Modal reconstruction of transverse mode-locked laser beams

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Abstract

Transverse mode-locking in an end-pumped solid state laser by amplitude modulation with an acousto-optic modulator was investigated. Using the stochastic parallel gradient descent algorithm the modal power coefficients and the modal phases of the transverse mode-locked (TML) laser beam were reconstructed from the measured spatial and spatio-temporal intensity distributions, respectively. The distribution of the reconstructed modal power coefficients revealed that the average mode order of the transverse mode-locking process could be increased by a factor of about 8 compared to previous works, corresponding to an increase in the normalized oscillation amplitude by a factor of about 3. Furthermore, we found that besides a non-Poissonian modal power distribution, strong aberrations of the modal phases occurred in the experiment, resulting in a deformation of the oscillating spot. Additionally, we demonstrated the generation of up to four spots oscillating simultaneously on parallel traces by operating the TML laser on a higher mode order in the orthogonal direction to the transverse mode-locking process. TML lasers are of interest, e.g., for beam scanning purposes, as they have the potential to enable spot resolving rates in the multi-GHz regime.

1 Introduction

The idea of transverse mode-locking was already suggested [1] and demonstrated [2] by Auston in 1968, shortly after the first demonstration of locking longitudinal laser modes in 1964 [3, 4]. Auston showed that by locking the phases of multiple Hermite–Gaussian (HG) modes a fast scanning beam, oscillating in the transverse direction of the optical axis, could be generated. Since this initial demonstration of transverse mode-locking in a He–Ne laser the successful locking of transverse modes was also realized in semiconductor [5] and solid-state lasers [6, 7]. However, in these experiments the transverse mode-locking was only obtained over limited time periods as the lasers were either Q-switched or pumped by a pulsed laser source. Haug et al. [8] demonstrated that by locking two Laguerre–Gaussian

modes instead of Hermite–Gaussian modes a time varying spot size of the laser beam could be achieved. Furthermore, work has been published demonstrating the simultaneous locking of longitudinal and transverse laser modes, either in frequency-degenerated free-space lasers [9–11] or in non-linear fiber lasers [12].

Transverse mode-locked (TML) lasers have the potential to achieve spot resolving rates in the multi-GHz regime and, therefore, are of interest for any application requiring a fast scanning beam. However, before TML lasers can be adapted for fast scanning applications a better understanding of the parameters affecting their spatio-temporal dynamics is required. Theoretical considerations predict that the modal power distribution as well as the modal phases have a significant influence on the spatio-temporal dynamics of TML lasers [7]. However, while in previous publications the spatio-temporal intensity distributions of TML lasers have been measured, no analysis of the modal power distribution or the modal phases have been performed [2, 5–7]. In most cases it was estimated, that a number of 2–7 modes had contributed to the mode-locking process, while the highest average mode order claimed was $\bar{n} = 4$ [2].

Here, we present transverse mode-locking in an end-pumped solid state laser with a wide aperture using a standard acousto-optic modulator (AOM). Based on measurements of the spatial and spatio-temporal intensity

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distribution of the laser, an advanced analysis of multiple transverse mode-locked states was provided. As part of this analysis, the modal power coefficients as well as the modal phases of the mode-locked states have been reconstructed using the stochastic parallel gradient descent (SPGD) algorithm [22, 23]. With our setup, the average mode order $\bar{n}$ of the TML laser could be adjusted to any value reaching up to $\bar{n} = 31$. Thus, a normalized oscillation amplitude of up to 7.93 ± 0.25 was achieved, while locking up to 12 transverse modes simultaneously. Furthermore, the dependence of the spatio-temporal intensity distribution on the distribution of the modal power coefficients and the modal phases was investigated by comparing the measurements with numerical calculations. Finally, by operating the laser on a higher mode order in the direction orthogonal to the mode-locking process the generation of up to four spots oscillating simultaneously on parallel traces could be demonstrated.

2 Theory and experimental setup

If the transverse modes of a laser are phase-locked with each other, the laser generates a periodic spatio-temporal intensity distribution $I(x, y, t)$ repeating itself with the frequency ν_T of the transverse mode spacing [1]. Here, x and y are the spatial coordinates in the horizontal and vertical directions, respectively. For a given set of modes, the spatio-temporal intensity distribution depends on the modal amplitudes $A_{m,n}$ and the modal phases $\phi_{m,n}$:

$$I(x, y, t) = \left| \sum_{m,n=0}^{\infty} A_{m,n} HG_{m,n}(x, y) e^{i\omega_{m,n}t + i\phi_{m,n}} \right|^2, \quad (1)$$

where $HG_{m,n}(x, y)$ represents the electric field distribution of the Hermite–Gaussian modes normalized such that

$$\iint |HG_{m,n}(x, y)|^2 dx dy = 1. \quad (2)$$

Thus, the sum of the modal power coefficients $|A_{m,n}|^2$ is equal to the total power P of the TML beam:

$$\sum_{m,n=0}^{\infty} |A_{m,n}|^2 = P. \quad (3)$$

The coefficients m and n indicate the mode orders in the horizontal and the vertical direction, respectively. The angular frequencies of the different modes are, therefore, defined by

$$\omega_{m,n} = 2\pi(\nu_0 + (m + n)\nu_T), \quad (4)$$

where ν_0 corresponds to the frequency of the fundamental mode.

