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REGULAR ARTICLE

Analytical models to determine room requirements in outpatient clinics

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Abstract Outpatient clinics traditionally organize processes such that the doctor remains in a consultation room while patients visit for consultation, we call this the Patient-to-Doctor policy (PtD-policy). A different approach is the Doctor-to-Patient policy (DtP-policy), whereby the doctor travels between multiple consultation rooms, in which patients prepare for their consultation. In the latter approach, the doctor saves time by consulting fully prepared patients. We use a queueing theoretic and a discrete-event simulation approach to provide generic models that enable performance evaluations of the two policies for different parameter settings. These models can be used by managers of outpatient clinics to compare the two policies and choose a particular policy when redesigning the patient process. We use the models to analytically show that the DtP-policy is superior to the PtD-policy under the condition that the

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doctor's travel time between rooms is lower than the patient's preparation time. In addition, to calculate the required number of consultation rooms in the DtP-policy, we provide an expression for the fraction of consultations that are in immediate succession; or, in other words, the fraction of time the next patient is prepared and ready, immediately after a doctor finishes a consultation. We apply our methods for a range of distributions and parameters and to a case study in a medium-sized general hospital that inspired this research.

Keywords Outpatient clinic · Health care · Queueing theory · Discrete-event simulation

1 Introduction

Demand for outpatient care is growing as a result of increasingly effective ambulatory care treatments and the overall growth of health care demand. Hence, managers of outpatient clinics are becoming increasingly aware of the importance of the efficient use of scarce resources, particularly doctor's time and facility space (Côté 1999). This results in many hospitals redesigning or rebuilding their outpatient clinics (e.g., the hospitals RIVAS Gorinchem, Reinier de Graaf Gasthuis, Haga Ziekenhuis, and Groene Hart Ziekenhuis).

In many hospitals, outpatient clinics are organized such that doctors remain in one consultation room, while patients visit for individual consultation. In this classic design, each doctor occupies one consultation room, which often doubles as the doctor's office (Vissers and Beech 2005). Patients wait in the waiting room until the doctor is available, and then enter the doctor's office for the consultation. We label this classic design Patient-to-Doctor policy (PtD-policy).

In a different approach, patients prepare themselves in separate, individual consultation rooms. Each patient is then visited by the doctor, who travels from room to room. We label this approach as Doctor-to-Patient policy (DtP-policy). The DtP-policy offers a potential decrease in total service time, given that doctors do not have to be present for patient preparation activities that require a consultation room, but do not require a doctor. We characterize these activities as pre-consultation (e.g., traveling to the room, undressing, blood pressure measures) and post-consultation (e.g., dressing, making appointments, leaving the room, cleaning the room). Nurses or assistants may be involved in these activities. In the DtP-policy, the doctor experiences travel time between each consultation, whilst traveling from room to room. Figure 1 illustrates the PtD-policy and the DtP-policy with two rooms.

In search of efficiency improvements in the outpatient clinic, managers are reconsidering the design of the outpatient clinic. Since differences in the outpatient process exist between different (specialties within) outpatient clinics, a policy efficient for one clinic may not be efficient for another. For example, when pre-consultation and/or post-consultation time are non-existent or relatively low in comparison with consultation time in a particular outpatient clinic process (e.g., psychology consultations), the DtP-policy may not result in savings of doctor time. Hence, before deciding to adopt a particular policy, it is important that an outpatient clinic manager understands

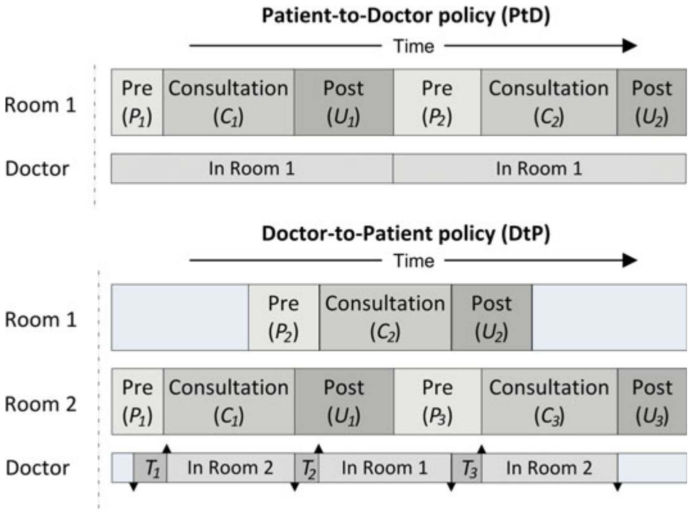

Fig. 1 An illustration of the PtD-policy and the DtP-policy with two rooms. Pre-consultation, consultation and post-consultation for patient n is indicated by P_n, C_n and U_n, respectively. T_n indicates the travel time of the doctor to patient n

which policy is most efficient and how many consultation rooms are required for the particular outpatient clinic's parameter settings. To support this decision making, we provide analytical models that can be used to rationally compare the two policies on several performance measures and to determine the required number of consultation rooms in a particular outpatient clinic setting. Our models provide quantitative arguments that facilitate a rational discussion about a proposed decision with stakeholders (e.g., hospital boards, doctors).

