Social Choice and Welfare (2023) 61:199–219
https://doi.org/10.1007/s00355-022-01446-y

ORIGINAL PAPER

Polarization and conflict among groups with heterogeneous members

Daniel Cardona[1] · **Jenny De Freitas**[1] · **Antoni Rubí-Barceló**[1]

Received: 14 July 2021 / Accepted: 14 December 2022 / Published online: 30 December 2022

Abstract
We analyze the choice of the policy platform that a group of heterogeneous challengers will support to confront the current policy in a subsequent contest between them and the status-quo defenders. The choice of this alternative policy will affect not only the incentives of challengers to get involved in the conflict (intra-group effect), but also the mobilization of status-quo defenders (inter-group effect). We disentangle these two effects and show that the degree of polarization (distance between the alternative and the status-quo policy) depends on how the efforts that groups exert in the contest affect their winning probabilities. Our results illustrate how the conflict resolution rules may affect the degree of polarization in political confrontations.

1 Introduction

The choice of a policy platform to compete against an opponent naturally reflects the dilemma between maximizing the winning probability or the winning utility.[1] In confrontations among individuals or homogeneous groups, Epstein and Nitzan (2004, 2007) have shown that this trade-off may lead to *strategic restraint*: Instead of supporting their ideal policy, competing individuals have incentives to support a

[1] This is the main trade-off in Wittman (1977).

We thank Pau Balart, Carmen Beviá, Luís Corchón, Socorro Puy, József Sákovics, two anonymous referees, an associate editor, and conference audiences at the PET Conference 2017, SAET 2017, UECE Lisbon Meetings 2018, SAEe 2018, CESC 2019, Contests: Theory and Evidence 2019, for their helpful comments. The financial support from the Spanish Ministerio de Ciencia y Educación and Ministerio de Universidades (Agencia Estatal de Investigación) through Grant PID2019-107833GB-I00/MCIN/AEI/10.13039/501100011033 is also acknowledged. An earlier version of this paper circulated under the title: 'Polarization or Moderation? Intra-group heterogeneity in endogenous-policy contests'.

Daniel Cardona
d.cardona@uib.cat

[1] Departament d'Economia Aplicada, Universitat de les Illes Balears, Campus UIB, 07122 Palma, Spain

platform closer to the opponent's ideal policy, thereby reducing polarization. The reason is that moderation involves both an effort saving—due to a lower conflict intensity—and a higher winning probability—because others reduce the conflict effort more than the agent who moderates—that offset the sacrifice of winning utility. These results bear on the assumption of internal homogeneity of groups, which is far from being a common feature in many situations. For instance, political parties are composed of different ideological trends; or in a labor dispute, workers' preferences are not necessarily perfectly aligned. This study aims to shed some light on the effect of intra-group heterogeneity on the choice of a common policy platform in confrontations among groups and analyzes how the degree of polarization depends on the conflict resolution rules.

We consider the canonical environment in which agents have single-peaked preferences over policies ranging in a one-dimensional space. These agents are heterogeneous regarding their peaks (ideal policies), and they are organized into two groups (lobbies) accordingly: the defenders of the status quo and its challengers. On behalf of challengers, a *representative* chooses the policy that would be considered as the alternative to the status quo. Then, challengers strive for this policy against status-quo defenders. This interaction is modeled as a contest (between two groups) where lobbyists select their efforts individually and non-cooperatively. We abstract away from the nature and details of the dispute between the two groups by assuming that a contest success function (hereafter, CSF) determines the groups' winning probabilities. As remarked by Skaperdas and Vaidya (2012), two CSFs can be suitably used to represent these types of disputes. In one of them, the winning probabilities depend on the ratio of groups' efforts; and, in the other, they depend on the difference in groups' efforts. These two types of CSFs have received most of the attention in the literature on contests and have been axiomatized by Skaperdas (1996). Our analysis includes both specifications.

Intuitively, the challengers' representative must weigh two different effects when choosing the policy platform to deal with the status quo. On the one hand, moderation causes a reduction in the stake of the status-quo defenders, which leads them to weaken their efforts (inter-group effect). On the other hand, this moderation also modifies the aggregate stake of challengers, which leads them to vary their efforts (intra-group effect). These two effects interact and lead to changes in the winning probability, the winning utility, and the cost of effort, which, in turn, will determine the optimal proposal of the challengers' representative. To disentangle these two effects, we proceed gradually. First, we analyze a setting where status-quo defenders do not observe the challengers' policy before deciding their effort in the contest. In such a way, we isolate the intra-group effect. This setting is reasonable in situations where the policy platform of the challengers is changeable secretly (from the point of view of status-quo defenders).[2] Second, we analyze the interaction between inter- and intra-group forces by assuming that the challengers' target policy is observable

[2] Similarly, Baik and Lee (2007) or Nitzan and Ueda (2011, 2018) consider contests where groups cannot observe the sharing rule of other groups.

by status-quo defenders so they can strategically tailor their efforts in the contest to that policy.

Our results show that the subsequent contest could induce the representative to select a policy platform closer to or farther from the status quo than her ideal policy; hence, the degree of polarization might decrease or increase.[3] We obtain that, under both types of CSF, the intra-group forces caused by the challengers' heterogeneity lead any representative's optimal target policy to lie between her ideal policy and the one inducing the highest group effort. This illustrates the dilemma mentioned above between maximizing the winning utility and the winning probability. Consequently, intra-group forces push relatively extremist representatives to moderate their demands and relatively centered representatives to polarize them. Additionally, inter-group forces encourage any representative to moderate her claim (as in Epstein and Nitzan 2004). We show that the result of the interaction between inter- and intra-group effects crucially depends on the nature of the conflict and, in particular, on the CSF used to model it: When the winning probability depends on the ratio of efforts, the degree of polarization always decreases, even when challengers outnumber status-quo defenders by an arbitrarily large number—a situation where intra-group effects would seem to be dominant. Instead, when the winning probability depends on the difference in efforts, the representative might select a more extreme target policy. This will happen when the group of status-quo defenders is sufficiently small (so that the inter-group benefits of moderation are limited) and, compared with the representative, most challengers are relatively extremist (so that the intra-group benefits of more extreme policies are prominent).

Our baseline model can be extended in different directions. In one of them, the challengers' target-policy is not selected by a representative but the outcome of a collective choice process. In that case, the preferred policy of the median member of a group would play a prominent role, as it would be the Condorcet-winner policy if this group had no opposition. Nevertheless, when the policy selection precedes a contest, as in our model, proving the existence of a Condorcet winner is challenging. We focus on the relative location of this Condorcet winner (whenever it exists) to show that our main results essentially hold. That is, challengers select a target policy that is more or less moderated than the ideal policy of the median challenger depending on similar conditions to those detailed in our model.

1.1 Related literature

Our analysis mostly contributes to the literature on lobbying and policy selection when lobbies can engage in strategic restraint. In this strand of the literature, the seminal contributions of Epstein and Nitzan (2004, 2007) have been followed by Münster (2006) and Cardona and Rubí-Barceló (2016). Besides, other contributions have studied strategic restraint in environmental policy conflicts (Heyes 1997;

[3] This is very much related to the concepts of convergence and divergence of policy platforms used in the political economy literature.