In the here presented experiments only modes in the vertical direction were phase-locked with each other, while the laser was operated on a single mode order in the horizontal direction. Therefore, the reduced expressions A_n and ϕ_n for the modal amplitudes and the modal phases will be used, respectively. For simplicity, furthermore field variations in the horizontal direction are neglected and the transverse coordinate in the vertical direction is replaced by the normalized coordinate $\xi = \sqrt{2}x/w_0$, with w_0 corresponding to the fundamental mode size. The output intensity distribution of an ideal TML beam is given by [1]

$$I(\xi, t) = \frac{1}{\sqrt{2}} \exp(-(\xi - \xi_0 \cos(2\pi\nu_T t))^2), \quad (5)$$

i.e., a Gaussian-shaped spot oscillating at the frequency of the transverse mode spacing along a cosine-shaped trajectory with a normalized amplitude of $\xi_0 = \sqrt{2\bar{n}}$. To realize such an ideal TML beam a Poisson distribution of the modal power coefficients $|A_n|^2$ is required [1] and the modal phase differences

$$\Delta\phi_n = \phi_n - \phi_{n+1} \quad (6)$$

have to be equal to zero.

For the realization of the TML laser the setup schematically depicted in Fig. 1 was used, which consisted of a pump beam shaping section, the TML laser itself, and a diagnostics section.

For the pump beam shaping the pump light emitted from a fiber-coupled laser diode (LD; K808DA5RN-35.00W by BWT Beijing LTD) at a wavelength of 808nm was focused onto the laser crystal using a combination of a cylindrical lens (CL) with a focal length of $f_{CL} = 156$ mm in the vertical direction and a lens (L1) with a focal length of $f_1 = 125$ mm. While L1 was used for mode matching purposes, CL was used to reshape the pump spot into a focal line, covering the full height of the laser crystal. Thus, in the horizontal direction the laser was limited to a single transverse mode order, while in the vertical direction the spatially broad gain distribution enabled the simultaneous operation of multiple transverse modes. The effective pump power incident on the laser crystal was determined by measuring the residual pump light that was reflected by a dichroic mirror (DM) which had a highly transmissive coating for the pump wavelength of 808nm and a highly reflective coating for the laser wavelength of 1064nm. In the later course of the setup the DM was used to separate the output of the laser from the incident pump light.

As a TML laser we chose an end-pumped Nd:YVO₄ laser with a plano-concave two-mirror cavity. The 1mm long YVO₄ crystal was a-cut and doped with 1 at.% Nd³⁺-ions. The outward facing side of the crystal had a highly reflective coating ($r = 99.89\%$) for the laser wavelength and served as a plane

Fig. 1 Schematic of the experimental setup for transverse mode-locking in a solid-state laser, consisting of a laser diode (LD), multiple lenses (L1–L4), a cylindrical lens (CL), a dichroic mirror (DM), two power meters (PWR), a Nd:YVO₄ laser crystal (Nd:YVO₄), an acousto-optic modulator (AOM), an adjustable slit, a curved mirror (CM),

a long-pass filter (LP), multiple beam splitters (BS), a neutral-density filter (ND), a CCD-camera (CCD), an isolator (ISO), and a photodiode (PD). Output A and output B represent the laser output via the curved and the plane (PM) mirror of the laser cavity, respectively

mirror (PM) of the laser cavity. A wide aperture was chosen for the crystal (6mm×6mm) to reduce the cutoff losses for higher-order modes.

The amplitude modulation for the active transverse mode-locking was realized with an acousto-optic modulator (AOM). The AOM was operated at a fixed modulation frequency $\nu_{\text{mod}} = 41.1\text{MHz}$ and had an aperture of 6.5mm×10mm. The strongest modulation of $13.87\% \pm 0.22\%$ was measured at the center of the AOM. For a beam displaced in the horizontal direction relative to the AOM the modulation strength was constant, while it decreased with an $1/e$ -width of 1.5mm for a displacement in the vertical direction. Therefore, the AOM provided an effective modulation area of 6.5 mm × 3mm. The average mode order $\bar{n}$ and, thus, the oscillation amplitude of the TML process was adjusted by the cavity alignment. Furthermore, a slit with an adjustable width was used to suppress higher-order modes that did not contribute to the mode-locking process.

The laser cavity was closed by a curved ($R = 2\text{m}$) and highly reflective ($r = 99.994\%$ at 1064nm) dielectric mirror (CM), resulting in an optical cavity length of $190.2 \pm 0.3\text{mm}$ with a measured longitudinal mode spacing of $\nu_L = 788 \pm 2\text{MHz}$. For a given ν_L the transverse mode spacing ν_T of the cavity is defined by

$$\nu_T = \frac{\Delta\zeta}{\pi} \nu_L, \quad (7)$$

where $\Delta\zeta$ is the Gouy-phase difference experienced by the beam after a single pass through the cavity [13]. Therefore, the transverse mode spacing was directly dependent on the cavity geometry and was calculated for the unpumped laser cavity to be 75MHz. However, due to the positive thermal optical coefficient of Nd:YVO₄ [14] the pump beam induced

a thermal focusing lens in the laser crystal. The thermal lens changed the cavity geometry, such that $\Delta\zeta$ was increased and ν_T shifted to higher frequencies. Due to the line shape of the pump focus the strength of the thermal lens deviated between the horizontal and vertical direction [15, 16], resulting in a different shift of ν_T for the two orthogonal directions. To match ν_T in the vertical direction to the optical modulation frequency $2\nu_{\text{mod}} = 82.2\text{MHz}$ of the AOM, a pump power of about 8W was required. Due to differences in the cavity alignment, the required pump power deviated for different mode-locking states by up to $\pm 1\text{W}$. The focal length of the thermal lens in the vertical direction was calculated to be 9.4m using the ABCD-matrix formalism [17, 18]. Correspondingly, the calculated fundamental mode size in the vertical direction of the pumped laser cavity was reduced to $w_0 = 419\mu\text{m}$ in comparison to a fundamental mode size of $437\mu\text{m}$ in case of the unpumped laser cavity. These theoretical considerations matched the vertical mode size of $422 \pm 7\mu\text{m}$ measured in the experiment. For the modes in the horizontal direction a fundamental mode size of $387 \pm 7\mu\text{m}$ was measured, indicating a thermal lens with a focal length of 2.5m. The corresponding transverse mode spacing was calculated to be 100MHz, such that a locking of modes in the horizontal direction was not possible.

