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Sourcing decisions with loss aversion under yield and demand randomness

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Abstract

Yield and demand randomness are common in the industry, and loss aversion has been regarded as an inherent behavior for decision-makers. We combine these two factors and investigate a retailer's ordering decisions under both yield and demand randomness with loss aversion. Before the selling season, the demand is unknown and the loss-averse retailer places an order from an unreliable supplier with an uncertain yield rate. After the demand and supply from the unreliable supplier are known, the retailer can carry out emergency replenishment from a spot market during the selling season. We characterize the retailer's optimal ordering decisions in three scenarios: (1) the retailer is risk-neutral; (2) the retailer is loss-averse and has a zero reference profit; (3) the retailer is loss-averse and has a nonzero fixed reference profit (FRP). We compare the retailer's order quantities in the three scenarios and find that the order quantity of the loss-averse retailer with a zero reference profit is always lower than that of the risk-neutral retailer. However, the order quantity of the loss-averse retailer with a nonzero fixed reference profit is higher than that of the risk-neutral retailer when the salvage value is sufficiently large. Interestingly, we find that the loss-averse retailer's optimal order quantity decreases with the reference profit and increases with the loss-averse degree and the maximum fulfillment rate from the unreliable supplier under some conditions. We investigate these conditions under a uniform demand distribution. We further study the ordering quantity of the retailer with a prospect-dependent reference point (PRP) and compare the results under FRP and PRP.

Keywords Loss aversion · Yield risk · Sourcing decision · Demand uncertainty

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1 Introduction

Yields of the production process in many industries are random and can be attributed to many factors, such as natural environment change, machine breakdown, material fluctuation and other supply chains or operations interruptions (Zare et al. 2019). For example, the natural environment is an important factor leading to the yield randomness of many manufacturers. The earthquake significantly affected the yields of manufacturers in Japan, while it is almost impossible to be forecasted (Gurnani et al. 2014). In the semiconductor industry, a small timing error in production or a small amount of dust content will influence the quality of the chips (He and Zhang 2008). A manufacturer of paper, Mazandaran Wood and Paper Industries from Iran, has dropped its production to 55 percent of the normal demand since 2012, because of the fluctuation of raw material (Zare et al. 2019). In the presence of random yield risks of suppliers, firms are prone to suffer risks from out-of-stock, overstock or production and delivery delays, which significantly undermines their reputation and financial performance. For instance, the sudden outbreak of COVID-19 forced Goodyear, one of the world's largest tire suppliers, to announce a temporary suspension of production. As a result, related automobile manufacturers and retailers suffered severe losses. Due to insufficient supply of the supplier, Ericsson, one of the world's largest mobile system companies suffered a \$400 million in lost sales (Yuan et al. 2020). Additionally, on the demand side, the variety of consumers' preferences for products leads to demand randomness (Hu and Feng 2017). The incorporation of the yield and demand randomness complicates the procurement decision-making for firms in practice. As such, it is worthy to take into account the two randomness in the decision-making model to provide guidance for firms to mitigate the supply–demand mismatch risk and obtain market competition advantages.

Faced with yield and demand randomness, firms usually prefer to hold a low inventory to avoid loss incurred by the overage. This decision-making bias behavior is incentivized by the firms' loss-averse attitude (Wei et al. 2019). The notion of loss aversion originated from prospect theory and states that people prefer avoiding losses to making gains (Wang and Wang 2018; Abdellaoui et al. 2007; Kőszegi and Rabin 2006). Meanwhile, the perception of losses and gains depends on a specific reference point, which is referred to as reference dependence (Wei et al. 2019). It is essential to allow for both loss-averse attitude and reference dependence when explaining decision-makers' decisions biases under supply and demand uncertainties (Long and Nasiry 2015). This paper aims to provide a "decision support" for the ordering party (i.e., the retailer) with a loss-averse attitude which is common in practice (Abdellaoui et al. 2007; Kőszegi and Rabin 2006). One problem of adopting the traditional expected utility theory is, clearly, that the predictions of decisions will be distorted due to behavioral factors including loss aversion. This is likely especially in the emerging market. In the emerging market, historical operational data are lacking. Hence, most decisions are made by individuals and affected by their behavioral factors and thus are usually unassisted by traditional standard decision support systems (Uppari & Hasija, 2019). Therefore, it is necessary to consider the loss-averse attitude of the ordering parties and provide the decision support for the retailers.

In this study, we attempt to address three main research questions in this paper: (1) Under both yield and demand randomness, what is the optimal ordering decision for a loss-averse retailer? What is the effect of loss aversion on a retailer's optimal order quantity? (2) How does the reference point affect a loss-averse retailer's decisions? (3) What are the effects of the supply characteristics on a loss-averse retailer's decisions?

To address these research issues, we consider a loss-averse retailer sourcing from an unreliable supplier and selling short-life products to meet stochastic demands, considering the reference profit. The loss-averse retailer has two ordering opportunities, one each in the preselling and selling seasons. Before the selling season, the retailer orders products from the unreliable supplier. Due to random production yield, the unreliable supplier would fulfill only part of the order that the retailer has placed, and the order fulfillment rate is uncertain. During the selling season, if the supplied products cannot satisfy the realized demand, the retailer will carry out replenishment from a spot market. The unsold items at the end of the selling season will be salvaged. This paper, respectively, presents the retailer's ordering decision models in three scenarios: (1) the retailer is risk-neutral; (2) the retailer is loss-averse and has a zero reference profit; and (3) the retailer is loss-averse and has a nonzero fixed reference profit (FRP). We characterize and compare the retailer's optimal ordering decisions in the three scenarios. It is found that the loss-averse retailer with a zero reference profit always orders less before the selling season than the risk-neutral retailer. However, the loss-averse retailer with a nonzero reference profit will order more than the risk-neutral retailer under some conditions. We investigate the effects of the reference profit, the loss-averse degree and the maximum fulfillment rate from the unreliable supplier on the retailer's optimal ordering decisions in this paper. We further study the ordering quantity of the retailer with a prospect-dependent reference point (PRP) and compare the results under FRP and PRP.

To the best of our knowledge, this paper is the first to provide guidance for the loss-averse retailer to determine optimal sourcing strategies under both yield and demand uncertainties considering the reference point. Some interesting results are obtained. First, the prior literature showed that the loss-averse decision-maker's order quantity always decreases with the degree of loss aversion (e.g., Wu et al. 2018; Li and Li 2018). However, it is found that the order quantity may become larger for a retailer with a higher loss-averse degree in this study where both uncertain yield and reference profit are considered. Under the yield randomness of the supplier, the received supply may be so small that the reference profit is hard to achieve, resulting in a low utility of the loss-averse retailer. Moreover, a low supply has a more detrimental effect on the utility of the retailer who has a higher loss-averse degree. As such, a retailer with a higher loss-averse degree may increase its order quantity to hedge against the risk of low supply. Second, the prior literature (e.g., Wu et al. 2018) that ignored yield uncertainty presented that a loss-averse decision-maker's order quantity always decreases with the reference profit to avoid the overage cost. Our study shows that a retailer with a higher reference profit should order more in some cases when yield uncertainty is considered. Under yield uncertainty, a retailer with a larger reference profit may increase its order quantity to avoid low supply, satisfy more demand and gain more profit. Third, this paper shows that the loss-averse retailer should always order more than the risk-neutral retailer when the salvage value is large under FRP, but this result does not hold under PRP. Fourth, it is found that as the unit procurement cost for the supplementary order increases, the order quantity of the loss-averse retailer before the selling season will increase under FRP, but may decrease under PRP. Fifth, the retailer's order quantity increases with the maximum fulfillment rate from the unreliable supplier under some conditions. Intuitively, the supplier with a larger maximum fulfillment rate can potentially fulfill more orders and will prompt the retailer to reduce the order quantity. However, a larger maximum fulfillment rate from the unreliable supplier reflects a higher variability of the supplier's fulfillment rate. Thus, to ensure the received supply reaches the target, the retailer may order more from the supplier. In this paper, we further investigate the conditions where the retailer's order quantity increases or decreases with the loss-averse degree, the profit reference, and the maximum fulfillment rate from the unreliable supplier under a uniform demand distribution.

The remainder of this paper is organized as follows: In Sect. 2, we briefly review the related literature. In Sect. 3, we introduce assumptions and the notation, model and formulate the problem and obtain the preliminary results. We conclude the results, summarize implications for the research and practice and identify directions for future research in Sect. 4. We put all the proofs in the Appendices.

2 Literature review

We mainly review two streams of research that are closely related to our work: One stream focuses on loss aversion in operational management, mainly by investigating how loss aversion affects decision-makers' decisions. The other stream mainly studies the supply risk management.

In the stream of research related to loss aversion, loss aversion is studied based on the prospect theory which is established by Kahneman and Tversky (1979). The prospect theory is widely investigated in academic fields of economics, marketing and finance. There is limited research related to the prospect theory in operations management, especially in the newsvendor model with loss aversion. To our knowledge, Schweitzer and Cachon (2000) are the first to incorporate prospect theory into newsvendor models.

The existing research on the newsvendor model with loss aversion can be divided into two streams: (i) zero reference point and (ii) nonzero reference point. In the research stream considering the zero reference point, Wang and Webster (2009) studied a single-period newsvendor problem and used loss aversion to model managers' decision-making behavior. Wang (2010) extended the standard newsvendor problem to a game setting in which multiple loss-averse newsvendors are competing for inventory from a risk-neutral supplier. Liu et al. (2013) investigated a newsvendor game where two different retailers (newsvendors) with loss-averse preferences sell two substitutable products. Using the loss-averse utility with a zero reference point, these studies all show that loss aversion always results in a decrease in order quantity.

With a zero reference point, Nagarajan and Shechter (2014) claimed that the prospect theory cannot explain observations of newsvendor's behavior. However, Long and Nasiry (2015) pointed out that Nagarajan and Shechter (2014) ignored a key factor, namely, the reference point, in their model. The reference point is an important feature since it determines how a decision-maker perceives gains and losses under a prospect and thus plays a significant role in explaining the decision-maker's attitude toward that prospect (Uppari and Hasija 2019; Wu et al. 2018). As the zero reference point is a special case in prospect theory, some researchers investigate how a nonzero reference point affects the order quantity of a loss-averse newsvendor. Long and Nasiry (2015) found that experimental observations such as asymmetry in ordering and the pull-to-center effect can be explained via studying the loss-averse newsvendor problem considering reference points. Wei et al. (2019) investigated a newsvendor's ordering decision under two exogenous reference points: a minimum requirement on profit and industry benchmark, and found the decision bias in relation to the loss attitude and reference profit of newsvendor. Uppari and Hasija (2019) considered the reference profit and investigated several prospect theory-based newsvendor models and provided a rigorous basis for choosing a model when characterizing a newsvendor's decision-making process. These studies focus on a loss-averse decision-maker's optimal order quantity with reference points under the demand uncertainty, and only one procurement stage is considered. Different from these papers, the retailer in our article is faced with both yield and demand uncertainties, and we further explore how the supply characteristics affect a loss-averse retailer's decisions. In addition, we investigate the retailer's ordering decisions in two stages. In the first stage, the retailer makes ordering decision under both yield and demand uncertainties. In the second stage, the retailer can conduct replenishment based on the revealed demand and received supply.

The second stream of related research studies supply risk management. Yield uncertainty may occur under many circumstances, such as natural disasters, equipment malfunctions, changes in government regulation and human-centered issues like labor strikes and fraud (Adhikari et al. 2020; Fattahi and Govindan 2018; Giri and Bardhan 2015; Bugert and Lasch 2018; He et al. 2015; Spieske and Birkel 2021; Bozorgi-Amiri et al. 2013; Lee and Chien 2014). Yield uncertainty management has attracted attention from practitioners and researchers in the supply chain management field. The stochastic-yield model is widely used in existing research on yield uncertainty, which assumes that the supply level is a function of the order quantity (Deo and Corbett 2009; Gao et al. 2014; Yano and Lee 1995). Shi et al. (2020) investigated a newsvendor's supply process improvement when the supplier's production capacity is random. Begen et al. (2016) examined the impacts of demand uncertainty, yield uncertainty and uncertainty reduction efforts on total cost and production quantity. Hu and Feng (2017) studied a one-supplier one-buyer supply chain with service requirements and revenue sharing contracts under yield and demand uncertainties. The stochastically proportional yield model is appropriate when the batch sizes are relatively large, or when the variance of the batch size inclines to be small. The stochastically proportional yield model is also suitable for situations in which yield losses occur due to limited production capabilities incurred by variations in materials or random environmental changes, etc. (Yano and Lee 1995). In this study, we also focus on the situations where the newsvendor orders products from a manufacturer possessing limited production capabilities incurred by variations in materials or random environmental changes. Thus, the stochastically proportional yield model is adopted in this study. Different from aforementioned studies only considering the yield risks, this paper allows for both yield and demand randomness, as well as the loss aversion and reference point of the decision-maker.

Scarce studies considered both loss aversion and yield risk. Ma et al. (2016) explored a loss-averse newsvendor's ordering decisions with zero reference point under both yield and demand uncertainties. Different from this paper, we consider both the reference point which can be nonzero, and the maximum fulfillment rate from the unreliable supplier, and obtain different results. Ma et al. (2016) stated that the loss-averse newsvendor's order quantity always decreases with its loss-averse degree. However, we find that the loss-averse newsvendor's order quantity may increase with its loss-averse degree, especially when the salvage value for each product is large. Ma et al. (2016) also illustrated that the loss-averse retailer always orders less than the risk-neutral retailer. Differently, this paper shows that the loss-averse retailer will order more than the risk-neutral retailer when the salvage value is large. We investigate not only the effects of the reference point but also the effects of the maximum fulfillment rate from the unreliable supplier, both of which are not discussed in Ma et al. (2016). We find that the order quantity of the loss-averse retailer with a zero reference profit always decreases with the maximum fulfillment rate under uniform demand distribution. In contrast, the loss-averse retailer's order quantity with a nonzero reference point may increase with the maximum fulfillment rate under uniform demand distribution.

To summarize, most previous literature studied the loss-averse newsvendor problem considering one of the factors of the reference point or yield uncertainty. There are scarce previous studies investigating a loss-averse newsvendor with both the reference point and yield uncertainty, and this paper fills this gap. A brief comparison among the aforementioned related literature is depicted in Table 1.

In summary, this paper's main contributions in the light of the prior literature are as follows:

(1) To the best of our knowledge, we are the first to investigate the optimal ordering problem for a retailer with a loss-averse attitude with a nonzero reference point under both yield and demand randomness, in consideration of emergency replenishment. The results in this paper provide guidelines for companies with loss-averse attitudes and nonzero reference profits on how to make ordering decisions to effectively deal with supply and demand uncertainties.
(2) We compare the optimal ordering decisions among the risk-neutral retailer, loss-averse retailer with a zero reference profit and loss-averse retailer with a nonzero fixed reference profit (FRP). We find that the loss-averse retailer with a zero reference profit always orders less than the risk-neutral retailer. However, it is found that the loss-averse retailer with a nonzero FRP orders more than the risk-neutral retailer when the salvage value is large.

Table 1 A brief comparison of related papers

References	Reference point		Yield uncertainty	Loss aversion
	Zero	Nonzero		
Wang and Webster (2009)	√			√
C. X. Wang (2010)	√			√
Liu et al. (2013)	√			√
Nagarajan and Shechter (2014)	√			
Long and Nasiry (2015)		√		
Uppari & Hasija (2019)		√		
Wu et al. (2018)	√	√		√
Wei et al. (2019)		√		√
Deo & Corbett (2009)			√	
Gao et al. (2014)			√	
Shi et al. (2020)			√	
Begen et al. (2016)			√	
Hu and Feng (2017)			√	
Ma et al. (2016)	√		√	√
This paper	√	√	√	√

(3) We investigate the effects of behavioral factors including the loss-averse degree and the reference profit on the retailer's ordering decisions. We find that with a zero reference profit, the retailer's order quantity always decreases with its loss-averse degree. However, with a nonzero reference profit, the retailer's order quantity may increase with its loss-averse degree, especially when the salvage value for each product is large. It is also found that the loss-averse retailer's order quantity may increase or decrease with its reference profit.

(4) We further investigate the ordering quantity of the retailer with a prospect-dependent reference point (PRP) and compare the results under FRP and PRP. Different from the result under the FRP model that the order quantity always increases with the unit order cost at the supplementary order, it is found that the order quantity may decrease with the unit order cost at the supplementary order under the PRP model. With a large salvage value, the loss-averse retailer under the FRP model will source more products than the risk-neutral retailer, and its order quantity always increases with the loss-averse degree, but these results may not hold under the PRP model.

(5) We also explore the effects of the maximum fulfillment rate from the unreliable supplier. We find that the order quantity of the loss-averse retailer with a zero reference profit always decreases with the maximum fulfillment rate under uniform demand distribution. In contrast, the loss-averse retailer's order quantity with a nonzero reference point may increase with the maximum fulfillment rate under uniform demand distribution.

3 Model

We consider a retailer's ordering problem, where the retailer sources products from an unreliable supplier and sells short-life products. The retailer procures products in two stages. In the first stage, i.e., before the selling season, the retailer places an order from an unreliable supplier at the unit cost c_1. It is noteworthy that the retailer only pays for the received supply from the unreliable supplier. In the second stage, i.e., during the selling season, the retailer determines whether to carry out replenishment from the spot market at a high unit cost $c_2(>c_1)$. It is assumed that the selling price for each product p is exogenously determined. On the demand side, the demand D is realized at the beginning of the selling season, while it is random before the selling season, and the cumulative distribution function $F(\cdot)$ of D is continuous and twice differentiable, and the corresponding p.d.f. is $f(\cdot)$. It is assumed that the maximum and minimum of the demand are $D_{\max}$ and $D_{\min}$, respectively. The retailer needs to determine the procurement quantity Q from the unreliable supplier, and the supply will arrive before the selling season. Meanwhile, the actual supply received from the supplier is uncertain because of its unreliability. Following the prior literature (Giri and Bardhan 2015; Xanthopoulos et al. 2012), we adopt the proportional yield model and assume that the retailer's received supply is given by $Q_r=\gamma Q$, where γ denotes the supplier's order fulfillment rate. We assume γ is uniformly distributed in the range $(0,\beta)$ and its probability density function is $g(\cdot)$ (Inderfurth 2004; Käki et al. 2015). β is the maximum fulfillment rate from the unreliable supplier. Note that β may be lower than 1, that is, the supplier may not be able to fully satisfy the orders even if there is no yield loss. This is because the supplier's order fulfillment rate may be limited by the preset production capacity or inventory level, which is common in practice (Closs et al. 2010; Rodrigues and Yoneyama 2020). In this paper, the demand D and supplier's order fulfillment rate γ are assumed to be stochastically independent. After the selling season comes, if the market demand can be fully satisfied by the retailer's inventory, the unsold item at the end of the selling season will be salvaged at the unit value s. However, if the demand is larger than the retailer's inventory, the retailer will place a supplementary order $Q_s=D-Q_r$ from the spot market. Figure 1 shows the timeline of the model, and Table 2 summarizes the notations used in this paper.

