i benefits by misreporting; hence, no stable mechanism is strategy-proof, finishing the proof. □

Proof of Proposition 3 Let us consider six students $i_1, .., i_6$ and three colleges c_1, c_2, c_3. Let $q_{c_1} = q_{c_2} = 2$, and $q_{c_3} = q^d_{c_3} = q^d_{c_1} = q^d_{c_2} = 1$.

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\rhd_{c_2}$
(c_3, N)	(c_2, D)	(c_2, D)	(c_2, N)	(c_1, D)	(c_1, N)	i_1	i_6	i_3	i_3
(c_1, D)	(c_3, N)	(c_3, N)	$\emptyset$	$\emptyset$	(c_2, D)	i_5	i_2	i_1	i_2
(c_1, N)	(c_2, N)	$\emptyset$	$\vdots$	$\vdots$	$\emptyset$	$\vdots$	i_3	$\vdots$	$\vdots$
$\emptyset$	$\emptyset$						$\vdots$		

In the problem above, DDA outcome, say μ, is such that $\mu_{i_1} = (c_1, N)$, $\mu_{i_2} = (c_2, N)$, $\mu_{i_3} = (c_3, N)$, $\mu_{i_4} = \emptyset$, $\mu_{i_5} = (c_1, D)$, and $\mu_{i_6} = (c_2, D)$. Let us next consider a false preferences for student i_1, say P'_{i_1}, where P'_{i_1} : $(c_1, D), \emptyset$. It is immediate to see that $DDA_{i_1}(P'_{i_1}, P_{-i_1}) = (c_1, D)$, benefiting student i_1, hence DDA is manipulable at P.

Let us next consider $SDDA$ at P. Its outcome, say μ', is such that $\mu'_{i_1} = (c_3, N)$, $\mu'_{i_2} = (c_2, N)$, $\mu'_{i_3} = (c_2, D)$, $\mu'_{i_4} = \emptyset$, $\mu'_{i_5} = (c_1, D)$, and $\mu'_{i_6} = (c_1, N)$. In $SDDA$, (c_2, D) is removed from the preferences of student i_2, causing a different matching from DDA's to emerge (it is the only preference change done in $SDDA$). As i_1, i_3, i_5, i_6 already obtain their top alternatives; they do not have an incentive to manipulate $SDDA$. On the other hand, it is easy to verify that none of the students i_2 and i_4 can manipulate $SDDA$. Hence, $SDDA$ is not manipulable at P, whereas DDA is.

Let us next consider a problem with three students, i_1, i_2, i_3, and two colleges, c_1, c_2. Let $q_{c_1} = 2$, $q_{c_2} = 1$, $q^d_{c_1} = 1$, and $q^d_{c_2} = 1$. The preferences and priorities are as follows:

P_{i_1}	P_{i_2}	P_{i_3}	$\succ_{c_1}$	$\succ_{c_2}$	$\rhd_{c_1}$
(c_2, N)	(c_1, D)	(c_1, D)	i_1	i_3	i_3
(c_1, D)	(c_2, N)	$\emptyset$	i_2	i_1	i_2
$\emptyset$	(c_1, N)	$\vdots$	$\vdots$	$\vdots$	$\vdots$

The DDA outcome at the problem, say μ, is such that $\mu_{i_1} = (c_2, N)$, $\mu_{i_2} = (c_1, D)$, and $\mu_{i_3} = \emptyset$. Student i_3 cannot manipulate DDA at P, and all the others already obtain their best alternative. Hence, DDA is not manipulable at P.

Let us next consider $SDDA$. Its outcome at P, say μ', is the same as μ above. Let us consider P'_{i_3} : $(c_1, D), (c_2, N), \emptyset$. We have $SDDA(P'_{i_3}, P_{-i_3}) = \mu''$ where $\mu''_{i_1} = (c_2, N)$, $\mu''_{i_2} = (c_1, N)$, and $\mu''_{i_3} = (c_1, D)$, benefiting the misreporting student i_3. Through misreporting, student i_3 causes student i_2 to receive (c_1, N), leading

$SDDA$ to remove (c_1, D) from her preferences. Thus, $SDDA$ is manipulable at P, whereas DDA is not. This finishes the proof. □

References

Abdulkadiroğlu A, Che Y-K, Pathak PA, Roth AE, Tercieux O (2020) Efficiency. Justified envy, and incentives in priority-based matching. Am Econ Rev Insights 2:425–42
Abizada A (2016) Stability and incentives for college admissions with budget constraints. Theor Econ 11:735–756
Afacan MO (2019) School choice with vouchers. J Econ Theory 179:57–72
Doğan B, Ehlers L (2021) Minimally unstable Pareto improvements over deferred acceptance. Theor Econ 16:1249–1279
Doğan B, Ehlers L (2022) Robust minimal instability of the top trading cycles mechanism. Am Econ J Microecon 14:556–82
Dur U, Gitmez AA, Yılmaz Ö (2019) School choice under partial fairness. Theor Econ 14:1309–1346
Ehlers L, Morrill T (2019) (Il)legal assignments in school choice. Rev Econ Stud 87:1837–1875
Gale D, Shapley LS (1962) College admissions and the stability of marriage. Am Math Mon 69:9–15
Hatfield JW, Milgrom P (2005) Matching with contracts. Am Econ Rev 95:913–935
Kesten O (2010) School choice with consent. Q J Econ 125:1297–1348
Pathak PA, Sönmez T (2013) School admissions reform in Chicago and England: comparing mechanisms by their vulnerability to manipulation. Am Econ Rev 103:80–106
Roth AE (1985) The college admission problem is not equivalent to the marriage problem. J Econ Theory 36:277–288
Sönmez T, Switzer TB (2013) Matching with (branch-of-choice) contracts at the United States Military Academy. Econometrica 81:451–488