The light of the TML laser was coupled out via both cavity mirrors, as the CM (output A) as well as the PM (output B) showed a remaining transmission at the laser wavelength of 0.006% and 0.11%, respectively. For the light from output A a longpass filter (LP) with a cut-off wavelength of 950nm was used to remove residual pump light. From the remaining laser light 10% were transmitted by a beam splitter (BS), and the lens L2 ($f_2 = 175\text{mm}$) imaged the plane of the laser crystal onto a CCD-camera, which was protected against saturation by a neutral density filter (ND). The remaining

90% of the light from output A were reflected by the beam splitter (BS) and detected by a power meter (PWR A).

The light from output B was separated from the incident pump light by the DM. Via a combination of two lenses (L3, $f_3 = 150\text{mm}$, and L4, $f_4 = 100\text{mm}$) the plane of the laser crystal was imaged into the plane of an amplified photodiode (PD; HSPR-X-I-2G-IN inverted by FEMTO Messtechnik GmbH), which was mounted on a 2D translation stage. To achieve a uniform reference frame the measurements of the CCD-camera and the photodiode were both rescaled to the spatial scale of the crystal plane. An isolator (ISO) was used to suppress back reflections from the photodiode which would otherwise disturb the laser oscillation. A fraction of 10% of the light was reflected by a beam splitter (BS) onto a power meter (PWR B) and served as a reference for the power coupled out via output B.

3 Results and discussion

The results will be presented in four separate sections: In the first section (Sect. 3.1) the time-averaged intensity distribution of the TML laser beam measured by the CCD-camera will be discussed, and the reconstruction of the modal power coefficients for different TML states will be given. In the second section (Sect. 3.2) the transverse mode-locking of the laser will be verified by evaluating the spatio-temporal intensity distribution measured with the photodiode. A detailed analysis of the measured spatio-temporal intensity distribution, including a reconstruction of the modal phases, will be given in the third section (Sect. 3.3). Finally, in the fourth section (Sect. 3.4) the simultaneous oscillation of multiple spots on parallel traces will be presented, which can be achieved by operating the laser on a higher mode order in the orthogonal direction to the transverse mode-locking process.

3.1 Time-averaged intensity distributions

With the above described setup a variety of TML beams were successfully excited by operating the laser on the fundamental mode order in the horizontal direction, while locking multiple mode orders in the vertical direction. The pump power required for the different TML beams was in average $8 \pm 1\text{W}$, while the combined output power of output A and B ranged from $26.3 \pm 0.6\text{mW}$ up to $71.1 \pm 0.69\text{mW}$.

The time-averaged intensity distributions of these TML beams were measured with the CCD-camera and are depicted in Fig. 2a–e. Different oscillation amplitudes were chosen for the measurements via the cavity alignment and the width of the adjustable slit. Therefore, the measured intensity distributions formed straight lines of different length, depending on the full range of the oscillation reaching from $1.4 \pm 0.2\text{mm}$ (Fig. 2a) to $4.7 \pm 0.2\text{mm}$ (Fig. 2e).

Fig. 2 a–e Normalized time-averaged intensity distributions $I(x, y)$ of different TML beams measured by the CCD-camera. The solid red lines indicate the central position of the AOM, while the dashed red lines correspond to the $1/e$ -width of the effective modulation area. f–j The red bars show the reconstructed modal power coefficients $|A_n|^2$ divided by the total power P of the corresponding TML beam. The blue bars show Poisson distributions calculated for the corresponding average mode numbers $\bar{n}$. The solid green lines indicate the reconstructed modal phases $\phi(\omega_n)$

In Fig. 2a–e the effective modulation area of the AOM is marked by the dashed red lines. It is noteworthy that for all measured TML states the center of the AOM and the center of the generated TML beam did not coincide. A hypothesis for this condition is that, for an asymmetric AOM position the TML beam reached outside the effective modulation area of the AOM and, therefore, the transverse mode-locked state was energetically favored in comparison to the unlocked state, as it enabled the laser beam to evade the time-dependent losses induced by the AOM. When the deflection of the AOM was strong the intensity distribution of the TML laser moved outside of the effective modulation area and returned when the deflection was weak.

A variety of different schemes have been developed, to provide a modal decomposition of multi-mode beams [19–25]. Yan et al. [25] even presented an approach including the degree of coherence between the contributing modes in addition to their modal amplitudes and phases. While these schemes were initially developed for the decomposition of single frequency multi-mode beams, the numerical analysis technique based on the SPGD algorithm [22–24] is also suited for the modal decomposition of a multi-mode beam with modes at different frequencies. Furthermore, the SPGD algorithm has proven in previous works to provide an unambiguous and precise modal decomposition of a multi-mode beam based on its measured intensity distribution [22, 23, 26]. Therefore, we have decided to use the SPGD algorithm to reconstruct the modal power coefficients $|A_n|^2$ from the measured time-averaged intensity distributions. Here, the measured intensity distributions shown in Fig. 2a–e were used for the reconstruction algorithm. As fitness parameter the correlation coefficient

$$\gamma = \frac{\iint (I(x, y) - \bar{I})(I'(x, y) - \bar{I}') dx dy}{\sqrt{\iint (I(x, y) - \bar{I})^2 dx dy \iint (I'(x, y) - \bar{I}')^2 dx dy}} \quad (8)$$

between the measured $I(x, y)$ and calculated $I'(x, y)$ time-averaged intensity distributions was used. In the course of the publication γ and γ' will represent the correlation coefficient for a calculated beam based on a Poissonian and the reconstructed distribution of the modal power coefficients, respectively. As in the horizontal direction the laser operation was limited to the fundamental mode order, only the modes in the vertical direction up to the $HG_{0,50}$ mode were considered for the reconstruction. Higher mode orders were neglected in favor of a reduced computational workload, as it was estimated that in sum they contributed less than 0.1% to the total power of the beams measured in the experiment. The size of the individual modes were calculated based on the above measured fundamental mode sizes in the vertical and horizontal direction, respectively. Furthermore, the measured time-averaged intensity distributions were aligned

with the axis of the modal basis set before the reconstruction algorithm was applied.