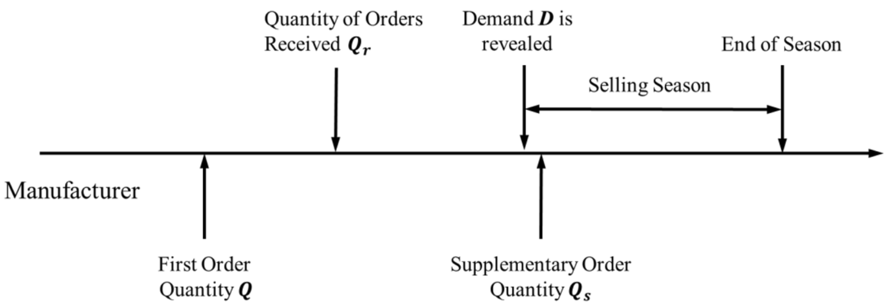

Fig. 1 Timeline of the model

Table 2 Notations

Notation	Description
D	Market demand
$D_{\min}$,$D_{\max}$	The minimum and maximum of the market demand
Q	The retailer's procurement quantity before the selling season
Q_r	The received supply before the selling season, $Q_r \leq Q$
Q_s	The retailer's supplementary order quantity during the selling season
β	The maximum fulfillment rate from the unreliable supplier
p	The selling price for each product
c_1	Unit procurement cost before the selling season, $c_1 < p$
c_2	Unit procurement cost for the supplementary order, $c_1 < c_2 < p$
s	Salvage value for each unsold product, $s < c_1$
γ	The supplier's order fulfillment rate, with the range $(0, \beta)$ and probability density function $g(\cdot)$
W	The retailer's profit at the end of the selling season
W_0	The reference profit
λ	Loss aversion coefficient

3.1 Benchmark (without loss aversion)

For benchmarking purposes, we first build the basic model without considering the retailer's loss-averse attitude. As noted before, the retailer needs to pay for the received supply Q_r instead of the order quantity, and its decision model on order quantity Q can be thus formulated as

$$\begin{aligned} &\max_{Q} E\left[W_m(Q)\right], \\ &W_m(Q) = pD - c_1\gamma Q + s(\gamma Q - D)^+ - c_2(D - \gamma Q)^+ \end{aligned} \tag{1}$$

where $E\left[W_m(Q)\right]$ corresponds to the retailer's expected profit, and $x^+ = \max\{x, 0\}$. In Eq. (1), the first term is the retailer's revenue, the second term is the procurement cost before the selling season, the third term is the salvage value for the leftover items, and the last term is the supplementary procurement cost.

Based on Eq. (1), the optimal order quantity Q^* satisfies:

$$\int_0^\beta \gamma F(\gamma Q^*)\mathrm{d}\gamma = \frac{\beta^2\left(c_2 - c_1\right)}{2\left(c_2 - s\right)} \tag{2}$$

The similar proof of the above equation can be found in the prior literature (He and Zhang 2008; Xie et al. 2021; Henig and Gerchak 1990). According to Eq. (2), we get Lemma 1. To simplify the expression of Lemma 1, we denote $B_1 = F(\beta Q^*) - \frac{c_2 - c_1}{c_2 - s}$.

Lemma 1 The risk-neutral retailer's order quantity is invariant of the selling price p, increases with the maximum fulfillment rate from the unreliable supplier β if $B_1 < 0$ and decreases with β if $B_1 \geq 0$.

Different from the results in prior papers (He and Zhang 2008; Xie et al. 2021; Henig and Gerchak 1990), it is found that the retailer's order quantity is invariant of the selling price p. This is because we consider the replenishment during the selling season, as well as the maximum fulfillment rate from the unreliable supplier are considered. The retailer with a higher selling price p has the same incremental marginal profit no matter the product is sourced before or during the selling season. Hence, the retailer's optimal procurement quantity will not change with p. The retailer's order quantity may increase or decrease with the maximum fulfillment rate from the unreliable supplier β. An interesting finding is that there are two opposing forces acting on the retailer's procurement quantity due to the maximum fulfillment rate from the unreliable supplier β. It is noteworthy that the retailer tends to order more than the quantity actually needed to hedge against the risk of supply uncertainty. On the one hand, the supplier with a larger β can potentially meet more orders and thus will induce the retailer to reduce procurement quantity. On the other hand, a larger β implies a higher variability of the supplier's order fulfillment rate. To ensure that the received supply reaches the target quantity, the retailer may order more from the supplier when β is larger. The overall effect on the retailer's procurement quantity regarding β will depend on which of these impacts predominates. As a result, the retailer's procurement quantity may increase or decrease with β, as shown in Lemma 1.

3.2 Loss aversion

3.2.1 Model description

In this section, we investigate a loss-averse retailer's optimal procurement decision. Following Wu et al. (2018), we use a piecewise-linear form of the loss-averse utility function expressed as follows:

$$U\left(W_m\right) = \begin{cases} W_m - W_0, & W_m \geq W_0 \\ \lambda\left(W_m - W_0\right), & W_m < W_0 \end{cases}, \tag{3}$$

where the coefficient $\lambda(> 1)$, indicating the degree of the retailer's loss aversion, and W_0 is the reference profit. As shown in Fig. 2, when the profit W_m is larger than the reference profit W_0 (i.e., $W_m - W_0 > 0$), the retailer feels at a gain and its utility is positive. When W_m is smaller than W_0 (i.e., $W_m - W_0 < 0$), the retailer feels at a loss, and its utility is negative. In this case, the slope of the utility function is sharper than that when the retailer feels at a gain. This result means that the retailer is more sensitive to the gap between the profit and the reference point (i.e., $W_m - W_0$) when it is at a loss, and reflects the retailer's loss-averse attitude. A larger λ implies a higher degree of loss aversion. When $\lambda = 1$, the retailer is risk-neutral.

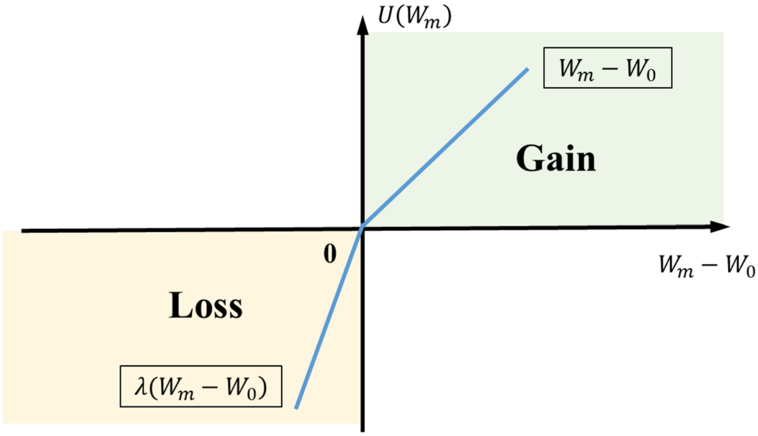

Fig. 2 The loss-averse piecewise-linear utility function

In the next section, we investigate a loss-averse retailer's ordering decisions with two different reference points, respectively: a fixed reference point (FRP) and a prospect-dependent reference point (PRP).

3.2.2 Optimal ordering decisions with a fixed reference point (FRP)

In this section, we consider a loss-averse retailer's ordering decisions with a fixed reference point (FRP) W_0^{FRP}. The fixed reference points commonly used include the status quo (i.e., zero payoff) and the industry average level (Hall, 1980; Wei Ying et al., 2019). With a fixed reference point (FRP), the retailer's profit $W_m(Q)$ can be rewritten as

$$W_m(Q) = \begin{cases} W_-(Q) = (p-s)D + \left(s - c_1\right)\gamma Q, D < \gamma Q \\ W_+(Q) = \left(p - c_2\right)D + \left(c_2 - c_1\right)\gamma Q, D \geq \gamma Q \end{cases}, \tag{4}$$

where $W_-(Q)$ is the retailer's profit when demand is lower than the received supply and $W_+(Q)$ is the profit when demand exceeds the received supply. To get the retailer's utility function, we need to compare the retailer's profit $W_m(Q)$ and reference profit W_0^{FRP}. The comparison is summarized in Lemma 2.

Lemma 2

1. If $\gamma < \frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q}$,

   when the demand $D < q_1^N(Q)$, the profit $W_m(Q) < W_0^{\mathrm{FRP}}$;

   when $D \geq q_1^N(Q)$, $W_m(Q) \geq W_0^{\mathrm{FRP}}$, where $q_1^N(Q) = \frac{W_0^{\mathrm{FRP}} - (c_2 - c_1)\gamma Q}{p - c_2}$.
   Otherwise,

when $D < q_2^N(Q)$, $W_m(Q) < W_0^{\mathrm{FRP}}$,
when $D \geq q_2^N(Q)$, $W_m(Q) \geq W_0^{\mathrm{FRP}}$, where $q_2^N(Q) = \frac{(c_1-s)\gamma Q + W_0^{\mathrm{FRP}}}{p-s}$

2. $q_1^N(Q) > \gamma Q$, and $q_2^N(Q) \leq \gamma Q$

Based on Lemma 2, the retailer's utility function when $\gamma < \frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q}$ is as follows:

$$U_1(W_m) = \begin{cases} \lambda * \left[(p-s)D + \left(s-c_1\right)\gamma Q - W_0^{\mathrm{FRP}}\right], D < \gamma Q \\ \lambda * \left[\left(p-c_2\right)D + \left(c_2-c_1\right)\gamma Q - W_0^{\mathrm{FRP}}\right], \gamma Q \leq D < q_1^N(Q) \\ \left(p-c_2\right)D + \left(c_2-c_1\right)\gamma Q - W_0^{\mathrm{FRP}}, D \geq q_1^N(Q) \end{cases}$$

When $\gamma \geq \frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q}$, the retailer's utility function is:

$$U_2(W_m) = \begin{cases} \lambda * \left[(p-s)D - \left(c_1-s\right)\gamma Q - W_0^{\mathrm{FRP}}\right], D < q_2^N(Q) \\ (p-s)D - \left(c_1-s\right)\gamma Q - W_0^{\mathrm{FRP}}, q_2^N(Q) \leq D < \gamma Q \\ \left(p-c_2\right)D + \left(c_2-c_1\right)\gamma Q - W_0^{\mathrm{FRP}}, D \geq \gamma Q \end{cases}$$

Hence, the retailer's expected utility function is provided by:

$$E\left[U\left(W_m\right)\right] = \int_0^{\frac{W_0^{FRP}}{(p-c_1)Q}} E\left[U_1\left(W_m\right)\right]g(\gamma)d\gamma + \int_{\frac{W_0^{FRP}}{(p-c_1)Q}}^{\beta} E\left[U_2\left(W_m\right)\right]g(\gamma)d\gamma,$$

where

$$\begin{aligned} E\left[U_1\left(W_m\right)\right] = &\lambda \int_0^{\gamma Q} \left((p-s)D + \left(s-c_1\right)\gamma Q - W_0^{FRP}\right)f(D)dD \\ &+ \lambda \int_{\gamma Q}^{\frac{W_0^{FRP}-(c_2-c_1)\gamma Q}{p-c_2}} \left(\left(p-c_2\right)D + \left(c_2-c_1\right)\gamma Q - W_0^{FRP}\right)f(D)dD \\ &+ \int_{\frac{W_0^{FRP}-(c_2-c_1)\gamma Q}{p-c_2}}^{+\infty} \left(\left(p-c_2\right)D + \left(c_2-c_1\right)\gamma Q - W_0^{FRP}\right)f(D)dD, \end{aligned}$$

and

$$E\left[U_2\left(W_m\right)\right]=\lambda\int_0^{\frac{W_0^{FRP}+(c_1-s)\gamma Q}{p-s}}\left((p-s)D+\left(s-c_1\right)\gamma Q-W_0^{FRP}\right)f(D)dD$$
$$+\int_{\frac{W_0^{FRP}+(c_1-s)\gamma Q}{p-s}}^{\gamma Q}\left((p-s)D+\left(s-c_1\right)\gamma Q-W_0^{FRP}\right)f(D)dD$$
$$+\int_{\gamma Q}^{+\infty}\left(\left(p-c_2\right)D+\left(c_2-c_1\right)\gamma Q-W_0^{FRP}\right)f(D)dD$$

Lemma 3 The loss-averse retailer's expected utility function $E\left[U\left(W_m\right)\right]$ is concave with respect to the order quantity Q.

Lemma 3 shows the concavity of the loss-averse retailer's objective function when the reference profit is nonzero. According to Lemma 1, we can obtain the retailer's optimal order quantity Q_λ^* with a reference profit W_0^{FRP}, as shown in Proposition 1. In order to simplify the expression of Proposition 1, we denote

$$B_2=\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}\frac{\left(c_2-c_1\right)\gamma}{p-c_2}f\left(\frac{W_0^{FRP}-\left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2}\right)d\gamma$$
$$-\int_{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}^{\beta}\frac{\left(c_1-s\right)\gamma}{p-s}f\left(\frac{W_0^{FRP}+\left(c_1-s\right)\gamma Q_\lambda^*}{p-s}\right)d\gamma,$$
$$B_3=c_2-c_1-\left(c_2-s\right)F\left(\beta Q_\lambda^*\right)-(\lambda-1)\left(c_1-s\right)F\left(\frac{W_0^{FRP}+\left(c_1-s\right)\beta Q_\lambda^*}{p-s}\right)$$

where Q_λ^* satisfies Eq. (5).

Proposition 1 (5)

1. The optimal order quantity Q_λ^* of the retailer under FRP satisfies

$$\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}\left(\left(c_2-c_1\right)\gamma+(\lambda-1)\left(c_2-c_1\right)\gamma F\left(\frac{W_0^{FRP}-\left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2}\right)+\lambda\left(s-c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)\right)d\gamma$$
$$+\int_{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}^{\beta}\left(\left(c_2-c_1\right)\gamma+\left(s-c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)+(\lambda-1)\left(s-c_1\right)\gamma F\left(\frac{W_0^{FRP}+\left(c_1-s\right)\gamma Q_\lambda^*}{p-s}\right)\right)d\gamma=0$$

2. The optimal order quantity Q_λ^* always increases with unit order cost for the supplementary order c_2. Q_λ^* increases with the reference profit W_0^{FRP} if $B_2 > 0$ and decreases with W_0^{FRP} if $B_2 \le 0$. Q_λ^* increases with the maximum fulfillment rate from the unreliable supplier β if $B_3 > 0$ and decreases with β if $B_3 \le 0$.
3. The optimal order quantity Q_λ^* of the retailer under FRP will increase with its loss-averse degree, if the salvage value s is very large.

Proposition 1 indicates that a higher unit order cost for the supplementary order c_2 will make the speculative sourcing in the preselling season more appealing. Hence, the retailer with a higher c_2 will increase the order quantity from the supplier before the selling season. It is expected that the retailer faced with a larger maximum fulfillment rate from the unreliable supplier β will reduce the order quantity, since the supplier with a larger maximum fulfillment rate can potentially fulfill more orders. However, Proposition 1–2 indicates that the retailer with a larger β may increase its order quantity in some cases. This is because a larger maximum fulfillment rate from the unreliable supplier reflects a higher variability of the supplier's fulfillment rate. Thus, the retailer with a larger β may order more from the supplier to ensure the received supply achieves the target.

Proposition 1–2 also illustrates that the retailer's order quantity may increase or decrease with the reference profit W_0^{FRP}. To further investigate the effect of the reference profit W_0^{FRP} on the loss-averse retailer's procurement decision, we conduct a numerical study and the parameters are as follows: $p = 70$, $c_1 = 38$, $c_2 = 50$, $\lambda = 3$, $s = 5$, $\beta = 1$. The retailer's expected profit when $W_0^{\mathrm{FRP}} = 0$ is about 13,000. Based on the value (which is taken as the center of the range), the range of the reference profit W_0^{FRP} is assumed to be [8000, 18000]. In Fig. 3, we can know that the retailer's procurement quantity Q first decreases then increases with the reference profit W_0^{FRP}.

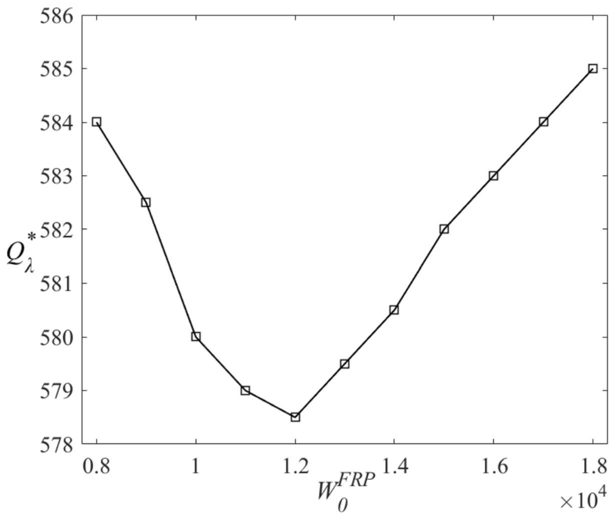

Fig. 3 Variation of the optimal order quantity Q^* in the reference profit W_0^{FRP}

Intuitively, a retailer in practice with a larger reference profit should order less to avoid loss incurred by the excessive inventory. However, it is found that the retailer with a larger reference profit may order more from the supplier under yield uncertainty. As the reference profit increases, an outcome is more likely to be regarded as a loss. Hence, the retailer with a larger reference profit may tend to order more products to hedge against the risk of supply insufficiency, sell more products and gain a higher profit. Figure 3 indicates that when the reference profit W_0^{FRP} is relatively large, the optimal order quantity increases with the reference profit W_0^{FRP}. In contrast, when W_0^{FRP} is small, the optimal order quantity decreases with W_0^{FRP}. If the salvage value s is very large, retailers will not undertake too much loss from unsold products. As a result, a retailer with a higher loss-averse degree may speculatively order more products to hedge against the supply risk, satisfy larger demand and gain a higher profit, as shown in Proposition 1–3.