Liston-Heyes 2001; Friehe 2013; Cardona et al. 2021) and in voting contexts with policy-motivated candidates (Lindbeck and Weibull 1993).

Closely related to our paper are Hirata and Kamada (2020) and De Freitas (2011). Both papers study how contributions by donors shape policy proposals. In De Freitas (2011), the focus is on the interplay between an endogenous economic issue and an exogenous ideological issue when ideological voters contribute to parties' political campaigns (private system). Parties are Downsian, and policy convergence is obtained. Nevertheless, this policy is different from the median's preferred one, which makes the private system more or less attractive to voters. In a one-dimensional policy setting, Hirata and Kamada (2020) study how the presence of donors affects the policy proposals of two parties. They show that, although only extreme donors contribute in equilibrium, this does not necessarily cause polarization. This result bears on three assumptions that differ from ours. First, political parties are sufficiently office motivated; second, the winning probability is determined only by the donors' contributions; and third, there are effectively only two extreme donors.[4] In our paper, the representative is policy motivated, effort costs are not linear so that all agents in the group choose positive efforts and, consequently, the relative sizes of the groups might play a role. Additionally, in our model the representative also exerts effort.

As the choice of a particular public policy entails some distribution of surplus within the group, it can be interpreted as the selection of an internal sharing rule, which certainly affects the aggregate effort of the group. From this viewpoint, our study is related to the literature analyzing the effects of an internal sharing rule on the outcome of contests (e.g., Nitzan and Ueda 2011, 2018; Kolmar and Wagener 2013; Flamand and Troumpounis 2015; Balart et al. 2016; Kobayashi and Konishi 2021). In another respect, selecting a target policy can be interpreted as the choice of a delegate that would interact on behalf of a principal or a group (Baik and Kim 1997; Schoonbeek 2004; Baik 2007). Nevertheless, in our framework the delegation would be partial, as efforts are individually chosen by all the group members.

The two-stage selection of policy and effort in our setting is closely related to valence models of political competition (Groseclose 2001; Aragonès and Palfrey 2002; Aragonès and Xefteris 2012). In particular to those where valence is endogenously determined (Schofield 2006; Herrera et al. 2008; Meirowitz 2008; Ashworth and Bueno de Mesquita 2009; Hirsch 2016; Balart et al. 2022).[5] However, the forces influencing political competition in those studies differ from ours. The first main difference is that polarization in our setting is a consequence of intra-party forces. Heterogeneity among party members is not considered in this specific literature, where a candidate, rather than a party, selects the policy platform. Schofield (2006) recognizes the role activists play in pulling equilibrium policy toward the extreme, without explicitly accounting for the distribution of party members across the policy space. Concerning the inter-party forces, there are other differences. In our setting,

[4] The last is a consequence of assuming linear costs. In Sect. 4.2 we discuss the implications of this cost function specification.

[5] We could interpret total effort exerted in the contest as 'campaign valences' (Carter and Patty 2015) that increase a party's probability of winning the election.

groups are policy-motivated parties *à la Wittman*, in contrast to the Downsian party approach in Ashworth and Bueno de Mesquita (2009) and Meirowitz (2008). In our study, moderation is a consequence of the strategic interaction of effort choices in the contest game (strategic restraint) rather than a consequence of uncertainty in elections (Wittman 1983; Calvert 1985). In a setting with ideological voters, Herrera et al. (2008) also found that a policy moderation decreases campaigning costs and increases the winning probability. However, this increase comes from narrowing the gap between the proposed policy and the expected median voter position, and not from a strategic interaction with the opponent.

The baseline model is presented in the next section, and in Sect. 3 we derive the results. Some extensions are discussed in Sect. 4. Section 5 concludes.

2 The model

Our model is based on Epstein and Nitzan (2004). There are two interest groups, C and D, with cardinality c and d, respectively. The members of these groups will engage in lobbying activities in a two-stage public-policy contest. In particular, the members of C (challengers) are interested in replacing the status quo, whereas the members of D (status-quo defenders) aim to keep it. The timing is as follows: Firstly, a self-interested representative $r \in C$ sets a target policy $x \in [0, 1]$. Secondly, there is a contest between groups C and D to determine whether policy x replaces the status-quo policy $y = 1$. In this contest, all the members of C and D (including r) decide simultaneously, individually, and non-cooperatively, the costly effort they will exert to support the target policy of their group.

The probability that the challengers' target policy x replaces the status-quo depends on the aggregate efforts of groups C and D, denoted by A and B, respectively. We consider *linear impact functions* so that $A = \sum_{j\in C} a_j$ and $B = \sum_{j\in D} b_j$, where a_j and b_j are the individual efforts of the members of C and D, respectively. Let $p(A, B)$ denote the CSF determining the winning probability of challengers. Thus, with probability $1 - p(A, B)$ group D wins and the status-quo policy remains. Our analysis includes two alternative specifications of the CSF within the power and logit functional forms axiomatized by Skaperdas (1996):

Ratio (ρ)*:* $p(A, B) = f_\rho\left(Q_\rho\right)$ where $Q_\rho = A/B$ and $f_\rho(\cdot)$ satisfies (i) $f_\rho(\cdot) \in [0, 1]$, (ii) $f'_\rho(\cdot) \geq 0$, and (iii) $f''_\rho(\cdot) < 0$.

Difference (δ)*:* $p(A, B) = f_\delta\left(Q_\delta\right)$ where $Q_\delta = A - B$ and $f_\delta(Q_\delta) = e^{\alpha Q_\delta}/\left(1 + e^{\alpha Q_\delta}\right)$, where $\alpha \in \left(0, \min_{z\in\Re} \sqrt{\frac{(1+e^z)^3}{e^z}}\right) \simeq (0, 2.5981)$.[6]

[6] This restriction on α guarantees (i) $1 - f''_\delta(\cdot) > 0$ and (ii) $f_\delta(\cdot) - f'_\delta(\cdot)f'_\delta(\cdot) > 0$. Condition (i) is necessary and sufficient for the second-order condition of the maximization problem analyzed in Sect. 3 to hold. For larger values of α, the quasi-concavity of the utility function defined in that section is not guaranteed. Additionally, (i) and (ii) are both required to characterize the strategic behavior of the representative in Propositions 1 and 2.

The preferences of any lobbyist $j \in C \cup D$ satisfy the vNM axioms, and they are represented by the elementary (Bernoulli) utility function $v(w, z) = u_j(w) - c(z)$, where $w \in [0, 1]$ represents a policy and $z \in \mathfrak{R}_+$ is the effort exerted in the contest. Specifically, we consider quadratic costs, $c(z) = z^2/2$, and quadratic utility functions over policies, $u_j(w) = 1 - (w - j)^2$, where j will denote both a lobbyist and her *peak* or ideal policy.[7] Let μ, ν, and Λ denote the average of the peaks in C, D, and $C \cup D$, respectively. That is, $\mu \equiv \frac{\sum_{j \in C} j}{c}$, $\nu \equiv \frac{\sum_{j \in D} j}{d}$, and $\Lambda \equiv \frac{c\mu + d\nu}{c+d}$. We assume that the members of C are (possibly) heterogeneous and have their peaks in $\left[0, 1/2\right]$, whereas all the members of D are identical and have their peak at 1.[8] We say that polarization decreases when the challengers' target policy is closer to the status-quo than the representative's ideal policy, *i.e.* when $x > r$. Accordingly, polarization increases when $x < r$.

