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A Faraday laser operating on Cs 852 nm transition

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Abstract

We demonstrate a Faraday laser at Cs-D2 resonance line 852 nm using a Cs Faraday optical filter as frequency-selecting element. In contrast to typical diode laser with high stability of the emission frequency by additional control systems, our Faraday laser offers stable output frequency exactly set by the peak transmission frequency of the Cs 852 nm Faraday optical filter. The system works stably over a range of laser diode (LD) current from 60 to 130 mA and the LD temperature from 14 to 35 °C, as well as the 48-h wavelength fluctuation range of no more than ± 2 p.m. A most probable linewidth of 17 kHz with Lorentz fitting is obtained by beating between two identical laser systems. The wavelength of our system is stabilized within transmission frequency region of 852 nm Faraday optical filter, and the peak of the transmission is corresponding to Cs atomic Doppler broadened line at the cell temperature of 41 °C and the magnetic field of 330 G, making it suitable for laser-pumped Cs gas-cell and atomic beam frequency standard, etc. Moreover, this scheme is firstly used on Cs atom, opening new doors in research of Faraday laser and its applications.

1 Introduction

Stable diode laser with narrow linewidth is required for atomic physics experiments [1, 2]. There are several typical diode lasers, such as grating laser [3, 4], Fabry–Perot etalon laser [5, 6], DFB laser [7], and DBR laser [8]. The frequency of all these lasers is sensitive to the laser diode (LD) current and LD temperature. Therefore, to obtain high stability of the emission frequency of diode laser, and stabilize the frequency of a laser to a particular atomic transition line, there are various methods of laser frequency stabilization in practical applications, including saturated absorption spectroscopy [9–12], polarization spectroscopy [13, 14], modulation transfer spectrum [15, 16], etc [17, 18]. However, it is difficult to suppress long-term frequency drifts and mode instability of lasers for many applications in atomic physics [19]. Moreover, once the laser loses lock, the frequency drifts may be very far. Usually a wavelength meter is needed to find the laser wavelength corresponding to the atomic transition. Therefore, in general, due to the diode laser frequency drifts and instability, there is a serious limitation for long-term operation of laser-pumped atomic clock [20–23], fountain clock [24, 25], atomic magnetometer [26, 27], atomic gravimeter [28, 29].

Since Faraday anomalous dispersion optical filter was introduced in 1956 [30], Faraday optical filters composed of different elements have been developed [31–44]. Faraday optical filters have been used as frequency-selecting element and applied to various applications, including Na guide-star [45], Mollow triplet filtering [46], Faraday geometry [47], Doppler lidar [48] and active optical clock [49]. In 1969, Sorokin et al. proposed frequency locking of organic dye lasers to atomic resonance lines [50]. Then, frequency locking of a CW dye laser to the center of the sodium D lines by a Faraday filter was demonstrated [51]. In 1993, Choi et al. reported their technique for optical feedback locking a single-mode commercial diode laser using a cesium Faraday optical filter at 852 nm [52]. But the output frequency of the "locked" laser changes with diode current or temperature, and does not correspond to atomic Doppler broadened line, rather than different Faraday transmitted spectra [53–55]. Until

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2011, our group reported a 780 nm "Faraday laser" using an antireflection-coated laser diode (ARLD) for the first time, in which the influence of the internal cavity mode on the output frequency was eliminated [56]. The output frequency corresponds to one of the transmitted spectra of the Faraday optical filter, and does not change with the current or temperature of the diode.

In recent years, the concept of Faraday laser is formally proposed and named [57–60]. The Faraday laser system uses the ARLD as the gain medium and the Faraday optical filter as the frequency selective device. The laser wavelength operates on the center of the highest transmission peak of Faraday optical filter. Although significant progress has been made in the study of Faraday laser, the Faraday laser operating on Cs atomic Doppler broadened line, which could be easily locked on Cs atomic sub-Doppler transition lines, and widely used in Cs gas cell and atomic beam frequency standard [20], has never been realized.