In Fig. 2f–j the reconstructed modal power coefficients $|A_n|^2$ divided by the total power P of the corresponding TML beams, are displayed as red bar plots. The correlation values γ' between the reconstructed and measured intensity distributions were for all TML beams above 0.992, representing a good match. As an estimation for the reliability of the reconstructed modal power coefficients, each reconstruction of the two beams with the smallest (Fig. 2f) and the largest (Fig. 2j) oscillation amplitude was repeated ten times. For both beams the standard deviation of each reconstructed modal power coefficient was less than 0.6% of the total beam power P . Considering the random nature of the SPGD algorithm and these low deviations, it can be assumed that the reconstructed modal power distributions were close to their global optimum.

It can be seen that with increasing scanning range of the TML beams the distributions of the modal power coefficients were shifted to higher mode orders, as predicted by the theory [1]. By calculating the first momentum of the modal power distributions the corresponding average mode numbers $\bar{n}$ were determined. For the measured TML beams the values of $\bar{n}$ reached from 3 (Fig. 2a + f) up to 31 (Fig. 2e + j), demonstrating that $\bar{n}$ could be varied over an order of magnitude using the presented setup. The value of $\bar{n} = 31$ exceeded the previous reported maximum value of $\bar{n} = 4$ by a factor of 7.75 [2].

In Fig. 2f–j, additionally to the reconstructed distributions, the Poisson distributions calculated for the corresponding average mode order $\bar{n}$ are plotted as blue bars. When comparing both distributions with each other, it can be seen that for the reconstructed modal power distributions the power was concentrated in a reduced number of modes. As a measure for the number N of modes that contributed to the transverse mode-locking process the full- 2σ width of the modal power distribution was used. Based on this definition, a number of $N = 5$ (Fig. 2f) up to $N = 12$ (Fig. 2j) individual modes contributed to the measured TML beams (see Table 1). For most of the TML beams this was about half of the modes calculated for the corresponding Poisson distribution. In case of an actively mode-locked laser the spectral bandwidth, i.e., the number of excited modes, depends on the modulation depth of the modulator [27]. In analogy, in a TML laser the number of contributing modes compared to a Poisson distribution can be reduced due to an insufficient modulation strength of the AOM, i.e., resulting in an insufficient mode-locking force prohibiting the exploitation of the available gain bandwidth. Therefore, the number of contributing modes might be increased by using an AOM with improved modulation strength and by applying additional gain shaping methods [28–32], e.g., to tailor the spatial gain of the TML laser in favor of a Poissonian

modal power distribution. However, so far the available gain shaping methods are either limited by the pump power that can be applied [28] or by the number of modes that can be excited [29–32].

As a representative example for the modal reconstruction, Fig. 3 shows a comparison between the time-averaged intensity distributions of the TML beam with $m = 0$ and $\bar{n} = 31$ either measured with the CCD-camera (Fig. 3a) or calculated based on the reconstructed (Fig. 3b) and the Poissonian (Fig. 3c) modal power distributions. All three intensity distributions showed a similar shape of a straight line with two spots of increased intensity at the vertical endpoints. This shape is a result of the cosine-shaped trajectory of the oscillating spot (see Eq. 5). In analogy to a pendulum the spot moved fast at the center and slow at the outer turning points of the beam, decreasing and increasing the time-averaged intensity, respectively. The non-Poissonian modal power distributions of the measured and reconstructed beams resulted in a deformation of the beam shape and increased the time-averaged intensity at the outer turning points by a factor of 1.4 compared to a beam with a Poissonian modal power distribution. Except for interference fringes caused by the cover glass of the CCD-camera, a good agreement between the measured and reconstructed intensity distribution can be observed with a correlation coefficient of $\gamma' = 0.993$.

While the above presented reconstruction of the time-averaged intensity distributions showed which modes were excited in the laser, it did not provide any information about the phase relations between these modes. Thus, to verify the successful transverse mode-locking of the laser, its

Fig. 3 Normalized time-averaged intensity distributions $I(x, y)$ of a TML beam with $m = 0$ and $\bar{n} = 31$ either **a** measured with the CCD-camera or **b**, **c** calculated based on the reconstructed and the Poissonian modal power distribution, respectively. The coefficients γ and γ' represent the correlation value of the corresponding intensity distributions with the measured intensity distribution shown in **a**

spatio-temporal intensity distribution was measured and the results are presented in the following section.

3.2 Measurement of the spatio-temporal intensity distribution

To measure the spatio-temporal intensity distributions of the TML laser beams, the photodiode (PD) was scanned in steps of $100 \mu\text{m}$ through the x - y -plane of the laser beams and the measured time traces were synchronized with each other via the trigger signal of the AOM driver. As the gain bandwidth of the laser covered multiple longitudinal modes, sporadic perturbations due to longitudinal mode competition could occur. However, the corresponding radio frequency signal of these perturbations was typically -19dB below the radio frequency signal of the transverse mode-locking process. Nevertheless, for a better signal-to-noise ratio the time traces of the following measurements were averaged over ten subsequent traces. The animation 1 in the supplementary material shows the temporal development of the spatial intensity distributions for each of the measured beams, respectively.