We analyze two different information structures of the above-mentioned game, depending on x being private information for the challengers (Sect. 3.1) or common knowledge (Sect. 3.2).

When the policy choice of the challenger group x is unobservable for the status-quo defenders, the ensuing contest is not a proper subgame, as none of the members of D can specify the payoff function of all contenders. Hence, subgame perfection does not guarantee that the effort choices best-reply the policy choice in any continuation game. For this reason, we use the notion of the Perfect Bayesian Equilibrium (PBE), as in Nitzan and Ueda (2011, 2018) or Trevisan (2020), even though we do not have an incomplete information game. For simplicity, we focus on pure strategies. A pure-strategy PBE consists of an assessment, i.e., a strategy profile $\sigma = \left(a_1(x), \ldots, \left(x, a_r(x)\right), \ldots, a_c(x), b_1, \ldots, b_d\right)$ and a system of beliefs specifying a probability distribution over the nodes at any information set, such that (i) strategies are sequentially rational given the equilibrium beliefs, and (ii) beliefs are consistent with the equilibrium strategy profile. Since x can be observed by challengers, this sequential rationality involves that their effort choice must best-reply the policy choice x in any continuation game. Notice that, when x is private information for the challengers, reaching a specific node of an information set of player $i \in D$ depends on both the challengers' target policy and the effort of those players preceding i in the extensive-form representation of the game. This creates a significant number of specificities depending on the order of play in this representation. Following the approach of Nitzan and Ueda (2011), we circumvent these problems by restricting beliefs to be defined over the selected target policy x only. Let x^e denote the expected target policy according to these beliefs.

When x is observed by status-quo defenders, PBE strategies coincide with Subgame Perfect Equilibrium (SPE) strategies, as there is a different subgame for any x. In this case, we use the SPE concept, so that a strategy profile is $\sigma = \left(a_1(x), \ldots, \left(x, a_r(x)\right), \ldots, a_c(x), b_1(x), \ldots, b_d(x)\right)$.

[7] Other functional forms are discussed in Sect. 4.2.

[8] This is a simplifying assumption that guarantees that any member of D prefers y to x. In Sect. 4.3, we discuss the implications of assuming an heterogeneous group of defenders. For this reason, we opt to present our results in terms of ν.

3 Results

When the representative's policy selection is public information, the strategic effect of this choice is twofold, as it influences the incentives for exerting effort in the contest of both challengers and status-quo defenders. These are referred to as the intra- and inter-group effects, respectively. Instead, this policy choice only affects the challengers' efforts when it is unobservable for the status-quo defenders. Therefore, in this case the target-policy choice is affected only by the intra-group effect.

3.1 Unobservable target policy: intra-group effect

At any information set following the choice of the target policy x, sequential rationality implies that, taking $B = \sum_{k\in D} b_k$ and $A_{-j} = \sum_{k\in C-\{j\}} a_k$ as given, lobbyist $j \in C$ chooses a_j to maximize her expected utility

$$f_i\left(Q_i\right)u_j(x) + \left[1 - f_i\left(Q_i\right)\right]u_j(1) - a_j^2/2 = f_i\left(Q_i\right)S_j(x) + u_j(1) - a_j^2/2,$$

where $i \in \{\rho, \delta\}$ denotes the CSF specification and $S_j(x) = u_j(x) - u_j(1) \geq 0$ denotes the *stake* of lobbyist $j \in C$.

Differentiating the expected utility function, we obtain the first-order condition (FOC) as[9]

$$f_i'\left(Q_i\right)\frac{\partial Q_i}{\partial A}S_j(x) - a_j \equiv 0, \text{ for all } j \in C. \tag{1}$$

Notice that, due to the quadratic effort costs, all lobbyists in C will exert a positive effort. In this sense, although exerting effort generates positive externalities on the other group members, there are no *strong free-riders*. Adding up the previous conditions, we obtain

$$f_i'\left(Q_i\right)\frac{\partial Q_i}{\partial A}S_C(x) - A \equiv 0 \tag{2}$$

where $S_C(x) = \sum_{j\in C} S_j(x)$. Note that the (equilibrium) optimal efforts satisfy

$$\frac{a_j}{a_k} = \frac{S_j(x)}{S_k(x)} \text{ and } \frac{a_j}{A} = \frac{S_j(x)}{S_C(x)} \tag{3}$$

for all $j, k \in C$. That is, the relative stake of any challenger determines her relative equilibrium effort in the contest.

[9] The second-order condition requires $f_i''\left(Q_i\right) \cdot \left[\frac{\partial Q_i}{\partial A}\right]^2 \cdot S_j(x) < 1$, which is satisfied for any $i \in \{\rho, \delta\}$. In particular, the condition under the difference CSF is $\alpha < \min_{z\in\Re} \sqrt{\frac{(1+e^z)^3}{e^z(1-e^z)}}$, which is guaranteed under our assumptions.

Members of D choose efforts according to the expected target policy x^e. Hence, proceeding as before we obtain

$$-f_i'\left(Q_i\right)\frac{\partial Q_i}{\partial B}S_j(x^e) - b_j \equiv 0 \tag{4}$$

and

$$-f_i'\left(Q_i\right)\frac{\partial Q_i}{\partial B}S_D(x^e) - B \equiv 0 \tag{5}$$

where $S_D(x) = \sum_{j\in D} S_j(x)$, with $S_j(x) = u_j(1) - u_j(x)$, $\forall j \in D$.

Using (2), (5), and $Q_\rho = A/B$, we obtain

$$Q_\rho^* = q_\rho(x;x^e) = \left[\frac{S_C(x)}{S_D(x^e)}\right]^{1/2}, \tag{6}$$

which is unique for any (x, x^e).

Likewise, using (2), (5), and $Q_\delta = A - B$, we get

$$f_\delta'\left(Q_\delta\right)\left[S_C(x) - S_D(x^e)\right] - Q_\delta \equiv 0. \tag{7}$$

From (7) we uniquely obtain $Q_\delta^* = q_\delta(x;x^e)$. To see this, note that (i) $Q_\delta^* \geq 0 \Leftrightarrow S_C(x) - S_D(x^e) \geq 0$; (ii)$f''\left(Q_\delta\right) \geq 0 \Leftrightarrow Q_\delta \leq 0$; (iii) $\lim_{Q_\delta\to-\infty} f_\delta'\left(Q_\delta\right)\left[S_C(x) - S_D(x^e)\right] - Q_\delta > 0$ and (iv) $\lim_{Q_\delta\to+\infty} f_\delta'\left(Q_\delta\right)\left[S_C(x) - S_D(x^e)\right] - Q_\delta < 0$.

Consequently, for any $(x, x^e) \in [0, 1]^2$ and any specification of the CSF, Eqs. (1) and (4) yield a unique equilibrium effort profile $\left\{a_j^*(x), b_k^*\right\}_{j\in C,k\in D}$, and the corresponding aggregate equilibrium efforts $A^*(x)$ and B^*. We can now characterize the equilibrium winning probability of challengers in terms of the proposed policy (and beliefs). All proofs are relegated to the Appendix.