In queueing terminology, the PtD-policy resembles a $G/G/1$ queueing model, under the assumption that patients are seen on a first-come, first-served basis (FCFS). The DtP-policy seems to resemble a polling system (Levy and Sidi 1990; Takagi 1998), where the server travels between multiple customer queues. However, as the outpatient clinic has a single queue of patients only, this analogy cannot be applied to evaluate the DtP-policy. The queueing model that most closely resembles the DtP-policy is a Production Authorization Card system (PAC-system). In a PAC-system, the number of jobs (patients) at a station (the doctor) is bounded by the number of PACs (rooms). Therefore, the departure of a job (patient exits) initiates demand for new jobs (a patient enters the empty room). The PAC-system, and thus the DtP-policy, is a typical 'pull' system, used in popular management philosophies such as Just-In-Time and Kanban. The PtD-policy is a 'push' system, whereby patients arrive in a buffer (the waiting room) and are pushed through the system. For results in queueing theory on push and pull systems, (see Boucherie et al. 2003; Kopzon et al. 2009). The exact and approximative solution approaches for PAC-systems are based on steady state queueing results (Buitenhek 1998). Since appointment schedules have a finite number of customers, and thus do not reach steady state (Robinson and Chen 2003; Ho and Lau 1992; Cayirli and Veral 2003), these solution approaches are inappropriate to analyze the DtP-policy and the PtD-policy.

There is a significant body of literature on resource planning in outpatient clinics, particularly related to outpatient scheduling. For a comprehensive review of the literature on outpatient scheduling, (see Cayirli and Veral 2003). The design and capacity dimensioning of outpatient clinics has received less attention in the literature. Different process set-ups for an emergency department are compared with a Multi-Class Open Queueing Network (MC-OQN) in Jiang and Giachetti (2008). The authors conclude that parallel processing of, for example, treatment and diagnostic tests, rather than serial processing, results in a shorter patient sojourn time under certain conditions. Other examples of successful process redesigns in outpatient clinics are Zonderland et al. (2009), Chand et al. (2009). Simulation is used to find the required number of examination rooms in an outpatient clinic (Côté 1999), an obstetrics outpatient center (Isken et al. 1999), a radiology department (Johnston et al. 2009), an emergency department (Baesler et al. 2003; Duguay and Chetouane 2007) and a family practice (Swisher et al. 2001; Swisher and Jacobson 2002). A combination of simulation and function estimation is used to design a transfusion center (De Angelis et al. 2003). All described papers use simulation to find the required number of rooms for a specific setting. In this paper, we develop analytical models of a generic outpatient clinic to compare the PtD-policy with the DtP-policy, and to determine the required number of rooms in the DtP-policy.

The performance measures we consider are doctor utilization, access time, and patient waiting time. Doctor utilization is the fraction of time the doctor is actually consulting a patient. Access time is the time between the request for an appointment and the realization of the appointment. Patient waiting time is the time between the scheduled starting time of the appointment and the actual starting time of the appointment. Increased doctor utilization leads to decreased access time, but also to increased patient waiting time, given that more patients are scheduled per time unit. Managers of outpatient clinics strive for high doctor utilization and low access times, even at the cost of some patient waiting time (Brahimi and Worthington 1991). This may be explained by three factors: doctors are considered expensive resources, service level agreements on access times may exist and low access times may attract more patients.

This paper is organized as follows. Section 2 introduces the model and presents expressions for the recursion of the time that the doctor finishes a consultation in both the PtD-policy and the DtP-policy. Section 3 compares these recursions analytically, and introduces an expression for the fraction of consultations that are in immediate succession, to calculate the required number of consultation rooms in the DtP-policy. Section 4 presents the results for a range of distributions and parameters, and a case study at a medium-size hospital. Section 5 discusses main conclusions.

2 Model

In Sects. 2.1 and 2.2, we develop expressions for the time the doctor finishes the consultation of the nth patient in the PtD-policy (F_n) and the DtP-policy (F'_n). These expressions are used in Sect. 3.1, to compare the PtD-policy and the DtP-policy, and to develop an expression for the fraction of consultations that are in immediate succession to calculate the required number of rooms in the DtP-policy. We first introduce notation and assumptions that apply to both policies.

