The Gömböc Pill

Gábor Domokos

Editor's note: An article in the New York Times *on February 7, 2019, discusses the Gömböc insulin capsule without naming the shape, and its relation to the turtle without referring to this journal. I've invited Gábor Domokos to give our readers that background. – MS*

The cover of the 2006/4 issue of the *Mathematical Intelligencer* (Fig. 1) featured a curious object alongside a turtle. The shape, later dubbed the Gömböc,[1] is the first known physical example of a class of convex homogeneous bodies called mono-monostatic, having just one stable and one unstable point of static equilibrium. The existence of this class was conjectured by Vladimir Arnold in 1995 [3]; with Péter Várkonyi, we proved his conjecture by describing a continuum of mono-monostatic shapes. These shapes are, however, too close to the sphere to be manufactured, so we also showed a physical example, commonly called the Gömböc [11]. The name is of Hungarian origin (a spheroidal figure in a folk tale) and refers to the proximity of the Gömböc to the sphere. One curious aspect of the Gömböc is that despite being homogeneous, when placed on a horizontal support surface it behaves like a children's roly-poly toy: no matter how you push it down, it always returns spontaneously to its single stable position. This property led us to study the behavior of tropical turtle species that evolved shells closely reminiscent of the Gömböc; the shape of the shell is their principal tool facilitating self-righting [6]. These findings were reported in the media, including the *New York Times* [10] and BBC,[2] and ever since, the Gömböc has proved to be a source of inspiration in science, art, and engineering.

In recent years, the Gömböc has motivated research into the evolution of natural shapes: while Gömböc-shaped pebbles are rare, the connection between geometric shape and the number of static balance points appears to be a key to understanding natural shape evolution [4]. Experimental and numerical evidence indicates that the number of static equilibrium points of sedimentary particles is reduced by natural abrasion. This observation helped us identify the geometric partial differential equations governing this process, and these models provided key evidence not only for the provenance of Martian pebbles [9] but also for the shape of the interstellar asteroid 'Oumuamua [5].

Beyond science, the Gömböc has also inspired artists: the award-winning short movie *Gömböc*, directed by Ulrike Vahl, is a character sketch about four misfits who in their fight against everyday reversals and obstacles have one thing in common: if they fall down, they get back on their feet. The characteristic shape of the gömböc is curiously reflected in the critically acclaimed novel *Climbing Days* [8], by Dan Richards, as he describes a scene: "All over Montserrat the landscape reared as gömböc domes and pillars." A recent solo exhibition of the work of conceptual artist Ryan Gander revolving around the theme of self-righting featured seven large Gömböc shapes gradually covered by black volcanic sand (Fig. 2). The Gömböc has also appeared around the globe in art galleries as a recurrent motive in the paintings of Vivien Zhang [2] (Fig. 3).

Tied to both science and literature, the book *Birth of a Theorem,* by Cedric Villani [12], is testimony to the spirit of mathematical research. The author refers to the Gömböc as a symbol of that spirit.

As a recent report in *Science* [1] indicates, the Gömböc has inspired not only scientists and artists but also engineers, and it may have contributed to a breakthrough in the pharmaceutical industry.

[1]More on the history of the Gömböc can be found online in the Wikipedia article "Gömböc."
[2]*QI*, Episode 8, Series F (February 13, 2009).

https://doi.org/10.1007/s00283-019-09891-x

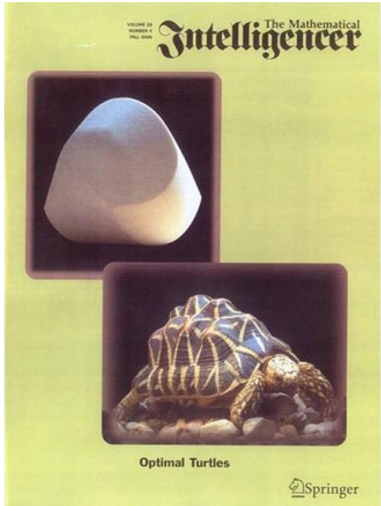

Figure 1. The cover of the 2006/4 issue of the *Mathematical Intelligencer.*

Figure 2. Ryan Gander: "The Self Righting of All Things," 2018. Portland stone. Exhibition at Lisson Gallery, London. (*Courtesy of Lisson Gallery. Photography by Jack Hems.*)