Lemma 1 *When the target policy x is private information for challengers, their winning probability increases as x approaches the average of the peaks in C, μ.*

The proof shows that the aggregate effort of group C increases when x moves towards the policy that maximizes the aggregate utility of group C. Under quadratic preferences, this policy is μ, the mean of the ideal policies of the members of group C.

The winning probability is undoubtedly a variable the representative r must consider when choosing the target policy —if r was exclusively office-motivated, that would be the unique variable she would consider (see e.g. Trevisan (2020), or Kobayashi and Konishi 2021). Nevertheless, since r is ideological, there is a trade-off between maximizing the winning probability and the winning utility, unless the ideal policy of the representative r coincides with μ. The equilibrium target policy x^*, described by the following result, will reflect this trade-off.

Proposition 1 *If x is private information for challengers then x^* lies between the ideal policy of the representative and the policy maximizing the aggregate utility of group C (μ), when $r \neq \mu$. Otherwise, $x^* = r = \mu$.*

The intra-group forces caused by the heterogeneity of group C push the representative to free-ride on other members' efforts by moving the target policy away from her peak r towards μ. This produces a reduction in the relative effort of lobbyist r—because her relative individual stake decreases—and also raises the aggregate effort of the group. For this reason, the policy selected by the representative generally differs from her ideal one. Specifically, polarization decreases if $r < \mu$ and increases if $r > \mu$.[10]

3.2 Observable target policy: intra- and inter-group effects

When the target policy is observed by the status-quo defenders, they strategically tailor their efforts to this policy. Hence, the representative's policy choice would weigh the intra-group forces described above and the following inter-group effect: Moderating the target policy (i.e., choosing a policy closer to the status-quo) would decrease the stake of the status-quo defenders, and this would reduce their incentives to exert effort in the contest, increasing the challengers' winning probability. Therefore, when $r < \mu$, intra- and inter-group forces are aligned to induce moderation. Otherwise, when $r > \mu$, these two forces affect the representative's optimal policy choice in opposite directions: the inter-group effect induces moderation whereas the intra-group effect pushes the representative to select a more extreme policy, intensifying polarization.

Before analyzing the result of this trade-off, we describe the effects of the target-policy choice on the equilibrium winning probabilities. As, in the present setting, PBE strategies coincide with Subgame Perfect Equilibrium strategies, we start by characterizing the Nash equilibrium at any subgame starting with a fixed and commonly known $x \in [0, 1]$. Following the same steps of Sect. 3.1 (now fixing $x^e = x$), we obtain that the equilibrium aggregate efforts, A^* and B^*, solve

$$f_i'\left(Q_i^*\right)\frac{\partial Q_i^*}{\partial A}S_C(x) - A^* \equiv 0, \tag{8}$$

and

$$-f_i'\left(Q_i^*\right)\frac{\partial Q_i^*}{\partial B}S_D(x) - B^* \equiv 0. \tag{9}$$

As the choice of the target policy by the representative generates both intra- and inter-group effects, now the decision to moderate or not might depend on the

[10] Notice that the representative would select policy μ if her objective was to maximize the aggregate expected utility.

specification of the CSF. For this reason, hereafter we will distinguish between $q_\rho(x) = A^*(x)/B^*(x)$ and $q_\delta(x) = A^*(x) - B^*(x)$. Using (8) and (9), we obtain

$$q_\rho(x) = -\frac{\partial Q_\rho/\partial A}{\partial Q_\rho/\partial B} \cdot \frac{S_C(x)}{S_D(x)} \tag{10}$$

and

$$q_\delta(x) = f'_\delta\left(Q_\delta\right)\left[\frac{\partial Q_\delta}{\partial A}S_C(x) + \frac{\partial Q_\delta}{\partial B}S_D(x)\right]. \tag{11}$$

Equation (10) uniquely determines

$$q_\rho(x) = \left[\frac{S_C(x)}{S_D(x)}\right]^{1/2}. \tag{12}$$

Likewise, from (11) we obtain

$$q_\delta(x) = f'_\delta\left(q_\delta(x)\right)\left[S_C(x) - S_D(x)\right], \tag{13}$$

which uniquely determines $q_\delta(x)$, as shown in Section 3.1.

Equations (12) and (13) highlight the importance of group sizes in determining $q_i(x)$, and therefore the probability of replacing the status quo. Nevertheless, we next show that, under the ratio CSF specification, this probability always increases after moderation, independently of group sizes. This is not true when the CSF depends on the difference in efforts, where this probability increases when x moves towards the policy maximizing the aggregate utility of the population. With quadratic preferences, this policy is Λ, the mean of the ideal policies of all contestants. Thus, the effects of moderation on the winning probability would be positive or negative depending on the sizes of the groups (and the mean of the peaks of their members).

Lemma 2 *If the target policy x is observed by status-quo defenders, then the challengers' winning probability increases when x approaches the status quo in case of a ratio CSF, or when x approaches the population average of the peaks, Λ, in case of a difference CSF.*

Apart from affecting the winning probability, the policy choice of the representative also affects her winning utility and her equilibrium effort cost. The optimal target policy x^*, described next, arises from the interaction of these three effects.

Proposition 2 *If the target policy is observed by status-quo defenders, then the representative always moderates (i.e. $x^* > r$) when the CSF has a ratio form. In case of a difference CSF, x^* lies between the ideal policy of the representative r and the population average of the peaks Λ if $r \neq \Lambda$. Otherwise, $x^* = r = \Lambda$.*

When the winning probability depends on the ratio of efforts, the representative always reduces polarization. This result reinforces Epstein and Nitzan's (2004), as strategic restraint still arises after adding the new forces generated by intra-group heterogeneity. However, when the winning probability depends on the difference in groups' efforts, polarization might increase or decrease depending on the relative location of the representative's peak with respect to the policy threshold Λ.[11] In particular, r moderates her policy claim when her ideal policy is farther from the status quo than Λ. On the contrary, r polarizes her policy claim when her ideal policy is closer to the status-quo than Λ.

4 Discussion

In order to simplify the presentation and at the same time obtain clearer results, our baseline model is built upon standard assumptions in terms of decision-making mechanisms within the group of challengers, functional forms, and players' heterogeneity. In this section, we discuss the robustness of our results beyond these assumptions.

4.1 Collective choice

Besides the interaction between intra- and inter-group effects as the main trade-off affecting the policy platform choice, when challengers are heterogeneous there is also a conflict among them when collectively deciding their (common) policy platform.

It is obvious that if the group of challengers had no opposition for implementing any policy, then the ideal policy of its median member, $m \in C$, would play a prominent role, as it would be the Condorcet winner.[12] However, when the selected policy must confront the defenders of the status quo in a subsequent contest, proving the single-peak or single-crossing properties of the (indirect) utility functions is challenging. Although many numerical simulations show that this is the case, we are unable to provide a general proof of the existence of a Condorcet-winner policy.[13] Instead, we are able to characterize the relative location of a Condorcet-winner policy, whenever it exists.

Proposition 3 *If a Condorcet-winner challenging policy exists then it cannot be more extreme than* m *when the CSF has a ratio form. In case of the difference CSF, if a Condorcet-winner challenging policy exists then it cannot be more extreme than* m *when* $m < \Lambda$ *and it cannot be more centered than* m *when* $m > \Lambda$.