Assume that at time zero the doctor is free. Patients arrive according to a stochastic process at time points $\{A_n, n = 1, 2, \ldots, N\}$, thus the first patient arrives at time A_1. The nth patient leaves the system after finishing pre-consultation (P_n), consultation with the doctor (C_n) and post-consultation (U_n), where P_n, C_n, U_n are random variables with $P_n, C_n, U_n \geq 0$, for $n = 1, 2, \ldots, N$. The nth patient leaves at time $D_n = F_n + U_n$ in the PtD-policy, and at time $D'_n = F'_n + U_n$ in the DtP-policy. Let R be the number of rooms and T_n the random variable for the doctor's travel time to the nth patient. We assume that $T_n, n = 1, 2, \ldots, N$, is an independent and identically distributed (i.i.d.) sequence of random variables, thus not connected to the sequence with which the doctor visits the rooms, and that the travel time of the doctor (T_n) is not longer than the travel time of the patient (included in P_n). We base the latter assumption on our experience that consultation rooms are located adjacently and the waiting room is at a further distance.

Assumption 1 $T_n \leq P_n$, for $n = 1, 2, \ldots, N$.

Throughout this paper, inequalities in expressions and equations for random variables are with probability one, i.e., $T_n \leq P_n \Leftrightarrow \Pr(T_n \leq P_n) = 1$. The following two assumptions imply that patients enter rooms and are consulted by the doctor in the sequence they arrive.

Assumption 2 Patients enter rooms on an FCFS basis. Hence, when a room is empty, the patient who has waited the longest in the queue is admitted.

Assumption 3 The doctor consults patients on an FCFS basis, thus in the sequence in which the patients enter rooms.

The following assumption deals with the doctor's travel in the DtP-policy after finishing consultation with a patient.

Assumption 4 When the doctor finishes consultation with the $(n-1)$th patient, and the nth patient has not entered a room yet, the doctor travels to an empty room when one becomes available, and waits there for the nth patient.

Under Assumption 4, the doctor either knows which room to go to after finishing consultation of a patient, or the doctor waits until a patient leaves and a room becomes available.

2.1 Recursion of the time the doctor finishes a consultation in the PtD-policy

We obtain the following expression for the recursion of the time that the doctor finishes the consultation of a patient in the PtD-policy.

Lemma 5 $F_n = \max\{A_n, F_{n-1} + U_{n-1}\} + P_n + C_n$, *where* $n = 1, 2, \ldots, N$ *and* $F_0 = 0$.

We prove Lemma 5 in Appendix A.

2.2 Recursion of the time the doctor finishes a consultation in the DtP-policy

Since the processes in the DtP-policy and the PtD-policy are identical when $R = 1$, we focus on $R > 1$ in the DtP-policy. The lemma presented in this section thus holds for any $R > 1$.

The exiting time for patients may not be in the same order as the arrivals, because it is possible for the $(n + 1)$th patient to exit before the (n)th patient (due to the randomness in U_n). To accommodate this, we define the $s(n)$th patient as the patient who is succeeded by the nth patient in a room. Thus when the $s(n)$th patient exits a room, the nth patient enters that room. We obtain the following expression for the recursion of the time that the doctor finishes the consultation of a patient in the DtP-policy.

Lemma 6 $F'_n = \begin{cases} \max\{A_n + P_n, F'_{n-1} + T_n\} + C_n, & \text{if } n \le R \\ \max\{\max\{F'_{s(n)} + U_{s(n)}, A_n\} + P_n, & \\ \max\{F'_{s(n)} + U_{s(n)}, F'_{n-1}\} + T_n\} + C_n, & \text{if } n > R \end{cases}$,
where $n = 1, 2, \ldots, N$ *and* $F'_0 = 0$.

We prove Lemma 6 in Appendix B.

3 Analytical models for performance evaluation

We use Lemmas 5 and 6 obtained in Sect. 2 to compare the DtP-policy with the PtD-policy in Sect. 3.1. In Sect. 3.2 we develop an expression for the fraction of consultations that are in immediate succession to calculate the required number of rooms in the DtP-policy.

3.1 Analytical comparison of the recursion of the finishing time for the doctor under both policies

In this section, we show that the time that the doctor finishes the consultation of a patient in the DtP-policy is not later than the time the doctor finishes consultation with that patient in the PtD-policy, under Assumptions 1–4, i.e.,

Theorem 7 $F'_n \le F_n$*, for* $n = 1, 2, \ldots, N$.

Since $F'_n \le F_n$, for $n = 1, 2, \ldots, N$, this also means $D'_n \le D_n$, for $n = 1, 2, \ldots, N$. Therefore, the departure of the nth patient never occurs later in the DtP-policy than the departure of that same patient in the PtD-policy.

We prove Theorem 7 in Appendix C.

