Power in the Council of the EU: organizing theory, a new index, and Brexit

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Abstract

We aim to estimate the power distribution in the Council of the European Union—both a priori and a posteriori. With respect to the latter, our analysis suggests that several previously used indices are ill-suited for this application. By introducing minimal modifications, we propose a new index and compare it with previous constructions in a unified framework. Empirically, we find that that all countries gain a priori voting power in the Council as a result of Brexit. We rely on data from the Chapel Hill Expert survey to compute a posteriori power and find that it is more unequally distributed than a priori power. Specifically, a posteriori power is almost exclusively held by relatively few rather populous states (yet not the United Kingdom). As regards Brexit, France appears as the main benefactor in terms of gaining a posteriori power; Poland loses substantive power in several areas but remains one of the most powerful EU member states.

Keywords A posteriori voting power · Council of the European Union · Brexit

1 Introduction

We study the voting power distribution in decision-making bodies facing binary ‘yay-nay’ decisions with a particular emphasis on the effect of the removal of a member. Such situations are ubiquitous—and similarly general will be our methods—but the subject is particularly topical in the context of the European Union (EU), be it due to potential voting right suspensions in the wake of recent infringement proceedings (Poland, Hungary) or due to the exit of one of its largest members [United Kingdom (UK)]. Given its significance, we focus here on the latter.

Loosely speaking, voting power is defined as the ability of each individual member (‘player’) of a decision-making body to change the outcome of a vote.

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Specifically, *a posteriori* power takes explicit account of players' ideological positions, to which *a priori* power applies the principle of insufficient reason. The former concept is sometimes seen as an enhancement of the latter, but, rather, the two just serve different purposes. A priori power is useful for normative assessments of voting rules. A posteriori power, by contrast, is descriptive and constitutes a first step towards estimating practical, 'actual', power.¹

We focus on one of the EU's main decision-making bodies, viz. the Council of the EU (Council of Ministers, henceforth 'the Council'). Having a concrete application in mind is crucial from a conceptual vantage point. We shall argue that several existing power indices—most of which previously applied to the Council—are unsuitable as they measure inapplicable or even convoluted notions of power and/or are based on implausible behavioral assumptions. With respect to a priori power, this lets us pick the Banzhaf index² over the Shapley–Shubik index (1954). As regards a posteriori power, we propose a new index, labelled 'status quo index', as we were not able to find a suitable existing index for our application. To illustrate why, we review the classical indices by Owen (1971) and Shapley (1977) and two more recent indices (Passarelli and Barr 2007; Álvarez-Mozos et al. 2013). By introducing the notion of a status quo, our own index essentially simply redefines the notion of default voting underlying the index by Passarelli and Barr (2007). All approaches, including our own, are cast in a unified framework that allows for the incorporation of agenda setting effects ('political winds'; Shapley (1977)) as well as uncertainty over players' positions. The latter also enables us to establish a connection with a priori power indices by making the above-mentioned principle of insufficient reason with respect to players' positions explicit (Proposition 1).

Enlargements of the EU and adaptations of the Council's standardly applied voting rule over the last decades have attracted various scholars to study their effect from an a priori angle [e.g. Laruelle and Widgrén (1998), Felsenthal and Machover (2001b), Leech (2002), Felsenthal Felsenthal and Machover (2004a), Napel and Widgrén (2006)]. We find that each member state's a priori power increases after Brexit, but significantly more so for very populous countries. When normalized, our results share similarities with recent work by Kóczy (2016), but we are led to different conclusions in view of our conceptual discussion.

A posteriori power computations for the Council have been tackled by fewer authors, most prominently perhaps by Widgrén (1995), Passarelli and Barr (2007), Barr and Passarelli (2009), Benati and Marzetti (2013); Mercik and Ramsey (2017) use an approach based on pre-coalitions and represent the only descriptive power study we are aware of that focuses explicitly on Brexit.³ Besides the difficulty to

¹ Actual power also entails additional factors such as strategic sophistication, bargaining skills, sub-coalition formation etc.

² Banzhaf (1964), and later Coleman (1971), independently reinvented the index originally discovered by Penrose (1946, (1952)). We nevertheless refer to it as 'Banzhaf index' as this is the most common terminology in the literature.

³ Staal (2016), upon inspection, is not a study of power but one of voting satisfaction, which is the probability that an individual player's decision agrees with that of the entire voting body (Brams and Lake 1978; Straffin et al. 1981).

select an appropriate power index, the estimation of each member states' political position poses a serious empirical challenge. While seemingly the most natural starting point, past voting behavior in the Council is difficult to analyze as most published results indicate unanimous acceptance. And even where they do not, such data serves as an indicator of satisfaction rather than power (cf. footnote 3). In the present paper, we rely on data from the Chapel Hill Expert Survey (CHES), which regularly interviews experts across Europe regarding their assessment of political positions of national parties in several policy dimensions. As opposed to macroeconomic or Eurobarometer data, which are often used to compute a posteriori power, this data set is more directly linked to the actors in the Council, i.e. national ministers delegated by their respective country's government. For the available policy dimensions we find that, overall, less populous member states usually wield no power at all. As for more populous states, France and Germany are typically most powerful, but also Poland exerts considerable power despite its smaller size. The latter's influence however generally diminishes after Brexit. Thus, as opposed to a priori power, power reversals occur. The UK, Italy and Spain, by contrast, are—both before and after Brexit—less powerful. This is a result of their relatively more extremal positions in the policy areas considered.

The rest of this paper is organized as follows. Section 2 introduces notation and discusses a priori power. Section 3 introduces our proposal, followed by four a posteriori indices from the literature. Section 4 argues why we choose the Banzhaf and status quo index, respectively, for our application. Readers primarily interested in our new index and our empirical results may content themselves with an understanding of Definition 2 and directly move to Sect. 5 which is devoted to applications to the Council. Section 6 concludes.

2 A priori power

If all that is known of a given voting situation is the voting rule itself, the Banzhaf and the Shapley–Shubik index are arguably the most well-known power indices. We present them here and introduce some useful notation along the way.

Consider the set $\mathcal{G}$ of voting games $g = (N, v)$ where N is a finite set of players with cardinality $n = |N|$ and $v : 2^N \rightarrow \{0, 1\}$ is a voting rule defined on sets $Y \subset N$, where Y is the yay-set, i.e. the set of players who vote in favor.⁴ A vote passes if and only if $v(Y) = 1$; by convention $v(\emptyset) = 0$ and $v(N) = 1$. If a player $i \in N$ can change the outcome of a vote for a given yay-set Y , i.e. if $v(Y \cup \{i\}) - v(Y) = 1$, she is called *pivotal* (for Y). Moreover, let Ω_N denote the set of permutations on N , here defined as bijections $\omega : N \rightarrow \{1, \dots, |N|\}$, and let P_i^ω be the set of predecessors of i under ω , $P_i^\omega = \{j \in N : \omega(j) < \omega(i)\}$. We call a player $i \in N$ also *pivotal* (for ω) if she is pivotal for P_i^ω . A power index (or value) is a function $\phi : \mathcal{G} \rightarrow [0, 1]^n$.

⁴ We purposefully use the letter Y instead of the often encountered C for 'coalition' because v need *not* have the interpretation of a 'utility/value' that is 'shared' among players in a coalition, see Sect. 4. By the same token, the usage of the word 'game' is—albeit standard—somewhat unfortunate as a denotation.

Definition 1 (*Banzhaf and Shapley–Shubik index*) For $g = (N, v) \in \mathcal{G}$ and $i \in N$, the (absolute) Banzhaf index (BI) is defined by

$$\phi_i^{\text{BI}}(g) = \frac{1}{2^{n-1}} \sum_{Y \subset N \setminus \{i\}} [v(Y \cup \{i\}) - v(Y)].$$

The Shapley–Shubik index (SSI) is defined by

$$\phi_i^{\text{SSI}}(g) = \frac{1}{n!} \sum_{\omega \in \Omega_N} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)].$$

Note that the SSI is normalized while the BI is not. The above formulas have the following intuitive interpretation. The BI measures the probability to be pivotal if all other player vote ‘yay’ with probability $\frac{1}{2}$ (making all yay-sets equally likely). The SSI measures the probability of being pivotal if players successively join a yay-set in random order. A conceptual discussion is deferred to sect. 4.

Formally, the SSI and the BI both represent *probabilistic values* (Weber 1988),

$$\phi_i(g) = \sum_{Y \subset N \setminus \{i\}} \pi_i(Y) [v(Y \cup \{i\}) - v(Y)], \quad (1)$$

with different probability measures π_i on $2^{N \setminus \{i\}}$: $\pi_i^{\text{SSI}}(Y) = \frac{1}{n} \binom{n-1}{|Y|}^{-1}$ and $\pi_i^{\text{BI}}(Y) = \frac{1}{2^{n-1}}$ for all $Y \in 2^{N \setminus \{i\}}$.⁵

Since players’ preferences in a priori settings are behind a ‘veil of ignorance’ (Rawls 1971; Harsanyi 1953, 1955), the BI and SSI only make statements about the voting rule itself. To elaborate, consider a weighted voting system $[q; w_1, \dots, w_n]$ with quota q and weights $w_1, \dots, w_n$ inducing a voting rule by $v(Y) = 1$ if and only if $\sum_{i \in Y} w_i \geq q$ where $Y \subset N$.

Example 1 Let $g = (N, v) \in \mathcal{G}$ be induced by $[q; w_1, w_2, w_3] = [3; 2, 1, 2]$. Then,

$$\begin{aligned} \phi_i^{\text{SSI}}(g) &= \frac{1}{3} \quad \text{for all } i \in N \\ \phi_i^{\text{BI}}(g) &= \frac{1}{2} \quad \text{for all } i \in N. \end{aligned}$$

Even though player 2 has only half the weight of other players in Example 1, her a priori power is identical, which is relevant e.g. for a fairness assessment of the voting rule. The incorporation of constraints beyond the voting rule itself leads to more tangible notions of power. Specifically, in Example 1, the players could be located on a one-dimensional political spectrum with players 1 and 3 being so far on the left and right respectively, that they never vote for the same policy proposal. All actual

⁵ SSI: note that each position of player i in a random permutation is equally likely, meaning that a predecessor set having length $|Y|$ occurs with equal probability $\frac{1}{n}$. For a fixed position $|Y| + 1$, all predecessor sets thus occur equally likely with probability $\binom{n-1}{|Y|}^{-1}$.

power then resides with player 2, as she is the only one able to tip the scales. Incorporating players' political positions leads to the concept of a posteriori power.

3 A posteriori power

We consider situations where—in addition to the voting rule itself—each player's political position is known. We develop a framework within which we present a new a posteriori index, the status quo index (SQI), and review existing proposals in the literature. A conceptual comparison is deferred to Sect. 4.

3.1 A new index

Even though the status quo is a relevant benchmark in many voting situations, it is mostly neglected in power analysis. We are only aware of a study by Widgrén and Napel (2013) who construct a strategic a priori index that incorporates the status quo. The underlying idea of our own model is that, by default, a player votes for a proposal if she regards it as an improvement of the status quo. For a given 'test player' we then ask, whether she can change the outcome of the vote by deviating from her original position, possibly at some cost.