[11] Having the average of all the peaks, Λ, as the reference point comes from the fact that, in the difference CSF case, the marginal impact of a lobbyist's effort does not depend on her group identity.

[12] For the sake of simplicity, statements about majority preference assume an odd number of challengers.

[13] The existence of a Condorcet-winner policy can be proved in simpler environments. For instance, when $p(A, B) = 1/2 + s(A - B)$, where $s > 0$ is restricted in order to guarantee the existence of a pure-strategy equilibrium (see Cardona et al. 2018).

In other words, when the CSF has a ratio form, the confrontation against status-quo defenders pushes the challengers to moderate their collective decision with respect to what they would choose if they had no opposition. The predominant role of the inter-group effect illustrated in the previous section underlies the result. However, when the CSF has a difference form and m is sufficiently centered ($m > \Lambda$), the subsequent confrontation leads challengers to select a policy more extreme than m. In that case, the positive effects of polarization in terms of stimulating the challengers' aggregate effort overbalance the negative impact that this polarization generates by also stimulating the efforts of the status-quo defenders.

4.2 Other functional specifications

In our baseline model, we consider quadratic costs of effort and a quadratic disutility of the policy component. These assumptions are commonly used, as they simplify the aggregation of the individual first-order conditions. Cardona et al. (2018) show that there are no major variations for some of the results (basically, for those referred to the unobservable target policy case) in a setting with quadratic costs and a more general utility function with a strictly convex disutility of the policy component.[14] Dropping the strict convexity assumptions will crucially modify our results. For example, in a setting with a linear disutility of the policy component, the main driving force leading to strategic restraint (Epstein and Nitzan 2004) will be neutralized, as shown in Cardona and Rubí-Barceló (2016).

Likewise, results might change under a linear specification of the cost function, as this may affect the set of active lobbyists. Under linear effort costs, only the extreme challenger(s) might exert a positive effort (see Hirata and Kamada 2020). Hence, moderation does not alter the (zero) effort cost of any representative r different from the extreme challenger and it reduces the effort cost of the extreme challenger(s). For the ratio-form CSF, it can be shown that any representative $r \in C$ will moderate her policy claim (*i.e.* $x_r^* > r$),[15] as the benefits from reducing the incentives of the opponents to become involved in the conflict (inter-group effect) offset the winning-utility loss from that moderation. For the difference CSF, we can apply the results of Baik (1998) to conclude that there does not exist a pure-strategy equilibrium in which both extreme lobbyists exert a positive effort. Moreover, if one lobbyist exerts a positive effort in a pure-strategy SPE, then she must be the extreme challenger, as long as the target policy x is more moderated than her peak, $\underline{x}$. In this case, as there is no inter-group effects, we can conclude that any representative r will select a target policy lying in the interval between her ideal policy r and the policy maximizing the challengers' effort, $\underline{x}$. This leads to polarization.

[14] The utility function in Cardona et al. (2018) is such as $u_j(x) = 1 - \theta(|x - j|)$, where $\theta(0) = 0$, $\theta'(0) = 0$, $\theta'(z) > 0$, and $\theta''(z) > 0$, for $z > 0$.

[15] See Cardona et al. (2018).

4.3 Heterogeneous defenders

In our baseline model, the group of lobbyists defending the status-quo policy $y = 1$ is assumed to be homogeneous—specifically, all the members of D have their peak at 1. This assumption is very convenient to derive our results. First, it allows us to easily aggregate the behavior of defenders; and second, it implies that any member of D always prefers the status quo to any other policy proposal. Introducing heterogeneity would not alter our results (Lemma 2 and Proposition 2) as far as any defender prefers the status-quo policy to the proposed challenging policy x, when the CSF has the ratio form. Unfortunately, for any $j \in D$ such that $j < 1$, there always exists an $l_j \in (0, j)$ such that $u_j(x) > u_j(1)$ for any $x \in \left(l_j, 1\right)$.[16] Hence, unless $j \geq 1$ for all $j \in D$, there will be members of D preferring the challenging policy in some subgames (*i.e.,* after some policy choices of r). Consistently, these lobbyists would never exert a positive effort supporting the status quo. Since the composition of groups is fixed in our model, there are two different alternatives to accommodate the behavior of these lobbyists: either not participating or sabotaging; that is, exerting a negative effort (see Chowdhoury and Gürtler, 2015 for a survey on sabotage in contests). None of these extensions can be easily adapted to our proofs for both CSF specifications and are left for future research.

5 Concluding remarks

We studied the situation where the representative of a group of heterogeneous lobbyists (challengers) sets a policy to deal with the status quo defended by an opposing group. This interaction is modeled as a contest where the members of the two groups individually decide their efforts to increase their group's probability of winning. The selected policy may urge challengers to exert more effort (intra-group effect) and will also affect the incentives of the opponents to become involved in the contest (inter-group effect). In this study, we focused on analyzing the interaction between these effects to shed some light on how they shape the policy reform proposals. Our results highlight the importance of the mechanism of conflict resolution in determining the degree of polarization among the positions defended by the competing groups.

For the sake of exposition, our results were presented gradually. First, we considered a situation where status-quo defenders cannot observe the challengers' target policy. Under this assumption, the inter-group effect is neutralized. As a consequence, the representative faces the trade-off between selecting a target policy closer to her peak or to the policy maximizing the aggregate effort of her group. Things change when the challengers' target policy is observed before the contest, so both intra- and inter-group forces affect this policy choice. In this case, we showed that the result of this interaction crucially depends on the nature of the conflict and, in particular, on the CSF resolving this conflict. When the

[16] Under quadratic preferences, $l_j = 2j - 1$.

winning probabilities depend on the ratio of groups' efforts, moderation always arises, thereby reducing the polarization. This reinforces the result of Epstein and Nitzan (2004) regarding the incentives of contestants to strategically restraint their claims before the conflict. However, polarization might increase when the winning probabilities depend on the difference in groups' efforts. Specifically, the challengers' representative may select a policy more extreme than her peak when he or she is more centered than the mean of the peaks of the whole population of contestants. In these cases, a more extreme policy induces a higher aggregate effort in her group, which overbalances the higher effort of the opponents. Hence, the endogenous policy choice increases polarization in these cases.

Proofs

First, notice that under quadratic preferences the following holds:

$$
\begin{aligned}
S_j(x) =& (1-x)(1+x-2j)\\
S_C(x) =& c\cdot(1-x)(1+x-2\mu) = cS_\mu(x)\\
S_D(x) =& d\cdot(1-x)(2\nu-1-x)
\end{aligned}
$$

Hereafter, let $i \in \{\rho, \delta\}$ denote the case under consideration.

In order to reflect the dependence of the equilibrium variables on x (when the case), we will refer to them by the corresponding functions: $Q_i^* = q_i(x;x^e)$, $a_j^* = a_j(x;x^e)$ for $j \in C$, $b_k^* = b_k(x^e;x)$ for $k \in D$, $A^* = A(x;x^e)$ and $B^* = B(x^e;x)$.