Remark 8 Under our FCFS assumptions, Assumptions 2 and 3, the modeled DtP-policy performs worse than a real-life DtP-policy, where the doctor may consult patients according to a dynamic sequence. The FCFS ordering may result in a waste of doctor's capacity, since the doctor may be waiting for the nth patient to finish pre-consultation, while the $(n + 1)$th patient is already finished with pre-consultation. In addition, Assumption 4 also causes waste of capacity, since the doctor waits until knowing which room to travel to next. This suggests that the ordering of the DtP-policy and the PtD-policy also holds when Assumptions 2–4 are relaxed.

Remark 9 When Assumption 1 is replaced by the weaker assumption $\Pr(T_n \leq s) \geq \Pr(P_n \leq s)$, for $n = 1, 2, \ldots, N$, we can show that $\Pr(F'_n \leq t) \geq \Pr(F_n \leq t)$, for $n = 1, 2, \ldots, N$, which implies that $\mathbb{E}F' \leq \mathbb{E}F$.

3.2 Analytical expression to calculate the required number of rooms

In a PtD-policy, the required number of rooms per doctor is one. In a DtP-policy, the required number of rooms is more than one. In this section we develop an expression for the fraction of consultations that are in immediate succession to calculate the required number of rooms in the DtP-policy.

To minimize access time of patients, health care managers aim to minimize idle time experienced by the doctor. To this end, the doctor's wait for the next available patient should be minimized (Harper and Gamlin 2003), or in other words, the *fraction of consultations that take place in immediate succession* should be maximized. After leaving a room, the doctor should return to this room after the next patient has finished pre-consultation. During the time that the doctor is away from a specific room ($U_{s(n)} + P_n$), the doctor performs $R - 1$ consultations in the other rooms and R travels (including the travel to the nth patient). Hence, we obtain the following expression, where the number of rooms (R) is chosen such that the fraction of consultations in immediate succession is larger than α, where $0 \leq \alpha \leq 1$.

$$\Pr\left(\sum_{k=n-R}^{n-1} C_k + \sum_{k=n-R}^{n} T_k \geq U_{s(n)} + P_n\right) \geq \alpha. \tag{1}$$

Example We evaluate Eq. (1) for gamma and normal distributed service times. The average duration of a process is given by μ_i and its variance is given by σ_i^2, where $i \in \{P, C, U, T\}$.

The gamma distribution is a frequently reported distribution for outpatient clinic consultation times (Cayirli and Veral 2003). Let the pre-consultation, the post-consultation, and the travel times be deterministic, and the consultation times be i.i.d. gamma distributed. The convolution of v i.i.d. gamma distributed variables with parameters (k, θ) is again a gamma distribution with parameters $(v \cdot k, \theta)$. Hence, the number of rooms, R, is obtained from

$$\int_{U+P-R\cdot T}^{\infty} x^{(R-1)\cdot(k-1)} \frac{e^{-\frac{x}{\theta}}}{\theta^{(R-1)\cdot k} \cdot \Gamma(R \cdot k)} \, \mathrm{d}x \geq \alpha, \tag{2}$$

where $\theta = \frac{\sigma_C^2}{\mu_C}$ and $k = \frac{\mu_C}{\theta}$ are parameters of the gamma distribution and $\Gamma(a)$ is the standard gamma function with parameter a.

When all service processes are i.i.d. normal distributed, its convolution results in a normal distribution with parameters (μ, σ). Hence, the number of rooms, R, is obtained from

$$\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}\,\mathrm{d}x \geq \alpha, \tag{3}$$

where $\mu = (R-1)\cdot\mu_C + R\cdot\mu_T - \mu_U - \mu_P$ and $\sigma^2 = (R-1)\cdot\sigma_C^2 + R\cdot\sigma_T^2 + \sigma_P^2 + \sigma_U^2$.

4 Results

Sections 4.1 and 4.2 describe the comparison of the two policies and the calculation of the required number of rooms. Section 4.3 describes the application of our methods at a pediatric outpatient clinic.

4.1 Comparison of the PtD-policy and the DtP-policy

In Theorem 7, we showed that the doctor finishes consultation with a patient earlier in the DtP-policy than in the PtD-policy under Assumptions 1–4. Hence, more patients can be consulted per time unit in the DtP-policy. In Remark 8, we indicated that the ordering of the DtP-policy and the PtD-policy may remain the same when Assumptions 2–4 are relaxed. Below, we use discrete-event simulation to study the ordering when Assumption 1 is relaxed.

The discrete-event simulation is a model of an outpatient clinic, where a consultation session lasts 8 h per day and patients arrive at the time they are scheduled. The Bailey–Welch rule (Bailey 1952) is used for the patient schedule. The rule states that when blocks of the size of the expected consultation time are used to schedule the patients, the last block is deleted and the first block holds two patients. We assume a coefficient of variation (CV $= \frac{\mu}{\sigma}$) of 0.6, which is within the range of 0.35–0.85 reported in the literature (Cayirli and Veral 2003). The length of each simulation run is one business day. With the replication/deletion approach (Law 2009), we find that 1,000 replications (days) appear to be sufficient for a confidence level of 99.9% with a relative error of 0.1% with respect to the number of consultations per week.