Baseline model In addition to a voting game $(N, v) \in \mathcal{G}$, consider a policy space $P \subset \mathbb{R}^m$ ($m \geq 1$) in which are located: the respective political position (bliss point) p_i of each player $i \in N$, a policy proposal p , and the status quo s . Each player makes the following default voting decision: she votes in favor of the proposal if and only if it is closer to her political position than the status quo. Mathematically, this means that a player $i \in N$ votes by default for p if and only if

$$\begin{aligned} p_i \in F_{ps} &= \{x \in P : \|x - p\|_m < \|x - s\|_m\} \\ &= \left\{x \in P : \left\langle x - \frac{p+s}{2}, s - p \right\rangle < 0\right\}, \end{aligned}$$

where $\|\cdot\|_m / \langle \cdot, \cdot \rangle$ denotes the standard Euclidean norm/scalar product on $\mathbb{R}^m$. Thus, F_{ps} is the intersection of P with the half space in $\mathbb{R}^m$ containing those positions that are in favor of p given s , see Fig. 1 for an illustration.

In this basic setting, a player is said to have status quo power if she is pivotal assuming that all other players vote by default. In formal terms, this amounts to a player i 's status quo power index to be given by

$$\phi_i^{\text{SQI}}(g_{\mathbf{p}p}) = v\left(Y_{\mathbf{p}pi}^s \cup \{i\}\right) - v\left(Y_{\mathbf{p}pi}^s\right). \quad (2)$$

In (2), $\mathbf{p}$ is short for the n -tuple of players' positions $(p_j)_{j \in N}$ ($\mathbf{p} = (p_1, \dots, p_n)$ if N is enumerated), $g_{\mathbf{p}p}$ denotes the above-described voting situation (formally defined below) and $Y_{\mathbf{p}pi}^s = \{j \in N \setminus \{i\} : p_j \in F_{ps}\}$ is the set of players other than i that vote for the proposal by default. Note that $Y_{\mathbf{p}pi}^s$ does not depend on the position p_i of test player i . The right hand side of (2) equals 1 if player i is pivotal and 0 otherwise.

Fig. 1 Example of a policy space P with player set $N = \{1, 2, 3, 4, 5\}$, respective positions $p_1, \dots, p_5$, proposal p , and status quo s . The bold line indicates the separating hyperplane of points having equal distance from both s and p . Players 1 and 2 are in $A_{ps} = P \setminus F_{ps}$ and thus vote against the proposal p , players 3, 4 and 5 are in F_{ps} and therefore vote for the proposal p

Proposals with uncertainty As a first extension, we incorporate ‘political winds’ (Shapley 1977): often, either a proposal cannot be positioned exactly—e.g. because there is (ex ante) uncertainty about an unstrategic agenda setter’s position—or one is interested in a distribution of proposals over a legislative period. Both cases can be modeled by allowing for ‘vagueness’ of proposals, a common feature of all a posteriori indices presented below. Formally, we describe proposals by a general probability measure ν on P ($\nu(P) = 1$; a fixed proposal p corresponds to a delta measure δ_p). For a voting situation $g_{\nu p}$ with proposal measure ν and policy proposal p , expression (2) generalizes to

$$\phi_i^{\text{SQI}}(g_{\nu p}) = \int_P \left[\nu(Y_{\text{ppi}}^s \cup \{i\}) - \nu(Y_{\text{ppi}}^s) \right] d\nu(p). \quad (3)$$

The right hand side of (3) is the probability of being pivotal given by ν . The next example allows the reader to familiarize him-/herself with the above ideas and gives some basic intuition for this preliminary version of the SQI.

Example 2 We consider voting situations $g_{\nu p}$ with $P = [0, 1]$, $\mathbf{p} = (p_1, p_2, p_3)$, $0 = p_1 \leq p_2 \leq s \leq p_3 = 1$, and $\nu = \nu_u$ being the uniform measure on P .

- (a) Consider the unanimity voting game $[q; w_1, w_2, w_3] = [3; 1, 1, 1]$. By default, player 1 votes for all $p \in [0, s]$, player 2 for all $p \in (2p_2 - s, s) \cap P$, and player 3 for $p \in (s, 1]$. Since a player can only be pivotal for a proposal if both other players vote for it by default, Players 1 and 2 have no power. By contrast, player 3 is pivotal for all $p \in (2p_2 - s, s) \cap P$ and thus

$$\phi^{\text{SQI}}(g_{\nu p}) = (0, 0, \nu_u((2p_2 - s, s) \cap P)).$$

- (b) Consider the majority voting game $[q; w_1, w_2, w_3] = [2; 1, 1, 1]$ (or, equivalently, $[3; 2, 1, 2]$, cf. Example 1). Default voting is as in (a), but a player is now pivotal for a proposal if exactly one other player votes it. For example, for $p \in [0, 2p_2 - s] \cap P$, we deduce that $Y_{\text{pp1}} = \emptyset$, whereas for $p \in (2p_2 - s, s) \cap P$, $Y_{\text{pp1}} = \{2\}$ and for $p \in (s, 1]$, $Y_{\text{pp1}} = \{3\}$, i.e. player 1 is pivotal if

$p \in (2p_2 - s, 1] \cap P \setminus \{s\}$. Similarly, player 2 is pivotal for all $p \in P \setminus \{s\}$ and player 3 for all $p \in [0, 2p_2 - s] \cap P$. It follows that

$$\phi^{\text{SQI}}(g_{\text{pv}}) = (v_u((2p_2 - s, 1) \cap P, 1, v_u([0, 2p_2 - s] \cap P)).$$

Player 1's power is largest if player 2's position is close to her own: $\phi_1^{\text{SQI}}(g_{\text{pv}}) = 1$ if $p_2 \leq \frac{s}{2}$, similarly for player 3. Player 1's is generically the most powerful.

Finally, we introduce two further generalizations that are not featured by the existing a posteriori indices discussed below.

Positions with uncertainty Similarly to the proposal p we also want to allow the player's positions $\mathbf{p}$ to be described by probability measures $\{\mu_j\}_{j \in N}$ ($\mu_j(P) = 1$ for all $j \in N$; a fixed position p_j corresponds to a delta measure δ_{p_j}).⁶ We denote the product measure of the $\{\mu_j\}_{j \in N}$, defined on the n -fold Cartesian product P^n of P , by $\mu = \times_{j \in N} \mu_j$. It captures situations where the exact position of each player is not entirely known or the position itself is fuzzy (e.g. being the political position of a non-monolithic entity). This will be relevant for our application but also has a theoretical appeal, as it will allow for an explicit connection of a priori indices with a posteriori indices (Proposition 1).

Flexibility As mentioned when giving the underlying intuition, the ability to change the outcome of a vote may not merely be attached to a player's pivotality but can also depend on exogenous constraints/cost that may prevent a player from exercising her pivotality. For example, a fiscally conservative party would—ceteris paribus—find it impossible to vote for a strong tax increase as it would lose the support of its voter base. We next incorporate such flexibility limitation of the test player in our model.⁷ Specifically, we require a test player to be sufficiently flexible to vote both for and against a given proposal in order to exercise power. Using *flexibility parameters* $f_j \geq 0$ ($j \in N$) we model this by means of simple *flexibility functions* $\{f_{jx}\}_{j \in N, x \in P}$ that will be multiplied with the basic expressions (2) and (3) for power computations:

$$f_{ix}(p, s) = \begin{cases} 1 & \text{if } B_{x_{f_i}} \cap F_{ps} \neq \emptyset \text{ and } B_{x_{f_i}} \cap (N \setminus F_{ps}) \neq \emptyset, \\ 0 & \text{else.} \end{cases} \quad (4)$$

The extension to more sophisticated flexibility functions is immediate, but we will adhere to the above simple form throughout this paper for simplicity. In Definition (4), $B_{x_{f_i}} = \{y \in P : \|y - x\|_m \leq f_i\}$ is the closed ball with center x and radius

⁶ A related idea of positional randomization also appears in Benati and Marzetti (2013).

⁷ This violates the central premise by Braham and Holler (2005) who argue that 'the exercise of an ability is not to be conflated with its possession'. While true in essence, we think that 'there is no clear boundary between physical and preference-based constraints', as pointed out in a rebuttal by Napel and Widgrén (2005): we view flexibility limitations of a party due to preferences of its voter base as real and important constraints.

Fig. 2 For a simple majority vote, player 1 is pivotal (player 2 is against p , player 3 is for p), yet not sufficiently flexible to deviate from her default vote. Player 2 is pivotal and sufficiently flexible since the interior of $B_{p_2 f_2}$ contains the midpoint $\frac{p+s}{2}$. Finally, player 3 is sufficiently flexible, yet not pivotal. It follows that $\phi^{SQI}(g_{pp}) = (0, 1, 0)$ in this situation, cf. Definition 2

$f_i \geq 0$ and $\emptyset$ denotes the empty set. The set $B_{x f_i}$ is the set of positions that a player with position x finds acceptable. She is sufficiently flexible ($f_{ix}(p, s) = 1$) if that set includes both positions to vote for and against a proposal p . In particular, if $m = 1$, this is the case if and only if a neighborhood of the midpoint $\frac{p+s}{2}$ lies in $B_{x f_i}$ since this midpoint divides P into F_{ps} and A_{ps} . As a special case, note that if it is very costly for a player i to deviate from her preferred position p_i , i.e. if f_{ip_i} is extremely localized, it is very unlikely that she can take opposing stances on a proposal; specifically, if $f_i = 0$, then $\phi_i^{SQI}(g_{pp}) = 0$. The SQI is thus not only non-normalized but also potentially non-normalizable. Figure 2 illustrates the idea of flexibility.

For a complete definition of the SQI, let $\mathcal{SG}^{sq, f}$ be the set of *spatial voting games with status quo and flexibility*, $g = (N, v, P, \{\mu_j\}_{j \in N}, v, s, \{f_j\}_{j \in N})$.

Definition 2 (*Status quo index*) The status quo index (SQI) is for $g = (N, v, P, \{\mu_j\}_{j \in N}, v, s, \{f_j\}_{j \in N, x \in P}) \in \mathcal{SG}^{sq, f}$ and $i \in N$ defined by

$$\phi_i^{SQI}(g) = \int_{P^n} \left(\int_P v \left(\left(Y_{\mathbf{p} p_i}^s \cup \{i\} \right) \right) - v \left(Y_{\mathbf{p} p_i}^s \right) \right) f_{ix}(p, s) dv(p) d\mu(\mathbf{p}).$$

The SQI equals the probability that player i is pivotal given our assumptions on default voting, her flexibility, and the uncertainty induced by $\{\mu_j\}_{j \in N}$ and v . Note that the SQI is a probabilistic value with

$$\pi_i^{SQI}(Y) = \int_{\{(\mathbf{p}, p) \in P^{n+1} : Y = Y_{\mathbf{p} p_i}^s\}} f_{ip_i}(p, s) d(\mu \times v)(\mathbf{p}, p), \quad (5)$$

cf. Eq. (1).