Based on Proposition 1–1, we can derive that when the reference profit $W_0^{\mathrm{FRP}} = 0$, the optimal order quantity Q_M^* satisfies

$$\begin{aligned} &-(c_2 - s)\int_0^{\beta} \gamma F(\gamma Q_M^*)\mathrm{d}\gamma + \frac{\beta^2}{2}(c_2 - c_1) \\ &-(\lambda-1)(c_1 - s)\int_0^{\beta} \gamma F\left(\frac{(c_1 - s)\gamma Q_M^*}{p-s}\right)\mathrm{d}\gamma = 0 \end{aligned} \tag{6}$$

Denote that.

$$\begin{aligned} Z^{FRP}\left(W_0^{FRP}\right) = &\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q^*}} \left((c_2 - c_1)\gamma F\left(\frac{W_0^{FRP} - (c_2 - c_1)\gamma Q^*}{p - c_2}\right) - (c_2 - s)\gamma F(\gamma Q^*)\right) d\gamma \\ &- \int_{\frac{W_0^{FRP}}{(p-c_1)Q^*}}^{\beta} (c_1 - s)\gamma F\left(\frac{W_0^{FRP} + (c_1 - s)\gamma Q^*}{p-s}\right) d\gamma, \end{aligned}$$

where Q^* satisfies Eq. (2).

Comparing the optimal order quantities in the three scenarios: (1) the retailer is risk-neutral; (2) the retailer is loss-averse and has a zero reference profit; (3) the retailer is loss-averse and has a nonzero reference profit, we can obtain the following proposition.

Proposition 2

1. With a zero reference profit, the loss-averse retailer always orders less than the risk-neutral retailer, i.e., $Q_M^* < Q^*$.

2. With a nonzero reference profit, the loss-averse retailer will order more than the risk-neutral retailer, i.e., $Q_\lambda^* > Q^*$, if one of the following conditions holds:

   (1) The reference profit W_0^{FRP} satisfies $Z^{\mathrm{FRP}}\left(W_0^{\mathrm{FRP}}\right) > 0$.
   (2) The salvage value s is sufficiently large.

Proposition 2 illustrates that the order quantity of the loss-averse retailer with a zero reference profit is always smaller than that of the retailer without loss aversion. This is because a loss-averse retailer inclines to reduce the order quantity to avoid the loss incurred by excessive inventory. However, when W_0^{FRP} is nonzero and sufficiently large, it is found that the loss-averse retailer may order more than the risk-neutral retailer. The results shown in Proposition 2–2 are different from Ma et al. (2016) who stated that the order quantity of the loss-averse retailer is always lower than that of the risk-neutral retailer, since they only considered the scenario where the reference profit $W_0^{\mathrm{FRP}} = 0$. Under a large reference profit W_0^{FRP}, an outcome is likely to be regarded as a loss. Thus, the retailer may tend to sell more products to gain more profit, resulting in a high-order quantity before the selling season. Proposition 2–2 provides the condition for reference profit W_0^{FRP} where the loss-averse retailer orders more than the risk-neutral retailer. Under a large salvage value s, the loss-averse retailer tends to increase the order quantity to hedge against the risk of low supply, satisfy more demand and gain more profit, without undertaking too much loss due to overstock, as shown in Proposition 2–2. This result implies that the loss-averse retailer under a nonzero fixed reference profit should order more than the risk-neutral retailer when the salvage value is large.

3.2.3 Numerical experiment

3.2.3.1 Sensitivity analysis The impacts of the reference profit W_0^{FRP} and unit order cost for the supplementary order c_2 on the retailer's optimal order quantity have been investigated in detail in Sect. 3.2.2. We further investigate the effects of other key parameters, such as the loss aversion coefficient (λ), salvage value for each unsold product (s) and unit procurement cost before the selling season (c_1). Numerical studies are conducted with the following base parameters:

$p = 70, c_1 = 38, c_2 = 50, \beta = 1, s = 5, \lambda = 3, W_0 = 13000.$

The retailer's expected profit when the reference profit $W_0^{\mathrm{FRP}} = 0$ is about 13,000. Based on the value (which is taken as the center of the range), the range of the reference profit W_0^{FRP} is assumed to be [8000, 18000]. The loss aversion coefficient λ is set to be 3 and thus falls in the range of [1.4, 5] which is often used by previous studies considering the loss aversion coefficient (Booij and Van de Kuilen 2009; Abdellaoui et al. 2007; Liu et al. 2019). The values of the selling price p, unit procurement cost before the selling season c_1, unit the supplementary order cost c_2 and salvage value s satisfy $p > c_2 > c_1 > s$ and thus in line with the actual operating conditions. The demand is supposed to follow normal distribution N (500,40) in the basic setting, which is one of the mostly used types of demand distributions in practice. A full factorial design of possible combinations of the key parameters is conducted to

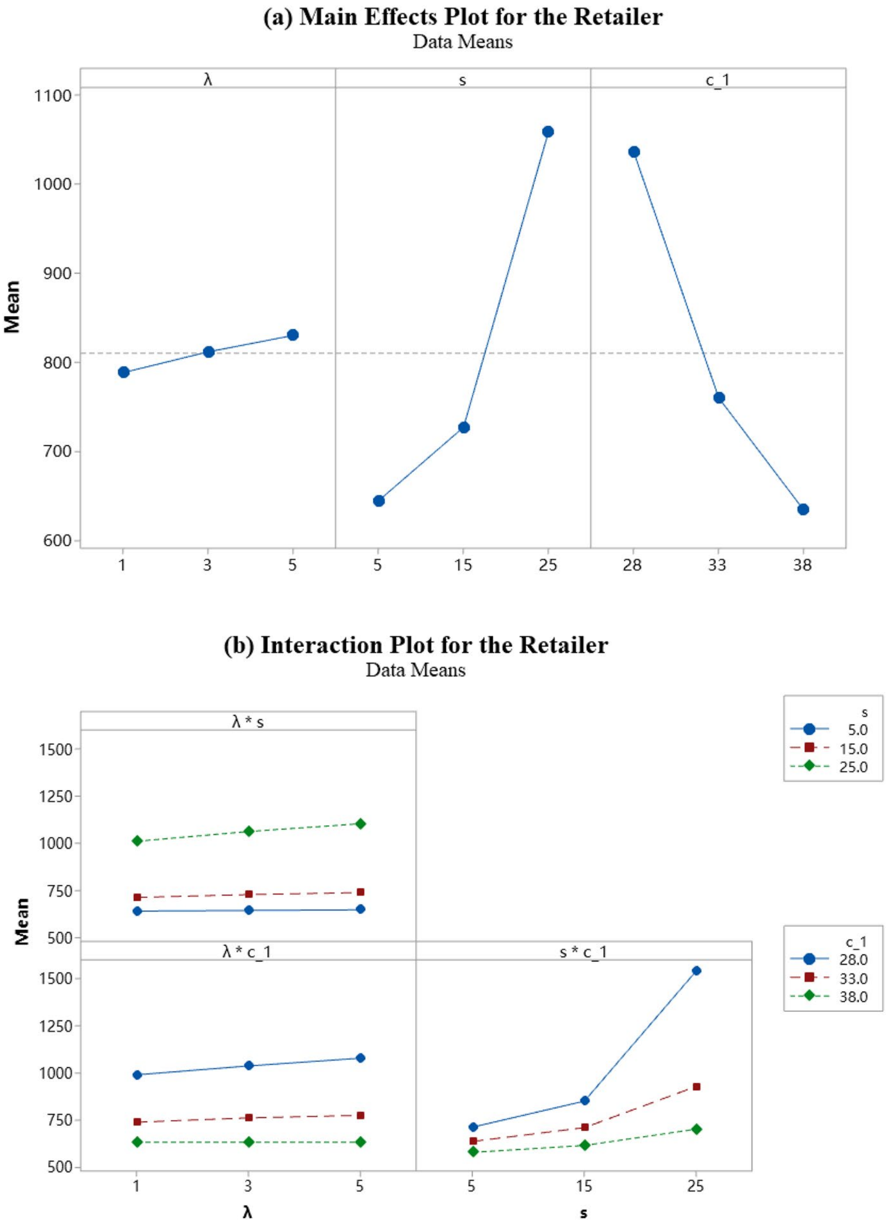

Fig. 4 The sensitivity of optimal order quantity to the experimental factors

evaluate the effects of these parameters on the retailer's ordering decisions. The factorial experiment is composed of three factors, and the levels of each factor are set according to the base case and feasible region:

1 Loss aversion coefficient (λ): 1, 3 and 5;
2 Salvage value for each unsold product (s): 5, 15 and 25;
3 Unit procurement cost before the selling season (c_1): 28, 38 and 48;

We run all the possible combinations of the three factors and levels. There are 27 possible combinations. Figure 4 presents the sensitivity of the retailer's optimal order quantity to the different factors. The figure is composed of the main effects of the factors (see, Fig. 4(a)) and of the interaction effects between the different factors (see, Fig. 4(b)).

Figure 4 (a) depicts that the loss-averse retailer's optimal order quantity increases with the salvage value for each unsold product s and decreases with unit procurement cost before the selling season c_1. A higher unit procurement cost before the selling season c_1 will make the speculative sourcing in the preselling season less appealing, and the retailer will reduce the order quantity from the supplier before the selling season. The retailer with a larger salvage value s undertakes a smaller loss for unsold products and thus inclines to adopt a more speculative procurement strategy. As a result, the retailer's procurement quantity increases with s. The strong interaction between c_1 and s is evident in Fig. 4 (b). It is shown that when c_1 is low, the retailer's order quantity before the selling season will be sensitive to s. This result is a good motivation for retailers in practice to consider the salvage value s more seriously when making ordering decisions, especially when the unit ordering cost is low. Figure 4 (b) also indicates that when the salvage value for each unsold product s is high ($s = 25$), the retailer's order quantity increases with the loss-averse degree λ. However, it is shown that when s is low ($s = 5$), as λ increases, the order quantity is relatively stable and has a slight downward trend.

We further conduct sensitivity analysis to explore how the retailer's order quantity changes with the retailer's loss-averse degree λ. It is found that the retailer's order quantity decreases with the retailer's loss-averse degree λ when the salvage value

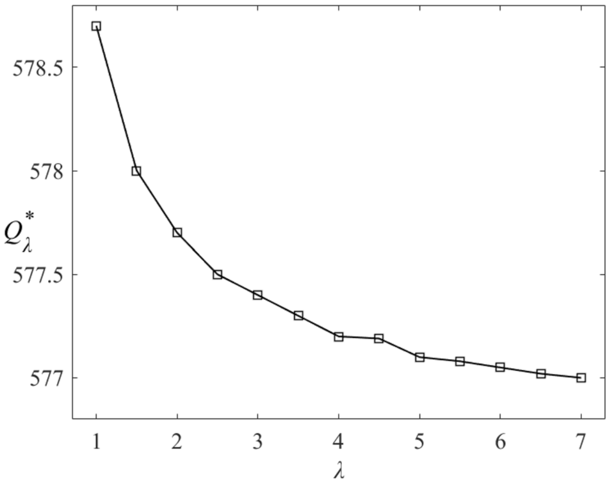

Fig. 5 Variation of the optimal order quantity Q_λ^* in the loss-averse degree λ in Situation 1

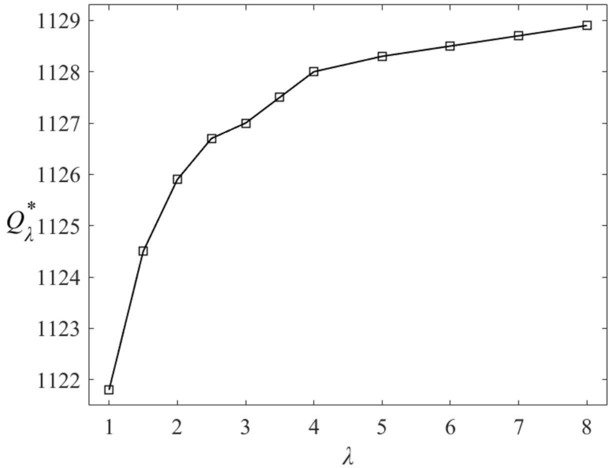

Fig. 6 Variation of the optimal order quantity Q_λ^* in the loss-averse degree λ in Situation 2

for each unsold product s is low. To present this finding, we consider two situations. In Situation 1: the salvage value for each unsold product s is low and equals 2. In Situation 2:s is high and equals 35. The results are shown in Figs. 5 and 6. We can find that in Situation 1, the retailer's order quantity decreases with the loss-averse degree λ, whereas it increases with λ in Situation 2. In Situation 1 where the salvage value s is very small, the overstock cost incurred by unsold products is quite high. Hence, a retailer with a higher loss-averse degree will reduce both procurement quantities to avoid the excessive inventory in Situation 1. However, in Situation 2 where the salvage value s is large, retailers will not bear too much loss from unsold products.

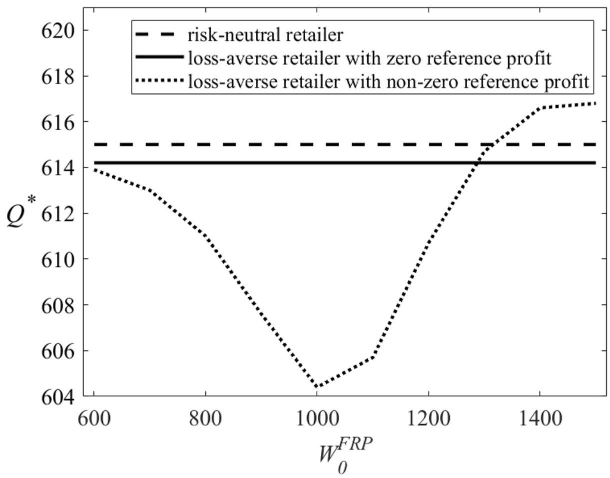

Fig. 7 Variation of the optimal order quantity Q^* in the reference profit W_0^{FRP} in different scenarios ($p = 52, c_1 = 48, c_2 = 50, \beta = 1, s = 44, \lambda = 15, D \sim N(500, 40)$)

Consequently, a retailer with a higher loss-averse degree tends to increase its procurement quantity to satisfy larger demand and improve the profit in Situation 2.

3.2.3.2 Comparative analysis Next, we compare the retailer's optimal order quantity in different scenarios: (1) the retailer is risk-neutral; (2) the retailer is loss-averse and has a zero reference profit; (3) the retailer is loss-averse and has a nonzero reference profit. Figure 7 demonstrates that the order quantity of the loss-averse retailer with a zero reference profit is always lower than that of the risk-neutral one. In contrast, when the reference profit W_0^{FRP} is sufficiently large, the order quantity of the loss-averse retailer is higher than that of the risk-neutral one. As W_0^{FRP} increases, an outcome is more likely to be perceived as a loss. Hence, the retailer with a large W_0^{FRP} inclines to sell more products to gain more profit, resulting in a high-order quantity. In addition, the yield risk also prompts the loss-averse retailer to purchase more from the supplier to avoid the shortage. As a result, when W_0^{FRP} is sufficiently large, under the supply uncertainty, the loss-averse retailer's order quantity is quite high and exceeds the risk-neutral retailer's order quantity.

In Fig. 8(a), when the salvage value is small, the ordering quantity decreases with the loss-averse degree λ in both the cases with a zero reference profit and a nonzero reference profit. However, it is found that the curve of the retailer's order quantity with a nonzero reference profit is steeper than that of the retailer with a zero reference profit. This is because compared with the loss-averse retailer with a zero reference profit, the loss-averse retailer with a nonzero reference profit is more likely to feel at a loss. Hence, the loss-averse attitude has a stronger impact on the ordering decision of a retailer with a nonzero reference profit. In addition, it is found that the risk-neutral retailer orders more than the loss-averse retailer with a zero reference profit, and the loss-averse retailer with a zero reference profit orders more than that with a nonzero reference profit.

Figure 8(b) indicates that the order quantity of the loss-averse retailer with a zero reference profit always decreases with the loss-averse degree λ. In contrast, the order quantities of the loss-averse retailers with nonzero reference profits will increase with λ. Note that the result is different from Ma et al. (2016) who illustrated that a higher loss-averse degree always leads the loss-averse newsvendor to reduce order quantity. This is because we consider the nonzero reference profit, and when the salvage value s is large ($s = 44$), retailers will not undertake too much loss from unsold products. As a result, a retailer with a higher loss-averse degree tends to increase its order quantity to satisfy more demand, improve the profit and achieve the nonzero reference profit. It is also found that the loss-averse retailer with a nonzero reference profit orders more than the risk-neutral retailer. However, the risk-neutral retailer orders more than the loss-averse retailer with a zero reference profit, which is different from the results shown in Fig. 8(a). This is because in this case, the reference profit is large ($W_0^{FRP} = 1400$), and thus an outcome is likely to be regarded as a loss. As such, the retailer tends to sell more products to gain more profit, resulting in a high-order quantity.

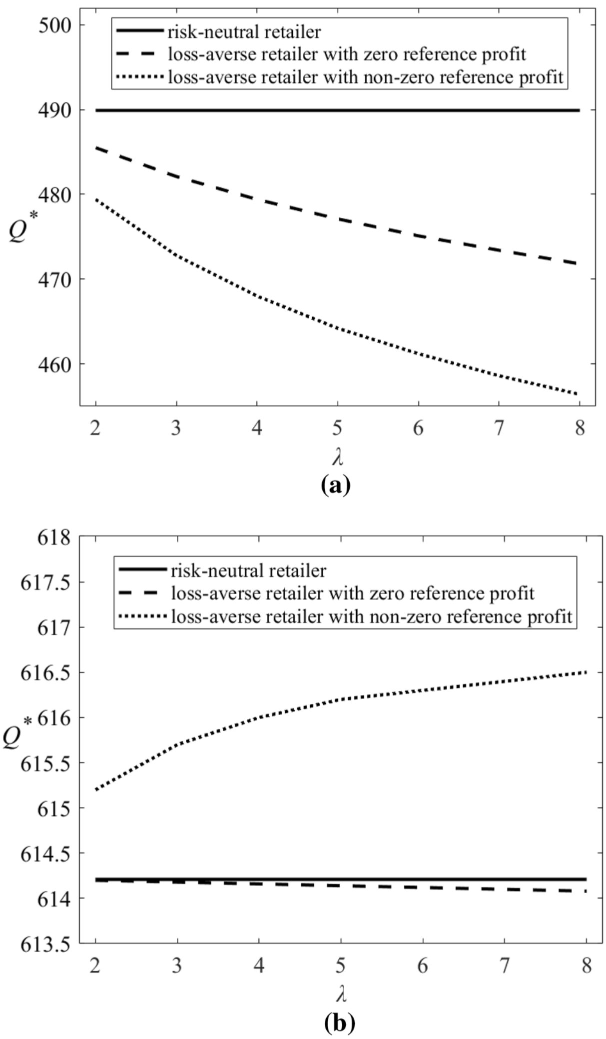

Fig. 8 Variation of the optimal order quantity Q^* in the loss-averse degree λ in different scenarios: (**a**) $s = 5,\ W_0^{\mathrm{FRP}} = 1000$; (**b**) $s = 44,\ W_0^{\mathrm{FRP}} = 1400$ $(p = 52, c_1 = 48, c_2 = 50, \beta = 1, D \sim N(500, 40))$

3.2.4 Special case: uniform demand distribution

In this section, we further investigate the special case in which the demand D in addition to order fulfillment rate γ are assumed to follow the uniform distribution (Inderfurth 2004; Käki et al. 2015). The range of D is assumed to be $[0, b]$.