As noted, due to their closeness to the sphere, all mono-monostatic shapes have very small tolerance for imperfections, and even for the physical Gömböc design, this tolerance is daunting (< 0.01%). Nevertheless, if we drop the requirement of homogeneity, the Gömböc design serves as a good starting geometry for finding the optimal shape for self-righting objects carrying bottom weights. This has already inspired engineers [7] designing Gömböc-like cages for drones exposed to midair collisions. Now a team from MIT and Harvard have proposed [1] a Gömböc-inspired capsule (Fig. 4) that releases insulin in the stomach (Fig. 5) and could replace injections for patients with diabetes.

The key element of the new capsule is its ability to find a unique position in the stomach. This ability is based on its bottom weight and its overall geometry, optimized for self-righting. According to the article, after studying our results [11, 6], the authors ran an optimization that produced a mono-monostatic capsule with a contour almost identical

Figure 3. Vivien Zhang: "Interrobang." Oil and spray on canvas, 140 × 160 cm, 2016. (*Courtesy of the artist.*)

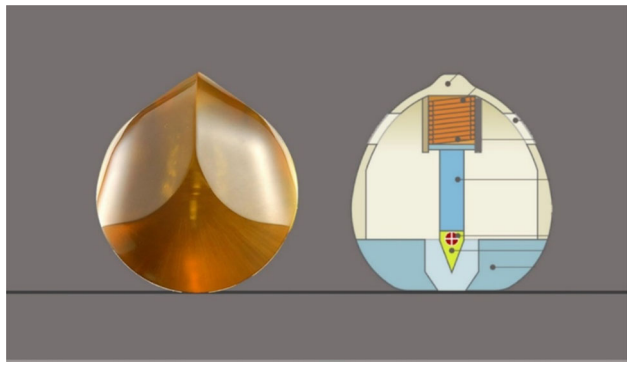

Figure 4. The Gömböc (left) and a design drawing of the insulin capsule proposed in [1] (right). (*Gömböc photo by István Oravecz. Capsule graphics based on a drawing in* [1]. *Reprinted with permission from AAAS.*)

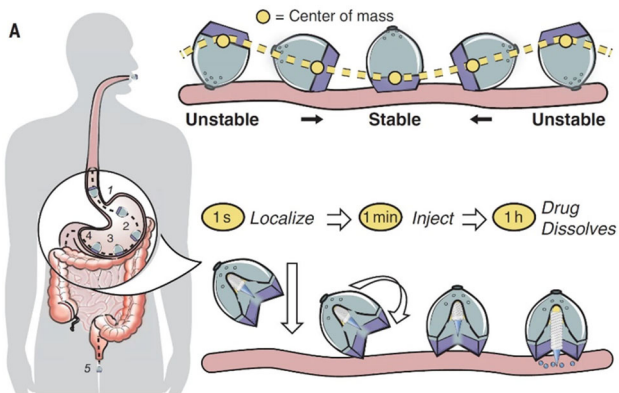

Figure 5. Working scheme of the Gömböc-inspired insulin capsule. (From [1]. Reprinted with permission from AAAS.)

to the frontal view of the Gömböc (cf. Fig. 4), showing once again that mathematical results may contribute to innovation in the most unexpected manner.

MTA-BME Morphodynamics Research Group
and Department of Mechanics, Materials, and Structures
Budapest University of Technology and Economics
Muegyetem rkp 3, K261
1111 Budapest
Hungary
e-mail: domokos@iit.bme.hu

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