Figure 2 shows the switching curve when all processes are gamma distributed. The switching curve from the PtD-policy to the DtP-policy depends on the ratio of doctor travel time to pre-consultation time and post-consultation time, and is insensitive to changes in the average consultation time and the CV. Also, the ratio of pre-consultation to post-consultation has only negligible impact on the choice for a policy; it is their sum that influences the superiority of a policy.

When ρ is varied ($\rho = \lambda E[C]$, where λ is the number of patients scheduled per time unit, and $E[C]$ is the expected consultation time), the switching curve for the DtP-policy is identical to the curve in Fig. 2 for $\rho \geq 0.7$. For $\rho < 0.7$, the DtP-policy performs better at even higher average travel times, but the potential benefit of the DtP-policy is relatively low, as can be seen in Fig. 3. Also, Fig. 3 illustrates that the potential benefit of the DtP-policy decreases as the ratio of consultation time versus pre-consultation time and post-consultation time decreases. This is caused by the fact that decreasing pre-consultation and post-consultation time per patient

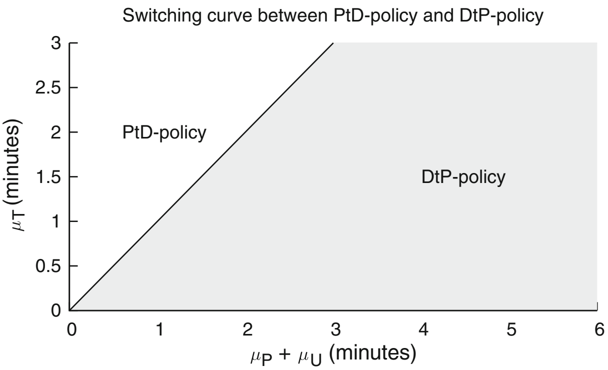

Fig. 2 The switching curve between the DtP-policy and the PtD-policy, where all processes are gamma distributed with CV $= 0.6$. A policy is superior to the other policy, when average doctor utilization is higher. The number of rooms is chosen with Eq. (1), with $\alpha = 0.90$

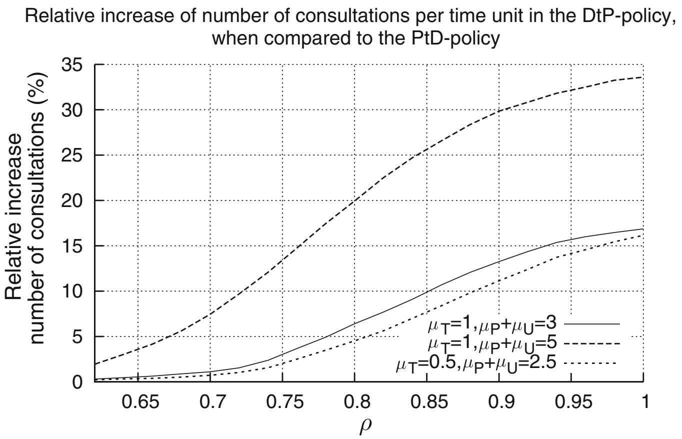

Fig. 3 The effect of varying ρ on the relative increase of the number of consultations per time unit in the DtP-policy, when compared to the PtD-policy. All processes are gamma distributed with CV $= 0.6$, $\mu_C = 10$, and $R = 2$

while keeping consultation time constant, leads to lower potential savings of doctor time.

4.2 Evaluation of the required number of rooms

The fraction (P_{succ} in Table 1) of consultations that are in immediate succession, left-hand side in Eq. (1), is evaluated numerically with Monte Carlo simulation for the gamma, lognormal and exponential distribution. For the normal distribution, we use Eq. 3. To compare the fraction with a performance measure, such as doctor utilization (Util. in Table 1), we use the discrete-event simulation introduced in Sect. 4.1. Table 1 presents both the fraction results and the doctor utilization for a given number of rooms, and it shows the effect of choosing a certain α. For example, when $\alpha = 0.90$,

Table 1 The results for the fraction of consultations that are in immediate succession, where $\mu_T = 1$ and CV $= 0.6$ for the gamma, lognormal and normal distributions, and CV $= 1$ for the exponential distribution