The retailer's optimal order quantity can be obtained in different scenarios: (1) Scenario 1: the retailer is risk-neutral; (2) Scenario 2: the retailer is loss-averse and has a zero reference profit; (3) Scenario 3: the retailer is loss-averse and has a nonzero reference profit. The retailer's optimal order quantities in Scenarios 1, 2 and

3 are denoted by Q^*_{U1}, Q^*_{U2} and Q^*_{U3}, respectively. According to Eqs. (2) and (5), we can obtain $Q^*_{U1} = \frac{3(c_2-c_1)b}{2\beta(c_2-s)}$ and $Q^*_{U2} = \frac{3(p-s)(c_2-c_1)b}{2\beta\left((\lambda-1)(c_1-s)^2+(p-s)(c_2-s)\right)}$. Based on Proposition 1–1, Q^*_{U3} satisfies

$$L_1\left(Q^*_{U3}\right) + L_2\left(Q^*_{U3}\right) + L_3\left(Q^*_{U3}\right) = 0,$$

where

$$L_1\left(Q^*_{U3}\right) = \frac{(\lambda-1)W_0^{\mathrm{FRP}}}{2b}\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q^*_{U3}}\right)^2\left(\frac{c_2-c_1}{p-c_2} - \frac{s-c_1}{p-s}\right),$$

$$L_2\left(Q^*_{U3}\right) = \frac{Q^*_{U3}}{3b}\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q^*_{U3}}\right)^3\left(\frac{-(\lambda-1)(c_2-c_1)^2}{p-c_2} - (c_2-s) + \frac{(\lambda-1)(c_1-s)^2}{p-s}\right),$$

$$\begin{aligned} L_3\left(Q^*_{U3}\right) =& \frac{(c_2-c_1)\beta^2}{2} + \frac{(s-c_2)Q^*_{U3}\beta^3}{3b} \\ &+ \frac{W_0^{\mathrm{FRP}}\beta^2(\lambda-1)(s-c_1)}{2b(p-s)} \\ &- \frac{(\lambda-1)(c_1-s)^2 Q^*_{U3}\beta^3}{3b(p-s)} \end{aligned}$$

We further investigate the effects of reference profit W_0^{FRP} on the retailer's order quantity. To simplify the expression, we denote that

$$\begin{aligned} L =& \frac{(p-c_2)(c_1-s)\beta^2(\lambda-1)}{2b\left((p-s)(c_2-c_1)+(c_1-s)(p-c_2)\right)}\left(\frac{c_2-c_1}{p-c_2} - \frac{s-c_1}{p-s}\right) \\ &+ \frac{(p-c_2)(c_1-s)\beta^2}{3b(p-c_1)\left((p-s)(c_2-c_1)+(c_1-s)(p-c_2)\right)} \\ &\left(\frac{-(c_2-c_1)^2(\lambda-1)}{p-c_2} + c_2 - s + \frac{(\lambda-1)(c_1-s)^2}{p-s}\right) \\ &+ \frac{\beta^3}{3b(p-c_1)}\left(s-c_2-\frac{(c_1-s)^2(\lambda-1)}{p-s}\right) \\ &\sqrt{\frac{(p-s)(c_2-c_1)+(c_1-s)(p-c_2)}{(p-c_2)(c_1-s)\beta^2}} + \frac{(\lambda-1)(s-c_1)\beta^2}{2b(p-s)} \end{aligned}$$

and then obtain Proposition 4.

Proposition 4 1. If $L \geq 0$, the optimal order quantity Q^*_{U3} increases with the reference profit W_0^{FRP}.

2. If $L < 0$, and

(1) $W_0^{\mathrm{FRP}} > \frac{(c_2-c_1)\beta^2}{-2L}$, the optimal order quantity Q^*_{U3} increases with W_0^{FRP};

(2) $W_0^{\mathrm{FRP}} \leq \frac{(c_2-c_1)\beta^2}{-2L}$, the optimal order quantity Q^*_{U3} decreases with W_0^{FRP}.

Proposition 4 indicates that a larger W_0^{FRP} has two opposite effects on the loss-averse retailer's order quantity Q^*_{U3}, which is consistent with the findings in Fig. 3. Proposition 4 provides the conditions in which Q^*_{U3} increases or decreases with the reference profit W_0^{FRP}. When $L \geq 0$, the retailer with a higher reference profit W_0^{FRP} will always increase the order quantity before the selling season. However, when $L < 0$, a higher reference profit W_0^{FRP} may induce the retailer to procure more or fewer products from the supplier. Specifically, if the reference profit W_0^{FRP} is relatively large, the optimal order quantity will increase with the reference profit W_0^{FRP}. In contrast, if W_0^{FRP} is small, the optimal order quantity will decrease with W_0^{FRP}.

We then investigate the effects of maximum fulfillment rate from the unreliable supplier β on the retailer's order quantity in different scenarios. To simplify the expressions of the ensuing proposition, we denote that

$$
\begin{aligned}
M = {} & \frac{(\lambda-1)W_0^{\mathrm{FRP}}\beta^2}{2b}\left(\frac{W_0^{\mathrm{FRP}}\left(\left(c_2-s\right)(p-s)+(\lambda-1)\left(c_1-s\right)^2\right)}{\left(p-c_1\right)\left(\left(c_2-c_1\right)b(p-s)+(\lambda-1)\left(s-c_1\right)W_0^{\mathrm{FRP}}\right)}\right)^2\left(\frac{c_2-c_1}{p-c_2}+\frac{c_1-s}{p-s}\right) \\
& + \left(\frac{W_0^{\mathrm{FRP}}\left(\left(c_2-s\right)(p-s)+(\lambda-1)\left(c_1-s\right)^2\right)}{\left(p-c_1\right)\left(\left(c_2-c_1\right)b(p-s)+(\lambda-1)\left(s-c_1\right)W_0^{\mathrm{FRP}}\right)}\right)^2\frac{W_0^{\mathrm{FRP}}\beta^2}{3b\left(p-c_1\right)} \\
& \times\left(\frac{-(\lambda-1)\left(c_2-c_1\right)^2}{p-c_2}+c_2-s+\frac{(\lambda-1)\left(c_1-s\right)^2}{p-s}\right) \\
& + \frac{\left(c_2-c_1\right)\beta^2}{2}+\beta^2\left(s-c_2-\frac{(\lambda-1)\left(c_1-s\right)^2}{(p-s)}\right)\frac{\left(c_2-c_1\right)b(p-s)+(\lambda-1)\left(s-c_1\right)W_0^{\mathrm{FRP}}}{3b\left(\left(c_2-s\right)(p-s)+(\lambda-1)\left(c_1-s\right)^2\right)} \\
& + \frac{\beta^2W_0^{\mathrm{FRP}}(\lambda-1)\left(s-c_1\right)}{2b(p-s)}
\end{aligned}
$$

Proposition 5

1. For a risk-neutral retailer or loss-averse retailer with a zero reference profit, the optimal order quantity always decreases with the maximum fulfillment rate from the unreliable supplier β;

2. For a loss-averse retailer with a nonzero reference profit,
   if $M \leq 0$, the optimal order quantity Q_{U3}^{*} will increase with the maximum fulfillment rate from the unreliable supplier β;
   Otherwise, the optimal order quantity Q_{U3}^{*} will decrease with β.

Proposition 5–1 shows that when the retailer is risk-neutral or loss-averse with a zero reference profit, a higher maximum fulfillment rate from the unreliable supplier β will always result in a lower-order quantity under the uniform distribution. As noted before, the retailer tends to order more than the quantity actually needed to hedge against the risk of supply insufficiency. The supplier with a larger β can potentially fulfill more orders, and thus a larger β will prompt the retailer to reduce procurement quantity. However, when the reference profit is nonzero, the loss-averse retailer's order quantity may increase with β. This is because compared with the loss-averse retailer with a zero reference profit, the loss-averse retailer with a nonzero reference profit is more likely to feel at a loss and is more unwilling to face supply uncertainty. Hence, considering that a larger β implies a higher variability of the supplier's order fulfillment rate, the retailer with a nonzero reference profit may order more from the supplier to ensure the received supply reaches the target quantity. Proposition 5–2 provides the conditions in which the retailer's order quantity increases or decreases with the maximum fulfillment rate from the unreliable supplier β.

Next, we compare the optimal procurement quantity in the three scenarios. The results are shown in Proposition 6. To simplify the expressions of this proposition, we denote that

$$Z_1 = \frac{2\left(c_2 - s\right)W_0^{\mathrm{FRP3}}\left((\lambda - 1)\left(c_1 - s\right)^2 + (p - s)\left(c_2 - s\right)\right)^2}{27\left(p - c_2\right)\left(p - c_1\right)(p - s)^2\left(c_2 - c_1\right)^2\beta^2} + \frac{W_0^{\mathrm{FRP}}\left(s - c_1\right)}{2(p - s)};$$

$$Z_2 = \frac{2\beta^2 W_0^{\mathrm{FRP3}}\left(c_2 - s\right)^3}{27(p - s)\left(p - c_2\right)\left(p - c_1\right)\left(c_2 - c_1\right)^2 b^2} - \frac{\left(c_1 - s\right)\left(\left(W_0^{\mathrm{FRP}} - a(p - s)\right)\left(c_2 - s\right) + \left(c_2 - c_1\right)b\left(c_1 - s\right)\right)}{2\left(c_2 - s\right)(p - s)}$$

Proposition 6

1. If $Z_1 \le 0$, $Q^*_{U1} \ge Q^*_{U3}$; Otherwise, $Q^*_{U2} < Q^*_{U1} < Q^*_{U3}$.
2. If $Z_2 \ge 0$, $Q^*_{U2} \le Q^*_{U3}$; Otherwise, $Q^*_{U3} < Q^*_{U2} < Q^*_{U1}$.

Proposition 6 illustrates that the order quantity of the loss-averse retailer with a zero reference profit is always smaller than that of the retailer without loss aversion. As explained before, the loss-averse retailer reduces the order quantity to avoid excessive inventory. Proposition 6–1 (or 6–2) provides the conditions in which the order quantity of the loss-averse retailer with reference profit $W_0^{\mathrm{FRP}} \neq 0$ is larger or smaller than that of the retailer without loss aversion (or the loss-averse retailer with the reference profit $W_0^{FRP} = 0$).

3.2.5 Extension: optimal ordering decisions with a prospect-dependent reference point (PRP)

In this section, we investigate a loss-averse retailer's ordering decisions with a prospect-dependent reference point (PRP) W_0^{PRP}. The PRP model discussed in this section is the weighted average of the maximum and minimum profits associated with ordering Q, which is the model used by Long and Nasiry (2015):

$$W_0^{\mathrm{PRP}}(Q) = \alpha W_m^{\max}(Q) + (1-\alpha) W_m^{\min}(Q),$$

where $W_m^{\max}(Q)$ and $W_m^{\min}(Q)$ are, respectively, the maximum and minimum profits regarding the order quantity Q and $\alpha \in [0,1]$ is the optimism level of the newsvendor. A low α reflects that the retailer has a low expectation for the final profit, whereas a high value of α implies that the retailer anchors more on the best-case scenario. The reference point W_0^{PRP} under PRP model is provided in the ensuing lemma:

Lemma 4 The reference profit $W_0^{\mathrm{PRP}}(Q)$ is given by the following formula:

$$\begin{aligned} W_0^{\mathrm{PRP}}(Q) = {} & \alpha\big(\big(p-c_2\big)D_{\max} + \big(c_2-c_1\big)\min\big\{\beta Q, D_{\max}\big\}\big) \\ & + (1-\alpha)\Big(\big(p-c_2\big)D_{\min} - \big[\big(c_1-s\big)\beta Q - \big(c_2-s\big)D_{\min}\big]^+\Big) \end{aligned}$$

With the reference point $W_0^{\mathrm{PRP}}(Q)$, similar to the analyses in Sect. 3.2.2, the retailer's utility function when $\gamma < \frac{W_0^{\mathrm{PRP}}(Q)}{(p-c_1)Q}$ is as follows:

$$U_1^{\mathrm{PRP}}\big(W_m\big) = \begin{cases} \lambda * \big[(p-s)D + \big(s-c_1\big)\gamma Q - W_0^{\mathrm{PRP}}(Q)\big], D < \gamma Q \\ \lambda * \big[\big(p-c_2\big)D + \big(c_2-c_1\big)\gamma Q - W_0^{\mathrm{PRP}}(Q)\big], \gamma Q \le D < q_1^N(Q) \\ \big(p-c_2\big)D + \big(c_2-c_1\big)\gamma Q - W_0^{\mathrm{PRP}}(Q), D \ge q_1^N(Q) \end{cases}.$$

When $\gamma \ge \frac{W_0^{\mathrm{PRP}}(Q)}{(p-c_1)Q}$, the retailer's utility function is

$$U_2^{\mathrm{PRP}}\left(W_m\right)=\begin{cases}\lambda * \left[(p-s)D+\left(c_1-s\right)\gamma Q-W_0^{\mathrm{PRP}}(Q)\right], D<q_2^N(Q)\\ (p-s)D-\left(c_1-s\right)\gamma Q-W_0^{\mathrm{PRP}}(Q), q_2^N(Q)\leq D<\gamma Q\\ \left(p-c_2\right)D+\left(c_2-c_1\right)\gamma Q-W_0^{\mathrm{PRP}}(Q), D\geq \gamma Q\end{cases}.$$

Hence, the retailer's expected utility function under a prospect-dependent reference point $W_0^{\mathrm{PRP}}(Q)$ is provided by:

$$v(Q)=\int_0^{\frac{W_0^{PRP}(Q)}{(p-c_1)Q}} E\left[U_1^{PRP}\left(W_m\right)\right]g(\gamma)d\gamma+\int_{\frac{W_0^{PRP}(Q)}{(p-c_1)Q}}^{\beta} E\left[U_2^{PRP}\left(W_m\right)\right]g(\gamma)d\gamma, \tag{7}$$

where

$$E\left[U_1^{PRP}\left(W_m\right)\right]=\lambda\int_0^{\gamma Q}\left((p-s)D+\left(s-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD$$

$$+\lambda\int_{\gamma Q}^{\frac{W_0^{PRP}(Q)-(c_2-c_1)\gamma Q}{p-c_2}}\left(\left(p-c_2\right)D+\left(c_2-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD$$

$$\int_{\frac{W_0^{PRP}(Q)-(c_2-c_1)\gamma Q}{p-c_2}}^{+\infty}\left(\left(p-c_2\right)D+\left(c_2-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD,$$

and

$$E\left[U_2^{PRP}\left(W_m\right)\right]=\lambda\int_0^{\frac{W_0^{PRP}(Q)+(c_1-s)\gamma Q}{p-s}}\left((p-s)D+\left(s-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD$$

$$+\int_{\frac{W_0^{PRP}(Q)+(c_1-s)\gamma Q}{p-s}}^{\gamma Q}\left((p-s)D+\left(s-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD$$

$$+\int_{\frac{W_0^{PRP}(Q)+(c_1-s)\gamma Q}{p-s}}^{\gamma Q}\left((p-s)D+\left(s-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD$$

$$+\int_{\gamma Q}^{+\infty}\left(\left(p-c_2\right)D+\left(c_2-c_1\right)\gamma Q-W_0^{PRP}(Q)\right)f(D)dD$$

Denote the first derivative of the formula (7) with respect to the order quantity Q by

$$R\left[Q, W_0^{PRP}(Q)\right] = \int_0^{\frac{W_0^{PRP}(Q)}{(p-c_1)Q}} \left((\lambda-1)\left(c_2-c_1\right)(\gamma-\alpha\beta)F\left(\frac{W_0^{PRP}(Q)-\left(c_2-c_1\right)\gamma Q}{p-c_2}\right) + \lambda\left(s-c_2\right)\gamma F(\gamma Q)\right)\frac{1}{\beta}d\gamma$$
$$+\int_{\frac{W_0^{PRP}(Q)}{(p-c_1)Q}}^{\beta} \left(\left(s-c_2\right)\gamma F(\gamma Q) + (\lambda-1)\left(\left(s-c_1\right)\gamma - \alpha\left(c_2-c_1\right)\beta\right)F\left(\frac{W_0^{PRP}(Q)+\left(c_1-s\right)\gamma Q}{p-s}\right)\right)\frac{1}{\beta}d\gamma$$
$$+\left(c_2-c_1\right)\beta\left(\frac{1}{2}-\alpha\right)$$

The retailer's expected utility function (7) is different under different forms of the reference point $W_0^{\mathrm{PRP}}(Q)$. Hence, we use different notations to represent the optimal order quantity of the retailer, i.e., the solution of $R\left[Q, W_0^{\mathrm{PRP}}(Q)\right]=0$, under different forms of the reference point $W_0^{\mathrm{PRP}}(Q)$:

If $W_0^{\mathrm{PRP}}(Q)=\alpha\left(\left(p-c_2\right)D_{\max}+\left(c_2-c_1\right)\beta Q\right)+(1-\alpha)\left(p-c_2\right)D_{\min}$, the solution is denoted by Q_A;

If $W_0^{\mathrm{PRP}}(Q)=\alpha\left(p-c_1\right)D_{\max}+(1-\alpha)\left(p-c_2\right)D_{\min}$ and $\frac{c_2-s}{c_1-s}D_{\min}>D_{\max}$, or $W_0^{PRP}(Q)=\alpha\left(\left(p-c_2\right)D_{max}+\left(c_2-c_1\right)\beta Q\right)+(1-\alpha)\left(\left(p-c_2\right)D_{min}-\left(c_1-s\right)\beta Q+\left(c_2-s\right)D_{min}\right)$ and $\frac{c_2-s}{c_1-s}D_{\min}\le D_{\max}$, the solution is denoted by Q_B;

If $W_0^{\mathrm{PRP}}(Q)=\alpha\left(p-c_1\right)D_{\max}+(1-\alpha)\left(\left(p-c_2\right)D_{\min}-\left(c_1-s\right)\beta Q+\left(c_2-s\right)D_{\min}\right)$, the solution is denoted by Q_C.

Let $K=min\left\{\frac{D_{\max}}{\beta}, \frac{(c_2-s)D_{\min}}{(c_1-s)\beta}\right\}$,$L=max\left\{\frac{D_{max}}{\beta}, \frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right\}$. We can derive the retailer's optimal order quantity under the PRP model, which is provided in Proposition 7.