Here, we present a Faraday laser system operating on Cs 852 nm transition, our system offers stable output frequency exactly set by the peak transmission frequency of the Cs 852 nm Faraday optical filter. Just like a helium–neon (He–Ne) laser [61], because the gain curve of a He–Ne laser at neon 633 nm has a width of the Doppler broadened line of about 1.5 GHz, and the output laser frequency is located at the gain profile. The output frequency of the Faraday laser is filtered by Faraday optical filter and located within transmission frequency region of about 800 MHz, and the peak of the transmission spectrum is corresponding to Cs atomic Doppler broadened line. At any time, we do not need a wavelength meter to calibrate or find the laser wavelength corresponding to Cs 852 nm transition. The Faraday laser works at frequencies corresponding to the highest transmission frequency region of Cs 852 nm Faraday optical filter, neither the LD current nor the LD temperature of the laser diode has significant influence on the output frequency. The 48-h wavelength fluctuation range is also measured. The Faraday laser system realized here provides a stable laser source with the frequency operating on Cs atomic Doppler broadened line for many practical applications. In addition, this scheme is firstly used on Cs atom, opening new doors in research of Faraday laser and its applications.

2 Experiment setup

2.1 Faraday laser

The experimental setup is schematically shown in Fig. 1a. The Faraday laser with the length of optical cavity of 40 cm is composed by the Cs 852 nm Faraday optical filter, ARLD, high-reflection mirror (Rc) and piezoelectric ceramic tube (PZT). The Faraday optical filter includes a Cs vapor cell, a pair of permanent magnets (M1 and M2), and a pair of crossed Glan–Taylor prisms (G1 and G2). The shape of the Cs cell in the Faraday optical filter is a cylinder with diameter of 15 mm and length of 30 mm. Pure Cs vapor is adopted as cell vapor. The light emitted from the 852 nm ARLD (Eagleyard EYP-RWL-0850-00100-1500-SOT12-0000) is filtered by the Faraday optical filter and fed back into the ARLD by Rc. The laser beam outputs from the seconed Glan–Taylor prism (G2). The temperature of the Cs cell is controlled at 41 °C by an electric heating element that allows to maintain the cell temperature within 0.1 °C. The permanent magnets create a static axial magnetic field of 330 G across the Cs vapor cell with the inhomogeneity of the magnetic field of less than 7%, which can be ignored, the magnetic field is measured in the absence of the Cs cell and there was no magnetic shielding.

The relevant energy levels of Cs atom are shown in Fig. 1b. Because the ground state $6S_{1/2}$ has two hyperfine structure lines [41], e.g., $F = 4$ and $F = 3$, they always exhibit two Faraday optical filter spectra, in which the central frequencies are separated by 9.2 GHz. To measure the performance of the Faraday optical filter, another homemade external cavity diode laser is used for transmittance detection. The transmitted spectrum of Cs 852 nm Faraday optical filter is interrogated by an external cavity diode laser with a mode-hop-free tuning range of 6 GHz.

2.2 Faraday optical filter

Figure 2 shows the transmitted spectra of the Faraday optical filter at Cs 852 nm transition corresponding to ground state

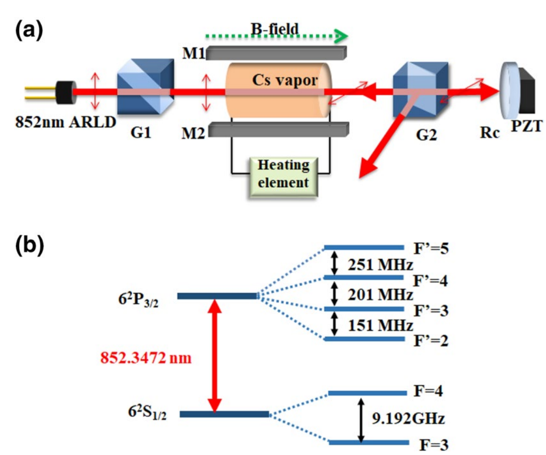

Fig. 1 **a** Schematic of the Faraday laser operating on Cs 852 nm transition. ARLD, anti-reflection coated laser diode; G1 and G2, a pair of Glan–Taylor prisms; M1 and M2: a pair of permanent magnets, Rc, high-reflection mirror for 852 nm; PZT: piezoelectric ceramic tube. **b** Relevant Cs energy level diagram

$F = 4$. The upper line is the saturated absorption spectrum of Cs. The five bottom lines are the transmitted spectra of the Cs 852 nm Faraday optical filter at cell temperatures of 39, 41, 43, 46, and 50 °C, respectively. The line shapes of the transmission spectra change with the cell temperature [62–64]. It can be seen that the Cs 852 nm Faraday optical filter works at frequencies corresponding to atomic Doppler broadened line at special cell temperatures. Figure 3 shows the transmitted spectra of the Faraday optical filter at Cs 852 nm transition corresponding to ground state $F = 3$. The upper line is the saturated absorption spectrum of Cs. Similarly, the five bottom lines, which are varied in shape, are the transmitted spectra of the Cs Faraday optical filter at cell temperatures of 39, 41, 43, 46, and 50 °C, respectively. It also can be seen that the Cs Faraday optical filter works at frequencies corresponding to atomic Doppler broadened line at special cell temperatures. The location of 0 detuning is corresponding to transition of $F = 4 \rightarrow F' = 5$ in Fig. 2, and transition of $F = 3 \rightarrow F' = 2$ in Fig. 3. The magnetic field across the Cs vapor cell is 330 G in all cases.