R	(μ_P, μ_C, μ_U)	Gamma		Lognormal		Normal		Exponential	
		P_{succ}	Util. (%)	P_{succ}	Util. (%)	P_{succ}	Util. (%)	P_{succ}	Util. (%)
2	(3, 15, 3)	0.920	91.2	0.946	91.4	0.879	91.1	0.781	86.6
3	(3, 15, 3)	0.998	91.6	0.996	91.6	0.981	91.8	0.960	87.6
4	(3, 15, 3)	1.000	91.6	0.997	91.6	0.997	91.8	0.993	87.7
2	(3, 15, 6)	0.800	89.8	0.823	90.3	0.791	89.6	0.674	84.6
3	(3, 15, 6)	0.985	91.5	0.979	91.6	0.963	91.7	0.909	87.4
4	(3, 15, 6)	0.999	91.6	0.988	91.6	0.993	91.8	0.976	87.7
2	(3, 15, 9)	0.675	87.0	0.687	87.6	0.680	87.0	0.591	81.9
3	(3, 15, 9)	0.953	91.4	0.945	91.5	0.933	91.5	0.852	87.0
4	(3, 15, 9)	0.995	91.6	0.973	91.6	0.987	91.8	0.949	87.6
2	(6, 15, 3)	0.800	89.4	0.823	89.7	0.791	89.1	0.674	84.1
3	(6, 15, 3)	0.985	91.0	0.979	91.0	0.963	91.2	0.909	86.8
4	(6, 15, 3)	0.999	91.0	0.988	91.0	0.993	91.3	0.976	87.0
2	(6, 15, 6)	0.671	86.8	0.684	87.3	0.685	86.6	0.580	81.5
3	(6, 15, 6)	0.958	90.8	0.952	90.9	0.937	91.0	0.853	86.4
4	(6, 15, 6)	0.997	91.0	0.978	91.0	0.988	91.2	0.953	87.0
2	(6, 15, 9)	0.551	83.0	0.550	83.5	0.571	83.0	0.509	78.2
3	(6, 15, 9)	0.911	90.5	0.903	90.6	0.896	90.6	0.794	85.6
4	(6, 15, 9)	0.989	90.9	0.961	91.0	0.978	91.2	0.921	86.9
2	(9, 15, 3)	0.675	86.1	0.687	86.6	0.680	85.9	0.591	81.0
3	(9, 15, 3)	0.953	90.2	0.945	90.3	0.933	90.4	0.852	85.8
4	(9, 15, 3)	0.995	90.3	0.973	90.4	0.987	90.7	0.949	86.4
2	(9, 15, 6)	0.551	82.6	0.550	83.1	0.571	82.5	0.509	77.8
3	(9, 15, 6)	0.911	89.9	0.903	90.0	0.896	89.9	0.794	85.0
4	(9, 15, 6)	0.989	90.3	0.961	90.4	0.978	90.6	0.921	86.3
2	(9, 15, 9)	0.449	78.3	0.434	78.7	0.466	78.4	0.446	74.3
3	(9, 15, 9)	0.853	89.2	0.844	89.4	0.843	89.2	0.739	84.0
4	(9, 15, 9)	0.975	90.3	0.944	90.4	0.963	90.6	0.887	86.1
5	(9, 15, 9)	0.996	90.4	0.960	90.4	0.992	90.7	0.953	86.4

The half-length of the 99.9% confidence interval for the doctor utilization is between 0.011% and 0.096%

four rooms are required when $\mu_P = \mu_U = 9$ and all processes lognormal distributed. In that case, the doctor utilization found with the simulation is 90.4%. The results in Table 1 show that doctor utilization increases with the fraction of consultations that are in immediate succession.

The stochastic nature of the consultation process should be considered when the required number of rooms is determined. When all processes are considered to be deterministic, three rooms are required in the example of Fig. 4. The graph shows that more rooms are required when CV increases.

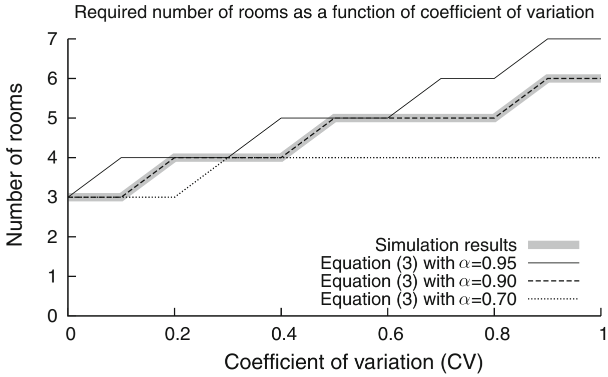

Fig. 4 The required number of rooms when CV increases. All processes are gamma distributed, with $\mu_P = 10, \mu_C = 10, \mu_U = 10, \mu_T = 1$. The number of rooms in the simulation is chosen such that the doctor utilization cannot increase more than 0.5% with an additional room. The simulation results coincide with Eq. (1), where $\alpha = 0.90$

Table 2 Duration parameters (minutes), retrieved from data for 1875 patients of the pediatric outpatient clinic in 2009

Process	Distribution	Average	Std. deviation
Pre-consultation	Gamma	5.90	6.06
Consultation	Gamma	15.57	8.12
Post-consultation	–	–	–

4.3 Case study at a medium-sized hospital

We apply our methods at the pediatric outpatient clinic of the ‘Groene Hart Ziekenhuis’ hospital (GHZ) in Gouda, the Netherlands. GHZ has 450 beds and over 2,000 employees [GHZ Website (2011)], and the seven doctors at the pediatric outpatient clinic consult 12,000 patients per year. We focus on a single doctor, who consults patients for 9 h per week. Patients are planned in time slots of 15 min. The parameters in Table 2 are the result of extensive data gathering.