Proof of Lemma 1 Differentiating (7) with respect to x yields

$$f_\delta''\left(Q_\delta\right)Q_\delta'(x)\left[S_C(x)-S_D(x^e)\right]+f_\delta'\left(Q_\delta(x)\right)S_C'(x)-Q_\delta'(x)=0.$$

Rearranging,

$$Q_\delta'(x)\left\{1-f_\delta''\left(Q_\delta\right)\left[S_C(x)-S_D(x^e)\right]\right\}=f_\delta'\left(Q_\delta\right)S_C'(x).$$

As $f_\delta''\left(Q_\delta\right)\left[S_C(x)-S_D(x)\right] \le 0$, it is immediate that

$$Q_\delta'(x) \ge 0 \Leftrightarrow S_C'(x) \ge 0.$$

For the ratio-form CSF, differenting Eq. (6) we obtain

$$q_\rho'(x) = \frac{1}{2}\left[\frac{S_C(x)}{S_D(x^e)}\right]^{-1/2} S_C'(x) \ge 0 \Leftrightarrow S_C'(x) \ge 0.$$

Under quadratic preferences, $S_C(x)$ is concave and has a maximum at $x=\mu$ since $S_C(x)=\sum_{j\in C}S_j(x)=c(1-x)(1+x-2\mu)=cS_\mu(x)$. As $f_i'(\cdot)>0$, the statement of Lemma 1 immediately follows. □

Proof of Proposition 1 The indirect utility function of the representative can be written as

$$V_r(x) = f_i\big(q_i(x)\big)S_r(x) + u_r(1) - \frac{a_r^2(x)}{2}.$$

Differentiation yields

$$V_r'(x) = f_i'\big(q_i(x)\big)S_r(x)q_i'(x) + f_i\big(q_i(x)\big)S_r'(x) - a_r(x)a_r'(x). \tag{14}$$

In case ρ, from Eqs. (1) and (2) we obtain the equilibrium values of $a_r(x)$ and $A(x)$. Specifically,

$$a_r(x) = f_\rho'\big(q_\rho(x)\big)B^{-1} \cdot S_r(x) = A(x)\frac{S_r(x)}{S_C(x)} = Bq_\rho(x)\frac{S_r(x)}{S_C(x)}.$$

Thus,

$$a_r'(x) = f_\rho''\big(q_\rho(x)\big)q_\rho'(x)B^{-1}S_r(x) + f_\rho'\big(q_\rho(x)\big)B^{-1}S_r'(x).$$

Hence, Eq. (14) can be written as

$$V_r'(x) = a_r(x)B\left[1 - \frac{f_\rho''\big(q_\rho(x)\big)S_r(x)}{B^2}\right]q_\rho'(x) + \left[f_\rho\big(q_\rho(x)\big) - \frac{S_r(x)}{S_C(x)}f_\rho'\big(q_\rho(x)\big)q_\rho(x)\right]S_r'(x).$$

As $f_\rho'' < 0$ and therefore $f_\rho\big(q_\rho(x)\big) > f_\rho'\big(q_\rho(x)\big)q_\rho(x)$, we obtain that $S_r'(x) \cdot q_\rho'(x) < 0$.

In case δ, Eqs. (1) and (2) yield $a_r(x) = f_\delta'\big(q_\delta(x)\big)S_r(x)$ and $A(x) = f_\delta'\big(q_\delta(x)\big)S_C(x)$. In this case, Eq. (14) can be written as

$$\begin{aligned} V_r'(x) &= f_\delta'\big(q_\delta(x)\big)S_r(x)q_\delta'(x) + f_\delta\big(q_\delta(x)\big)S_r'(x) \\ &\quad - f_\delta'\big(q_\delta(x)\big)S_r(x)\big[f_\delta''\big(q_\delta(x)\big)S_r(x)q_\delta'(x) + S_r'(x)f_\delta'\big(q_\delta(x)\big)\big] \\ &= S_r(x)f_\delta'\big(q_\delta(x)\big)\big[1 - f_\delta''\big(q_\delta(x)\big)S_r(x)\big]q_\delta'(x) \\ &\quad + \big[f_\delta\big(q_\delta(x)\big) - f_\delta'\big(q_\delta(x)\big)f_\delta'\big(q_\delta(x)\big)S_r(x)\big]S_r'(x). \end{aligned}$$

Our assumptions on α imply that (i) $1 - f_\delta''\big(q_\delta\big) > 0$ and (ii) $f_\delta\big(q_\delta\big) - f_\delta'\big(q_\delta\big)f_\delta'\big(q_\delta\big) > 0$.[17] Thus, as $S_r(x) \le 1$, $V_r'(x) = 0 \Rightarrow q_\delta'(x)S_r'(x) < 0$.

Given that $q_i'(x)S_r'(x) < 0$ for both $i = \rho$ and $i = \delta$, the statement of the proposition follows from Lemma 1. ☐

Remark 1 In the following proofs, defenders are allowed to be heterogeneous. In exchange, we assume that when the CSF has the ratio form then any defender prefers the status-quo policy to the proposed challenging policy x. When defenders

---

[17] In particular, $\alpha < \sqrt{\frac{(e^{\alpha q_\delta}+1)^3}{e^{\alpha q_\delta}(1-e^{\alpha q_\delta})}}$ and $\alpha < \sqrt{\frac{(e^{\alpha q_\delta}+1)^3}{e^{\alpha q_\delta}}}$ are sufficient conditions for (i) and (ii), respectively.

are homogeneous, this is always the case. Nevertheless, the new assumption would allow us to extent Lemma 2 and Proposition 2 to the case with heterogeneous defenders when the CSF has the ratio form.

Proof of Lemma 2 Differentiating Eq. (12), we obtain

$$q'_\rho(x) = \frac{1}{2}q_\rho^{-1}(x)\left[\frac{S'_C(x)}{S_D(x)} - \frac{S_C(x)S'_D(x)}{S_D^2(x)}\right] = \frac{1}{2}q_\rho(x)\left[\frac{S'_C(x)}{S_C(x)} - \frac{S'_D(x)}{S_D(x)}\right].$$

Substituting $S_C(x)$ and $S_D(x)$ and using $S'_C(x) = 2c(\mu - x)$ and $S'_D(x) = -2d(\nu - x)$, we obtain

$$\frac{S'_C(x)}{S_C(x)} - \frac{S'_D(x)}{S_D(x)} = 2\frac{\mu - \nu}{(1 + x - 2\mu)(1 + x - 2\nu)}.$$

Since each of the defenders prefers the status-quo policy to x, $\mu - \nu < 0$ and $1 + x - 2\nu < 0$. Therefore, $q'_\rho(x) > 0$ for all $x \in [0, 1)$.