3.2 Existing indices

Several approaches have been proposed to measure a posteriori voting power, some of which have been applied to the Council [see e.g. Passarelli and Barr (2007), Barr and Passarelli (2009)]. Here, we present four existing proposals: the Passarelli–Bar

index (PBI; 2007) as it is closest to our approach, the Owen index (OI; 1971) and the (Owen–)Shapley index (OSI; 1977; used by Barr and Passarelli (2009)) as they are most widely used, and, finally, the spectrum value (here referred to as ‘index’) (SI; Álvarez-Mozos et al. 2013), as it represents a nicely contrasting purely ordinal proposal. All four approaches can be cast into our framework allowing for some generalization and enabling a clearer conceptual comparison with the SQI and a priori indices.

3.2.1 Passarelli and Barr (2007)

Let $\mathcal{SG}^t$ be the set of *spatial voting games with tolerance*, $g = (N, \nu, P, \{\mu_j\}_{j \in N}, \nu, \{t_{jx}\}_{j \in N, x \in P})$ where $N, \nu, P, \{\mu_j\}_{j \in N}, \nu$ are defined as above, and the $t_{jx} : P \rightarrow [0, 1]$ are *tolerance functions* on P . The existence of tolerance functions is necessary to define default voting in $\mathcal{SG}^t$ where the absence of an explicit status quo prevents us from re-using the definition given for $\mathcal{SG}^{\text{sq}, f}$. Specifically, $t_{jx}(p)$ is the probability that a player with political position x votes for a proposal p ($j \in N$). Akin to the SQI, the PBI for arbitrary games can be formulated as an averaged version of the PBI for games with fixed positions and proposal,

$$g = g_{\mathbf{pp}} = (N, \nu, P, \{\delta_{p_j}\}_{j \in N}, \delta_p, \{t_{jx}\}_{j \in N, x \in P}) \in \mathcal{SG}^t.$$

Definition 3 (*Passarelli–Barr index*) The Passarelli–Barr index (PBI) is for every $g = (N, \nu, P, \{\mu_j\}_{j \in N}, \nu, \{t_{jx}\}_{j \in N, x \in P}) \in \mathcal{SG}^t$ and $i \in N$ defined by

$$\phi_i^{\text{PBI}}(g) = \int_{P^n} \left(\int_P \phi_i^{\text{PBI}}(g_{\mathbf{pp}}) d\nu(p) \right) d\mu(\mathbf{p}),$$

where

$$\phi_i^{\text{PBI}}(g_{\mathbf{pp}}) = \sum_{Y \subset N \setminus \{i\}} [v(Y \cup \{i\}) - v(Y)] t_{\mathbf{pp}}(Y). \quad (6)$$

Here, $t_{\mathbf{pp}}(Y) = \prod_{j \in Y} t_{jp_j}(p) \prod_{j \notin Y \cup \{i\}} (1 - t_{jp_j}(p))$ is the probability that the players in $N \setminus \{i\}$ who vote for proposal $p \in P$ by default form the yay-set Y .

For computational simplicity in expository examples, we will use a simple form for the $\{t_{jx}\}_{j \in N, x \in P}$,

$$t_{ix}(p) = \begin{cases} 1 & \text{if } \|p - x\|_m \leq t_i, \\ 0 & \text{else,} \end{cases} \quad (7)$$

where $t_i \geq 0$ is the radius of the ‘tolerance ball’ around player i ’s political position. Player i simply votes for all proposals within (outside) that ball with probability 1 (0). Passarelli and Barr consider a smoother transition of ‘yay’ to ‘nay’ by using bell-shaped functions of the form $t_{ix}(p) = e^{-\pi(\|x-p\|_m)^2}$. The SQI can be seen as a variation of the PBI that takes into account a status quo and extends it by

introducing flexibility parameters. Note that the $\{f_j\}_{j \in N}$ and the $\{t_j\}_{j \in N}$ have no connection: the former are relevant for the test player, while the latter determine default voting of others.

3.2.2 Owen (1971)

Let $\mathcal{SG}$ be the set of *spatial voting games*, $g = (N, v, P, \{\mu_j\}_{j \in N}, v)$, where all constituents are defined as above but with the following additional regularity assumption for the measure $\mu = \times_{j \in N} \mu_j$ and v :

$$\mu \times v(\{(\mathbf{p}, p) \in P^{n+1} : \|p_i - p\|_m = \|p_j - p\|_m \text{ for some } i \neq j \in N\}) = 0. \quad (8)$$

Events where players are equally distant from a proposal have measure zero.

Definition 4 (*Owen index*) The (generalized) Owen index (OI) is for every $g = (N, v, P, \{\mu_j\}_{j \in N}, v) \in \mathcal{SG}$ and $i \in N$ defined by

$$\phi_i^{\text{OI}}(g) = \int_{P^n} \left(\int_P \phi_i^{\text{OI}}(g_{\mathbf{p}p}) dv(p) \right) d\mu(\mathbf{p}),$$

where $g_{\mathbf{p}p} = (N, v, P, \{\delta_{p_i}\}_{i \in N}, \delta_p)$ and $\phi_i^{\text{OI}}(g_{\mathbf{p}p}) = v(P_i^{\omega_{\mathbf{p}p}^{\text{OI}}} \cup \{i\}) - v(P_i^{\omega_{\mathbf{p}p}^{\text{OI}}})$. Here, $\omega_{\mathbf{p}p}^{\text{OI}}$ is the permutation that orders players by increasing distance of their political position from p , i.e. $\omega_{\mathbf{p}p}^{\text{OI}}(i) < \omega_{\mathbf{p}p}^{\text{OI}}(j)$ if and only if $\|p_i - p\|_m < \|p_j - p\|_m$.

By condition (8) ϕ_i^{OI} is well defined, as configurations for which $\omega_{\mathbf{p}p}^{\text{OI}}$ does not exist have measure zero. The Owen index awards power to the pivotal player, when players are ordered by decreasing support for a proposal, as measured by Euclidean distance. The original definition in (Owen 1971) only considers the case where $\{\mu_j\}_{j \in N} = \{\delta_{p_j}\}_{j \in N}$ are point measures, the policy space is the standard unit sphere ($P = S^{m-1} = \{p \in \mathbb{R}^m : \|p\|_m = 1\}$), and $v = v_u$ is uniformly distributed on S^{m-1} .

3.2.3 Shapley (1977)

The OSI is very similar to the OI but defined on the set $\mathcal{SG}^d$ of *spatial voting games with directional proposals* $g = (N, v, P, \{\mu_j\}_{j \in N}, v)$, where, in contrast to $\mathcal{SG}$, the probability measure v is defined on the unit sphere S^{m-1} instead, corresponding to the view that proposals represent directions rather than locations in policy space. The regularity assumption (8) is replaced by

$$\mu \times v(\{(\mathbf{p}, p) \in P^n \times S^{m-1} : \langle p_i, p \rangle = \langle p_j, p \rangle \text{ for some } i \neq j \in N\}) = 0, \quad (9)$$

where $\langle \cdot, \cdot \rangle$ denotes that standard scalar product on $\mathbb{R}^m$. Thus, events where players have equal projections on a proposal have measure zero.

Definition 5 (*Owen–Shapley index*) The (generalized) Owen–Shapley index (OSI) is for every $g = (N, \nu, P, \{\mu_j\}_{j \in N}, \nu) \in \mathcal{SG}^d$ and $i \in N$ defined by

$$\phi_i^{\text{OSI}}(g) = \int_{P^m} \left(\int_{S^{m-1}} \phi_i^{\text{OSI}}(g_{\mathbf{p}p}) d\nu(p) \right) d\mu(\mathbf{p}),$$

where $g_{\mathbf{p}p} = (N, \nu, P, \{\delta_{p_i}\}_{i \in N}, \delta_p)$ and $\phi_i^{\text{OSI}}(g_{\mathbf{p}p}) = \nu(P_i^{\omega_{\mathbf{p}p}^{\text{OSI}}} \cup \{i\}) - \nu(P_i^{\omega_{\mathbf{p}p}^{\text{OSI}}})$. Here, $\omega_{\mathbf{p}p}^{\text{OSI}}$ is the permutation that orders all players by decreasing value of the projection of their political position on p , i.e. $\omega_{\mathbf{p}p}^{\text{OSI}}(i) < \omega_{\mathbf{p}p}^{\text{OSI}}(j)$ if and only if $\langle p_i, p \rangle > \langle p_j, p \rangle$.

The OSI is well defined by the regularity assumption (9), and the interpretation is similar to that of the OI, the only difference being how orderings of players are obtained. Shapley (1977) only considers the case of a uniform distribution ν on S^{m-1} , which Barr and Passarelli (2009) generalize to their application to the Council, Benati and Marzetti (2013) suggest an extension that amounts to allowing for arbitrary measures $\{\mu_j\}_{j \in N}$ (cf. footnote 6). The OI and the OSI are closely related: consider $g \in \mathcal{SG}$ or $g \in \mathcal{SG}^d$ respectively, with $R > r > 0$, $P = B_{0,R}$, and localized support of the position measures, $\text{supp}(\mu_i) \subset B_{0,r}$ ($i \in N$). Moreover, assume that proposals are uniformly distributed, i.e. $\nu = \nu_u$ on P and S^{m-1} respectively. It then holds that

$$\lim_{R \rightarrow \infty} \phi^{\text{OI}}(g) = \phi^{\text{OSI}}(g). \quad (10)$$

This is readily verified for $m = 1$ since, as R approaches infinity, proposals to the left and to the right of $B_{0,r}$ equally share all the mass while proposals inside of $B_{0,r}$ become irrelevant. The only relevant permutations $\omega_{\mathbf{p}p}^{\text{OSI}}$ are those given by an ordering from left to right and from right to left. But these are exactly the orderings that share all mass in the construction of the OSI in this setting. A general proof is given in Martin et al. (2017). For a beautiful geometric correspondence involving the Owen–Shapley index for $m = 2$, see Owen and Shapley (1989).

3.2.4 Álvarez-Mozos et al. (2013)

We complement the distance-based SQI, PBI, OI, and OSI with an ordinal approach recently developed by Álvarez-Mozos et al. (2013), and show how it fits into our framework. Let $\mathcal{PG}$ be the set of *voting games with spectrum*, $g = (N, \nu, \mu, \nu)$. In this setting, μ is no longer a product measure on some underlying political space P , but a probability measure on Ω_N , the set of permutations on N (hence the name $\mathcal{PG}$), and ν is a probability measure on N . That is, positional configurations of players are described by permutations and proposals correspond to a position in N . Note also that this is a one-dimensional approach. Any permutation $\sigma \in \Omega_N$ defines a natural ordering of players (the spectrum) by $i \prec^\sigma j$ if and only if $\sigma(i) < \sigma(j)$ for $i, j \in N$. A yay-set $Y \subset N$ is *connected* with respect to $\prec^\sigma$ if for all players $i, j \in Y$: $i \prec^\sigma k \prec^\sigma j$ implies that $k \in Y$. A permutation ω is called *admissible* with respect to the ordering

$<^\sigma$ if the set of predecessors P_i^ω is connected with respect to $<^\sigma$ for all $i \in N$. The set of admissible permutations is denoted by $\Omega_N^{<^\sigma}$. Using delta measures on Ω_N and N , a game $g = g_{\sigma p} = (N, v, \delta_\sigma, \delta_p) \in \mathcal{PG}$ in this setting is one with fixed positions (given by σ) and fixed proposal (given $p \in N$, i.e. corresponding to some player's policy position).