Proposition 7 The optimal order quantity Q^*_{PRP} can be derived from one of the following cases:

(1) If $Q_A\in[0,K], Q_B\in(K,L], Q_C\in(L,+\infty)$, then $Q^*_{PRP}=\underset{Q^*_{PRP}\in\{Q_A,Q_B,Q_C\}}{arcmax}\, v\left(Q^*_{PRP}\right)$;

(2) If $Q_A\in[0,K], Q_B\in(K,L], Q_C\in[0,L]$, then $Q^*_{PRP}=\underset{Q^*_{PRP}\in\{Q_A,Q_B\}}{arcmax}\, v\left(Q^*_{PRP}\right)$;

(3) If $Q_A\in[0,K], Q_B\notin(K,L], Q_C\in(L,+\infty)$, then $Q^*_{PRP}=\underset{Q^*_{PRP}\in\{Q_A,Q_C\}}{arcmax}\, v\left(Q^*_{PRP}\right)$;

(4) If $Q_A\in(K,+\infty), Q_B\in(K,L], Q_C\in(L,+\infty)$, then $Q^*_{PRP}=\underset{Q^*_{PRP}\in\{Q_B,Q_C\}}{arcmax}\, v\left(Q^*_{PRP}\right)$;

(5) If $Q_A\in[0,K], Q_B\in[0,K), Q_C\in[0,L]$, then $Q^*_{PRP}=Q_A$;

(6) If $Q_A\in(K,+\infty), Q_B\in(K,L], Q_C\in[0,L]$, then $Q^*_{PRP}=Q_B$;

(7) If $Q_A\in(K,+\infty), Q_B\in(L,+\infty), Q_C\in(L,+\infty)$, then $Q^*_{PRP}=Q_C$;

Proposition 7 indicates the optimal order quantity of a loss-averse retailer with a prospect-dependent reference point (PRP) in different cases. Based on Proposition 7, we investigate the effects of the unit order cost for the supplementary order, loss-averse degree and the maximum fulfillment rate from the unreliable supplier on the retailer's ordering decisions. The results are provided in Proposition 8. To simplify the expressions of Proposition 8, some complex formulas are represented by B_4 and B_5, and their specific forms are provided in the proof of Proposition 8.

Proposition 8.

1. Under the PRP model, the optimal order quantity Q^*_{PRP} increases with the unit order cost for the supplementary order c_2 if $B_4 > 0$ and decreases with c_2 if $B_4 \leq 0$. Q^*_{PRP} increases with the maximum fulfillment rate from the unreliable supplier β if $B_5 > 0$, and decreases with β if $B_5 \leq 0$.
2. Under the PRP model, the retailer with a higher loss-averse degree may increase or decrease the order quantity, if the salvage value s is very large.

Proposition 8–1 illustrates the effects of the unit order cost for the supplementary order c_2 and the maximum fulfillment rate from the unreliable supplier β on the retailer's order quantity. It is shown that the optimal order quantity may increase or decrease with β, which is consistent with the results under the fixed reference point (FRP). Intuitively, the retailer confronted with a higher supplementary order cost will order more before the selling season in practice, which has been verified under the FRP model. However, it is found that the order quantity may decrease with the unit order cost for the supplementary order c_2 under the PRP model. As c_2 increases, on the one hand, a larger c_2 may prompt the retailer to order more before the selling season to reduce total order costs. On the other hand, a lower-order quantity before the selling season will lead to a lower reference profit of the retailer under the PRP model, which may result in a larger expected utility. Hence, as c_2 increases, the latter effect may overwhelm the former effect, and the retailer may reduce the order quantity to achieve a larger expected utility under the PRP model. Proposition 8–2 shows that the retailer under the PRP model with a higher loss-averse degree may increase or decrease the order quantity when the salvage value s is very large. This result is different from the result under the FRP model that a higher loss-averse degree always leads to a higher-order quantity when the salvage value s is large. This is because different from the FRP model where the reference profit is fixed, the retailer under the PRP model with a lower-order quantity will own a lower reference profit, which may result in its higher probability of being in the gain state and thus a larger expected utility. As such, under the PRP model, the retailer with a higher loss-averse degree may reduce the order quantity, which will result in a lower reference profit, and thus can hedge against the risk of state of loss and achieve a larger expected utility.

We then compare the order quantities of the loss-averse retailer under the PRP model and the risk-neutral retailer, and the result is summarized in Proposition 9. Denote that

$$Z^{PRP}\left(W_0^{PRP}\right) = \int_0^{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}} \left((\lambda - 1)\left[\left(c_2 - c_1\right)\gamma - N\right] F\left(\frac{W_0^{PRP}(Q^*) - \left(c_2 - c_1\right)\gamma Q^*}{p - c_2} \right) \right.$$
$$\left. -(\lambda - 1)\left(c_2 - s\right)\gamma F(\gamma Q^*) - N \right) d\gamma$$
$$+ \int_{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}}^{\beta} \left((\lambda - 1)\left[\left(s - c_1\right)\gamma - N\right] F\left(\frac{W_0^{PRP}(Q^*) + \left(c_1 - s\right)\gamma Q^*}{p - s} \right) - N \right) d\gamma$$

where $N_1 = \frac{\partial W_0^{\mathrm{PRP}}(Q^*_{\mathrm{PRP}})}{\partial Q^*_{\mathrm{PRP}}}$ and Q^* is the optimal order quantity of the risk-neutral retailer and satisfies Eq. (2).

Proposition 9 Under the PRP model,

1. the loss-averse retailer will order more than the risk-neutral retailer when ss$Z^{\mathrm{PRP}}\left(W_0^{\mathrm{PRP}}\right) > 0$.
2. The loss-averse retailer may order more or less than the risk-neutral retailer, if the salvage value s is sufficiently large.

Ma et al. (2016) stated that the order quantity of the loss-averse retailer is always lower than that of the risk-neutral retailer. However, Proposition 9–1 provides the condition where the loss-averse retailer orders more than the risk-neutral retailer under the PRP model. This is because Ma et al. (2016) only considered the scenario of zero reference profit. When the reference profit is nonzero and large, an outcome, clearly, is likely to be regarded as a loss. Hence, the retailer may incline to sell more products to gain more profit, which leads to a high-order quantity. The result shown in Proposition 9–2 is different from that in Proposition 2 where the retailer has a FRP. Under FRP models, a large salvage value s will lead the retailer to source more products to satisfy more demand and gain a larger profit without bearing too much loss from overstock. The reference profit of the retailer under the PRP model, however, increases with its order quantity. Hence, the retailer under the PRP model may decide a lower-order quantity than the risk-neutral retailer, which will cause a lower reference profit and lower risk of loss state, and get a larger expected utility, when the salvage value s is sufficiently large.

4 Conclusions

In this paper, we rigorously model and address the procurement issue of a loss-averse retailer with a reference profit under both supply and demand uncertainties. The loss-averse retailer makes procurement decisions from the unreliable supplier who will partly fulfill the order under the demand uncertainty before the selling season. During the selling season, the demand reveals, and if the products are stocked out, the retailer has an emergency replenishment opportunity from the spot market to satisfy unmet demand. We investigate the retailer's ordering decisions in three scenarios: (1) the retailer is risk-neutral; (2) the retailer is loss-averse and has a zero reference profit; (3) the retailer is loss-averse and has a nonzero fixed reference profit (FRP). We obtain and compare the retailer's optimal ordering decisions in these three scenarios. It is found that compared with the risk-neutral retailer, the loss-averse retailer with a zero reference profit always orders less before the selling season. However, the loss-averse retailer with a nonzero reference profit will order more than the risk-neutral retailer under some conditions. This paper investigates the effects of the reference profit, the loss-averse degree and the maximum fulfillment rate from the unreliable supplier on the retailer's optimal ordering decisions. The special case in which the demand follows uniform distribution is further explored. This paper further studies the retailer's ordering decision with a prospect-dependent reference point (PRP) and compares the results under FRP and PRP. Some interesting results are obtained, which can provide guidance for the practitioners:

1. It is expected that a retailer with a higher loss-averse degree will order less to avoid the loss incurred by the excessive inventory in practice. However, we find that a retailer with a higher loss-averse degree and a fixed reference point should increase the order quantity to hedge against the risk of low supply, when the salvage value is large.
2. Under a higher reference profit, an outcome is more likely to be regarded as a loss, and intuitively, the retailer tends to decrease the order quantity to reduce the loss brought by the excessive inventory. However, we find that when yield uncertainty is considered, the retailer with a higher reference profit should order more to hedge against the risk of insufficient supply, satisfy more demand and gain a higher profit under some conditions. These conditions are provided in this paper.
3. It is expected that the loss-averse retailer always orders less than the risk-neutral retailer. However, it is found that the loss-averse retailer with yield uncertainty should order more than the risk-neutral retailer to hedge against the supply risk, when the salvage value is large.
4. Intuitively, a larger supplementary order cost prompts the retailer to order more before the selling season to save the total order cost. However, it is found that the retailer with a prospect-dependent reference profit will reduce its order quantity when the supplementary order cost becomes higher in some cases. The conditions where the retailer with a higher supplementary order cost should order more or less than before are provided.

5. The retailer with a larger maximum fulfillment rate from the unreliable supplier should order more under some conditions provided in this paper. Intuitively, the supplier with a larger maximum fulfillment rate can potentially fulfill more orders and will prompt the retailer to reduce the order quantity. However, a larger maximum fulfillment rate reflects a higher variability of the fulfillment rate. Hence, the retailer may order more from the supplier to ensure that the received supply reaches the target.

The optimal ordering quantity is derived and analyzed for a loss-averse retailer under both yield and demand randomness in this study. Additionally, the above interesting results obtained would provide guidance and managerial insights for practitioners on how to make ordering decisions in different scenarios. In practice, companies also need to know how to obtain the parameters to apply our results. The parameters used in this paper are composed of two types: operational parameters and loss aversion parameters. Operational parameters, such as the unit ordering cost and the selling price, are independent of individuals and can be collected according to practical operating conditions. There are two important loss aversion parameters in this paper: the reference point and loss-averse coefficient. There are several different types of reference points. Fixed reference points (FRP) include the status quo (i.e., zero payoff), the minimum payoff, the maximum payoff and the industry average level (Hall 1980; Wei et al. 2019; Uppari and Hasija 2019). Prospect-dependent reference points (PRP) include the expected profit and the weighted average of the maximum and minimum profit (Uppari and Hasija 2019; Kőszegi and Rabin 2006). This paper provides the loss-averse retailer's optimal ordering decisions under both FRP and PRP. The practitioners can choose one of the reference point models to use according to their actual reference point type. The loss-averse coefficients used by previous studies fall in [1.4,5] (Abdellaoui et al. 2007; Booij and Van de Kuilen 2009; Liu et al. 2019). This range of the loss-averse coefficient can provide a reference for practitioners' selection of loss-averse coefficients. In addition, it is noteworthy that the loss aversion parameters can also be obtained by practitioners through prospect theory elicitation methods (Abdellaoui et al. 2007; Kemel and Paraschiv 2013).

In addition, this paper provides some implications for the research. We investigate a loss-averse retailer's ordering decisions considering both the effects of reference point and yield uncertainty of the supplier. Our theoretical results can act as a testable input for future behavioral research on supply risks (Gurnani et al. 2014; Ancarani et al. 2013; Käki et al. 2015; Sarkar and Kumar 2015; Xue et al. 2021). In addition, the results in this paper can provide a research basis for designing contracts in supply chains with behavioral phenomena. Behavioral phenomena ought to be considered when designing coordination contracts (Becker-Peth et al. 2013; Becker-Peth and Thonemann 2016; Castañeda et al. 2019; Wu and Chen 2014; Johnsen et al. 2021; Schiffels and Voigt 2021). The contracts design between retailers and suppliers requires the investigation of the ordering decisions of the retailer. The results in this study, namely the optimal ordering decisions of a loss-averse retailer, can be an input for future research on coordination contracts design between retailers and suppliers considering loss aversion.

There are several directions for future research. Our model merely considers the loss-averse retailer's procurement decisions. Future research can further investigate the decisions of the supplier and the game between the loss-averse retailer and supplier and explore the channel coordination mechanism under both supply and demand uncertainties. Moreover, the study can be extended by taking the competition between two suppliers or retailers into account.

Appendix

Proof of Lemma 1

Note that the optimal order quantity Q^* satisfies $\int_0^\beta \gamma F(\gamma Q^*)\mathrm{d}\gamma = \frac{\beta^2(c_2-c_1)}{2(c_2-s)}$, we denote $A = \int_0^\beta \gamma F(\gamma Q^*)\mathrm{d}\gamma - \frac{\beta^2(c_2-c_1)}{2(c_2-s)}$. Hence,

$$\frac{\partial A}{\partial Q^*} = \int\limits_0^\beta \gamma^2 f(\gamma Q^*)\mathrm{d}\gamma, \frac{\partial A}{\partial \beta} = \beta F(\beta Q^*) - \frac{\beta(c_2-c_1)}{c_2-s} \text{ and } \frac{\partial Q^*}{\partial \beta} = -\frac{\frac{\partial A}{\partial \beta}}{\frac{\partial A}{\partial Q^*}} = -\frac{\beta F(\beta Q^*) - \frac{\beta(c_2-c_1)}{c_2-s}}{\int_0^\beta \gamma^2 f(\gamma Q^*)d\gamma}$$

Whether $\frac{\partial Q^*}{\partial \beta}$ is positive or negative is uncertain. Thus, the risk-neutral retailer's order quantity may increase or decrease with β.

Similarly, we can prove that the risk-neutral retailer's order quantity is invariant of the selling price p. □

Proof of Lemma 2.

According to Eq. (4), $W_-(Q) < (p-c_1)\gamma Q$ and $W_+(Q) \geq (p-c_1)\gamma Q$. Hence, when $\gamma < \frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q}$, that is, $W_0^{\mathrm{FRP}} > (p-c_1)\gamma Q$, $W_-(Q)$ is always smaller than W_0^{FRP}, whereas $W_+(Q)$ may be larger or smaller than W_0^{FRP}. To find the critical value of demand D_t where the retailer's reference profit $W_m(Q)$ equals the reference profit W_0^{FRP}, let $W_+(Q) = W_0^{\mathrm{FRP}}$,$D_t = \frac{W_0^{\mathrm{FRP}}-(c_2-c_1)\gamma Q}{p-c_2} = q_1^N(Q) > \gamma Q$. As a result, when the demand $D < D_t = q_1^N(Q)$, the profit $W_m(Q) < W_0^{\mathrm{FRP}}$; otherwise, $W_m(Q) \geq W_0^{\mathrm{FRP}}$.

Similarly, when $\gamma \geq \frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q}$, we can prove that when $D < \frac{(c_1-s)\gamma Q + W_0^{\mathrm{FRP}}}{p-s} = q_2^N(Q)$, $W_m(Q) < W_0^{\mathrm{FRP}}$; otherwise, $W_m(Q) \geq W_0^{\mathrm{FRP}}$. □

Proof of Lemma 3.

Taking the first- and second-order derivatives of $E[U(W_m)]$ yields

$$\frac{\partial E\left[U\left(W_m\right)\right]}{\partial Q}=\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q}}\frac{\partial E\left[U_1\left(W_m\right)\right]}{\partial Q}\frac{1}{\beta}d\gamma+\int_{\frac{W_0^{FRP}}{(p-c_1)Q}}^{\beta}\frac{\partial E\left[U_2\left(W_m\right)\right]}{\partial Q}\frac{1}{\beta}d\gamma$$

$$\begin{aligned}\frac{\partial^2 E\left[U\left(W_m\right)\right]}{\partial Q^2}=&\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q}}\left(\frac{-\gamma^2}{p-c_2}(\lambda-1)\left(c_2-c_1\right)^2 f\left(\frac{W_0^{FRP}-\left(c_2-c_1\right)\gamma Q}{p-c_2}\right)+\lambda\left(s-c_2\right)\gamma^2 f(\gamma Q)\right)\frac{1}{\beta}d\gamma\\&+\int_{\frac{W_0^{FRP}}{(p-c_1)Q}}^{\beta}\left(\left(s-c_2\right)\gamma^2 f(\gamma Q)-\frac{\left(c_1-s\right)^2}{p-s}(\lambda-1)\gamma^2 f\left(\frac{W_0^{FRP}+\left(c_1-s\right)\gamma Q}{p-s}\right)\right)\frac{1}{\beta}d\gamma<0.\end{aligned}$$

Hence, $E\left[U\left(W_m\right)\right]$ is a concave function. □

Proof of Proposition 1.

1. According to Lemma 3, and the optimal order quantity Q_λ^* satisfies $\frac{\partial E[U(W_m)]}{\partial Q_\lambda^*}=0$, that is,

$$\begin{aligned}&\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}\left(\left(c_2-c_1\right)\gamma+(\lambda-1)\left(c_2-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}-\left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2}\right)+\lambda\left(s-c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)\right)\mathrm{d}\gamma\\&+\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^{\beta}\left(\left(c_2-c_1\right)\gamma+\left(s-c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)+(\lambda-1)\left(s-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}+\left(c_1-s\right)\gamma Q_\lambda^*}{p-s}\right)\right)\mathrm{d}\gamma=0\end{aligned}$$

2. We assume that

$$\begin{aligned}H=&\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}\left(\left(c_2-c_1\right)\gamma+(\lambda-1)\left(c_2-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}-\left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2}\right)+\lambda\left(s-c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)\right)\\&\frac{1}{\beta}\mathrm{d}\gamma+\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^{\beta}\left(\left(c_2-c_1\right)\gamma+\left(s-c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)+(\lambda-1)\left(s-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}+\left(c_1-s\right)\gamma Q_\lambda^*}{p-s}\right)\right)\frac{1}{\beta}\mathrm{d}\gamma=0\end{aligned}$$

Hence,

$$\frac{\partial H}{\partial Q_\lambda^*} = \int_0^{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}} \left(\frac{-\gamma^2}{p-c_2}(\lambda-1)\left(c_2-c_1\right)^2 f\left(\frac{W_0^{\mathrm{FRP}} - \left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2} \right) + \lambda\left(s-c_2\right)\gamma^2 f\left(\gamma Q_\lambda^*\right) \right) \frac{1}{\beta}\mathrm{d}\gamma$$

$$+ \int_{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}}^{\beta} \left(\left(s-c_2\right)\gamma^2 f\left(\gamma Q_\lambda^*\right) - \frac{\left(c_1-s\right)^2}{p-s}(\lambda-1)\gamma^2 f\left(\frac{W_0^{\mathrm{FRP}} + \left(c_1-s\right)\gamma Q_\lambda^*}{p-s} \right) \right) \frac{1}{\beta}\mathrm{d}\gamma < 0$$

$$\frac{\partial H}{\partial c_2} = \int_0^{\frac{W_0^{FRP}}{(p-c_1)Q_\lambda^*}} \left(\gamma + (\lambda-1)\gamma F\left(\frac{W_0^{\mathrm{FRP}} - \left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2} \right) + \frac{W_0^{\mathrm{FRP}} - \left(p-c_1\right)\gamma Q_\lambda^*}{\left(p-c_2\right)^2} \right.$$

$$\left. (\lambda-1)\left(c_2-c_1\right)\gamma f\left(\frac{W_0^{\mathrm{FRP}} - \left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2} \right) - \lambda\gamma F\left(\gamma Q_\lambda^*\right) \right) \frac{1}{\beta} d\gamma$$

$$+ \int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^{\beta} \left(\gamma - \gamma F\left(\gamma Q_\lambda^*\right)\right) \frac{1}{\beta}\mathrm{d}\gamma > 0$$

$$\frac{\partial Q_{\lambda 2}^*}{\partial c_2} = -\frac{\frac{\partial H}{\partial c_2}}{\frac{\partial H}{\partial Q_\lambda^*}} > 0.$$

$$\frac{\partial H}{\partial W_0^{FRP}} = \int_0^{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}} \frac{(\lambda-1)\left(c_2-c_1\right)\gamma}{\beta\left(p-c_2\right)} f\left(\frac{W_0^{\mathrm{FRP}} - \left(c_2-c_1\right)\gamma Q_\lambda^*}{p-c_2} \right) d\gamma$$

$$- \int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^{\beta} \frac{(\lambda-1)\left(c_1-s\right)\gamma}{\beta(p-s)} f\left(\frac{W_0^{\mathrm{FRP}} + \left(c_1-s\right)\gamma Q_\lambda^*}{p-s} \right) \mathrm{d}\gamma$$

$\frac{\partial Q_\lambda^*}{\partial W_0^{\mathrm{FRP}}} = -\frac{\frac{\partial H}{\partial W_0^{\mathrm{FRP}}}}{\frac{\partial H}{\partial Q_\lambda^*}}$, and whether it is positive or negative is unknown.

$$\frac{\partial H}{\partial \beta} = \left(c_2-c_1\right)\beta - \left(c_2-s\right)\beta F\left(\beta Q_\lambda^*\right) - (\lambda-1)\left(c_1-s\right)\beta F\left(\frac{W_0^{FRP} + \left(c_1-s\right)\beta Q_\lambda^*}{p-s} \right)$$

$\frac{\partial Q_\lambda^*}{\partial \beta} = -\frac{\frac{\partial H}{\partial \beta}}{\frac{\partial H}{\partial Q_\lambda^*}}$, and whether it is positive or negative is unknown.