To represent the spatio-temporal intensity distributions of the TML beams in two-dimensional images their projection onto the t - y -plane was calculated. As a representative example Fig. 4e displays a false color image of the spatio-temporal intensity distribution of the TML beam with $m = 0$ and $\bar{n} = 31$. In this representation, the spatial intensity distribution of the beam in the vertical direction is plotted along the vertical axis, while the temporal intensity distribution of the beam is plotted along the horizontal axis. The resulting spatio-temporal intensity distribution of the beam shows that with increasing time the TML beam performed a periodic oscillation in the vertical direction, thus, confirming the successful transverse mode-locking of the laser. The trajectory of the oscillating beam closely resembled a cosine-shaped path, as given by Eq. 5. The dashed white line in Fig. 4e represents a cosine-fit to the center of mass of the spatio-temporal intensity distribution. The standard deviation of the fit was 1.7% with respect to the oscillation amplitude of $2.35 \pm 0.15 \text{ mm}$. The corresponding normalized oscillation amplitude was $\xi_0 = 7.93 \pm 0.25$, which is in good agreement with the theoretically expected value of $\xi_0^{\text{theo}} = 7.87$ for $\bar{n} = 31$ and significantly exceeds the previously reported value ($\xi_0 = 2.8$) of reference [2]. The frequency of the oscillation matched the transverse mode spacing $\nu_T = 82.2\text{MHz}$ in the vertical direction of the laser cavity.

In Fig. 5 the normalized oscillation amplitude ξ_0 of the five TML beams presented above (see Fig. 2) are depicted by the red circles. The black crosses indicate the corresponding normalized widths of the slit, which limited the beam oscillation in the vertical direction. The measured values for ξ_0 were in excellent agreement with the normalized amplitudes

Fig. 4 Time-averaged intensity distributions (**a–d**) and spatio-temporal intensity distributions (**e–h**) of a TML beam with $m = 0$ and $\bar{n} = 31$: **a, e** were obtained by photodiode measurements, **b, f** were calculated assuming a Poisson distribution of the modal power coefficients and flat phase ($\Delta\phi_n = 0$ for all n), **c, g** were calculated based on the reconstructed modal power coefficients $|A_n|^2$ and a flat phase, and **d, h** were calculated based on the reconstructed modal power coefficients $|A_n|^2$ and the reconstructed modal phases ϕ_n . For each of the calculated spatio-temporal intensity distributions (**f–h**) the corresponding correlation coefficient γ , γ' , and γ'' with the measured spatio-temporal intensity distribution **e**) is given. The solid red line indicates the central position of the AOM, while the dashed red lines correspond to the $1/e$ -width of the effective modulation area

$$\xi_0^{\text{theo}}(\bar{n}) = \sqrt{2\bar{n}}, \quad (9)$$

predicted by the theory (solid blue line in Fig. 5) [1].

Fig. 5 Normalized oscillation amplitude ξ_0 (red circles) deduced from the measured photodiode traces for different average mode numbers $\bar{n}$ and the theoretical values (blue line) predicted by Eq. 9. The black crosses indicate the normalized width of the slit that limited the beam oscillation in the vertical direction. The margin of error for the normalized slit width and the normalized oscillation amplitude ξ_0 are results of the measurement accuracy for the slit width and the size of the active photodiode area, respectively

To compare the scanning speed of the TML laser with common beam scanning techniques the rate at which multiple spots can be resolved is of interest. Let us assume a normalized oscillation amplitude of $\xi_0 = 7$ for an ideal TML laser, i.e., with a Poisson distribution of the modal power coefficients and a flat phase ($\Delta\phi_n = 0$ for all n). Then, the oscillating spot passes a number of 28 resolvable spots during one period. At an oscillation frequency of $\nu_T = 82.2$ MHz this would correspond to a rate of resolved spots of about 2.3GHz. As a comparison the resolvable spot rates of common beam scanning methods are typically below 100 MHz [33], while beam steering at a GHz rate was so far only achieved using phased arrays [34]. However, to utilize TML lasers for fast scanning applications a constant size of the oscillating spot is required. Therefore, in the following section, we will study the deviations of the oscillating spot size in the experiment by comparing the measured spatio-temporal intensity distribution with the spatio-temporal intensity distribution of a TML beam with a Poissonian modal power distribution. Furthermore, we will analyse the influence of the modal power distribution and the modal phases on the time-dependent size of the oscillating spot.

3.3 Analysis of the spatio-temporal intensity distribution

Significant differences can be observed, when comparing the spatio-temporal intensity distribution of the measured TML beam ($m = 0$ and $\bar{n} = 31$; see Fig. 4 e)) with the calculated

spatio-temporal intensity distribution of an ideal beam with a Poisson distribution of the modal power coefficients and a flat phase (see Fig. 4f). While for the calculated beam the oscillating spot size was equal to the fundamental mode size at all times, the vertical size of the measured spot changed periodically. At the center of the oscillation the vertical spot size was stretched by a factor of 4.47 ± 0.58 compared to the spot size of the ideal beam (see Fig. 4f), while it was squeezed by a factor of 0.66 ± 0.21 at the turning points. As can be seen in Fig. 4e the time-dependence of the spot size caused an increase of the peak intensity at the outer turning points and a decrease at the center of the oscillation. Thus, the correlation between the measured and calculated spatio-temporal intensity distributions was quite low with $\gamma = 0.760$.