Definition 6 (*Spectrum index*) The (generalized) spectrum index (SI) is for every $g = (N, v, \mu, \nu) \in \mathcal{PG}$ and $i \in N$ defined by

$$\phi_i^{\text{SI}}(g) = \int_{\Omega_N} \left(\int_N \phi_i^{\text{SI}}(g_{\sigma p}) d\nu(p) \right) d\mu(\sigma),$$

where

$$\phi_i^{\text{SI}}(g_{\sigma p}) = \binom{n-1}{p-1}^{-1} \sum_{\substack{\omega \in \Omega_N^{<^\sigma} \\ \omega(\sigma^{-1}(p)) = 1}} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)]. \quad (11)$$

In order to understand Eq. (11), observe that $\binom{n-1}{p-1}$ is the number of admissible permutations with respect to $<^\sigma$ that start with the player at position p i.e. with player $\sigma^{-1}(p) \in N$. It follows that $\phi_i^{\text{SI}}(g_{\sigma p})$ is the probability that player i is pivotal for an admissible permutation that starts with player $\sigma^{-1}(p)$, assuming all such permutations are equally likely. Any distribution of proposals can be written as $\nu = \sum_{p \in N} \nu_p \delta_p$, with $\nu_p \geq 0$ and $\sum_{p \in N} \nu_p = 1$. For games $g = g_\sigma = (N, v, \delta_\sigma, \nu)$ with fixed order we write $\phi_i^{\text{SI}}(g_\sigma) = \sum_{p \in N} \nu_p \phi_i^{\text{SI}}(g_{\sigma p})$ where $g_{\sigma p} = (N, v, \delta_\sigma, \delta_p)$. Specifically, if $\nu_p = \frac{1}{2^{n-1}} \binom{n-1}{p-1}$ (a non-uniform, symmetric distribution with moderately centered mass; $\sum_{p \in N} \binom{n-1}{p-1} = 2^{n-1}$), it follows that

$$\phi_i^{\text{SI}}(g_\sigma) = \frac{1}{2^{n-1}} \sum_{\omega \in \Omega_N^{<^\sigma}} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)], \quad (12)$$

which is the case considered by Álvarez-Mozos et al. (2013)—akin to the SSI, but with the summation being restricted to admissible permutations.

Example 3 Consider the setup of Example 2. With some abuse of notation, we denote by g an element of $\mathcal{SG}^t$, $\mathcal{SG}$, $\mathcal{SG}^d$, or $\mathcal{PG}$, with the obvious identifications of constituents wherever possible. In particular, let μ be the product measure of fixed positions $\mu = \delta_p = \times_{j \in N} \delta_{p_j}$ translating to $\mu = \delta_\sigma$ in the $\mathcal{PG}$ context where $\sigma(i) < \sigma(j)$ if and only if $p_i < p_j$. Moreover, let $\nu = \nu_u$ be the uniform measure on P and S^{m-1} , respectively for $\mathcal{SG}^\#$, and $\nu = \frac{1}{2^{n-1}} \sum_{p \in N} \binom{n-1}{p-1} \delta_p$ for $\mathcal{PG}$. Finally, let $\{t_{jx}\}_{j \in N, x \in P}$ be given as in (7) assuming $t_1, t_2, t_3 \leq 1$, and define $P_1 = [0, t_1]$, $P_2 = [p_2 - t_2, p_2 + t_2] \cap P$, $P_3 = [1 - t_3, 1]$ where, in case of the SI we also assume $p_2 \neq 0, 1$. Finally, let $A \triangle B = (A \setminus B) \cup (B \setminus A)$ denote the symmetric difference between two sets A and B . It holds that

$$\begin{aligned}
 \text{a) } \phi^{\text{PBI}}(g) &= (\nu_u(P_2 \cap P_3), \nu_u(P_1 \cap P_3), \nu_u(P_1 \cap P_2)) \\
 &= \left(\nu_u([\max\{1 - t_3, p_2 - t_2\}, p_2 + t_2] \cap P), \nu_u([1 - t_3, t_1]), \right. \\
 &\quad \left. \nu_u([p_2 - t_2, \min\{t_1, p_2 + t_2\}] \cap P) \right), \\
 \phi^{\text{OI}}(g) &= \phi^{\text{OSI}}(g) = \phi^{\text{SI}}(g) = \left(\frac{1}{2}, 0, \frac{1}{2} \right). \\
 \text{b) } \phi^{\text{PBI}}(g) &= (\nu_u(P_2 \triangle P_3), \nu_u(P_1 \triangle P_3), \nu_u(P_1 \triangle P_2)), \\
 \phi^{\text{OI}}(g) &= \left(\frac{1 - p_2}{2}, \frac{1}{2}, \frac{p_2}{2} \right), \\
 \phi^{\text{OSI}}(g) &= (0, 1, 0), \\
 \phi^{\text{SI}}(g) &= \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4} \right).
 \end{aligned}$$

4 Which index for the Council?

We specify voting power as decisiveness, how it relates to the above indices and why we use the BI and SQI in our application to the Council.

4.1 Voting power as decisiveness

We define voting power as the ability of a player to change the outcome of a vote. This is for instance relevant if one is interested in knowing which player an outsider should bribe in order to effectively influence a vote. Operationally, such power is measured by determining a player's probability of being pivotal/decisive for a given voting rule, default voting of others, flexibility, and other potential constraints in more sophisticated models. This definition is featured prominently in the books by Felsenthal and Machover (1998), where it is referred to as *I-power*, and Morriss (2002) and it equals what Laruelle and Valenciano (2005) call '*decisiveness*' in their parsimonious voting framework.⁸ The latter only assumes probability distributions over *yay*-sets, for which our constructions of the SQI and PBI are explicit models. If voter preferences are subject to the principle of insufficient reason, the BI emerges, cf. Proposition 1 below and Sect. 7.2 in Laruelle and Valenciano (2005). *I-power*/decisiveness is connected to but distinct from the concepts of 'success' (= voting satisfaction, cf. footnote 3) and 'luck'.⁹

⁸ These authors abstain from using the expression 'voting power' in order to 'avoid any argument about the use of words'. We hope the same will not be necessary here.

⁹ A player's 'success' is the probability that the outcome of the vote coincides with her own decision and a player's 'luck' is the probability to be successful without being being decisive (i.e. pivotal). This yields Barry's equation: 'Success' = 'Decisiveness' + 'Luck', see Laruelle and Valenciano (2005, p 177) for a mathematical formulation.

The SSI can also be constructed probabilistically, but the underlying distribution makes it an unnatural measure of *I*-power/decisiveness (Laruelle and Valenciano 2005, Proposition 3). Order-based indices like the OI/OSI/SI/SSI are perhaps more promising for measuring *P*-power,¹⁰ i.e. a player's expected relative share in a unit prize which is only available to the winners, e.g. the distribution of ministries in a coalition government (Felsenthal and Machover 1998).¹¹

Unfortunately, what is being measured is often unclear, as pointed out by Felsenthal et al. (1998), Felsenthal and Machover (2001a), Felsenthal and Machover (1998), Felsenthal and Machover (2004b), Felsenthal and Machover (2005), Coleman (1971), Morriss (2002). Indeed, early papers on the SSI (Shapley and Shubik 1954; Shapley and Riker 1966) seem to ineptly refer to *I*-power/decisiveness, much like the original references of the OI/OSI and their application to the Council (Barr and Passarelli 2009) or comparable situations (e.g. O'Neill (1996)). Even the inventors of the SI initially seem to refer to a *I*-power/decisiveness, but then more adequately apply their index to a government formation process.

The following proposition conveniently summarizes this discussion by giving a mathematically explicit description of the principle of insufficient reason: for each a posteriori index we give a condition for the positional measure μ allowing us to identify it with either the BI (SQI/PBI) or the SSI (OI/OSI/SI). The proof is straightforward and carried out in Appendix 1.

Proposition 1 *Let $(N, v) \in \mathcal{G}$ be the voting game underlying g in the following contexts. The following equalities hold:*

1. *If $g \in \mathcal{SG}^{\text{sq},f}$ with $P = S^{m-1}$ ($m \geq 2$), $\mu_j = \mu_u$ uniform, $v(\{s\}) = 0$ and $f_j > 2$ for all $j \in N$, then $\phi^{\text{SQI}}(g) = \phi^{\text{BI}}(N, v)$.*
2. *If $g \in \mathcal{SG}^l$ with either (i) $t_{jx} \equiv \frac{1}{2}$ for all $j \in N$ or (ii) $P = S^{m-1}$ for $m \geq 2$, $\mu_j = \mu_u$ uniform, and t_{jx} as in (7) with $t_j = \sqrt{2}$ for all $j \in N$, then $\phi^{\text{PBI}}(g) = \phi^{\text{BI}}(N, v)$.*
3. *If $g \in \mathcal{SG}$ with identical $\mu_j = \mu^*$ ($j \in N$), then $\phi^{\text{OI}}(g) = \phi^{\text{SSI}}(N, v)$.*
4. *If $g \in \mathcal{SG}^d$ with identical $\mu_j = \mu^*$ ($j \in N$), then $\phi^{\text{OSI}}(g) = \phi^{\text{SSI}}(N, v)$.*
5. *If $g \in \mathcal{PG}$ with $\mu = \mu_u$ uniform on Ω_N , then $\phi^{\text{SI}}(g) = \phi^{\text{SSI}}(N, v)$.*

4.2 Significance of the status quo

It remains to choose between the SQI and the PBI, which differ by the incorporation of the status quo. Ignoring the latter has strong implications on voting behavior: such

¹⁰ Felsenthal and Machover (1998) and Laruelle and Valenciano (2005) voice conceptual reservations regarding the measurability of *P*-power; see Morriss (2002, p. xlvii) for a counterargument.

¹¹ A rationale can easily be given for the SI, where a proposal p can be thought of as denoting a player who initiates a coalition by successively including adjacent players. It is less clear in case of the OI and OSI.

players oppose any proposal not close enough to their own position, even if they view it as an improvement of the status quo (protest). Extremely stubborn/principled players (strongly localized tolerance functions) will even vote against *any* generic proposal. Conversely, if such a player's position is close to the status quo, she will even vote in favor of proposals that she regards as inferior to it, as long as they are close enough to her own position. Assuming such default voting seems implausible to us.¹²

The next example illustrates how existing a priori indices can differ and how these differences can be interpreted.

Example 4 Consider the setting of Examples 2 and 3 with $f_1 = f_2 = f_3 = 1$, $1 \geq t_1 = t_2 = t_3 = t > 0$, and $s = 1$.