Thus, the optimal order quantity increases with unit order cost for the supplementary order c_2 and may increase or decrease with the supplier's maximum yield rate β. □

Proof of Proposition 2.

Considering that Q^* satisfies $\int_0^\beta \gamma F(\gamma Q^*)d\gamma = \frac{\beta^2(c_2-c_1)}{2(c_2-s)}$ and Q_M^* satisfies

$$W\left(Q_M^*\right) = -\left(c_2 - s\right)\int_0^\beta \gamma F\left(\gamma Q_M^*\right)d\gamma + \frac{\beta^2}{2}\left(c_2 - c_1\right) - (\lambda - 1)\left(c_1 - s\right)\int_0^\beta \gamma F\left(\frac{\left(c_1 - s\right)\gamma Q_M^*}{p - s}\right)d\gamma = 0,$$

substituting Q_M^* by Q^*, we get

$$W(Q^*) = \int_0^\beta \left(\left(c_2 - c_1\right)\gamma + \left(s - c_2\right)\gamma F(\gamma Q^*) + (\lambda - 1)\left(s - c_1\right)\gamma F\left(\frac{\left(c_1 - s\right)\gamma Q^*}{p - s}\right)\right)\frac{1}{\beta}d\gamma$$
$$= \int_0^\beta (\lambda - 1)\left(s - c_1\right)\gamma F\left(\frac{\left(c_1 - s\right)\gamma Q^*}{p - s}\right)\frac{1}{\beta}d\gamma < 0.$$

It is easy to obtain that $\frac{\mathrm{d}W(Q)}{\mathrm{d}Q} < 0$. Hence, $Q_M^* < Q^*$. According to Proposition 1–1, for a loss-averse retailer with a nonzero reference profit, its optimal order quantity satisfies

$$H\left(Q_\lambda^*\right) = \int_0^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q_\lambda^*}} \left(\left(c_2 - c_1\right)\gamma + (\lambda - 1)\left(c_2 - c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}} - \left(c_2 - c_1\right)\gamma Q_\lambda^*}{p - c_2}\right) + \lambda\left(s - c_2\right)\gamma F\left(\gamma Q_\lambda^*\right)\right)\mathrm{d}\gamma +$$
$$\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^\beta \left(\left(c_2 - c_1\right)\gamma + \left(s - c_2\right)\gamma F\left(\gamma Q_\lambda^*\right) + (\lambda - 1)\left(s - c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}} + \left(c_1 - s\right)\gamma Q_\lambda^*}{p - s}\right)\right)d\gamma = 0 \tag{8}$$

Substituting Q_λ^* by Q^* in Eq. (8), we can obtain

$$H(Q^*) =(\lambda-1)\left\{\int_0^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q^*}}\left((c_2-c_1)\gamma F\left(\frac{W_0^{\mathrm{FRP}}-(c_2-c_1)\gamma Q^*}{p-c_2}\right)-(c_2-s)\gamma F(\gamma Q^*)\right)\right.$$
$$\left. \mathrm{d}\gamma - \int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q^*}}^{\beta}(c_1-s)\gamma F\left(\frac{W_0^{\mathrm{FRP}}+(c_1-s)\gamma Q^*}{p-s}\right)d\gamma\right\} = (\lambda-1)Z^{\mathrm{FRP}}\left(W_0^{\mathrm{FRP}}\right) \tag{9}$$

If $Z^{\mathrm{FRP}}\left(W_0^{\mathrm{FRP}}\right)>0$, $H(Q^*)>0$. Considering that $\frac{\mathrm{d}H(Q)}{\mathrm{d}Q}<0$, $Q_\lambda^*>Q^*$. Moreover, note that $H(Q)$ is a continuous function, we can derive that $Q_\lambda^*>Q^*$ when the reference profit W_0^{FRP} satisfies $Z^{\mathrm{FRP}}\left(W_0^{\mathrm{FRP}}\right)>0$.

According to Eq. (9), when s is large and approaches c_1, $H(Q^*)$ approaches.

$$(\lambda-1)(c_2-c_1)\int_0^{\frac{W_0^{FRP}}{(p-c_1)Q^*}}\gamma\left(F\left(\frac{W_0^{FRP}-(c_2-c_1)\gamma Q^*}{p-c_2}\right)-F(\gamma Q^*)\right)d\gamma>0$$

Considering that $\frac{\mathrm{d}H(Q)}{\mathrm{d}Q}<0$, $Q_\lambda^*>Q^*$. Since $H(Q)$ is a continuous function, it can be derived that $Q_\lambda^*>Q^*$ when the reference profit s is sufficiently large. □

Proof of Proposition 3

According to Eq. (8),

$$\frac{\partial H(Q_\lambda^*)}{\partial Q_\lambda^*}=\int_0^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q_\lambda^*}}\left(\frac{-\gamma^2}{p-c_2}(\lambda-1)(c_2-c_1)^2 f\left(\frac{W_0^{\mathrm{FRP}}-(c_2-c_1)\gamma Q_\lambda^*}{p-c_2}\right)+\lambda(s-c_2)\gamma^2 f(\gamma Q_\lambda^*)\right)\frac{1}{\beta}\mathrm{d}\gamma+$$
$$\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^{\beta}\left((s-c_2)\gamma^2 f(\gamma Q_\lambda^*)-\frac{(c_1-s)^2}{p-s}(\lambda-1)\gamma^2 f\left(\frac{W_0^{\mathrm{FRP}}+(c_1-s)\gamma Q_\lambda^*}{p-s}\right)\right)\frac{1}{\beta}\mathrm{d}\gamma<0$$

$$\frac{\partial H(Q_\lambda^*)}{\partial\lambda}=\int_0^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q_\lambda^*}}\left((c_2-c_1)\gamma F\left(\frac{W_0^{\mathrm{FRP}}-(c_2-c_1)\gamma Q_\lambda^*}{p-c_2}\right)+(s-c_2)\gamma F(\gamma Q_\lambda^*)\right)\mathrm{d}\gamma$$
$$+\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_\lambda^*}}^{\beta}\left((s-c_1)\gamma F\left(\frac{W_0^{\mathrm{FRP}}+(c_1-s)\gamma Q_\lambda^*}{p-s}\right)\right)\mathrm{d}\gamma$$

when s is large and approaches c_1, $\frac{\partial H(Q_\lambda^*)}{\partial\lambda}$ approaches.

$$\int_0^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q_\lambda^*}} (c_2-c_1)\gamma\left(F\left(\frac{W_0^{\mathrm{FRP}-(c_2-c_1)\gamma Q_\lambda^*}}{p-c_2}\right)-F\left(\gamma Q_\lambda^*\right)\right)\mathrm{d}\gamma>0$$

In this case, $\frac{\partial Q_\lambda^*}{\partial\lambda}=-\frac{\frac{\partial H(Q_\lambda^*)}{\partial\lambda}}{\frac{\partial H(Q_\lambda^*)}{\partial Q_\lambda^*}}>0$. Hence, when s is large and approaches c_1, the loss-averse retailer's order quantity will increase with the loss-averse degree λ. □

Proof of Proposition 4

We assume that $H_0=\frac{\partial E[U(W_m)]}{\partial Q_{U3}^*}$

$$\frac{\partial H_0}{\partial Q_{U3}^*}=\left(\frac{-1}{p-c_2}(\lambda-1)(c_2-c_1)^2+\lambda(s-c_2)\right)\frac{1}{b-a}\frac{1}{3\beta}\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U3}^*}\right)^3+\left((s-c_2)\right.$$
$$\left.-\frac{(c_1-s)^2}{p-s}(\lambda-1)\right)\frac{1}{b-a}\frac{1}{3\beta}\left(\beta^3-\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U3}^*}\right)^3\right)<0$$

$$\frac{\partial H_0}{\partial c_2}=\frac{1}{\beta}\left(\frac{1}{2}\left(\frac{W_0^{FRP}}{(p-c_1)Q_{U3}^*}\right)^2\left(1+(\lambda-1)\frac{\frac{W_0^{FRP}}{p-c_2}-a}{b-a}+\frac{W_0^{FRP}(\lambda-1)(c_2-c_1)}{(b-a)(p-c_2)^2}+\lambda\frac{a}{b-a}-\frac{b}{b-a}\right)\right.$$
$$+\frac{1}{3}\left(\frac{W_0^{FRP}}{(p-c_1)Q_{U3}^*}\right)^3\left(\frac{(c_2-c_1)Q_{U3}^*(\lambda-1)}{(b-a)(p-c_2)}-\frac{(p-c_1)Q_{U3}^*(\lambda-1)(c_2-c_1)}{(p-c_2)^2(b-a)}+\frac{Q_{U3}^*(1-\lambda)}{b-a}\right)$$
$$\left.+\frac{b}{b-a}\frac{1}{2}\beta^2-\frac{Q_{U3}^*}{b-a}\frac{1}{3}\beta^3\right)>0$$

$$\frac{\partial H_0}{\partial W_0^{\mathrm{FRP}}}=\frac{(\lambda-1)}{(b-a)\beta}\left(\frac{(c_2-c_1)}{p-c_2}\frac{1}{2}\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U3}^*}\right)^2+\frac{(s-c_1)}{p-s}\frac{1}{2}\left(\beta^2-\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U3}^*}\right)^2\right)\right)$$

$$\frac{\partial Q}{\partial W_0^{FRP}}=-\frac{\frac{\partial H_0}{\partial W_0^{FRP}}}{\frac{\partial H_0}{\partial Q_{U3}^*}}$$
$$=-\frac{\frac{(\lambda-1)}{(b-a)\beta}\left(\frac{(c_2-c_1)}{p-c_2}\frac{1}{2}\left(\frac{W_0^{FRP}}{(p-c_1)Q_{U3}^*}\right)^2+\frac{(s-c_1)}{p-s}\frac{1}{2}\left(\beta^2-\left(\frac{W_0^{FRP}}{(p-c_1)Q_{U3}^*}\right)^2\right)\right)}{\left(\frac{-1}{p-c_2}(\lambda-1)(c_2-c_1)^2+\lambda(s-c_2)\right)\frac{1}{b-a}\frac{1}{3\beta}\left(\frac{W_0^{FRP}}{(p-c_1)Q_{U3}^*}\right)^3+\left((s-c_2)-\frac{(c_1-s)^2}{p-s}(\lambda-1)\right)\frac{1}{b-a}\frac{1}{3\beta}\left(\beta^3-\left(\frac{W_0^{FRP}}{(p-c_1)Q_{U3}^*}\right)^3\right)}$$

Hence, when $Q_{U3}^* \le \frac{W_0^{\mathrm{FRP}}}{(p-c_1)}\sqrt{\frac{(p-s)(c_2-c_1)+(c_1-s)(p-c_2)}{(p-c_2)(c_1-s)\beta^2}}$, $\frac{\partial Q_{U3}^*}{\partial W_0^{FRP}} \ge 0$; When $Q_{U3}^* > \frac{W_0^{\mathrm{FRP}}}{(p-c_1)}\sqrt{\frac{(p-s)(c_2-c_1)+(c_1-s)(p-c_2)}{(p-c_2)(c_1-s)\beta^2}}$, $\frac{\partial Q_{U3}^*}{\partial W_0^{\mathrm{FRP}}} < 0$. We denote $A_1 = \frac{W_0^{\mathrm{FRP}}}{(p-c_1)}\sqrt{\frac{(p-s)(c_2-c_1)+(c_1-s)(p-c_2)}{(p-c_2)(c_1-s)\beta^2}}$. If $Q_{U3} = A_1$, $L_1(Q_{U3}) + L_2(Q_{U3}) + L_3(Q_{U3}) = W_0^{\mathrm{FRP}}L + \frac{1}{2}(c_2 - c_1)\beta^2$. Hence, if $L \ge 0$, $L_1(Q_{U3}) + L_2(Q_{U3}) + L_3(Q_{U3}) \ge 0$, and thus $Q_{U3} \ge A_1$. The optimal order quantity Q_{U3}^* always decreases with the reference profit W_0^{FRP} when $L \ge 0$. If $L < 0$ and $L_1(Q_{U3}) + L_2(Q_{U3}) + L_3(Q_{U3}) = W_0^{\mathrm{FRP}}L + \frac{1}{2}(c_2 - c_1)\beta^2 < 0$, the optimal order quantity Q_{U3}^* increases with the reference profit W_0^{FRP}. If $L < 0$ and $W_0^{\mathrm{FRP}}L + \frac{1}{2}(c_2 - c_1)\beta^2 > 0$, the optimal order quantity Q_{U3}^* decreases with the reference profit W_0^{FRP}. □

Proof of Proposition 5

It can be obtained that

$$\frac{\partial H_0}{\partial \beta} = \left((c_2 - c_1) + (s - c_2)\frac{\beta Q_{U3}^* - a}{b - a} + (\lambda - 1)(s - c_1)\frac{\frac{W_0^{\mathrm{FRP}} + (c_1 - s)\beta Q_{U3}^*}{p-s} - a}{b - a} \right)$$

$$\frac{\partial Q_{U3}^*}{\partial \beta} = -\frac{\frac{\partial H_0}{\partial \beta}}{\frac{\partial H_0}{\partial Q_{U3}^*}}$$

$$= \frac{\left((c_2 - c_1) + (s - c_2)\frac{\beta Q_{U3}^* - a}{b-a} + (\lambda - 1)(s - c_1)\frac{\frac{W_0^{\mathrm{FRP}} + (c_1 - s)\beta Q_{U3}^*}{p-s} - a}{b-a} \right)}{\left(\frac{-1}{p-c_2}(\lambda - 1)(c_2 - c_1)^2 + \lambda(s - c_2) \right)\frac{1}{b-a}\frac{1}{3\beta}\left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U3}^*} \right)^3 + \left((s - c_2) - \frac{(c_1 - s)^2}{p-s}(\lambda - 1) \right)\frac{1}{b-a}\frac{1}{3\beta}\left(\beta^3 - \left(\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U3}^*} \right)^3 \right)}$$

Hence, when $Q_{U3}^* \le \frac{(c_2-c_1)(b-a)+(c_2-s)a+(\lambda-1)(s-c_1)\left(\frac{W_0^{\mathrm{FRP}}}{p-s}-a\right)}{\beta\left(c_2-s+\frac{(\lambda-1)(c_1-s)^2}{p-s}\right)}$, $\frac{\partial Q_{U3}^*}{\partial \beta} \ge 0$; when $Q_{U3}^* > \frac{(c_2-c_1)(b-a)+(c_2-s)a+(\lambda-1)(s-c_1)\left(\frac{W_0^{\mathrm{FRP}}}{p-s}-a\right)}{\beta\left(c_2-s+\frac{(\lambda-1)(c_1-s)^2}{p-s}\right)}$, $\frac{\partial Q_{U3}^*}{\partial \beta} < 0$. We denote $A_2 = \frac{(c_2-c_1)(b-a)+(c_2-s)a+(\lambda-1)(s-c_1)\left(\frac{W_0^{\mathrm{FRP}}}{p-s}-a\right)}{\beta\left(c_2-s+\frac{(\lambda-1)(c_1-s)^2}{p-s}\right)}$. If $Q_{U3} = A_2, L_1(Q_{U3}) + L_2(Q_{U3}) + L_3(Q_{U3}) = M$. If $M \le 0$, the optimal order quantity Q_{U3}^* will increase with the maximum fulfillment rate from the unreliable supplier β; If $M > 0$, the optimal order quantity Q_{U3}^* will decrease with β. □

Proof of Proposition 6

1. $Q_{U1}^{*}=\frac{3(c_2-c_1)b}{2\beta(c_2-s)}$. Q_{U3}^{*} satisfies

$$\int_{0}^{\frac{W_0^{FRP}}{(p-c_1)Q_{U2}^{*}}}\left(\left(c_2-c_1\right)\gamma+(\lambda-1)\left(c_2-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}-\left(c_2-c_1\right)\gamma Q_{U2}^{*}}{p-c_2}\right)+\lambda\left(s-c_2\right)\gamma F\left(\gamma Q_{U2}^{*}\right)\right)\mathrm{d}\gamma$$
$$+\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U2}^{*}}}^{\beta}\left(\left(c_2-c_1\right)\gamma+\left(s-c_2\right)\gamma F\left(\gamma Q_{U2}^{*}\right)+(\lambda-1)\left(s-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}+\left(c_1-s\right)\gamma Q_{U2}^{*}}{p-s}\right)\right)\mathrm{d}\gamma=0 \tag{10}$$

Substitute Q_{U1}^{*} into the function (10),

$$\int_{0}^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q_{U1}^{*}}}\left(\left(c_2-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}-\left(c_2-c_1\right)\gamma Q_{U1}^{*}}{p-c_2}\right)+\left(s-c_2\right)\gamma F\left(\gamma Q_{U1}^{*}\right)\right)d\gamma$$
$$+\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q_{U1}^{*}}}^{\beta}\left(s-c_1\right)\gamma F\left(\frac{W_0^{\mathrm{FRP}}+\left(c_1-s\right)\gamma Q_{U1}^{*}}{p-s}\right)d\gamma$$
$$=\frac{\beta^2}{b-a}\left(\begin{array}{c}\left(\frac{W_0^{\mathrm{FRP}}}{\left(p-c_1\right)}\right)^3\left(\frac{\left((\lambda-1)\left(c_1-s\right)^2+(p-s)\left(c_2-s\right)\right)}{3(p-s)\left(c_2-c_1\right)b}\right)^2 \\ \frac{2\left(p-c_1\right)^2\left(c_2-s\right)}{3(p-s)\left(p-c_2\right)}+\frac{W_0^{\mathrm{FRP}}\left(s-c_1\right)}{2(p-s)}\end{array}\right)=\frac{\beta^2 Z_1}{b-a}$$

Hence, when $Z_1\geq 0, Q_{U1}^{*}\leq Q_{U3}^{*}$, and when $Z_1<0, Q_{U1}^{*}>Q_{U3}^{*}$.