If the reconstructed distribution of the modal power coefficients $|A_n|^2$ (red bars in Fig. 2j) was considered in the calculation of the spatio-temporal intensity distribution (see Fig. 4g) the correlation value increased to $\gamma' = 0.903$ (see Table 1). Furthermore, the updated calculation resulted in a time-dependent variation of the vertical spot size and peak intensity, analogous to the measurements. The calculated spot size was stretched in the vertical direction by a factor of 1.95 ± 0.31 compared to the spot size of the ideal beam (see Fig. 4f) and squeezed by a factor of 0.62 ± 0.20 at the turning points. While the spot size at the turning points of the oscillation was in a good agreement between the measurement and the reconstruction, the measured spot size at the center of the oscillation was still 2.29 ± 0.31 times broader than the reconstructed spot size. It was verified that the bandwidth of the photodiode ($\approx 2\text{GHz}$) was sufficient to resolve the broadening of the oscillating spot. Therefore, in a next step the modal phases ϕ_n were considered in the calculations, to explain the remaining deviations between measurement and calculation.

For a better comparability of the modal phases with the spectral phase of a longitudinally mode-locked laser, they were rewritten as a function of the angular frequency ω_n , taking into account that the transverse modes oscillated at different frequencies separated by ν_T . The modal phases were then expressed in form of a Taylor series expansion

$$\phi(\omega_n) = \sum_{k=0}^{\infty} \frac{p_k}{k!} (\omega_n - \omega_{\bar{n}})^k, \quad (10)$$

where $\omega_{\bar{n}}$ is the angular frequency of the mode with the average mode order $\bar{n}$ and the p_k are Taylor coefficients representing the different orders of the modal phases. The modal phases of the TML beams were then reconstructed from the measured spatio-temporal intensity distributions using the Taylor coefficients p_1 , p_2 , and p_3 as degrees of freedom for the SPGD algorithm. As a constant phase offset had no physical relevance for the spatio-temporal intensity distribution, the zeroth-order Taylor coefficient p_0 was set to zero. The correlation coefficient described in Eq. 8 was used as the fitness parameter for the SPGD algorithm, except that here $I(x, y)$ represents the measured and $I'(x, y)$ the reconstructed spatio-temporal intensity distribution. For the modal power distributions the values reconstructed in Sect. 3.1 from the time-averaged intensity distributions were used. While the beam was centered in the vertical direction, the upper turning point was defined as starting point of the oscillation. The correlation coefficient of the fully reconstructed beam, i.e., including both the reconstructed modal power distribution and the reconstructed modal phases, will be represented by γ'' .

The reconstructed phases of the different beams are plotted as solid green lines in Fig. 2f–j. The corresponding coefficients p_1 , p_2 , and p_3 of the different phase orders are shown in Table 1. Similar to the repeated reconstruction of the modal power coefficients presented above, the modal phases of the two beams with the smallest (Fig. 2f) and the largest (Fig. 2j) oscillation amplitude were reconstructed ten times to determine their standard deviation. As a result, it was found that the reconstructed coefficients p_1 , p_2 and p_3 of the beam with the smallest oscillation amplitude (Fig. 2f) deviated strongly by 11.4%, 5.5%, and 51.8%, respectively. However—as will be seen later—due to the low number of contributing modes, the modal phase of this beam had a weak influence on the spatio-temporal intensity distribution. Thus, the deviations of the phase coefficients had a negligible effect of less than 0.01% on

Table 1 Average mode numbers $\bar{n}$ and number of contributing modes N reconstructed for the measured TML beams

$\bar{n}$	N	γ'	p_1 in ns	p_2 in ns ²	p_3 in ns ³	γ''
3	5	0.986	− 0.12	− 0.33	0.24	0.990
8	6	0.983	0.12	0.49	0.24	0.988
14	7	0.967	0.13	0.22	0.05	0.971
24	9	0.906	0.18	0.66	0.02	0.973
31	12	0.903	0.19	0.41	− 0.04	0.971

The Taylor coefficients p_1 , p_2 , and p_3 of the modal phases, retrieved from the measured TML beams using the SPGD algorithm. The correlation coefficients γ' and γ'' between the measured and reconstructed TML beam calculated without and with the modal phases, respectively

the fitness parameter of the reconstruction algorithm. In contrast, the phase coefficients p_1 , p_2 , and p_3 that were reconstructed for the beam with the largest oscillation amplitude (Fig. 2j) and which was strongly affected by the modal phase, deviated by less than 0.6%, 0.06%, and 3.7%, respectively. These low deviations indicate that the reconstruction algorithm has converged close to the global optimum.

The modal phases of the TML laser had an analogous effect on the spatial intensity distribution of the oscillating spot as the spectral phase has on the temporal intensity distribution of a mode-locked pulse [35]. The linear phase term represented by the coefficient p_1 only shifted the spot in the spatial domain and, therefore, compensated the initial offset between the calculations and the measurements. The quadratic (p_2) and cubic (p_3) phase terms, corresponding to the chromatic group delay dispersion and third-order dispersion, respectively, had a direct impact on the shape of the oscillating spot. While the quadratic phase term stretched the spot spatially, the cubic phase term, depending on its sign, generated a number of leading or trailing satellite spots. Note, however, that due to the finite size of the transverse modes the analogy becomes invalid towards the outer turning points of the TML beam.

The modal phases of the TML laser beam changed, if the alignment of the laser cavity was changed. Since a realignment of the laser cavity was required to switch between the different TML states, the reconstructed modal phase coefficients p_2 and p_3 deviated between the different TML measurements. Although, the scope of this publication was the reconstruction of the modal power coefficients and the modal phases of the TML beams, we would like to shortly express our preliminary thoughts regarding the origin of the modal phases: In our opinion material dispersion can be ruled out as a cause of the measured modal phases, as the reconstructed group delay dispersion (p_2) was nine orders of magnitude larger than the estimated group delay dispersion of $15 \times 10^3 \text{fs}^2$ caused by the material within the laser cavity. Due to the spatial structure of the individual modes, we suppose that spatial effects, e.g., deviations of the thermal lens and the asymmetric position of the AOM with respect to the optical axis, are the origin for the modal dispersion. Further investigations and systematic parameter studies, which are beyond the scope of this paper, will be required to verify and further elaborate our hypothesis.