Implicit differentiation of Eq. (13) yields

$$q'_\delta(x) = \frac{f'_\delta\left(q_\delta(x)\right)\left[S'_C(x) - S'_D(x)\right]}{1 - f''_\delta\left(q_\delta(x)\right)\left[S_C(x) - S_D(x)\right]}. \tag{15}$$

Using $f''_\delta\left(Q_\delta\right)\left[S_C(x) - S_D(x)\right] < 0$, it is immediate that the denominator is positive. Thus,

$$q'_\delta(x) \geq 0 \Leftrightarrow S'_C(x) - S'_D(x) \geq 0.$$

Substituting $S'_C(x) = 2c(\mu - x)$ and $S'_D(x) = -2d(\nu - x)$, we obtain that

$$S'_C(x) - S'_D(x) \geq 0 \Leftrightarrow x \leq \frac{c\mu + d\nu}{c + d} \equiv \Lambda.$$

Hence,

$$q'_\delta(x) \geq 0 \Leftrightarrow x \leq \Lambda.$$

□

Proof of Proposition 2 The indirect utility function of the representative r is

$$V_r(x) = f_i\left(q_i(x)\right)S_r(x) + u_r(1) - \frac{1}{2}a_r^2(x). \tag{16}$$

Case ρ. Using (1), the definition of $q_\rho(x)$, and (3), we obtain

$$a_r^2(x) = f_\rho'\left(q_\rho(x)\right)q_\rho(x)S_r(x)\frac{S_r(x)}{S_C(x)}.$$

Differentiation of (16) yields

$$\begin{aligned}V_r'(x) =& f_\rho'\left(q_\rho(x)\right)q_\rho'(x)S_r(x) + f_\rho\left(q_\rho(x)\right)S_r'(x)\\&-\frac{1}{2}f_\rho''\left(q_\rho(x)\right)q_\rho(x)q_\rho'(x)\frac{S_r^2(x)}{S_C(x)} - \frac{1}{2}f_\rho'\left(q_\rho(x)\right)q_\rho'(x)\frac{S_r^2(x)}{S_C(x)}\\&-f_\rho'\left(q_\rho(x)\right)q_\rho(x)\frac{S_r(x)}{S_C(x)}S_r'(x) + \frac{1}{2}f_\rho'\left(q_\rho(x)\right)q_\rho(x)\frac{S_r^2(x)}{S_C^2(x)}S_C'(x).\end{aligned}$$

For $x \leq r$, $S_r'(x) \geq 0$. Moreover, $f_\rho\left(q_\rho(x)\right) > f_\rho'\left(q_\rho(x)\right)q_\rho(x)$, by assumption. Consequently,

$$f_\rho\left(q_\rho(x)\right)S_r'(x) - f_\rho'\left(q_\rho(x)\right)q_\rho(x)\left(\frac{S_r(x)}{S_C(x)}\right)S_r'(x) \geq 0$$

By Assumption, $f_\rho''\left(q_\rho(x)\right)q_\rho \leq 0$. Moreover, as

$$q_\rho'(x) = \frac{1}{2}q_\rho(x)\left[\frac{S_C'(x)}{S_C(x)} - \frac{S_D'(x)}{S_D(x)}\right] > 0,$$

we obtain that

$$V_r'(x) > \frac{1}{2}q_\rho(x)f_\rho'\left(q_\rho(x)\right)S_r(x)\left[\frac{S_C'(x)}{S_C(x)}\left(1 + \frac{1}{2}\frac{S_r(x)}{S_C(x)}\right) - \frac{S_D'(x)}{S_D(x)}\left(1 - \frac{1}{2}\frac{S_r(x)}{S_C(x)}\right)\right].$$

Since $S_C' > 0$ when $x < \mu$ and $S_D' = 2d(x-\nu) < 0$, $V_r'(x) > 0$ for any $x < \min\{r, \mu\}$. Therefore, it only remains to show that $V_r'(x) > 0$ for any $x \in (\mu, r)$ whenever $\mu < r$. Notice that $V_r'(x) > 0$ if

$$\frac{S_C'(x)/S_C(x)}{S_D'(x)/S_D(x)} < \frac{2S_C(x) - S_r(x)}{2S_C(x) + S_r(x)}. \tag{17}$$

First, let us focus on the right-hand side of (17). Under quadratic preferences, $cS_r < S_C$ when $\mu < r$. Therefore

$$\frac{2S_C(x) - S_r(x)}{2S_C(x) + S_r(x)} = \frac{2cS_C(x) - cS_r(x)}{2cS_C(x) + cS_r(x)} > \frac{2cS_C(x) - S_C(x)}{2cS_C(x) + S_C(x)} = \frac{2c-1}{2c+1}.$$

Hence, $\frac{2S_C(x)-S_r(x)}{2S_C(x)+S_r(x)} > 3/5$ when $c \geq 2$ (and $\frac{2S_C(x)-S_r(x)}{2S_C(x)+S_r(x)} \geq 5/7$ when $c \geq 3$).

Second, under quadratic preferences, the left-hand side of (17) can be written as

$$\frac{S_C'(x)/S_C(x)}{S_D'(x)/S_D(x)} = \frac{(x-\mu)(x-2\nu+1)}{(x-\nu)(x-2\mu+1)}.$$

As

$$\frac{\partial\left(\frac{(x-\mu)(x-2\nu+1)}{(x-\nu)(x-2\mu+1)}\right)}{\partial\mu}=\frac{(x-1)(x-2\nu+1)}{(x-\nu)(x-2\mu+1)^2}<0, \tag{18}$$

we can write that

$$\frac{S_C'(x)/S_C(x)}{S_D'(x)/S_D(x)}<\frac{x(x-2\nu+1)}{(x-\nu)(x+1)}.$$

Let us assume that $\nu\leq 1$. Since

$$\frac{\partial\left(\frac{x(x-2\nu+1)}{(x-\nu)(x+1)}\right)}{\partial\nu}=\frac{x(1-x)}{x^2(x+1)}>0,$$

we can conclude that $\frac{S_C'(x)/S_C(x)}{S_D'(x)/S_D(x)}<\frac{x}{x+1}<\frac{1}{3}$, because $x<r\leq 1/2$.

Assume that $\nu>1$ and $c=2$. Since $\mu\geq r/2\geq x/2$, (18) implies that

$$\frac{(x-\mu)(x-2\nu+1)}{(x-\nu)(x-2\mu+1)}<\frac{(x/2)(2\nu-1-x)}{(\nu-x)}.$$

As

$$\frac{\partial\left(\frac{(x/2)(2\nu-1-x)}{(\nu-x)}\right)}{\partial x}=-\frac{1}{2(x-\nu)^2}\left(-x^2+2x\nu-2\nu^2+\nu\right)>0,$$

we can write that

$$\frac{(x-\mu)(x-2\nu+1)}{(x-\nu)(x-2\mu+1)}<\frac{0.25(2\nu-1.5)}{(\nu-0.5)}<1/2.$$

Finally, consider that $\nu>1$ and $c\geq 3$. Given that $x<1/2$ and

$$\frac{\partial\left(\frac{x(x-2\nu+1)}{(x-\nu)(x+1)}\right)}{\partial x}=\frac{\nu}{(x-\nu)^2(1+x)^2}\left(x^2-2x+2\nu-1\right)>0,$$

we can conclude that

$$\frac{x(x-2\nu+1)}{(x-\nu)(x+1)}<\frac{0.5(2\nu-1.5)}{(\nu-0.5)1.5}<2/3.$$

By comparing the upper bounds of the left-hand side of the inequality (17) with the lower bounds of the right-hand side, it can be concluded that $V_r'(x)>0$ for any $x\in(\mu,r)$.