We measure the transmitted spectra of Cs 852 nm Faraday optical filter at different Cs cell temperatures. Because the transmitted spectrum of the Faraday optical filter is changed with the Cs cell temperature without regularity, in order to find the best temperature condition of the Faraday optical filter, we record the transmittance of the peak working at frequencies corresponding to Cs atomic Doppler broadened line. The transmittance of the peak at different Cs cell temperatures is shown in Fig. 4. The blue triangle shows the transmittance of the peak corresponding to ground state $F = 4$ as a function of the Cs cell temperature. The pink circle shows the transmittance of the peak corresponding to ground state $F = 3$ as a function of the Cs cell temperature. The red pentagram shows the absolute difference of two peak transmittances corresponding to ground state $F = 4$ and ground state $F = 3$ at same temperature condition. There are two criteria for the best temperature condition. First, one of the peak transmittance must be high. Second, the absolute difference of two peak transmittancesshould be as high as possible. By comprehensive consideration, we find that 41 °C is the best temperature condition. At the condition of temperature of 41 °C and magnetic field of 330 G, the transmittance of the peak corresponding to ground state $F = 4$ is about 85%, and the transmittance of the peak corresponding to ground state $F = 3$ is about 57%, which can

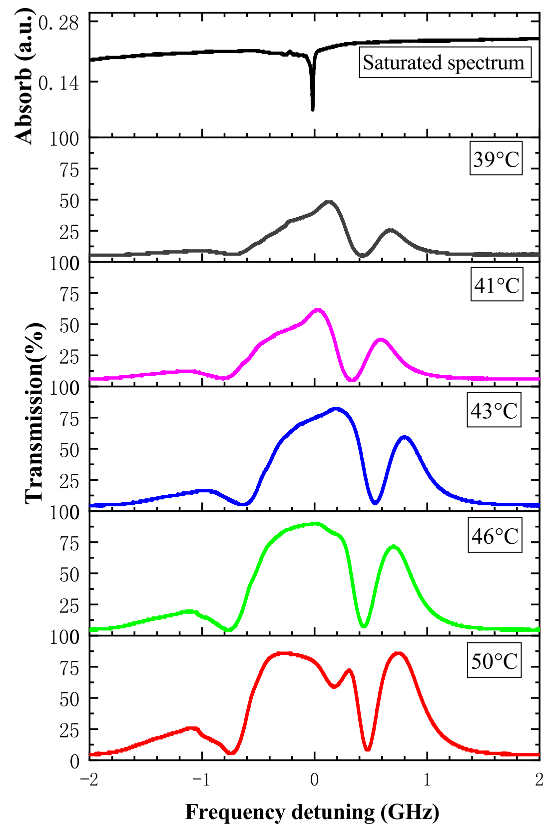

Fig. 3 Transmitted spectra of Cs 852 nm Faraday optical filter corresponding to ground state $F = 3$ under the same temperature condition with ground state $F = 4$

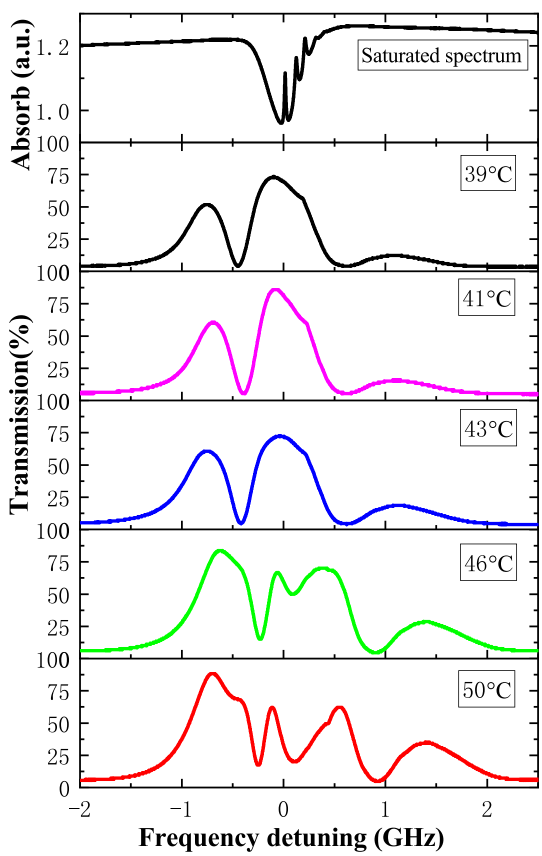

Fig. 2 Transmitted spectra of Cs 852 nm Faraday optical filter corresponding to ground state $F = 4$ at 330 G as well as temperatures of 39 °C, 41 °C, 43 °C, 46 °C, and 50 °C