2. $Q_{U2}^{*}=\frac{3(p-s)\left(c_2-c_1\right)b}{2\beta\left((\lambda-1)\left(c_1-s\right)^2+(p-s)\left(c_2-s\right)\right)}$. Substituting Q_{U2}^{*} into the function (10), we can get

$$\int_{0}^{\frac{W_0\mathrm{FRP}}{(p-c_1)Q}}\left(\left(c_2-c_1\right)\gamma\frac{\frac{W_0^{\mathrm{FRP}}-(c_2-c_1)\gamma Q}{p-c_2}-a}{b-a}+\left(s-c_2\right)\gamma\frac{\gamma Q-a}{b-a}+\left(c_1-s\right)\gamma\frac{\frac{(c_1-s)\gamma Q}{p-s}-a}{b-a}\right)d\gamma$$
$$+\int_{\frac{W_0^{\mathrm{FRP}}}{(p-c_1)Q}}^{\beta}\left(s-c_1\right)\gamma\left(\frac{\frac{W_0^{\mathrm{FRP}}+(c_1-s)\gamma Q}{p-s}-a}{b-a}-\frac{\frac{(c_1-s)\gamma Q}{p-s}-a}{b-a}\right)d\gamma$$
$$==\frac{\beta^2}{b-a}\left(\left(\frac{W_0^{\mathrm{FRP}}}{\left(p-c_1\right)}\right)^3\left(\frac{2\beta\left(c_2-s\right)}{3\left(\left(c_2-c_1\right)b\right)}\right)^2\frac{\left(p-c_1\right)^2\left(c_2-s\right)}{6(p-s)\left(p-c_2\right)}+\frac{\left(s-c_1\right)\left(\frac{W_0^{\mathrm{FRP}}}{p-s}-a\right)}{2}-\frac{\left(c_2-c_1\right)b\left(c_1-s\right)^2}{2\left(c_2-s\right)(p-s)}\right)=\frac{\beta^2}{b-a}Z_2$$

Hence, when $Z_2 \geq 0, Q^*_{U2} \leq Q^*_{U3}$, and when $Z_2 < 0, Q^*_{U2} > Q^*_{U3}$. □

Proof of Lemma 4

Given an order quantity Q, it is easy to derive that when the maximum received supply βQ is smaller than the maximum demand $D_{\max}$, i.e., $\beta Q < D_{\max}$, the maximum profit of the retailer is $\left(p - c_2\right)D_{max} + \left(c_2 - c_1\right)\beta Q$. Otherwise, the maximum profit of the retailer is $\left(p - c_1\right)D_{\max}$.

Additionally, when the maximum received supply βQ is smaller than $\frac{c_2-s}{c_1-s}D_{\min}$, the minimum profit of the retailer is $\left(p - c_2\right)D_{\min}$; otherwise, the minimum profit of the retailer is $\left(p - c_2\right)D_{\min} - \left(\left(c_1 - s\right)\beta Q - \left(c_2 - s\right)D_{\min}\right)$.

Based on above analyses, we can derive the reference profit under the following different conditions:

When $\frac{c_2-s}{c_1-s}D_{\min} > D_{\max}$,

if $\beta Q < D_{\max}$, $W_0^{\mathrm{PRP}}(Q) = \alpha\left(\left(p - c_2\right)D_{\max} + \left(c_2 - c_1\right)\beta Q\right) + (1 - \alpha)\left(p - c_2\right)D_{\min}$;

if $D_{\max} < \beta Q < \frac{c_2-s}{c_1-s}D_{\min}$, $W_0^{PRP}(Q) = \alpha\left(p - c_1\right)D_{max} + (1 - \alpha)\left(p - c_2\right)D_{min}$;

if $\beta Q > \frac{c_2-s}{c_1-s}D_{\min}$.

When $\frac{c_2-s}{c_1-s}D_{\min} \leq D_{\max}$,

if $\beta Q < \frac{c_2-s}{c_1-s}D_{\min}$, $W_0^{\mathrm{PRP}}(Q) = \alpha\left(\left(p - c_2\right)D_{\max} + \left(c_2 - c_1\right)\beta Q\right) + (1 - \alpha)\left(p - c_2\right)D_{\min}$;

if $\frac{c_2-s}{c_1-s}D_{\min} < \beta Q < D_{\max}$, $W_0^{\mathrm{PRP}}(Q) = \alpha\left(\left(p - c_2\right)D_{\max} + \left(c_2 - c_1\right)\beta Q\right) + (1 - \alpha)\left(\left(p - c_2\right)D_{\min} - \left(c_1 - s\right)\beta Q + \left(c_2 - s\right)D_{\min}\right)$;

if $\beta Q > D_{\max}$, $W_0^{\mathrm{PRP}}(Q) = \alpha\left(p - c_1\right)D_{\max} + (1 - \alpha)\left(\left(p - c_2\right)D_{\min} - \left(c_1 - s\right)\beta Q + \left(c_2 - s\right)D_{\min}\right)$.

To summarize, $W_0^{\mathrm{PRP}}(Q) = \alpha\left(\left(p - c_2\right)D_{\max} + \left(c_2 - c_1\right)\min\left\{\beta Q, D_{\max}\right\}\right) + (1 - \alpha)\Big(\left(p - c_2\right)D_{\min} - \left[\left(c_1 - s\right)\beta Q + \left(c_2 - s\right)D_{\min}\right]^+\Big)$. □

Proof of Proposition 7

Denote $\frac{\partial W_0^{\mathrm{PRP}}(Q)}{\partial Q}$ by N_1, we can derive that

$$\frac{\partial E\left[U^{\mathrm{PRP}}\left(W_m\right)\right]}{\partial Q} = \int_0^{\frac{W_0\mathrm{PRP}(Q)}{(p-c_1)Q}} \frac{\partial E\left[U_1^{\mathrm{PRP}}\left(W_m\right)\right]}{\partial Q}\frac{1}{\beta}d\gamma + \int_{\frac{W_0^{\mathrm{PRP}}(Q)}{(p-c_1)Q}}^{\beta} \frac{\partial E\left[U_2^{\mathrm{PRP}}\left(W_m\right)\right]}{\partial Q}\frac{1}{\beta}d\gamma$$

$$= \int_{0}^{\frac{W_0\mathrm{PRP}(Q)}{(p-c_1)Q}} \Bigg((c_2-c_1)\gamma + (\lambda-1)(c_2-c_1)\gamma F\left(\frac{W_0^{\mathrm{PRP}}(Q)-(c_2-c_1)\gamma Q}{p-c_2}\right)$$

$$+\lambda(s-c_2)\gamma F(\gamma Q) - N_1\left((\lambda-1)F\left(\frac{W_0^{\mathrm{PRP}}(Q)-(c_2-c_1)\gamma Q}{p-c_2}\right)+1\right)\Bigg)$$

$$\frac{1}{\beta}\mathrm{d}\gamma + \int_{\frac{W_0^{\mathrm{PRP}}(Q)}{(p-c_1)Q}}^{\beta} \Bigg((c_2-c_1)\gamma + (s-c_2)\gamma F(\gamma Q) + (\lambda-1)(s-c_1)\gamma F\left(\frac{W_0^{\mathrm{PRP}}(Q)+(c_1-s)\gamma Q}{p-s}\right)$$

$$-N_1\left((\lambda-1)F\left(\frac{W_0^{\mathrm{PRP}}(Q)+(c_1-s)\gamma Q}{p-s}\right)+1\right)\Bigg)\frac{1}{\beta}\mathrm{d}\gamma,$$

$$\frac{\partial^2 E\left[U^{\mathrm{PRP}}\left(W_m\right)\right]}{\partial Q^2} = \int_{0}^{\frac{W_0\mathrm{PRP}(Q)}{(p-c_1)Q}} \Bigg(-\frac{\left(N_1-(c_2-c_1)\gamma\right)^2}{p-c_2}$$

$$(\lambda-1)f\left(\frac{W_0^{\mathrm{PRP}}(Q)-(c_2-c_1)\gamma Q}{p-c_2}\right) - \lambda(c_2-s)\gamma^2 f(\gamma Q)\Bigg)\frac{1}{\beta}\mathrm{d}\gamma$$

$$+ \int_{\frac{W_0^{\mathrm{PRP}}(Q)}{(p-c_1)Q}}^{\beta} \Bigg(-(c_2-s)\gamma^2 f(\gamma Q) - \frac{\left(N_1+(c_1-s)\gamma\right)^2}{p-s}$$

$$(\lambda-1)f\left(\frac{W_0^{\mathrm{PRP}}(Q)+(c_1-s)\gamma Q}{p-s}\right)\Bigg)\frac{1}{\beta}\mathrm{d}\gamma < 0$$

Hence, the retailer's expected utility function $E\left[U^{\mathrm{PRP}}\left(W_m\right)\right]$ is a concave function. It is noteworthy that $\frac{\partial E[U^{\mathrm{PRP}}(W_m)]}{\partial Q}$ decreases with N_1.

Let

$$R\left[Q, W_0^{\mathrm{PRP}}(Q)\right] = \int_{0}^{\frac{W_0\mathrm{PRP}(Q)}{(p-c_1)Q}} \Bigg((c_2-c_1)\gamma + (\lambda-1)(c_2-c_1)\gamma F\left(\frac{W_0^{\mathrm{PRP}}(Q)-(c_2-c_1)\gamma Q}{p-c_2}\right)$$

$$+\lambda(s-c_2)\gamma F(\gamma Q) - N_1\left((\lambda-1)F\left(\frac{W_0^{\mathrm{PRP}}(Q)-(c_2-c_1)\gamma Q}{p-c_2}\right)+1\right)\Bigg)\frac{1}{\beta}d\gamma$$

$$+ \int_{\frac{W_0^{\mathrm{PRP}}(Q)}{(p-c_1)Q}}^{\beta} \Bigg((c_2-c_1)\gamma + (s-c_2)\gamma F(\gamma Q) + (\lambda-1)(s-c_1)\gamma F\left(\frac{W_0^{\mathrm{PRP}}(Q)+(c_1-s)\gamma Q}{p-s}\right)$$

$$-N_1\left((\lambda-1)F\left(\frac{W_0^{\mathrm{PRP}}(Q)+(c_1-s)\gamma Q}{p-s}\right)+1\right)\Bigg)\frac{1}{\beta}\mathrm{d}\gamma$$

and $Q_R\left[W_0^{\mathrm{PRP}}(Q)\right]$ is the order quantity that satisfies $R\left[Q, W_0^{\mathrm{PRP}}(Q)\right]=0$ given the reference point $W_0^{\mathrm{PRP}}(Q)$. We assume the $Q_R\left[W_0^{\mathrm{PRP}}(Q)\right]=Q_A$ when $W_0^{PRP}(Q)=\alpha\left(\left(p-c_2\right)D_{max}+\left(c_2-c_1\right)\beta Q\right)+(1-\alpha)\left(p-c_2\right)D_{min}$, and $Q_R\left[W_0^{\mathrm{PRP}}(Q)\right]=Q_C$ when $W_0^{\mathrm{PRP}}(Q)=\alpha\left(p-c_1\right)D_{\max}+(1-\alpha)\left(\left(p-c_2\right)D_{\min}-\left(c_1-s\right)\beta Q+\left(c_2-s\right)D_{\min}\right)$. Q_B denotes $Q_R\left[W_0^{\mathrm{PRP}}(Q)\right]$ when $W_0^{\mathrm{PRP}}(Q)=\alpha\left(p-c_1\right)D_{\max}+(1-\alpha)\left(p-c_2\right)D_{\min}$ and $\frac{c_2-s}{c_1-s}D_{\min}>D_{\max}$; Q_B denotes $Q_R\left[W_0^{PRP}(Q)\right]$ when $W_0^{PRP}(Q)=\alpha\left(\left(p-c_2\right)D_{max}+\left(c_2-c_1\right)\beta Q\right)+(1-\alpha)\left(\left(p-c_2\right)D_{min}-\left(c_1-s\right)\beta Q+\left(c_2-s\right)D_{min}\right)$ and $\frac{c_2-s}{c_1-s}D_{min}\le D_{max}$. Hence, according to lemma 1, it can be derived that when$\frac{c_2-s}{c_1-s}D_{min}>D_{max}$, if $Q_A\in[0,\frac{D_{max}}{\beta}]$, $Q_B\in\left(\frac{D_{\max}}{\beta},\frac{(c_2-s)D_{\min}}{(c_1-s)\beta}\right]$, $Q_C\in\left(\frac{(c_2-s)D_{\min}}{(c_1-s)\beta},+\infty\right)$, then the optimal order quantity $Q_{\mathrm{PRP}}^*=\underset{Q_{\mathrm{PRP}}^*\in\{Q_A,Q_B,Q_C\}}{\mathrm{arcmax}}\ v\left(Q_{\mathrm{PRP}}^*\right)$;

if $Q_A\in\left[0,\frac{D_{\max}}{\beta}\right]$, $Q_B\in\left(\frac{D_{\max}}{\beta},\frac{(c_2-s)D_{\min}}{(c_1-s)\beta}\right]$, $Q_C\in\left[0,\frac{(c_2-s)D_{\min}}{(c_1-s)\beta}\right]$, then $Q_{\mathrm{PRP}}^*=\underset{Q_{\mathrm{PRP}}^*\in\{Q_A,Q_B\}}{\mathrm{arcmax}}\ v\left(Q_{\mathrm{PRP}}^*\right)$;

if $Q_A\in\left[0,\frac{D_{max}}{\beta}\right]$, $Q_B\notin\left(\frac{D_{max}}{\beta},\frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_C\in\left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta},+\infty\right)$, then $Q_{PRP}^*=\underset{Q^*\in\{Q_A,Q_C\}}{arcmax}\ v\left(Q_{PRP}^*\right)$;

if $Q_A\in\left(\frac{D_{max}}{\beta},+\infty\right)$, $Q_B\in\left(\frac{D_{max}}{\beta},\frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_C\in\left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta},+\infty\right)$, then $Q_{PRP}^*=\underset{Q_{PRP}^*\in\{Q_B,Q_C\}}{arcmax}\ v\left(Q_{PRP}^*\right)$;

if $Q_A\in\left[0,\frac{D_{max}}{\beta}\right]$, $Q_B\in\left[0,\frac{D_{max}}{\beta}\right)$, $Q_C\in\left[0,\frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, then $Q_{PRP}^*=Q_A$;

if $Q_A\in\left(\frac{D_{max}}{\beta},+\infty\right)$, $Q_B\in\left(\frac{D_{max}}{\beta},\frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_C\in\left[0,\frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, then $Q_{PRP}^*=Q_B$;

if $Q_A\in\left(\frac{D_{max}}{\beta},+\infty\right)$, $Q_B\in\left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta},+\infty\right]$, $Q_C\in\left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta},+\infty\right)$, then $Q_{PRP}^*=Q_C$.

When $\frac{c_2-s}{c_1-s}D_{min}\le D_{max}$,

if $Q_A\in\left[0,\frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_D\in\left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta},\frac{D_{max}}{\beta}\right]$, $Q_C\in\left(\frac{D_{max}}{\beta},+\infty\right)$, then $Q_{PRP}^*=\underset{Q_{PRP}^*\in\{Q_A,Q_C,Q_D\}}{arcmax}\ v\left(Q_{PRP}^*\right)$;

if $Q_A \in \left[0, \frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_D \in \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, \frac{D_{max}}{\beta}\right]$, $Q_C \in \left[0, \frac{D_{max}}{\beta}\right]$, then $Q^*_{PRP} = \underset{Q^*_{PRP} \in \{Q_A, Q_D\}}{arcmax} v\left(Q^*_{PRP}\right)$;

if $Q_A \in \left[0, \frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_D \notin \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, \frac{D_{max}}{\beta}\right]$, $Q_C \in \left(\frac{D_{max}}{\beta}, +\infty\right)$, then $Q^*_{PRP} = \underset{Q^*_{PRP} \in \{Q_A, Q_C\}}{arcmax} v\left(Q^*_{PRP}\right)$;

If $Q_A \in \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, +\infty\right)$, $Q_D \in \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, \frac{D_{max}}{\beta}\right]$, $Q_C \in \left(\frac{D_{max}}{\beta}, +\infty\right)$, then $Q^*_{PRP} = \underset{Q^*_{PRP} \in \{Q_C, Q_D\}}{arcmax} v\left(Q^*_{PRP}\right)$;

if $Q_A \in \left[0, \frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right]$, $Q_D \in \left[0, \frac{(c_2-s)D_{min}}{(c_1-s)\beta}\right)$, $Q_C \in \left[0, \frac{D_{max}}{\beta}\right]$, then $Q^*_{PRP} = Q_A$;

if $Q_A \in \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, +\infty\right)$, $Q_D \in \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, \frac{D_{max}}{\beta}\right]$, $Q_C \in \left[0, \frac{D_{max}}{\beta}\right]$, then $Q^*_{PRP} = Q_D$;

if $Q_A \in \left(\frac{(c_2-s)D_{min}}{(c_1-s)\beta}, +\infty\right)$, $Q_D \in \left(\frac{D_{max}}{\beta}, +\infty\right]$, $Q_C \in \left(\frac{D_{max}}{\beta}, +\infty\right)$, then $Q^*_{PRP} = Q_C$.