- (a) It holds that $\phi^{\text{SQI}}(g) = (0, 0, \min\{1, 2 - 2p_2\})$ and $\phi^{\text{PBI}}(g) = (\min\{t, p_2 + 2t - 1\}, \max\{0, 2t - 1\}, \min\{t, 2t - p_2\})$; specifically, if $p_2 = \frac{1}{2}$ and $t \leq \frac{1}{2}$, then $\phi^{\text{SQI}}(g) = (0, 0, 1)$, yet $\phi^{\text{PBI}}(g) = (t, 0, t)$. With non-protesting behavior, player 1 has no *I*-power (contrary to the PBI result), since player 3 vetoes any proposal by default. By contrast, player 3 alone decides whether or not a vote passes as the other two players favor any proposal $p \neq 0, 1$ to the status quo (in agreement with the SQI). Order-based indices split all *P*-power equally among the two extremal players, capturing the idea that the central player is never pivotal in a bargaining process leading to the grand coalition.
- (b) It holds that $\phi^{\text{SQI}}(g) = (\min\{1, 2 - 2p_2\}, 1, \max\{0, 2p_2 - 1\})$. For simplicity, let $0 \leq p_2 - t \leq t \leq 1 - t \leq p_2 + t \leq 1$ so that $\phi^{\text{PBI}}(g) = (p_2, 2t, p_2)$; specifically, if $p_2 = \frac{1}{2}$, then $\phi^{\text{SQI}} = (1, 1, 0)$, yet $\phi^{\text{PBI}}(g) = (\frac{1}{2}, 2t, \frac{1}{2})$. With non-protesting behavior, player 3 has no *I*-power (contrary to the PBI result) as both other players view any proposal $p \neq 0, 1$ as an improvement of the status quo and will thus, by default, accept it regardless of player 3's decision (in agreement with the SQI). Order-dependent indices award most *P*-power to the central player, capturing the idea that she will be part of any majority coalition. Interestingly, if $p_2 < \frac{1}{2}$, then $\phi_1^{\text{OI}}(g) > \phi_3^{\text{OI}}(g)$, corresponding to the fact that closer neighbors are more likely to be in the same coalition.

5 An application: power in the Council before and after Brexit

We present our empirical results regarding the EU Council by computing the BI and the SQI.

¹² Note that disregarding a status quo is inconsequential in a priori settings by the principle of insufficient reason (Proposition 1).

5.1 Qualified majority voting rule

We formulate the usual voting rule in the Council—applied in about 80% of the votes (Consilium 2019)—using our notation (Appendix 2 contains the legal wording). To that end, let N_{EU} be the set of EU member states (EU27 or EU28), i.e. $n_{\text{EU}} = |N_{\text{EU}}| = 27$ or 28 depending on whether or not the UK is included. Let w_i denote the population of country $i \in N_{\text{EU}}$ and $w_{\text{EU}} = \sum_{j \in N_{\text{EU}}} w_j$ the total population of the EU. For simplicity, we assume that any member can only vote either for or against a given proposal (no abstentions). For any yay-set Y the voting rule in the Council can then be written as

$$v_{\text{EU}}(Y) = \begin{cases} 1 & \text{if } \left[\sum_{j \in Y} w_j \geq 0.65 \cdot w_{\text{EU}} \text{ and } |Y| \geq 0.55 \cdot n_{\text{EU}} \right] \\ & \text{or } [|Y| > n_{\text{EU}} - 4], \\ 0 & \text{else.} \end{cases} \quad (13)$$

Strictly speaking, (13) defines several voting rules at once, namely one for each membership and population configuration. However, we shall still refer to it as ‘the’ voting rule where no confusion can arise.

5.2 A priori power

To compute the Banzhaf index (BI), a direct application of Definition 1 results in testing up to $2^{n_{\text{EU}}-1} = 2^{27} \approx 1.3 \times 10^8$ subsets of N_{EU} which is computationally demanding. Computations were instead carried out with suitable applications of generating functions and implemented in Wolfram Mathematica. For computational details and numerical results, we refer the reader to Appendix 3. Here, we content ourselves with documenting relative changes in a priori I -power (as measured by the BI) due to Brexit.

Table 1 reveals that all countries become more powerful after Brexit but large countries more significantly so than smaller ones, and in fact in a non-monotonic manner, see Kirsch et al. (2018) for an analysis of this effect. Biggest beneficiary is Poland, followed by Spain and France. Intuitively, all countries are equally likely to be pivotal for a yay-set that already satisfies the population criterion in (13). If it does not, larger countries are more likely to be pivotal than smaller ones. The departure of the UK, one of the most populous countries in the EU, decreases the membership threshold ($0.55 \cdot n_{\text{EU}}$) less strongly than the population threshold ($0.65 \cdot w_{\text{EU}}$), thereby increasing the chances of being pivotal predominantly for larger states. A similar reasoning is given by Kóczy (2016), who employs a simplified version of (13) to compute the SSI for the Council before and after Brexit. The SSI, despite rather measuring P -power, delivers data comparable to the normalized BI. As suggested by the third column in Table 1, Kóczy concludes that several small countries lose power as a result of Brexit, even though influence is distributed among fewer

Table 1 Relative changes of the BI and the normalized BI as a result of Brexit in %

Country	BI	Normalized BI
Germany	+ 23.1	+ 17.0
France	+ 24.5	+ 18.2
UK	–	–
Italy	+ 22.9	+ 16.7
Spain	+ 29.4	+ 22.9
Poland	+ 34.5	+ 27.8
Romania	+ 12.5	+ 6.9
Netherlands	+ 12.1	+ 6.5
Belgium	+ 10.0	+ 4.5
Greece	+ 9.7	+ 4.2
Czech Rep.	+ 9.6	+ 4.1
Portugal	+ 9.4	+ 3.9
Sweden	+ 9.2	+ 3.8
Hungary	+ 9.1	+ 3.7
Austria	+ 8.5	+ 3.1
Bulgaria	+ 7.3	+ 2.0
Denmark	+ 6.2	+ 0.9
Finland	+ 6.1	+ 0.8
Slovakia	+ 6.0	+ 0.7
Ireland	+ 5.4	+ 0.2
Croatia	+ 4.9	– 0.4
Lithuania	+ 3.4	– 1.8
Slovenia	+ 2.6	– 2.5
Latvia	+ 2.5	– 2.7
Estonia	+ 1.6	– 3.5
Cyprus	+ 1.1	– 4.0
Luxembourg	+ 0.6	– 4.4
Malta	+ 0.5	– 4.5

member states (a similar conclusion is drawn in (Mercik and Ramsey 2017)). However, as seen in the second column, this putative decrease in a priori power is merely a relic of normalization: instead, all countries benefit from the departure of the UK in terms of a priori power.¹³

5.3 A posteriori power

We are forced to make a series of assumptions which we list in what follows.

¹³ Of course, it is in general still possible that the removal of a player in a voting scheme leads to a decrease of absolute power of a remaining player; consider e.g. player 1 after the removal of player 3 in a simple majority vote with weights $(w_1, w_2, w_3) = (2, 3, 4)$.

5.3.1 Assumptions

Policy space P and positional measures $\{\mu_j\}_{j \in N_{\text{EU}}}$. Countries in the Council are represented by national ministers (one for each government of an EU country) meeting in 10 different configurations depending on topic. To estimate government positions, we rely on data from the Chapel Hill Expert Survey (CHES), which regularly interviews a number of experts regarding their assessment of the positioning of national parties in various policy dimensions. Specifically, our analysis is based on the flash survey from 2017 (Marks et al. 2017), and complemented by data from the more comprehensive 2014 survey (Marks et al. 2014) for those countries that have not been surveyed in 2017. This leaves us with three concrete policy areas where qualified majority voting as in (13) is applied (additional areas are analyzed in Appendix 4; legal articles in brackets refer to the Treaty on the Functioning of the European Union):

BUD	EU authority over members' econ./budgetary policies (Art. 314),
IM	Immigration policy (Art. 79),
MULT	Integration of immigrants and asylum seekers (Art. 78 and 79).

Survey Likert scales range from 1 to 7 for BUD and from 0 to 10 for IM and MULT. As resulting policy spaces we define $P = [1, 7]$ and $P = [0, 10]$, respectively. In particular, they are one-dimensional. While many studies intend to give a single overall power index, we find it more transparent to consider single policy dimensions one-by-one. In reality, a given policy proposal rarely mixes different policy areas as the Council meets in explicitly different configurations.

The position of each government party is viewed as an independent normally distributed random variable (with average and variance according to expert assessments); the position μ_j ($j \in N_{\text{EU}}$) of the government as a whole is assumed to be induced by the average thereof, truncated to P (estimated means and standard deviations are given in Tables 3 and 4 in Appendix 4).¹⁴ The underlying idea behind this assumption is that all involved parties were pivotal in forming the government, thereby having an approximately equal share in determining the government's position.

Proposal measure v . We assume that the proposal measure v is given by the position of the European Commission. The reason is that the right to initiate legislative proposals in the EU is essentially monopolized by the Commission (Art. 17(2) TEU), which consists of 28 resp. 27 Commissioners (one for each country) all of which are bound by oath to act independently of their national government (Art. 245 TFEU). Analogous to the construction of the $\{\mu_j\}_{j \in N_{\text{EU}}}$, we assume the Commission's position to be the average of the positions of its members' national parties

¹⁴ The density of a distribution truncated to P is given by $f_P(x) = \frac{f(x)\mathbf{1}_P(x)}{\int_P f(x)dx}$, where f is the original distribution and $\mathbf{1}$ the indicator function for P .

(assumed again to be independent and normally distributed) truncated to P .¹⁵ The low standard deviations in Tables 3 and 4 indicate that the Commission speaks in a mostly unified voice in the policy areas considered here.

Status quo s. The status quo is assumed as the midpoint of the policy space P since it corresponds to the neutral position on the respective Likert scale. This amounts to assuming that ‘being in favor of X’ (as choosable in the survey) equals ‘appreciating if X is promoted with respect to the current state of affairs’. This correspondence is not perfect, but, we believe, a significantly better approximation to estimate default voting than Eurobarometer data.

Flexibility parameters $\{f_j\}_{j \in N}$. These are perhaps the least tangible parameters in our model. Yet flexibility in our view is an essential feature of power that simply cannot be ignored just on grounds of being hard to measure (cf. footnote 7). Instead we assume f_i to be given by the standard deviation of the position of country i as indicated in the CHES. The reasoning behind this linkage is that political parties oftentimes deliberately assume fuzzy positions so as to exert more flexibility—and hence power—in their decision-making process.

5.3.2 Results

Status quo power index calculations were implemented in Wolfram Mathematica using Monte Carlo simulations with 100,000 random picks according to the measures $\{\mu_j\}_{j \in N_{EU}}$ and v for each policy area. Since the SQI is non-negative and bounded by unity Hoeffding’s inequality (Hoeffding 1963) implies confidence intervals at the 99%-level given by ± 0.0051 resp. 0.51% around the values in Table 5 in Appendix 4.

We restrict ourselves here to graphical representations, briefly explain the underlying mechanisms that lead to the result, and mention the most striking features. In Figs. 3, 4 and 5, all indicated positions represent average values as given in Tables 3 and 4 and indicated midpoints are given by $\frac{p+s}{2}$, where p is the average position of the Commission (see also Example 2).

