Correction to: Asymptotic Formulas for Extreme Statistics of Escape Times in 1, 2 and 3-Dimensions

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In section 5.1 entitled “The shortest NEP from a bounded domain in $\mathbb{R}^{2,3}$ ” for dimension 3, we found an error in Eq. 97 of page 20. This leads to the conclusion that also in three dimensions, the mean time for the fastest $\tau^{(1)}$ depends on the reciprocal of $\log N$ and not on $1/\sqrt{\log N}$, as written previously. Here are the corrected computations:

$$\begin{aligned}\bar{\tau}^{(3dim)} &= \int_0^\infty [Pr\{t_1 > t\}]^N dt \\ &\approx \int_0^\infty \exp \left\{ N \log \left(1 - \frac{a^2}{\delta \sqrt{\pi}} t^{-\frac{1}{2}} e^{-\left(\frac{\delta}{\sqrt{4t}}\right)^2} \right) \right\} dt \quad (97)\end{aligned}$$

$$\begin{aligned}&\approx \int_0^\infty \exp \left\{ -N \frac{a^2}{\delta \sqrt{\pi}} t^{-\frac{1}{2}} e^{-\left(\frac{\delta}{\sqrt{4t}}\right)^2} \right\} dt \\ &\approx \frac{\delta^2}{4} \int_0^\infty \exp \left\{ -N' \frac{1}{\sqrt{u}} e^{-\frac{1}{u}} \right\} du \quad (98)\end{aligned}$$

where $N' = \frac{2Na^2}{\sqrt{\pi}\delta^2}$. Using the method developed in section 3.1 with the change of variable,

Zeev Schuss passed away in July 2018.

The original article can be found online at <https://doi.org/10.1007/s00332-018-9493-7>.

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Fig. 3 b MFPT of the fastest particles versus the number of particles N . The asymptotic solution (green curve) $\frac{\alpha}{\log(N)+\beta}$ fitted the stochastic simulations, obtained with 2000 runs

$$w = w(t) = \frac{1}{\sqrt{t}} e^{-1/t}, \quad w'(t) = \frac{1}{\sqrt{t}} e^{-1/t} \left(-\frac{1}{2t} + \frac{1}{4t^2} \right). \quad (99)$$

We have with $w' = 4w(\log(w))^2$

$$\bar{\tau}^{(3dim)} \approx \delta^2 \int_0^\infty \frac{\exp(-N'w)}{4w(\log(w))^2} du.$$

We then follow the computation of section 3.1 “Escape from a Ray”. When the diffusion coefficient is D , we obtain the formula

$$\bar{\tau}^{3dim} \approx \frac{\delta^2}{4D \log \left(2N \frac{a^2}{\pi^{1/2} \delta^2} \right)}. \quad (100)$$

When there are p windows, whose distances from the initial position of the Brownian particles are $d_k = \text{dist}(P_0, P_k)$, formula (100) changes to

$$\bar{\tau}^3 \approx \frac{\delta^2}{4D \log \left(2N \frac{a^2}{\pi^{1/2} \delta^2} \right)}, \quad (101)$$

where $\delta^2 = \min(d_1^2, \dots, d_p^2)$. The asymptotic formula (100) is compared with results of Brownian simulations and shows very good agreement (Fig. 3). Fig.3b of the main manuscript has to be replaced by Fig. 3, which fits the dependency of $\bar{\tau}^{(3dim)}$ with respect to the total number of particles N .

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