Number Theory 2

"Mathematics is the queen of the sciences, and number theory is the queen of mathematics." So supposedly asserted the German mathematician **Carl Friedrich Gauss** (1777–1855), who constructed by hand a list of all the prime numbers up to 3 million when he was just 15 and wrote a groundbreaking text, *Disquisitiones Arithmeticae* (Number-Theoretic Investigations), whose publication in 1801 transformed the subject. Gauss also gave a criterion in terms of "Fermat primes" for when a regular polygon with n sides can be constructed using only a straightedge and compass; the stamp below shows the case $n = 17$.

Sophie Germain (1776–1831) produced pioneering work on prime numbers that greatly impressed Lagrange, Legendre, and Gauss. In particular, she proved that if p is a prime number less than 100, then Fermat's equation $x^p + y^p = z^p$ has no positive integer solutions if x, y, and z are mutually prime to one another and to p.

Gauss proposed that $\pi(x)$, the number of primes up to a number x, is asymptotically equal to $x / \ln x$ (the *prime number theorem*), and this was eventually proved in 1896. Earlier, partial progress toward this aim had been made by the Russian mathematician **Pafnuty Chebyshev** (1821–1894), who proved in 1851 that if the ratio of $\pi(x)$ and $x / \ln x$ tends to a limit as x increases, then this limit must be 1.

In 1914, **Srinivasa Ramanujan** (1887–1920), one of the most intuitive mathematicians of all time, arrived in England to work with G. H. Hardy at Cambridge, and together they produced some spectacular work that led to an exact determination of the number $p(n)$ of partitions of any number n.

In a well-known story, Hardy remarked to Ramanujan that a taxi he had taken to visit him had the dull number 1729. "No," replied Ramanujan, "it is the smallest number expressible as the sum of two cubes in two different ways" (1728 + 1 and 1000 + 729).

Two well-known conjectures in number theory are *Goldbach's conjecture*, that every even number greater than 2 can be written as the sum of two primes, and the *twin prime conjecture*, that there are infinitely many pairs of primes of the form p, $p + 2$. In 1966, the Chinese mathematician **Chen Jingrun** proved the spectacular result that there are at most finitely many integers that cannot be represented as either the sum of two primes or the sum of a prime and the product of two primes. In 1973, he proved that there are infinitely many primes p for which $p + 2$ is either a prime or the product of two primes.

Gauss

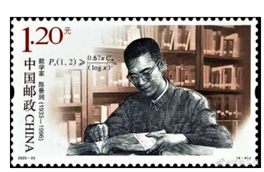
Germain

Chebyshev

Chebyshev

Ramanujan

Chen Jingrun



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