Case δ. As $\partial Q_\delta / \partial A = 1$, $a_r(x) = f'_\delta\left(q_\delta(x)\right)S_r(x)$. Consequently,

$$a'_r(x) = f''_\delta\left(q_\delta(x)\right)q'_\delta(x)S_r(x) + S'_r(x)f'_\delta\left(q_\delta(x)\right).$$

Differentiating (16) and substituting these expressions we obtain

$$\begin{aligned} V'_r(x) &= f'_\delta\left(q_\delta(x)\right)q'_\delta(x)S_r(x) + f_\delta\left(q_\delta(x)\right)S'_r(x) \\ -f'_\delta\left(q_\delta(x)\right)S_r(x)&\left[f''_\delta\left(q_\delta(x)\right)S_r(x)q'_\delta(x) + S'_r(x)f'_\delta\left(q_\delta(x)\right)\right] \\ &= q'_\delta(x)\left[1 - f''_\delta\left(q_\delta(x)\right)S_r(x)\right]f'_\delta\left(q_\delta(x)\right)S_r(x) \\ &+ S'_r(x)\left[f_\delta\left(q_\delta(x)\right) - f'_\delta\left(q_\delta(x)\right)f'_\delta\left(q_\delta(x)\right)S_r(x)\right] \end{aligned}$$

Our assumptions on α imply that (i) $1 - f''_\delta\left(q_\delta\right) > 0$ and (ii)$f_\delta\left(q_\delta\right) - f'_\delta\left(q_\delta\right)f'_\delta\left(q_\delta\right) > 0$. Thus, as $S_r(x) \leq 1$, $V'_r(x) = 0 \Rightarrow q'_\delta(x)S'_r(x) < 0$. Moreover, using $S'_r = 2(r - x)$ and Lemma 2, we can conclude that $V'_r(x) > 0$ when $x \leq \min\{r, \Lambda\}$ and $V'_r(x) < 0$ when $x \geq \max\{r, \Lambda\}$. Then, the statement of the proposition immediately follows. □

Proof of Proposition 3 We analyze the two cases in turn:

Case ρ. From the proof of Proposition 2 we know that, for any $j \in C$, $V'_j(x) > 0$ for any $x \leq j$. This implies that any challenger $i \geq m$ prefers m to x for any $x < m$. Hence, if a Condorcet winner policy exists it cannot be lower than m.

Case δ. Consider $m < \Lambda$. From the proof of Proposition 2, we know that (i) for any $j \in [m, \Lambda)$, $V'_j(x) > 0$, for any $x \leq j$ and (ii) for any $j \geq \Lambda$, $V'_j(x) > 0$, for any $x < \Lambda$. Hence, in this case, if a Condorcet winner policy exists it cannot be lower than m. Similarly, when $m > \Lambda$ it can be shown that if a Condorcet winner policy exists, then it cannot be higher than m.

□

Funding Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature.

References

Aragonès E, Palfrey TR (2002) Mixed equilibrium in a Downsian model with a favored candidate. J Econ Theory 103(1):131–161

Aragonès E, Xefteris D (2012) Candidate quality in a Downsian model with a continuous policy space. Games Econ Behav 75(2):464–480

Ashworth S, Bueno de Mesquita E (2009) Elections with platform and valence competition. Games Econ Behav 67(1):191–216

Baik K-H (1998) Difference-form contest success functions and effort levels in contests. Eur J Polit Econ 14(4):685–701

Baik K-H (2007) Equilibrium contingent compensation in contests with delegation. South Econ J 73(4):986–1002

Baik K-H, Kim I-G (1997) Delegation in contests. Eur J Polit Econ 13:281–298

Baik K-H, Lee S (2007) Collective rent seeking when sharing rules are private information. Eur J Polit Econ 23(3):768–776

Balart P, Casas A, Troumpounis O (2022) Technological change, campaign spending and polarization. J Public Econ 211, Article 104666

Balart P, Flamand S, Troumpounis O (2016) Strategic choice of sharing rules in collective contests. Soc Choice Welf 46:239–262

Calvert R (1985) Robustness of the multidimensional voting model: candidate motivations, uncertainty, and convergence. Am J Polit Sci 29(1):69–95

Cardona D, De Freitas J, Rubí-Barceló A (2021) Environmental policy contests: command and control versus taxes. Int Tax Public Financ 28:654–684

Cardona D, De Freitas J, Rubí-Barceló A (2018) Polarization or Moderation? Intra-group heterogeneity in endogenous-policy contests. DEA Working Paper No. 87, Universitat de les Illes Balears

Cardona D, Rubí-Barceló A (2016) Group-contests with endogenous claims. Eur J Polit Econ 44:97–111

Carter J, Patty JW (2015) Valence and campaigns. Am J Polit Sci 59(4):825–840

Chowdhury S, Gürtler O (2015) Sabotage in contests: a survey. Public Choice 164(1):135–155

De Freitas J (2011) Political support for a private system of financing political campaigns. FinanzArchiv 67(4):352–377

Epstein GS, Nitzan S (2004) Strategic restraint in contests. Eur Econ Rev 48(1):201–210

Epstein GS, Nitzan S (2007) Endogenous public policy and contests. Springer, Berlin

Flamand S, Troumpounis O (2015) Prize-sharing rules in collective rent seeking. In: Companion to the political economy of rent seeking. Edward Elgar Publishing

Friehe T (2013) The role of investments in environmental lobbying contests. Environ Resour Econ 56:337–352

Groseclose T (2001) A model of candidate location when one candidate has a valence advantage. Am J Polit Sci 45(4):862–886

Herrera H, Levine D, Martinelli C (2008) Policy platforms, campaign spending and voter participation. J Public Econ 92(3–4):501–513

Heyes AG (1997) Environmental regulation by private contests. J Public Econ 63:407–428

Hirata D, Kamada Y (2020) Extreme donors and policy convergence. Soc Choice Welf 55:149–176

Hirsch AV (2016) Policy contest. Unpublished manuscript

Kobayashi K, Konishi H (2021) Effort complementarity and sharing rules in group contests. Soc Choice Welf 56:205–221

Kolmar M, Wagener A (2013) Inefficiency as a strategic device in group contests against dominant opponents. Econ Inq 51(4):2083–2095

Lindbeck A, Weibull JW (1993) A model of political equilibrium in a representative democracy. J Public Econ 51:195–209

Liston-Heyes C (2001) Setting the stakes in environmental contests. J Environ Econ Manag 41:1–12

Meirowitz A (2008) Electoral contests, incumbency advantages, and campaign finance. J Polit 70(3):681–699

Münster J (2006) Lobbying contests with endogenous policy proposals. Econ Polit 18(3):389–397

Nitzan S, Ueda K (2011) Prize sharing in collective contests. Eur Econ Rev 55:678–687

Nitzan S, Ueda K (2018) Selective incentives and intragroup heterogeneity in collective contests. J Public Econ Theory 20(4):477–498

Skaperdas S (1996) Contest success functions. Econ Theor 7:283–290

Skaperdas S, Vaidya S (2012) Persuasion as a contest. Econ Theor 51(2):465–486

Schofield N (2006) Equilibria in the spatial stochastic model of voting with party activists. Rev Econ Des 10(3):183–203
Schoonbeek L (2004) Delegation in group contests. Eur J Polit Econ 420:263–272
Trevisan F (2020) Optimal prize allocations in group contests. Soc Choice Welf 55:431–451
Wittman D (1977) Candidates with policy preferences: a dynamic model. J Econ Theory 14:180–189
Wittman D (1983) Candidate motivation: a synthesis of alternative theories. Am Polit Sci Rev 77(1):142–157