A Direct Proof of the Steiner–Lehmus Theorem

VICTOR PAMBUCCIAN

In his article “An Elementary-Minded Mathematician” [1], which appeared in the summer 2021 issue of this journal, O. A. S. Karamzadeh mentions, regarding the possibility of a direct proof of the Steiner–Lehmus theorem, that “While investigating different proofs of this theorem, Conway and Ryba finally concluded—rightly, in my opinion—that the question was actually the wrong one.” He goes on to doubt the meaningfulness of the notion of a direct proof. The reader is left with the impression that the question regarding a direct proof is either meaningless or that, as asserted as early as 1852 by J. J. Sylvester, there can be no direct proof of the Steiner–Lehmus theorem.

In reality, the question is meaningful. It can be formalized, and in fact, Sylvester’s problem has been solved: there is a direct proof, even one inside intuitionistic logic, where

the double negation of a sentence is a strictly weaker statement than the sentence itself, and the contrapositive of an implication is weaker than the implication itself. The positive solution is the subject of [3], using the intuitionistic axiom system for Euclidean geometry proposed in [2].

Victor Pambuccian
School of Mathematical and Natural Sciences
Arizona State University, West Campus
Phoenix, AZ 85069-7100
USA
e-mail: pamb@asu.edu

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