Nevertheless, significant differences were measured, regarding how strong the spatio-temporal intensity distributions of the different TML beams were affected by the modal phases. In case of the TML beams with $\bar{n} = 3, 8$, and 14 the correlation increased by less than 0.5% if the modal phases were considered, compared to an increase of the correlation by more than 7.4% in case of the TML beams with $\bar{n} = 24$ and 31. For a fixed magnitude of the modal phases, their

influence on the spatio-temporal intensity distribution was limited by the bandwidth of the TML laser, i.e., the number N of modes contributing to the mode-locking process (see Table 1). The more modes contributed, the stronger was the influence of the higher-order phase terms p_2 and p_3 , which affected the shape of the oscillating spot. Furthermore, due to the relatively low bandwidth of the reconstructed TML beams the influence of the cubic phase on the spatio-temporal intensity distribution was low, as for the here reconstructed TML beams, the correlation value γ'' decreased by less than 0.15% if the cubic phase term p_3 was neglected in the calculations. Such a low impact of p_3 on the correlation coefficient demonstrates that the shape of the oscillating spot was dominated by the quadratic phase term p_2 and that it was justified to neglect even higher phase orders in the reconstruction algorithm.

If the reconstructed modal phases ϕ_n , in addition to the reconstructed modal power coefficients $|A_n|^2$ presented in Sect. 3.1, were considered in the calculations, a complete reconstruction of the TML beams was achieved. For all of the TML beams (see Table 1) the correlation coefficients γ'' were larger than 0.97 verifying the successful reconstruction of the TML beams to a high degree of detail. As a representative example in Fig. 4h the fully reconstructed spatio-temporal intensity distribution of the TML beam with $\bar{n} = 31$ is shown, which is in good agreement with the measured spatio-temporal intensity distribution displayed in Fig. 4e. In animation 2, provided in the supplementary material, an improved match between the intensity distribution of the measured and calculated oscillating spot can be observed, as the reconstructed modal power coefficients and the inter-modal phases are successively included in the calculations.

The presented reconstruction of the TML beams demonstrated that both the modal power distribution as well as the modal phases can have a significant impact on the spatio-temporal intensity distribution of the TML laser beam. Thus, to obtain a TML beam with a constant spot size, as desired for fast scanning applications, not only a Poissonian modal power distribution, but also a compensation of the higher-order modal phase terms is required. This is even more important, if the number of modes N , contributing to the transverse mode-locking process, would become even larger.

3.4 Multi-trace transverse mode-locking

In the measurements presented above, the TML beams were limited in the horizontal direction to the fundamental mode order ($m = 0$), resulting a single spot which oscillated on a straight vertical line. By aligning the laser such that it oscillated on a higher mode order in the horizontal direction ($m > 0$) and by locking multiple mode orders in the vertical direction the generation of mode-locked beams oscillating on multiple separate traces was achieved.

Fig. 6 Normalized time-averaged intensity distributions of the TML beam with **a** $m = 1$ and $\bar{n} = 14$, **b** $m = 2$ and $\bar{n} = 10$ and **c** $m = 3$ and $\bar{n} = 9$, measured with a CCD-camera. **d** Spatio-temporal intensity distribution of the TML beam of subfigure **b** measured with a photodiode. The solid red lines indicate the central position of the AOM, while the dashed red lines correspond to the $1/e$ -width of the effective modulation area

Figure 6a–c show the CCD-images measured for three different TML beams with the horizontal mode orders $m = 1$, $m = 2$, and $m = 3$, respectively. Corresponding to the horizontal mode order a clear splitting of the three oscillating beams in either 2, 3 or 4 vertical traces could be observed. The excitation of multi-trace beams with a higher horizontal mode order was limited by the decreasing overlap of the modes with the pumped gain area, resulting in an inefficient pumping of the modes. A possible way to solve this issue would be an improved matching of the pump beam to the profile of the targeted modes, e.g., by splitting the pump beam into multiple lines. This would increase the overlap of the modes with the pumped gain area as well as the pump efficiency [28].

Using the above described modal reconstruction method the average vertical mode orders $\bar{n}$ of the three beams were identified to be 14, 10 and 9, with a number of 8, 7 and 8 contributing modes, respectively.

As an example, in Fig. 6d the projection of the spatio-temporal intensity distribution measured for the TML beam (of Fig. 6b) with $m = 3$ and $\bar{n} = 9$ is shown. Similar to the single-trace TML beams the multi-trace beams oscillated along a cosine-shaped trajectory (dashed white line in Fig. 6d). The spots on the different traces oscillated synchronously, as can be seen in animation 3 provided in the supplementary material, showing the temporal development of the spatial intensity distributions for each of the measured multi-trace beams. Multi-trace TML beams could be applied to further increase the imaging speed in laser

scanning microscopy, similar to the multifocal scanning of the sample [36].

4 Conclusion and outlook

In summary, we have demonstrated the successful transverse mode-locking of an end-pumped solid-state laser with an acousto-optic modulator. Compared to previous publications [2, 5–7] we provided an advanced analysis of the transverse mode-locked (TML) laser by reconstructing the modal power coefficients and the modal phases of the generated laser beams with the stochastic parallel gradient descent algorithm. Based on these reconstructions, we found that with the presented setup the average mode order $\bar{n}$ could be freely adjusted up to a value of 31. The reconstructed values of $\bar{n}$ were in excellent agreement with the measured normalized oscillation amplitudes of the TML laser reaching from $\xi_0 = 2.41 \pm 0.25$ up to $\xi_0 = 7.93 \pm 0.25$. For comparison, in previously published TML lasers only average mode orders of $\bar{n} \leq 4$ have been claimed, resulting in normalized oscillation amplitudes of $\xi_0 \leq 2.8$ [2].