BUD The Commission favors more EU authority over national economic and budgetary policies, which by inspection is rejected by a robust qualified majority (13) in the Council by default both before and after Brexit. Thus, no single country has the ability to change the outcome of a corresponding vote resulting in zero status quo power. We may expect voting satisfaction to be high for member states opposing stronger budget authority by the EU.

IM Typical default voting leads to an approval of proposals by the Commission leading to higher voting satisfaction of member states favoring a more restrictive immigration policy. Most powerful at present is Germany followed by France, Poland and Spain. The latter two switch places after Brexit, where—contrary to the a priori case —Poland’s power decreases (power reversal).

MULT The Commission pushes for more assimilations, but according proposals will be narrowly rejected by default. Voting satisfaction is thus expected to be mixed with a slight advantage for member states favoring multiculturalism. As regards a

¹⁵ We have neglected the UK Commissioner who currently is an independent.

Fig. 3 EU authority over members' economic and budgetary policies; $P = [1, 7]$ with 1 = "strongly opposes" and 7 = "strongly favors". The horizontal axis is truncated

Fig. 4 Immigration policy; $P = [0, 10]$ with 0 = "fully opposed to a restrictive policy on immigration" and 10 = "fully in favor of a restrictive policy on immigration". The horizontal axis is truncated

posteriori power, the highest chances of being pivotal have France and Poland; yet, the former gains, the latter loses power with Brexit (power reversal). The UK's non-negligible power despite its distance from the midpoint is a result of its large positional standard deviation ($\sigma = 1.32$).

5.3.3 Discussion

While we believe that our approach (using the SQI and CHES data) has some advantages over existing a posteriori power analyses of the Council, substantial simplifications still had to be made. With this caveat in mind, Figs. 3, 4 and 5 show the

Fig. 5 Multiculturalism; $P = [0, 10]$ with 0 = “strongly favors multiculturalism” and 10 = “strongly favors assimilation”. The horizontal axis is truncated

following general patterns (cf. Example 2): a favorable positioning with respect to the midpoint between the status quo and the Commission’s proposals is essential to exert power. The farther away and the less populous, the higher the chances that a member state exerts no power at all. As a result, status quo power tends to be much more unequally distributed than a priori power except for when no state possesses any power (BUD).¹⁶

Together with analyses for three more general ideological areas in Appendix 4, the most striking country-specific findings summarize as follows. Hungary, the UK, and Italy most often take extremal positions and thus have comparably little power. Poland, by contrast, even though losing influence after Brexit, remains an overall influential member for its population size. This suggests that suspending Polish voting rights in the Council due to ongoing infringement proceedings could have substantial consequences on EU decision-making. France, arguably the most powerful member state, keeps or even gains influence in all considered areas after Brexit. Results are mixed for Germany, but by and large the country maintains substantial a posteriori power in the Council. By contrast, the UK, Spain and Italy, despite sizeable populations, predominantly exert little power both before and after Brexit. In particular, the UK—despite having almost identical population—wields significantly less power than France (or is equally powerless) in all considered areas.

¹⁶ Respective GINI coefficients are 0.71 (EU28) and 0.77 (EU27) for IM as well as 0.69 (EU28) and 0.69 (EU27) for MULT compared to 0.31 (EU28) and 0.34 (EU27) for the BI. See also Table 6 in Appendix 4 which reveals similar trends for additional policy areas.

6 Conclusion and outlook

We have argued that *I*-power/decisiveness is a natural power notion to be applied to the Council. From an a priori perspective, this makes the BI a natural choice. From an a posteriori perspective, order-based (SSI-like) indices may grasp *P*-power (SI), or capture a convoluted notion of power (OI/OSI), but appear inept for our application. Following a BI-like construction instead, the PBI seems more fitting, but suffers from implausible behavioral assumptions by disregarding the status quo. As a remedy, we have proposed a new index, the SQI.

Obviously, the SQI—or rather the underlying notion of power—is subject to important limitations. It simply measures the chances of being pivotal, given a player's flexibility and a model of default voting. As such, the SQI is applicable even if a vote is secret. In the Council, however, negotiations across different issues are everyday business, and players may deviate in a coordinated way from their default positions. What is more, the agenda setter (Commission) may take such processes into account when making proposals and adjust accordingly. In fact, many votes in the Council pass unanimously, since proposals that do not find the support of a qualified majority during a deliberation phase, will never be put to a formal vote. Conversely, even when a proposal appears to be backed sufficiently, a formal vote is often delayed in order to get the remaining opposing members on board (Trzaskowski 2016; Council of the European Union 2019). To incorporate such strategic sophistication, bargaining skills, procedural aspects, soft power, etc. into a plausible notion of power is notoriously tricky. On top of all that, the elicitation of political positions remains a difficult endeavor from an empirical point of view as existing surveys are usually not tailored to a posteriori power calculations, thereby necessitating additional assumptions on players, the status quo, and the policy proposer. All of this should be subject to further research.

As regards the application of the SQI to the Council, we find that, a priori, all member states become more powerful after Brexit, but populous countries more so than smaller ones. To compute a posteriori power, we have considered six policy areas as determined by the CHES from where positional data was taken. Most strikingly perhaps, we find several power reversals in the case of Poland as a result of Brexit, but sustained influence given the country's size.

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Proof of proposition 1

Throughout the proof we will make repeated use of Fubini's theorem,

$$\int_X \left(\int_Y \phi^\#(g_{xp}) dv(p) \right) d\mu(x) = \int_Y \left(\int_X \phi^\#(g_{xp}) d\mu(x) \right) dv(p),$$

whose conditions trivially holds since μ and ν are probability measures ($\# = \text{SQI, PBI, OI, OSI, SI}$; $X = P^n, \Omega_N$; $Y = P, S^{m-1}, N$).

SQI Recall that $F_{ps} = \{x \in P : \|x - p\|_m < \|x - s\|_m\}$ and let $A_{ps} = P \setminus F_{ps}$, where $P = S^{m-1}$. Without loss we may assume that $p \neq s$ is fixed since $\nu(\{s\}) = 0$: F_{ps} and A_{ps} are then exactly two half spheres, hence $\mu_u(F_{ps}) = \mu_u(A_{ps}) = \frac{1}{2}$, see Fig. 6. Using Eqs. (1) and (5) we obtain

$$\begin{aligned} \pi_i^{\text{SQI}}(Y) &= \int_{\{(\mathbf{p}, p) \in P^{n+1} : Y = Y_{\mathbf{p}pi}^s\}} f_{ip_i}(p, s) d(\mu \times \nu)(\mathbf{p}, p) \\ &= \int_{\{\mathbf{p}^{(i)} \in P^{n-1} : Y = Y_{\mathbf{p}pi}^s\}} d\mu^{(i)}(\mathbf{p}^{(i)}) \\ &= \mu^{(i)}(\{\mathbf{p}^{(i)} \in (S^{m-1})^{n-1} : Y = Y_{\mathbf{p}pi}^s\}) \\ &= \mu_u(F_{ps})^{|Y|} \mu_u(A_{ps})^{n-1-|Y|} \\ &= \left(\frac{1}{2}\right)^{n-1} = \pi_i^{\text{BI}}(Y). \end{aligned}$$

The second inequality above holds since p is fixed, since $f_j > 2$ for all $j \in N$ implies that $f_{ix} \equiv 1$ (every player is perfectly flexible), and since $Y_{\mathbf{p}pi}^s$ does not depend on p_i . Here, $\mathbf{p}^{(i)}$ denotes the tuple of positions of all $j \in N \setminus \{i\}$ and $\mu^{(i)} = \times_{j \neq i} \mu_j$ is the uniform product measure on the space of positional configurations without player i

Fig. 6 SQI: The policy space $P = S^{m-1}$ is divided into two half spheres F_{ps} and A_{ps} of equal measure. PBI: Indicated is the half sphere S_p of positions that vote in favor of p under assumption (ii) on the $\{t_{jk}\}_{j \in N, x \in P}$

($= P^{n-1}$). The third and fourth equation indicate the probability that those configurations are such that precisely $|Y|$ players vote in favor of p while the remaining $n - 1 - |Y|$ vote against it.

PBI As above we first fix a proposal p . If (i) holds then for all Y it follows that

$$t_{pp}(Y) = \prod_{j \in Y} t_{jp_j}(p) \prod_{j \notin Y \cup \{i\}} \left(1 - t_{jp_j}(p)\right) = \left(\frac{1}{2}\right)^{n-1}$$

regardless of player's positions, and the result is immediate. If (ii) holds, note that $S_p = \{x \in S^{m-1} : \|p - x\|_m < \sqrt{2}\}$ is exactly the half sphere centered at p , that is, for the given t_{jx} , if and only if a voter j 's position p_j is within that half sphere, does she vote in favor of proposal p , see again Fig. 6. Since positions are uniformly distributed, each player's probability to hold a position p_i in S_p is $\frac{1}{2}$. It follows that

$$\begin{aligned} \int_{P^n} t_{pp}(Y) d\mu(\mathbf{p}) &= \prod_{j \in Y} \int_P t_{jp_j}(p) d\mu_u(p_j) \prod_{j \notin Y \cup \{i\}} \int_P \left(1 - t_{jp_j}(p)\right) d\mu_u(p_j) \\ &= \left(\frac{1}{2}\right)^{n-1} \end{aligned}$$

for all $Y \subset N \setminus \{i\}$ and hence $\int_{P^n} \phi_i^{\text{PBI}}(g_{pp}) d\mu(\mathbf{p}) = \phi_i^{\text{BI}}(N, v)$ by Eq. (6).

OI Again, we fix a proposal p . Since each player's position is described by the very same probability measure, all orderings $\omega \in \Omega_N$ are equally likely. Explicitly, for an arbitrary $\omega \in \Omega_N$, we have (recall that $\mu = \times_{i=1}^n \mu^*$ is the product measure on P^n)

$$\begin{aligned} \mu(\{\mathbf{p} \in P^n : \omega_{pp}^{\text{OI}} = \omega\}) &= \mu(\{\mathbf{p} \in P^n : \|p_{\omega^{-1}(1)} - p\|_m < \dots < \|p_{\omega^{-1}(n)} - p\|_m\}) \\ &= \int_P \dots \int_P \mathbf{1}_{\{\mathbf{p} \in P^n : \|p_{\omega^{-1}(1)} - p\|_m < \dots < \|p_{\omega^{-1}(n)} - p\|_m\}} d\mu^*(p_1) \dots d\mu^*(p_n) \\ &= \int_P \dots \int_P \mathbf{1}_{\{\mathbf{p} \in P^n : \|p_{\omega^{-1}(1)} - p\|_m < \dots < \|p_{\omega^{-1}(n)} - p\|_m\}} d\mu^*(p_{\omega^{-1}(1)}) \dots d\mu^*(p_{\omega^{-1}(n)}) \\ &= \int_P \dots \int_P \mathbf{1}_{\{\mathbf{p} \in P^n : \|p_1 - p\|_m < \dots < \|p_n - p\|_m\}} d\mu^*(p_1) \dots d\mu^*(p_n). \end{aligned}$$

We have used Fubini's theorem in the third equality and relabelled variables in the fourth. Since the expression on the last line does not depend on ω and since there are $n!$ permutations in total, it follows that

$$\mu(\{\mathbf{p} \in P^n : \omega_{pp}^{\text{OI}}\}) = \frac{1}{n!}.$$

Finally, we compute

$$\begin{aligned}
\int_{P^n} \phi_i^{\text{OI}}(g_{\mathbf{p}p}) d\mu(\mathbf{p}) &= \int_{P^n} \left[v(P_i^{\omega_{\mathbf{p}p}^{\text{OI}}} \cup \{i\}) - v(P_i^{\omega_{\mathbf{p}p}^{\text{OI}}}) \right] d\mu(\mathbf{p}) \\
&= \sum_{\omega \in \Omega_N} \int_{\{\mathbf{p}: \omega_{\mathbf{p}p}^{\text{OI}} = \omega\}} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)] d\mu(\mathbf{p}) \\
&= \frac{1}{n!} \sum_{\omega \in \Omega_N} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)] \\
&= \phi_i^{\text{SSI}}(N, v).
\end{aligned}$$

OSI The proof is analogous to the case of the OI with the difference that proposals are elements of $\mathcal{S}^{m-1}$ and that we replace $\omega_{\mathbf{p}p}^{\text{OI}}$ by $\omega_{\mathbf{p}p}^{\text{OSI}}$.