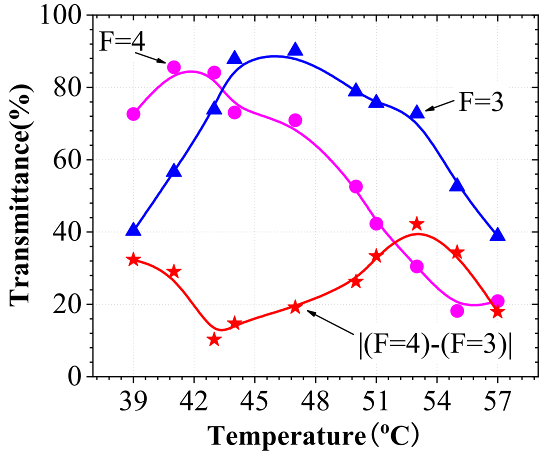

Fig. 4 The peak transmittance of 852 nm Faraday optical filter at different Cs cell temperatures; the system optical loss of 20% is not included

ensure the Faraday laser work at ground state $F = 4$ stably. Since the optical loss of cell windows is a constant and it has no effect on Faraday effect, in all cases, the system optical loss of 20% is not included.

3 Results

3.1 Wavelength characteristic

The laser wavelength is measured by a wavelength meter (WM, ADVANTEST TQ8325). A laser beam exits from G2 is coupled into the multi-mode optical fiber using a coupler and sent to the wavelength meter. Figure 5 reports the wavelength measurement results, under the condition of the LD temperature ranging from 14 to 35 °C and the LD current from 60 to 130 mA, the measured wavelength of the Faraday laser centers on 853.356 nm corresponding to the transmission spectrum of ground state $F = 4$ with maximum transmittance of 85%. In the upper subfigure, the blue square presents the wavelength of the Faraday laser at different LD temperature ranging from 14 to 35 °C with LD current of 94 mA. In the right subfigure, the red square shows the wavelength of the Faraday laser at different LD current ranging from 74 to 130 mA with LD temperature of 27 °C. The measured result of 48-h long-term wavelength fluctuations of the Faraday laser is shown in Fig. 6. Under the condition of LD current of 85 mA and LD temperature of 21.5 °C, the wavelength fluctuation range is about ± 2 p.m. within 48 h. The temperature of the Cs vapor cell remains at 41 °C in all cases, as well as the magnetic field across the Cs vapor cell of 330 G.

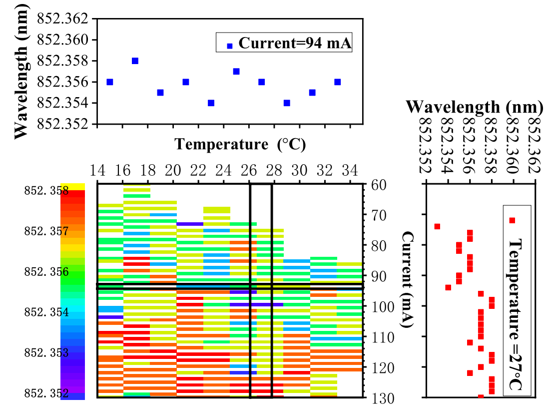

Fig. 5 The measured wavelengths of the Faraday laser at different LD current and temperature

In this experiment, because the length of optical cavity is 40 cm, which has a wide free spectrum range of 375 MHz. Mechanical vibration and air flow both can influence the wavelength instability. To reduce these influences, on the basis of previous studies [57, 58], the wavelength fluctuations are suppressed to a great degree using a longer fiber to extend the cavity. Hence, a fiber-based stable 852 nm Faraday laser with narrow linewidth can be realized.