Proposition 7 can be obtained by combining the above results. □

Proof of Proposition 8

1. Denote $N_1 = \frac{\partial W_0^{\mathrm{PRP}}(Q^*_{\mathrm{PRP}})}{\partial Q^*_{\mathrm{PRP}}}$, $N_2 = \frac{\partial W_0^{PRP}(Q^*_{PRP})}{\partial c_2}$, $N_3 = \frac{\partial W_0^{PRP}(Q^*_{PRP})}{\partial \beta}$;

$$
\begin{aligned}
B_4 = \int_0^{\frac{W_0^{PRP}(Q^*_{PRP})}{(p-c_1)Q^*_{PRP}}} & \Bigg((\lambda-1)\gamma\left(F\left(\frac{W_0^{PRP}(Q^*_{PRP}) - (c_2-c_1)\gamma Q^*_{PRP}}{p-c_2} \right) - F\left(\gamma Q^*_{PRP}\right) \right) \\
& + \frac{\left(\left(N_2 - \gamma Q^*_{PRP}\right)(p-c_2) + \left(W_0^{PRP}(Q^*_{PRP}) - (c_2-c_1)\gamma Q^*_{PRP}\right)\right)(\lambda-1) f\left(\frac{W_0^{PRP}(Q^*_{PRP}) - (c_2-c_1)\gamma Q^*_{PRP}}{p-c_2} \right)\left((c_2-c_1)\gamma - N_1\right)}{(p-c_2)^2} \\
& - \frac{\partial N_1}{\partial c_2}\left((\lambda-1) F\left(\frac{W_0^{PRP}(Q^*_{PRP}) - (c_2-c_1)\gamma Q^*_{PRP}}{p-c_2} \right) + 1 \right) \Bigg) d\gamma \\
& + \int_{\frac{W_0^{PRP}(Q^*_{PRP})}{(p-c_1)Q^*_{PRP}}}^{\beta} \Bigg(\gamma - \gamma F\left(\gamma Q^*_{PRP}\right) + \frac{N_2}{p-s}(\lambda-1)\left((s-c_1)\gamma - N_1\right) f\left(\frac{W_0^{PRP}(Q^*_{PRP}) + (c_1-s)\gamma Q^*_{PRP}}{p-s} \right) \\
& - \frac{\partial N_1}{\partial c_2}\left((\lambda-1) F\left(\frac{W_0^{PRP}(Q^*_{PRP}) + (c_1-s)\gamma Q^*_{PRP}}{p-s} \right) + 1 \right) \Bigg) d\gamma
\end{aligned}
$$

$$B_5 = \int_0^{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}} \left((\lambda-1)\left(\left(c_2-c_1\right)\gamma - N_1\right) f\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2-c_1\right)\gamma Q_{PRP}^*}{p-c_2} \right) \right.$$
$$\left. \frac{N_3}{p-c_2} - \frac{\partial N_1}{\partial \beta}\left((\lambda-1)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2-c_1\right)\gamma Q_{PRP}^*}{p-c_2} \right) + 1 \right) \right) d\gamma$$
$$+ \int_{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}}^{\beta} \left((\lambda-1)\left(\left(s-c_1\right)\gamma - N_1\right) f\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1-s\right)\gamma Q_{PRP}^*}{p-s} \right) \right.$$
$$\left. \frac{N_3}{p-s} - \frac{\partial N_1}{\partial \beta}\left((\lambda-1)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1-s\right)\gamma Q_{PRP}^*}{p-s} \right) + 1 \right) \right) d\gamma$$

$$R\left[Q_{PRP}^*, W_0^{PRP}\left(Q_{PRP}^*\right)\right]$$
$$= \int_0^{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}} \left(\left(c_2-c_1\right)\gamma + (\lambda-1)\left(c_2-c_1\right)\gamma F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2-c_1\right)\gamma Q_{PRP}^*}{p-c_2} \right) \right.$$
$$\left. +\lambda\left(s-c_2\right)\gamma F\left(\gamma Q_{PRP}^*\right) - N_1\left((\lambda-1)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2-c_1\right)\gamma Q_{PRP}^*}{p-c_2} \right) + 1 \right) \right) d\gamma$$
$$+ \int_{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}}^{\beta} \left(\left(c_2-c_1\right)\gamma + \left(s-c_2\right)\gamma F\left(\gamma Q_{PRP}^*\right) + (\lambda-1)\left(s-c_1\right)\gamma F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1-s\right)\gamma Q_{PRP}^*}{p-s} \right) \right.$$
$$\left. -N_1\left((\lambda-1)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1-s\right)\gamma Q_{PRP}^*}{p-s} \right) + 1 \right) \right) d\gamma = 0$$

We can obtain that

$$\frac{\partial R\left[Q_{PRP}^*, W_0^{PRP}\left(Q_{PRP}^*\right)\right]}{\partial Q_{PRP}^*} = \int_0^{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}} \left(-\frac{\left(N_1 - \left(c_2-c_1\right)\gamma\right)^2}{p-c_2}(\lambda-1) f\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2-c_1\right)\gamma Q_{PRP}^*}{p-c_2} \right) \right.$$
$$\left. -\lambda\left(c_2-s\right)\gamma^2 f\left(\gamma Q_{PRP}^*\right) \right) d\gamma + \int_{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}}^{\beta} \left(-\left(c_2-s\right)\gamma^2 f\left(\gamma Q_{PRP}^*\right) \right.$$
$$\left. -\frac{\left(N_1 + \left(c_1-s\right)\gamma\right)^2}{p-s}(\lambda-1) f\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1-s\right)\gamma Q_{PRP}^*}{p-s} \right) \right) d\gamma < 0$$

$$\begin{aligned}
&\frac{\partial R\left[Q_{PRP}^{*}, W_{0}^{PRP}\left(Q_{PRP}^{*}\right)\right]}{\partial c_{2}} \\
&= \int_{0}^{\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)}{(p-c_{1})Q_{PRP}^{*}}} \left((\lambda-1)\gamma\left(F\left(\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)-\left(c_{2}-c_{1}\right)\gamma Q_{PRP}^{*}}{p-c_{2}}\right)-F\left(\gamma Q_{PRP}^{*}\right)\right)\right. \\
&\quad+\frac{\left(\frac{\partial W_{0}^{PRP}\left(Q_{PRP}^{*}\right)}{\partial c_{2}}-\gamma Q_{PRP}^{*}\right)\left(p-c_{2}\right)+\left(W_{0}^{PRP}\left(Q_{PRP}^{*}\right)-\left(c_{2}-c_{1}\right)\gamma Q_{PRP}^{*}\right)}{\left(p-c_{2}\right)^{2}}(\lambda-1) f\left(\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)-\left(c_{2}-c_{1}\right)\gamma Q_{PRP}^{*}}{p-c_{2}}\right) \\
&\quad\left.\left(\left(c_{2}-c_{1}\right)\gamma-N_{1}\right)-\frac{\partial N_{1}}{\partial c_{2}}\left((\lambda-1) F\left(\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)-\left(c_{2}-c_{1}\right)\gamma Q_{PRP}^{*}}{p-c_{2}}\right)+1\right)\right) d\gamma \\
&\quad+\int_{\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)}{(p-c_{1})Q_{PRP}^{*}}}^{\beta}\left(\gamma-\gamma F\left(\gamma Q_{PRP}^{*}\right)+\frac{\frac{\partial W_{0}^{PRP}\left(Q_{PRP}^{*}\right)}{\partial c_{2}}}{p-s}(\lambda-1)\left(\left(s-c_{1}\right)\gamma-N_{1}\right) f\left(\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)+\left(c_{1}-s\right)\gamma Q_{PRP}^{*}}{p-s}\right)\right. \\
&\quad\left.-\frac{\partial N_{1}}{\partial c_{2}}\left((\lambda-1) F\left(\frac{W_{0}^{PRP}\left(Q_{PRP}^{*}\right)+\left(c_{1}-s\right)\gamma Q_{PRP}^{*}}{p-s}\right)+1\right)\right) d\gamma
\end{aligned} \tag{11}$$

According to Eq. (11), we can get $\frac{\partial Q_{\mathrm{PRP}}^{*}}{\partial c_{2}}=-\frac{\frac{\partial R\left[Q_{\mathrm{PRP}}^{*}, W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)\right]}{\partial c_{2}}}{\frac{\partial R\left[Q_{\mathrm{PRP}}^{*}, W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)\right]}{\partial Q_{\mathrm{PRP}}^{*}}}$ is positive if $B_4 > 0$, and negative if $B_4 \le 0$. As a result, under the PRP model, the optimal order quantity Q_{PRP}^{*} increases with the unit order cost for the supplementary order c_2 if $B_4 > 0$ and decreases with c_2 if $B_4 \le 0$.

Similarly, we can prove that Q_{PRP}^{*} increases with the maximum fulfillment rate from the unreliable supplier β if $B_5 > 0$ and decreases with β if $B_5 \le 0$.

2. It can be derived that

$$\begin{aligned}
\frac{\partial R\left[Q_{\mathrm{PRP}}^{*}, W_{0}^{\mathrm{PRP}}\left(Q_{PRP}^{*}\right)\right]}{\partial \lambda} &= \int_{0}^{\frac{W_{0}\mathrm{PRP}\left(Q_{\mathrm{PRP}}^{*}\right)}{(p-c_{1})Q_{\mathrm{PRP}}^{*}}}\left(\left(c_{2}-c_{1}\right)\gamma F\left(\frac{W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)-\left(c_{2}-c_{1}\right)\gamma Q_{\mathrm{PRP}}^{*}}{p-c_{2}}\right)\right. \\
&\quad\left.+\left(s-c_{2}\right)\gamma F\left(\gamma Q_{\mathrm{PRP}}^{*}\right)-N_{1} F\left(\frac{W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)-\left(c_{2}-c_{1}\right)\gamma Q_{\mathrm{PRP}}^{*}}{p-c_{2}}\right)\right) d\gamma \\
&\quad+\int_{\frac{W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)}{(p-c_{1})Q_{\mathrm{PRP}}^{*}}}^{\beta}\left(\left(s-c_{1}\right)\gamma F\left(\frac{W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)+\left(c_{1}-s\right)\gamma Q_{\mathrm{PRP}}^{*}}{p-s}\right)\right. \\
&\quad\left.-N_{1} F\left(\frac{W_{0}^{\mathrm{PRP}}\left(Q_{\mathrm{PRP}}^{*}\right)+\left(c_{1}-s\right)\gamma Q_{\mathrm{PRP}}^{*}}{p-s}\right)\right) d\gamma
\end{aligned} \tag{12}$$

According to Lemma 4, when $\beta Q < D_{\max}$ and $\left(c_1 - s\right)\beta Q < \left(c_2 - s\right)D_{\min}$, $W_0^{\mathrm{PRP}}(Q) = \alpha\left(\left(p - c_2\right)D_{\max} + \left(c_2 - c_1\right)\beta Q\right) + (1 - \alpha)\left(p - c_2\right)D_{\min}$ and $N_1 = \alpha\left(c_2 - c_1\right)\beta$. Substitute $W_0^{\mathrm{PRP}}(Q)$ and N_1 into Eq. (12), we can obtain

$$\frac{\partial R\left[Q_{PRP}^*, W_0^{PRP}\left(Q_{PRP}^*\right)\right]}{\partial \lambda} = \int_0^{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}} \left(\left(c_2 - c_1\right)(\gamma - \alpha\beta)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2 - c_1\right)\gamma Q_{PRP}^*}{p - c_2}\right) + \left(s - c_2\right)\gamma F\left(\gamma Q_{PRP}^*\right)\right)d\gamma$$
$$\int_{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}}^{\beta} \left(\left(s - c_1\right)\gamma - \alpha\left(c_2 - c_1\right)\beta\right)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1 - s\right)\gamma Q_{PRP}^*}{p - s}\right)d\gamma$$

When s is very large and approaches c_1, the above function approaches

$$\frac{\partial R\left[Q_{PRP}^*, W_0^{PRP}\left(Q_{PRP}^*\right)\right]}{\partial \lambda} = \int_0^{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}} \left(\left(c_2 - c_1\right)(\gamma - \alpha\beta)F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) - \left(c_2 - c_1\right)\gamma Q_{PRP}^*}{p - c_2}\right) + \left(c_1 - c_2\right)\gamma F\left(\gamma Q_{PRP}^*\right)\right)$$
$$d\gamma - \alpha\left(c_2 - c_1\right)\beta \int_{\frac{W_0^{PRP}\left(Q_{PRP}^*\right)}{(p-c_1)Q_{PRP}^*}}^{\beta} F\left(\frac{W_0^{PRP}\left(Q_{PRP}^*\right) + \left(c_1 - s\right)\gamma Q_{PRP}^*}{p - s}\right)d\gamma,$$

which may be positive or negative, and thus $\frac{\partial Q_{PRP}^*}{\partial \lambda} = -\frac{\frac{\partial R\left[Q_{PRP}^*, W_0^{PRP}\left(Q_{PRP}^*\right)\right]}{\partial \lambda}}{\frac{\partial R\left[Q_{PRP}^*, W_0^{PRP}\left(Q_{PRP}^*\right)\right]}{\partial Q_{PRP}^*}}$ may be a positive or negative value. Hence, the optimal order quantity may increase or decrease with λ, when the salvage value s is very large. □

Proof of Proposition 9

Considering that Q_{PRP}^* satisfies

$$
\begin{aligned}
&R\left[Q^*_{PRP}, W_0^{PRP}\left(Q^*_{PRP}\right)\right] \\
&= \int_0^{\frac{W_0^{PRP}\left(Q^*_{PRP}\right)}{(p-c_1)Q^*_{PRP}}} \left(\left(c_2 - c_1\right)\gamma + (\lambda - 1)\left(c_2 - c_1\right)\gamma F\left(\frac{W_0^{PRP}\left(Q^*_{PRP}\right) - \left(c_2 - c_1\right)\gamma Q^*_{PRP}}{p - c_2} \right) \right. \\
&\left. + \lambda\left(s - c_2\right)\gamma F\left(\gamma Q^*_{PRP}\right) - N_1\left((\lambda - 1)F\left(\frac{W_0^{PRP}\left(Q^*_{PRP}\right) - \left(c_2 - c_1\right)\gamma Q^*_{PRP}}{p - c_2} \right) + 1 \right) \right) d\gamma \\
&+ \int_{\frac{W_0^{PRP}\left(Q^*_{PRP}\right)}{(p-c_1)Q^*_{PRP}}}^{\beta} \left(\left(c_2 - c_1\right)\gamma + \left(s - c_2\right)\gamma F\left(\gamma Q^*_{PRP}\right) + (\lambda - 1)\left(s - c_1\right)\gamma F\left(\frac{W_0^{PRP}\left(Q^*_{PRP}\right) + \left(c_1 - s\right)\gamma Q^*_{PRP}}{p - s} \right) \right. \\
&\left. - N_1\left((\lambda - 1)F\left(\frac{W_0^{PRP}\left(Q^*_{PRP}\right) + \left(c_1 - s\right)\gamma Q^*_{PRP}}{p - s} \right) + 1 \right) \right) d\gamma = 0
\end{aligned}
$$

Substituting Q^*_{PRP} by Q^*, we can obtain

$$
\begin{aligned}
&R\left[Q^*, W_0^{PRP}\left(Q^*\right)\right] \\
&= \int_0^{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}} \left((\lambda - 1)\left[\left(c_2 - c_1\right)\gamma - N_1\right] F\left(\frac{W_0^{PRP}(Q^*) - \left(c_2 - c_1\right)\gamma Q^*}{p - c_2} \right) - (\lambda - 1)\left(c_2 - s\right)\gamma F\left(\gamma Q^*\right) \right. \\
&\left. - N_1 \right) d\gamma + \int_{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}}^{\beta} \left((\lambda - 1)\left[\left(s - c_1\right)\gamma - N_1\right] F\left(\frac{W_0^{PRP}(Q^*) + \left(c_1 - s\right)\gamma Q^*}{p - s} \right) - N_1 \right) d\gamma = Z^{PRP}\left(W_0^{PRP}\right)
\end{aligned}
$$

Hence, when $Z^{PRP}\left(W_0^{PRP}\right) > 0$, $R\left[Q^*, W_0^{PRP}(Q^*)\right] > 0$. Considering that $\frac{\mathrm{d}R\left[Q, W_0^{PRP}(Q)\right]}{\mathrm{d}Q} < 0$, $Q^*_{PRP} > Q^*$. Besides, note that $H(Q)$ is a continuous function, we can get that $Q^*_{PRP} > Q^*$ when the reference profit W_0^{PRP} satisfies $Z^{PRP}\left(W_0^{PRP}\right) > 0$.

When the salvage value s approaches c_1, $R\left[Q^*, W_0^{PRP}(Q^*)\right]$ approaches

$$
\begin{aligned}
&R'\left[Q^*, W_0^{PRP}\left(Q^*\right)\right] \\
&= \int_0^{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}} \left((\lambda - 1)\left(c_2 - c_1\right)\gamma \left(F\left(\frac{W_0^{PRP}(Q^*) - \left(c_2 - c_1\right)\gamma Q^*}{p - c_2} \right) - F\left(\gamma Q^*\right) \right) \right. \\
&\left. - N_1\left((\lambda - 1)F\left(\frac{W_0^{PRP}(Q^*) - \left(c_2 - c_1\right)\gamma Q^*}{p - c_2} \right) + 1 \right) \right) d\gamma \\
&+ \int_{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}}^{\beta} \left(-N_1\left((\lambda - 1)F\left(\frac{W_0^{PRP}(Q^*) + \left(c_1 - s\right)\gamma Q^*}{p - s} \right) + 1 \right) \right) d\gamma
\end{aligned}
$$

It is easy to prove that if $N_1 = \alpha\left(c_2 - c_1\right)\beta$,

$$
\begin{aligned}
& R'\left[Q^*, W_0^{PRP}(Q^*)\right] \\
&= \int_0^{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}} \left((\lambda-1)\left(c_2-c_1\right)(\gamma-\alpha\beta)\left(F\left(\frac{W_0^{PRP}(Q^*) - \left(c_2-c_1\right)\gamma Q^*}{p-c_2} \right) - F\left(\gamma Q^*\right) \right) \right. \\
&\quad \left. -\alpha\left(c_2-c_1\right)\beta\right) d\gamma + \int_{\frac{W_0^{PRP}(Q^*)}{(p-c_1)Q^*}}^{\beta} \left(-\alpha\left(c_2-c_1\right)\beta\left((\lambda-1)F\left(\frac{W_0^{PRP}(Q^*) + \left(c_1-s\right)\gamma Q^*}{p-s} \right) + 1 \right) \right) d\gamma
\end{aligned}
$$

When α is sufficiently large, $\gamma \le \alpha\beta$, and $R'\left[Q^*, W_0^{PRP}(Q^*)\right] < 0$. Hence, in this case, the loss-averse retailer order less than the risk-neutral retailer, if the salvage value s is sufficiently large. As a result, we can obtain Proposition 9. ☐

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