SI Note that $\mu = \mu_u$ assigns a weight of $\frac{1}{n!}$ to each permutation $\sigma \in \Omega_N$. For a fixed proposal $p \in N$ we compute

$$\begin{aligned}
\int_{\Omega_N} \phi_i^{\text{SI}}(g_{\sigma p}) d\mu(\sigma) &= \frac{1}{n!} \sum_{\sigma \in \Omega_N} \binom{n-1}{p-1}^{-1} \sum_{\substack{\omega \in \Omega_N^{<\sigma} \\ \omega(\sigma^{-1}(p)) = 1}} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)] \\
&= \frac{1}{n!} \sum_{\omega \in \Omega_N} [v(P_i^\omega \cup \{i\}) - v(P_i^\omega)] \\
&= \phi_i^{\text{SSI}}(N, v).
\end{aligned}$$

Since the first sum extends over all $\sigma \in \Omega_N$, each permutation ω appears equally often in the double sum. In fact, each permutation appears $\binom{n-1}{p-1}$ times in total, since that is the number of admissible permutations that start with player $\sigma^{-1}(p)$ for given σ . $\square$

Voting rule in the EU Council: legal text

Art. 16(4) of the Treaty on European Union (TEU) states

As from 1 November 2014,¹⁷ a qualified majority shall be defined as at least 55 % of the members of the Council, comprising at least fifteen of them and representing Member States comprising at least 65 % of the population of the Union.

A blocking minority must include at least four Council members, failing which the qualified majority shall be deemed attained.

The other arrangements governing the qualified majority are laid down in Article 205(2) of the Treaty on the Functioning of the European Union.

¹⁷ For the sake of accuracy, Art. 3(2) in Protocol 36 on transitional provision required that the Nice system be used upon request of a member during a transition period ending on March 31, 2017.

Note that $0.55 \times 28 = 15.4$ and $0.55 \times 27 = 14.85$ so that we interpret ‘comprising at least fifteen’ to be redundant. The criterion of requiring at least four members in a blocking minority was added, so that the largest three countries in the EU28 and EU27, in both cases comprising more than 35% of the EU’s population, cannot block policies (see also Art. 238(3)a TEU).

Computational details for a priori power

Generating functions algorithm for the BI

The following was already noted by Mann and Shapley (1962), in their computations of the SSI for weighted voting systems. Let $Y \subset N_{\text{EU}}$ and $0 \leq q \leq w_{\text{EU}}$ be arbitrary and let $g_{Y,q}(z) = \sum_{k=0}^n g_{Y,q}^k z^k$ be the coefficient of x^q in the polynomial

$$g_Y(x, z) = \prod_{i \in Y} (1 + zx^{w_i}).$$

Then, $g_{Y,q}^k$ equals the number of subsets of Y with exactly k members and a total population of q . The computation of the BI is given by Algorithm 1.

```

input     $g = (N_{\text{EU}}, v_{\text{EU}}); \{w_j\}_{j \in N_{\text{EU}}}; w^c = 0.65 \cdot w_{\text{EU}}; n^c = 0.55 \cdot n_{\text{EU}};$ 
 $Y_i = N \setminus \{i\};$ 
output   $\phi^{\text{BI}}(g);$ 
for  $i \in N$  do
     $h_1^{(i)}(z) \leftarrow \sum_{q=1}^{w^c - w_i - 1} g_{Y_i, q}(z) = \sum_{k=0}^{n_{\text{EU}}} h_{1,i}^k z^k;$ 
     $h_2^{(i)}(z) \leftarrow \sum_{q=w^c - w_i}^{w^c - 1} g_{Y_i, q}(z) = \sum_{k=0}^{n_{\text{EU}}} h_{2,i}^k z^k;$ 
     $h_3^{(i)}(z) \leftarrow \sum_{q=w^c}^{w_{\text{EU}} - w_i} g_{Y_i, q}(z) = \sum_{k=0}^{n_{\text{EU}}} h_{3,i}^k z^k;$ 
     $\phi_i^{\text{BI}}(g) \leftarrow \frac{1}{2^{n_{\text{EU}} - 1}} \left( h_{1,i}^{n_{\text{EU}} - 4} + \sum_{k=n^c - 1}^{n_{\text{EU}} - 4} h_{2,i}^k + h_{3,i}^{n^c - 1} \right);$ 
end

```

Algorithm 1: BI for v_{EU}

Note that the three summands in brackets in the last line comprise all instances where country i is pivotal in a mutually exclusive manner. First, $h_{1,i}^{n_{\text{EU}} - 4}$ equals the number of sets Y_i with $n_{\text{EU}} - 4$ members that do not meet the population quota w^c even after adding country i . Since $|Y_i \cup \{i\}| = n_{\text{EU}} - 4 + 1 > n_{\text{EU}} - 4$, country i is pivotal. Second, $h_{2,i}^k$ for $n^c - 1 \leq k \leq n_{\text{EU}} - 4$ equals the number of sets Y_i with k members and subcritical population that turn into winning sets after adding country i without meeting the criterion $|Y_i \cup \{i\}| > n_{\text{EU}} - 4$. Third, $h_{3,i}^{n^c - 1}$ equals the number of sets Y_i with population of at least w^c but with only $n^c - 1$ members so that country i is pivotal.

Numerical results

Table 2 Population numbers from are from 2017 (Eurostat 2019) and were rounded to multiples of 100,000 for a priori power calculations based on Algorithm 1

Country	Population	BI		Normalized BI	
		EU28	EU28	EU28	EU27
Germany	82,521,653	16.98	20.91	10.28	12.03
France	66,989,083	13.97	17.39	8.46	10.00
UK	65,808,573	13.75	–	8.32	–
Italy	60,589,445	12.85	15.79	7.78	9.08
Spain	46,527,039	10.22	13.22	6.19	7.60
Poland	37,972,964	8.37	11.26	5.07	6.48
Romania	19,644,350	6.17	6.94	3.73	3.99
Netherlands	17,081,507	5.74	6.44	3.48	3.70
Belgium	11,351,727	4.80	5.28	2.90	3.03
Greece	10,768,193	4.70	5.15	2.84	2.96
Czech Rep.	10,578,820	4.66	5.11	2.82	2.94
Portugal	10,309,573	4.61	5.05	2.79	2.90
Sweden	9,995,153	4.56	4.99	2.76	2.87
Hungary	9,797,561	4.53	4.95	2.74	2.84
Austria	8,772,865	4.37	4.74	2.64	2.72
Bulgaria	7,101,859	4.09	4.38	2.47	2.52
Denmark	5,748,769	3.85	4.09	2.33	2.35
Finland	5,503,297	3.82	4.05	2.31	2.33
Slovakia	5,435,343	3.80	4.03	2.30	2.32
Ireland	4,784,383	3.70	3.91	2.24	2.25
Croatia	4,154,213	3.60	3.78	2.18	2.17
Lithuania	2,847,904	3.37	3.49	2.04	2.01
Slovenia	2,065,895	3.25	3.34	1.97	1.92
Latvia	1,950,116	3.24	3.32	1.96	1.91
Estonia	1,315,635	3.12	3.17	1.89	1.82
Cyprus	854,802	3.05	3.09	1.85	1.77
Luxembourg	590,667	3.00	3.02	1.82	1.74
Malta	460,297	2.99	3.00	1.81	1.73

All power index numbers are in %

Computational details for a posteriori power

Input data

Population data is as described in Table 2. Positional data is constructed from Marks et al. (2014, (2017) as explained in Sect. 5.3. We also list all government parties as of February 08, 2019 (abbreviations are in alignment with the CHES codebook

unless stated otherwise). The following additional assumptions were necessary due to changes in the European party landscape since 2014 and 2017 respectively.

- *UK*: DUP is not considered, since—despite a common confidence and supply agreement with the Conservatives—it is formally not a government party.
- *Italy*: Independents are neglected due to unavailable data.
- *Poland*: ‘United Poland’ is neglected as it is directly associated with PiS.
- *Romania*: Positions of ALDE, for which no CHES data exists, are formed by merging expert opinions on its predecessor parties PLR and PC.
- *Hungary*: KDNP is not considered, since—despite being an official government coalition partner—it is a de facto satellite party of the governing Fidesz.
- *Bulgaria*: the three parties IMRO, NFSB, ATAKA that form the alliance ‘United Patriots’ are treated as parties, i.e. we view the government as a coalition of a total of four parties.
- *Finland*: values for Blue Reform are based on those of the Finn’s Party from which it emerged in 2017.
- *Ireland*: Fianna Fáil is not considered, since—despite a common confidence and supply agreement with Fine Gael— it is formally not a government party.

In Tables 3 and 4, μ denotes means, σ denotes standard deviations. Specifically, Table 4 represents data from three additional, more general ideological variables:

INT	Orientation towards European integration,
LR	Overall ideological stance,
DEM	Democratic rights.

For these more general positions we posit the interpretation that they serve as a proxy for a member state’s position whenever they can be clearly related to a proposal. Survey Likert scales range from 1 to 7 for INT and from 0 to 10 for LR and DEM, resulting in policy spaces $P = [0, 10]$ and $P = [1, 7]$, respectively.