3.2 Linewidth characteristic

To measure the linewidth of the 852 nm Faraday laser, we carry out a heterodyne beating experiment between two identical integrated systems. The two light beams of the Faraday lasers are combined for beating on a photodiode (HAMAMATSU, C5658), then the beat note is sent to a spectrum analyzer (Agilent, N9320B), which is recorded over a 40 ms period with the resolution bandwidth (RBW) being 16 kHz and span bandwidth being 7 MHz. The result of the heterodyne measurement is shown in Fig. 7a. The

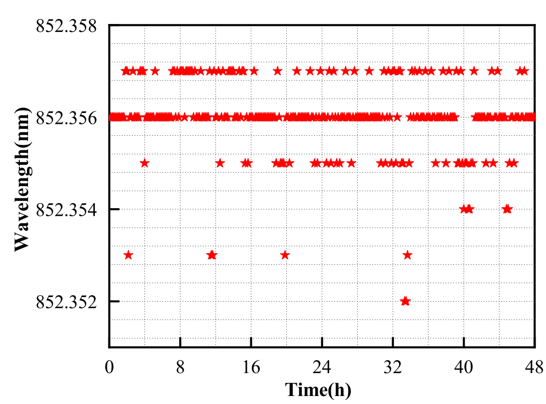

Fig. 6 The measured wavelengths of the faraday laser as a function of time (48 h), center on 852.356 nm

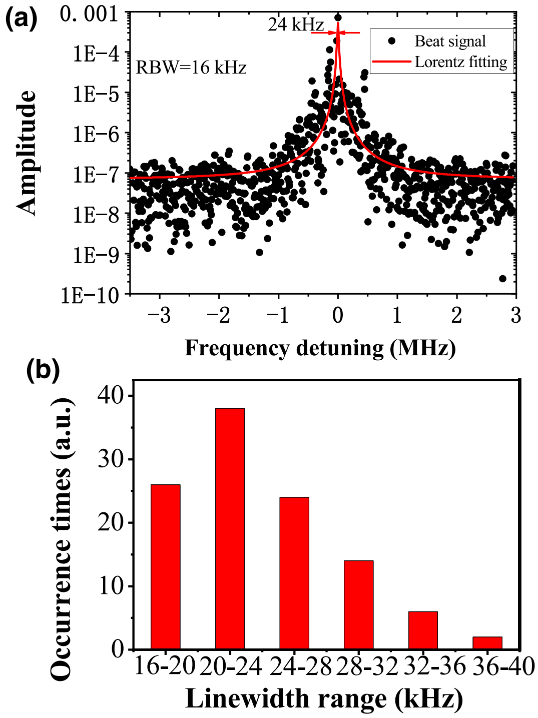

Fig. 7 **a** An example of the beating signal between two identical integrated 852 nm laser systems (black dots) and the Lorentz fitting (red solid curve). **b** Occurrence times of each linewidth

linewidth is measured using a Lorentzian fit, the most probable linewidth [65, 66] of the beat note is 24 kHz, corresponding to a linewidth of 17 kHz for each laser system by assuming that each laser contributes the same to the linewidth measurements. Figure 7b shows the linewidth distribution of approximately 110 groups of the beat-note spectra with a 5 kHz step, and the most probable linewidth distribution of the beat note is in the 20–24 kHz range.

4 Conclusion

In conclusion, we have developed a proof-of-principle Faraday laser system operating on Cs 852 nm transition. We take advantage of the transmission spectrum of the Faraday optical filter, which is corresponding to Csatomic Doppler broadened line, to reduce the need for external control system that is usually required for a wavelength-stabilized diode laser for operating on atomic transition. Besides, the Faraday optical filter works stably, and makes the Faraday laser immune to fluctuations of LD temperature and LD current. With the current design, we achieve the durable and compact wavelength-stabilized laser. We selected the Cs-D2 line ($6S_{1/2}$ to $6P_{3/2}$ transition) as a frequency reference to obtain a long-term stability in wavelength. The successful development of the 852 nm Faraday laser predicts potential possibility for the implementation of Faraday laser based on similar high-transmission Faraday optical filter. Therefore, this method can greatly expand the application of alternate operating wavelengths for the other alkali-metal atoms, including high melting point metals [38].

In the Faraday laser system, the magnetic field of 330 G is not a theoretical value, but obtained by experiment. In the whole system, different magnetic field condition needs different magnet structure, considering the volume and complexity of the Faraday laser, we choose the magnetic field of 330 G as the optimum value. In the future, we will make a theoretical calculation to find if there is better value.

Furthermore, the successful implementation of the low-cost Faraday laser can be applied to practical applications requiring for a high stability of the laser source operating on atomic transition. It will be possible to improve the long-term stability and the accuracy of the applications. Moreover, the simple and inexpensive Faraday laser also can be applied to submarine communication [36], optical communication in free space, and other areas.

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