We know that the DtP-policy outperforms the PtD-policy if we assume that the doctor’s travel time is always lower than the patient’s travel time. The simulation results indicate that the DtP-policy outperforms the PtD-policy, when the average travel time does not exceed 6 min. In estimating the number of rooms, we assume that travel time is 0.5 min on average, with CV = 0.6. Table 3 shows that three rooms are required, if $\alpha = 0.90$. The fraction of consultations that are in immediate succession (P_{succ} in Table 3) is evaluated numerically with Monte Carlo simulation, and the doctor utilization (Utilization in Table 3) is found with our discrete-event simulation.

5 Conclusion

Inspired by the hospitals ‘RIVAS Gorinchem’, ’Reinier de Graaf Gasthuis’ and ‘Groene Hart Ziekenhuis’, which were in the process of redesigning their outpatient clinic, this

Table 3 Results to determine the required number of rooms in the case study

Number of rooms	P_{succ}	Utilization (%)
2	0.883	92.5
3	0.984	93.3
4	0.998	93.3

The half-length of the 99.9% confidence interval for the doctor utilization is between 0.053% and 0.057%

paper has developed analytical and simulation models to compare different parameter settings in two policies for the organization of outpatient clinics. In the first policy, doctors remain in one consultation room, while patients visit for consultation. We call this the PtD-policy, and in this policy, the doctor attends the complete patient process: pre-consultation, consultation and post-consultation. In the second policy, patients prepare themselves in individual consultation rooms, with or without the aid of a nurse, while the doctor travels from room to room. We call this the DtP-policy, and in this policy, the doctor only attends the consultation, and experiences travel time to go from room to room.

We use the models to evaluate the two policies on doctor utilization, patient access time and patient waiting time. The models provide insight in the ordering of the PtD-policy and the DtP-policy in different parameter settings for different outpatient clinics. As a result, we show that an outpatient clinic should choose the DtP-policy, when for each patient the doctor's travel time is lower than the patient's pre-consultation time. We extend this result with a discrete-event simulation, which indicates that a DtP-policy should be chosen when the average doctor travel time is lower than the sum of the average pre-consultation time and the average post-consultation time.

We developed an expression for the fraction of consultations that are in immediate succession to calculate the required number of rooms in the DtP-policy. Using the developed expression as described in this paper results in choosing the required number of rooms such that the fraction of consultations in immediate succession is maximized and the idle time of the doctor is minimized.

To support decision making in outpatient clinics, we provide analytical models that can be used to compare the two policies on several performance measures and to determine the required number of consultation rooms in a particular outpatient clinic setting. Our experience in applying this research showed that our models are valuable for providing quantitative arguments to support the discussion of a proposed decision with stakeholders (e.g., hospital boards, doctors).

For the aforementioned hospitals we have successfully applied the insights obtained with our methods in the redesign of their outpatient clinics, based on data from their outpatient clinics. For the hospital managers, our results provided quantitative measures and formal proof to support their decision to redesign the outpatient clinic from a PtD-policy to a DtP-policy. With our models and the data, we also helped the hospitals to determine the required number of consultation rooms for each doctor in the DtP-policy.

Appendix A: Proof of Lemma 5

Consider the recursion of the departure process. We distinguish two cases:

(i) When $A_n \geq D_{n-1}$, the nth patient comes in after the $(n-1)$th patient has left, thus the doctor is available immediately upon arrival of the nth patient at A_n. Hence, $D_n = A_n + P_n + C_n + U_n$.

(ii) When $A_n < D_{n-1}$, the nth patient comes in while the doctor is occupied. The nth patient can start pre-consultation upon departure of the $(n-1)$th patient. Hence, $D_n = D_{n-1} + P_n + C_n + U_n$.

Combining (i) and (ii) obtains

$$D_n = \max\{A_n, D_{n-1}\} + P_n + C_n + U_n. \tag{4}$$

Since $D_n = F_n + U_n$, we have proven Lemma 5. □

Appendix B: Proof of Lemma 6

The recursion of the finishing time for the doctor is explained by examining the time both the patient and the doctor are ready for consultation. The nth patient is available for consultation after finishing pre-consultation. The doctor is available for the nth patient, after the consultation of the $(n-1)$th patient plus the travel to the nth patient. We distinguish two cases:

(i) When $n \leq R$, the number of customers in the system is smaller than the number of rooms. Hence, the nth patient enters a room immediately upon arrival and is ready for consultation after pre-consultation $(A_n + P_n)$. The doctor consults the patient after finishing consultation of the $(n-1)$th patient and the travel time $(F'_{n-1} + T_n)$. The moment consultation can start if $n \leq R$ is thus: $\max\{A_n + P_n, F'_{n-1} + T_n\}$.