Table 3 Specific policy areas: positional data for EU governments and the Commission

Country	Parties in government	BUD		IM		MULT	
		μ .	σ .	μ .	σ .	μ .	σ .
Germany	CDU, CSU, SPD	4.00	0.46	5.75	0.95	6.49	0.70
France	LREM	5.20	0.00	6.05	1.70	4.47	2.21
UK	Cons	1.31	1.08	8.00	1.30	7.08	1.32
Italy	M5S, LN, Independents	1.57	0.53	8.15	0.57	8.14	0.53
Spain	PSOE	4.58	0.47	4.00	1.52	4.00	0.82
Poland	PiS, PR, SP	3.17	0.79	5.49	0.70	5.25	0.83
Romania	PSD, ALDE	4.22	0.66	4.66	1.41	6.13	1.25
Netherlands	VVD, CDA, D66, CU	4.07	0.37	5.64	0.79	6.34	0.65
Belgium	NVA, CD&V, MR, VLD	6.15	0.36	6.45	0.69	6.40	0.84
Greece	SYRIZA, ANEL	2.69	0.98	4.85	1.17	5.01	1.07
Czech Rep.	ANO	2.15	1.07	7.64	1.34	7.79	1.53
Portugal	PS	4.91	0.52	3.17	1.47	3.73	1.56
Sweden	SAP, MP	3.03	0.78	4.61	1.15	3.47	1.06
Hungary	Fidesz	1.14	1.14	9.91	0.29	9.75	0.58
Austria	ÖVP, FPÖ	3.00	0.65	8.00	0.75	8.45	0.60
Bulgaria	GERB, United Patriots	2.73	0.49	8.55	0.88	8.17	0.77
Denmark	V, LA, KF	4.28	0.54	6.30	1.03	6.43	1.18
Finland	KESK, KOK, Blue Reform	3.42	0.34	6.58	0.68	6.96	0.60
Slovakia	Smer-SD, SNS, MH	4.08	0.55	7.91	0.77	7.74	1.04
Ireland	FG	4.38	0.73	6.17	1.60	5.00	0.00
Croatia	HDZ, HNS	4.88	0.33	5.07	0.53	5.00	0.77
Lithuania	LVZS, LSDP	4.30	0.67	5.48	1.15	5.51	1.08
Slovenia	SMC, SD, DeSUS	4.18	0.40	3.78	0.93	3.42	1.00
Latvia	V, ZZS, NA	4.69	0.47	6.76	1.12	6.10	1.55
Estonia	EK, SDE, IRL	4.51	0.37	4.98	1.19	4.73	0.83
Cyprus	DISY, Evroko	4.38	0.48	7.25	0.35	6.50	0.00
Luxembourg	DP, LSAP, Greng	5.17	0.67	5.67	0.58	6.17	0.78
Malta	PL	3.25	0.55	8.50	1.00	6.25	1.50
Commission		4.65	0.19	5.84	0.27	5.85	0.28

Table 4 General ideology: positional data for EU governments and the Commission

Country	INT		LR		DEM	
	μ .	σ .	μ .	σ .	μ .	σ .
Germany	5.73	0.46	5.60	0.42	5.64	0.59
France	7.00	0.00	5.47	0.61	2.45	1.47
UK	2.36	1.08	7.64	0.63	7.00	1.35
Italy	2.07	0.53	6.73	1.20	6.96	0.91
Spain	6.71	0.47	3.86	0.77	2.86	0.95
Poland	4.62	0.79	4.83	1.03	4.95	0.63
Romania	5.80	0.66	5.09	1.16	6.99	1.26
Netherlands	5.04	0.37	6.33	0.38	5.35	0.72
Belgium	6.13	0.36	6.80	0.31	4.30	0.49
Greece	4.58	0.98	5.35	0.61	5.27	0.68
Czech Rep.	4.07	1.07	4.92	0.95	5.14	1.70
Portugal	6.55	0.52	3.83	0.94	3.08	1.62
Sweden	4.94	0.78	3.50	0.58	3.01	1.11
Hungary	2.64	1.14	8.73	0.83	9.24	0.89
Austria	4.30	0.65	7.40	0.37	8.00	0.77
Bulgaria	3.64	0.49	6.20	1.62	7.85	0.79
Denmark	5.04	0.54	7.30	0.51	5.15	0.97
Finland	4.23	0.34	6.11	0.76	6.95	0.63
Slovakia	5.27	0.55	5.75	0.67	6.85	0.81
Ireland	6.44	0.73	6.63	0.52	6.38	1.30
Croatia	6.61	0.33	5.67	0.50	5.00	0.63
Lithuania	5.65	0.67	3.53	0.80	5.23	1.33
Slovenia	6.01	0.40	4.19	0.57	4.41	0.74
Latvia	5.83	0.47	7.03	0.68	6.44	0.95
Estonia	6.33	0.37	4.64	0.46	4.60	0.85
Cyprus	6.38	0.48	8.08	0.76	6.63	0.85
Luxembourg	6.33	0.67	4.00	0.00	0.67	0.33
Malta	5.60	0.55	5.00	2.71	2.25	0.96
Commission	6.11	0.12	5.78	0.19	5.13	0.26

Graphical output for additional variables

INT The Commission advocates a stronger European integration. A look at population numbers reveals that by default such proposals will normally be rejected. Our data indicates that in order to push for more integration in the EU after Brexit, Poland will be a key player to get on board. However, in view of the fact that there

Fig. 7 European integration; $P = [1, 7]$ with 1 = “strongly opposed”, 4 = “neutral”, and 7 = “strongly in favor”. The horizontal axis is truncated

are many member states with non-zero power, there may often be several players that are potentially be pivotal for a specific proposal. Note also that in practice, many policy proposals that imply stronger European integration are decided by unanimity voting.

LR The Commission is right-leaning, which, by default, may or may not be supported by a qualified majority (13) given that Germany’s and France’s average positions are very close to the midpoint. For the same reasons, those two countries are the most powerful. Poland, again, experiences a power reversal through Brexit. The UK, given its more extreme position, has no power to change the outcome of votes in this area.

DEM. The Commission’s position is very similar to the status quo, making it approximately equally likely for each country to be for or against a proposal by default. As opposed to the BI, countries’ default behavior is of course strongly correlated because of their political positions. Before Brexit, qualified majorities exist, if at all, in case the Commission’s proposal aims for a more authoritarian policy compared to the status quo, resulting in considerable power for Poland. After Brexit, most countries exert very little (single-player) power (see Figs. 7, 8 and 9).

Fig. 8 Overall ideological stance; $P = [0, 10]$ with 0 = “extreme left”, 5 = “center”, and 10 = “extreme right”. The horizontal axis is truncated

Fig. 9 Democratic rights; $P = [0, 10]$ with 0 = “libertarian/postmaterialist”, 5 = “center”, and 10 = “traditional/authoritarian”

Numerical output and power inequality

Tables 5 and 6 show SQI values for all considered policy areas as well as GINI coefficients. A comparison with the corresponding GINI coefficients for the BI based on Table 2 (0.31 for EU28 and 0.34 for EU27) confirms that a posteriori power tends to be substantially more unequally distributed than a priori power. This corroborates our graphical findings, whereby a posteriori power tends to be concentrated on relatively few member states. GINI coefficients for BUD are a relic of randomization effects: recall that in that area, no member states exerts significantly non-zero power.

Table 5 SQI for EU28 and EU27 in %: BUD, IM, MULT

Country	BUD		IM		MULT	
	EU28	EU27	EU28	EU27	EU28	EU27
Germany	0.01	0.56	38.53	38.32	13.34	12.10
France	0.00	0.00	25.77	31.59	33.82	47.30
UK	0.07	–	5.93	–	16.23	–
Italy	0.00	0.00	0.00	0.00	0.00	0.00
Spain	0.01	0.21	12.91	13.84	8.34	9.26
Poland	0.01	0.92	14.58	10.38	32.24	14.99
Romania	0.01	0.22	7.13	7.08	6.85	9.30
Netherlands	0.01	0.23	7.01	6.97	3.66	4.52
Belgium	0.00	0.00	2.31	2.04	3.48	4.60
Greece	0.01	0.12	4.16	3.94	4.39	7.57
Czech Rep.	0.01	0.10	1.89	1.67	2.54	3.16
Portugal	0.00	0.05	1.87	1.91	3.90	4.59
Sweden	0.00	0.12	3.41	3.38	1.74	1.90
Hungary	0.00	0.04	0.00	0.00	0.00	0.00
Austria	0.00	0.07	0.05	0.03	0.00	0.00
Bulgaria	0.00	0.01	0.03	0.03	0.04	0.04
Denmark	0.00	0.15	2.26	1.82	3.21	2.78
Finland	0.00	0.02	1.12	0.79	0.42	0.36
Slovenia	0.00	0.17	0.07	0.04	0.74	0.65
Ireland	0.00	0.13	2.46	1.72	0.00	0.00
Croatia	0.00	0.04	1.64	1.36	2.01	3.71
Lithuania	0.00	0.11	1.76	1.17	2.61	2.24
Slovakia	0.01	0.11	0.38	0.28	0.40	0.48
Latvia	0.00	0.09	1.09	0.62	1.97	1.78
Estonia	0.00	0.09	0.99	0.58	1.23	1.18
Cyprus	0.00	0.11	0.00	0.00	0.00	0.00
Luxembourg	0.00	0.05	0.88	0.45	1.74	1.15
Malta	0.00	0.03	0.05	0.03	1.68	1.24
GINI	0.79	0.54	0.71	0.77	0.69	0.69

Table 6 SQI for EU28 and EU28 in %: INT, LR, DEM

Country	INT		LR		DEM	
	EU28	EU27	EU28	EU27	EU28	EU27
Germany	1.54	13.10	27.24	24.60	8.51	3.31
France	0.00	0.00	27.71	30.56	6.37	8.42
UK	5.73	–	0.13	–	5.04	–
Italy	0.00	0.00	12.80	12.47	2.08	0.73
Spain	0.04	0.29	5.14	3.75	2.17	2.31
Poland	8.59	42.95	17.54	7.03	19.94	5.02
Romania	2.11	9.06	7.87	6.72	2.98	1.18
Netherlands	5.12	16.65	0.90	0.86	5.79	3.21
Belgium	0.08	0.33	0.00	0.00	1.60	1.53
Greece	3.55	10.66	6.82	5.91	3.95	2.79
Czech Rep.	2.94	8.84	5.80	4.82	4.20	3.20
Portugal	0.13	0.45	2.15	1.69	2.22	2.19
Sweden	3.17	9.55	0.11	0.08	1.04	0.96
Hungary	0.70	2.28	0.01	0.01	0.00	0.00
Austria	1.88	6.36	0.00	0.00	0.01	0.01
Bulgaria	0.12	0.38	5.82	4.60	0.04	0.02
Denmark	2.26	6.80	0.04	0.03	2.88	2.02
Finland	0.22	0.84	4.66	3.65	0.16	0.06
Slovenia	1.98	3.76	5.42	4.46	0.60	0.31
Ireland	0.45	1.13	0.82	0.64	2.21	1.11
Croatia	0.00	0.00	5.31	3.81	2.64	1.43
Lithuania	0.76	1.07	0.45	0.36	2.32	1.31
Slovakia	0.09	0.18	0.59	0.44	1.28	0.48
Latvia	0.31	0.51	0.75	0.51	1.34	0.41
Estonia	0.01	0.01	1.10	0.78	1.42	0.59
Cyprus	0.03	0.06	0.06	0.03	0.78	0.20
Luxembourg	0.17	0.27	0.00	0.00	0.00	0.00
Malta	0.35	0.50	4.27	2.39	0.03	0.01
GINI	0.66	0.72	0.67	0.69	0.57	0.57

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