By comparing numerical calculations with measurements we have investigated the influence of the modal power coefficients and the modal phases of a TML laser beam on its spatio-temporal intensity distribution. Our analysis confirmed the prediction [1] that deviations of the modal power distribution from a Poisson distribution resulted in a time-dependent variation of the oscillating spot size. Furthermore, we found that the modal phases had a significant influence on the time-dependent spot size. The modal phases affected the spatial shape of the oscillating spot at the center of the beam in the same way as spectral phase aberrations do affect the temporal shape of a pulse within a longitudinally mode-locked laser.

Finally, we demonstrated the simultaneous oscillation of up to four spots on parallel traces by operating the laser on a higher mode order in the orthogonal direction to the transverse mode-locking process.

An advanced control of the modal power coefficients, e.g., by spatial gain shaping [28–32], as well as the identification and compensation of the effects causing the cavity internal modal dispersion are the next steps required to adapt transverse mode-locking for real life applications. With the corresponding improvements TML lasers have a great potential for applications requiring fast scanning laser beams, enabling spot resolving rates in the multi-GHz regime.

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References

1. D. Auston, IEEE J. Quantum Electron. **4**, 420–422 (1968)
2. D. Auston, IEEE J. Quantum Electron. **4**, 471–473 (1968)
3. L.E. Hargrove, R.L. Fork, M.A. Pollack, Appl. Phys. Lett. **5**, 4–5 (1964)
4. P. Smith, M. Duguay, E. Ippen, Prog. Quantum Electron. **3**, 107–229 (1974)
5. S.S. Vyshlov, L.P. Invanov, A.S. Logginov, K.Y. Senatorov, ZhETF Pis. Red. **13**, 131–133 (1971)
6. A.M. Dukhovnyi, A.A. Mak, V.A. Fromzel, J. Exp. Theor. Phys. **32**, 636–642 (1971)
7. A.V. Agashkov, Sov. J. Quantum Electron. **16**, 497 (1986)
8. C. Haug, J. Whinnery, IEEE J. Quantum Electron. **10**, 406–408 (1974)
9. P. Smith, Appl. Phys. Lett. **13**, 235–237 (1968)
10. H.C. Liang, T.W. Wu, J.C. Tung, C.H. Tsou, K.F. Huang, Y.F. Chen, Laser Phys. Lett. **10**, 105804 (2013)
11. D. Côté, H.M. van Driel, Opt. Lett. **23**, 715–717 (1998)
12. L. Wright, D. Christodoulides, F. Wise, Science **358**, 94–97 (2017)
13. B.E.A. Saleh, M.C. Teich, *Fundamentals of Photonics*, 2nd edn. (Wiley, New Jersey, 2007), pp. 386–387
14. Z. Ma, D. Li, J. Gao, N. Wu, K. Du, Opt. Commun. **275**, 179–185 (2007)
15. L. Cini, J.I. Mackenzie, Appl. Phys. B **123**, 273 (2017)
16. J.L. Blows, J.M. Dawes, T. Omatsu, J. Appl. Phys. **83**, 2901–2906 (1998)
17. H. Kogelnik, T. Li, Appl. Opt. **5**, 1550–1567 (1966)
18. M. Erden, H. Ozaktas, J. Opt. Soc. Am. A **14**, 2190–2194 (1997)
19. T. Kaiser, D. Flamm, S. Schröter, M. Duparré, Opt. Express **17**, 9347–9356 (2009)
20. O. Shapira, A.F. Abouraddy, J.D. Joannopoulos, Y. Fink, Phys. Rev. Lett. **94**, 143902 (2005)
21. R. Brüning, P. Gelszinnis, C. Schulze, D. Flamm, M. Duparré, Appl. Opt. **52**, 7769–7777 (2013)
22. H. Lü, P. Zhou, X. Wang, Z. Jiang, Appl. Opt. **52**, 2905–2908 (2013)
23. L. Huang, S. Guo, J. Leng, H. Lü, P. Zhou, X. Cheng, Opt. Express **23**, 4620–4629 (2015)
24. L. Li, J. Leng, P. Zhou, J. Chen, Opt. Express **25**, 19680–19690 (2017)
25. W. Yan, X. Xu, J. Wang, Appl. Opt. **58**, 6891–6898 (2019)
26. M. Schnack, T. Hellwig, C. Fallnich, Opt. Lett. **41**, 5588 (2016)
27. A.E. Siegmann, *Lasers* (University Science Books, Mill Valley, 1986), p. 1067
28. F. Schepers, T. Bexter, T. Hellwig, C. Fallnich, Appl. Phys. B **5**, 75 (2019)
29. Y. Chen, T. Huang, C. Kao, C. Wang, S. Wang, IEEE J. Quantum Electron. **33**, 1025–1031 (1997)
30. K. Shimohira, Y. Kozawa, S. Sato, Opt. Lett. **36**, 4137–4139 (2011)
31. W. Kong, A. Sugita, T. Taira, Opt. Lett. **37**, 2661–2663 (2012)
32. T. Sato, Y. Kozawa, S. Sato, Opt. Lett. **40**, 3245–3248 (2015)
33. G.R.B.E. Römer, P. Bechtold, Phys. Procedia **56**, 29–39 (2014)
34. M. Jarrahi, R.F.W. Pease, D.A.B. Miller, T.H. Lee, Appl. Phys. Lett. **92**, 10–13 (2008)
35. R. Trebino, *Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses* (Springer Science + Business Media, New York, 2000), pp. 11–36
36. J. Bewersdorf, R. Pick, S.W. Hell, Opt. Lett. **23**, 655–657 (1998)

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