(ii) When $n > R$, the nth patient may have to wait for the exit of the $s(n)$th patient$(F'_{s(n)} + U_{s(n)})$ before entering a room, or the patient can enter a room immediately upon arrival (A_n), if a room is available. After entering a room, pre-consultation has to be finished before consultation can start. Hence, the patient is ready for consultation at $\max\{F'_{s(n)} + U_{s(n)}, A_n\} + P_n$. The doctor is ready for consultation after traveling to the room (T_n). The doctor can start traveling after the consultation of the $(n-1)$th patient (F'_{n-1}), and, due to Assumption 4, the $s(n)$th patient must have exited $(F'_{s(n)} + U_{s(n)})$. Therefore, the doctor is available for the consultation of the nth patient at $\max\{F'_{s(n)} + U_{s(n)}, F'_{n-1}\} + T_n$. The moment consultation can start if $n > R$ is thus $\max\{\max\{F'_{s(n)} + U_{s(n)}, A_n\} + P_n, \max\{F'_{s(n)} + U_{s(n)}, F'_{n-1}\} + T_n\}$.

We combine (i) and (ii) to obtain Lemma 6. □

Appendix C: Proof of Theorem 7

We prove Theorem 7 by induction. Clearly $F'_1 \leq F_1$, since in an initial (empty) system, the process is identical, because we have Assumption 1. The induction hypothesis is $F'_j \leq F_j$, for $j = 1, 2, \ldots, n-1$. It remains to prove that $F'_n \leq F_n$.

Observe from Assumptions 2 and 3 that $F_{n-1} \leq F_n$ and $F'_{n-1} \leq F'_n$. In addition, the $s(n)$th patient is the patient that leaves a room before the nth patient can enter that room. Therefore, it is certain that the $s(n)$th patient has entered a room before the nth patient, so that

$$F'_{s(n)} + U_{s(n)} \leq F'_{n-1} + U_{n-1}, \quad \text{for } n = 1, 2, \ldots, N. \tag{5}$$

It is sufficient to consider the case $n > R$, since for $n \leq R$ by definition we have $F'_{s(n)} + U_{s(n)} = 0$.

For the case $A_n \leq F'_{s(n)} + U_{s(n)}$, we obtain:

$$\begin{aligned}
F'_n &= \max\{F'_{s(n)} + U_{s(n)} + P_n, \\
&\quad \max\{F'_{s(n)} + U_{s(n)}, F'_{n-1}\} + T_n\} + C_n \quad (\text{Lemma 6}, n > R) \\
&\leq \max\{F'_{n-1} + U_{n-1} + P_n, \\
&\quad \max\{F'_{n-1} + U_{n-1}, F'_{n-1}\} + T_n\} + C_n \quad [\text{Eq. (5)}] \\
&= F'_{n-1} + U_{n-1} + \max\{P_n, T_n\} + C_n \\
&\leq F_{n-1} + U_{n-1} + \max\{P_n, T_n\} + C_n \quad (\text{Induction hypothesis}) \\
&\leq F_{n-1} + U_{n-1} + P_n + C_n \quad (\text{Assumption 1}) \\
&\leq \max\{A_n, F_{n-1} + U_{n-1}\} + P_n + C_n = F_n \quad (\text{Lemma 5})
\end{aligned}$$

For the case $A_n \geq F'_{s(n)} + U_{s(n)}$, we obtain:

$$\begin{aligned}
F'_n &= \max\{A_n + P_n, \max\{F'_{s(n)} + U_{s(n)}, F'_{n-1}\} + T_n\} + C_n \quad (\text{Lemma 6}, n > R) \\
&\leq \max\{A_n + P_n, \max\{A_n, F'_{n-1}\} + T_n\} + C_n \\
&= \max\{A_n + \max\{P_n, T_n\}, F'_{n-1} + T_n\} + C_n \\
&\leq \max\{A_n + \max\{P_n, T_n\}, F_{n-1} + T_n\} + C_n \quad (\text{Induction hypothesis}) \\
&\leq \max\{A_n + P_n, F_{n-1} + P_n\} + C_n \quad (\text{Assumption 1}) \\
&\leq \max\{A_n, F_{n-1} + U_{n-1}\} + P_n + C_n = F_n \quad (\text{Lemma 5})
\end{aligned}$$

From the above, it follows that if $F'_j \leq F_j$, for $j = 1, 2, ..., n-1$, then $F'_n \leq F_n$. This proves Theorem 7. □

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